Velocity of Neutrino

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Abstract.
The properties of neutrinos (the electric charge, the neutrino mass, rhe neutrino velocity, the oscillation of neutrinos) are obtained from the neutrino matricies representation.

Introduction

Wolfgang Pauli postulated the neutrino in 1930 to explain the energy spectrum of beta decays, the decay of a neutron into a proton and an electron. Clyde Cowan, Frederick Reines found the neutrino experimentally in 1955. Enrico Fermi[1] developed the first theory describing neutrino interactions and denoted this particles as *neutrino* in 1933. In 1962 Leon M. Lederman, Melvin Schwartz and Jack Steinberger showed that more than one type of neutrino exists. Bruno Pontecorvo[2] suggested a practical method for investigating neutrino masses in 1957, over the subsequent 10 years he developed the mathematical formalism and the modern formulation of vacuum oscillations...

1. Matricies

Let 1_n be an identical $n \times n$ matrix and 0_n is a $n \times n$ zero matrix. If A and all $AB_{j,s}$ are $n \times n$ matricies then [1, pp.36-39]

$$A\begin{bmatrix} B_{0,0} & B_{0,1} & \cdots & B_{0,n} \\ B_{1,0} & B_{1,1} & \cdots & B_{1,n} \\ \cdots & \cdots & \cdots & \cdots \\ B_{m,0} & B_{m,1} & \cdots & B_{m,n} \end{bmatrix} := \begin{bmatrix} AB_{0,0} & AB_{0,1} & \cdots & AB_{0,n} \\ AB_{1,0} & AB_{1,1} & \cdots & AB_{1,n} \\ \cdots & \cdots & \cdots & \cdots \\ AB_{m,0} & AB_{m,1} & \cdots & AB_{m,n} \end{bmatrix}$$

and

[1] Enrico Fermi (29 September 1901 28 November 1954) was an Italian-born, naturalized American physicist particularly known for his work on the development of the first nuclear reactor, Chicago Pile-1, and for his contributions to the development of quantum theory, nuclear and particle physics, and statistical mechanics.

[2] Bruno Pontecorvo (Marina di Pisa, Italy, August 22, 1913 – Dubna, Russia, September 24, 1993) was an Italian-born atomic physicist, an early assistant of Enrico Fermi and then the author of numerous studies in high energy physics, especially on neutrinos.

$$\boxed{\left[\begin{array}{cccc} B_{0,0} & B_{0,1} & \cdots & B_{0,n} \\ B_{1,0} & B_{1,1} & \cdots & B_{1,n} \\ \cdots & \cdots & \cdots & \cdots \\ B_{m,0} & B_{m,1} & \cdots & B_{m,n} \end{array}\right] A := \left[\begin{array}{cccc} B_{0,0}A & B_{0,1}A & \cdots & B_{0,n}A \\ B_{1,0}A & B_{1,1}A & \cdots & B_{1,n}A \\ \cdots & \cdots & \cdots & \cdots \\ B_{m,0}A & B_{m,1}A & \cdots & B_{m,n}A \end{array}\right]}. \tag{1}$$

If A and all $B_{j,s}$ are $k \times k$ matrices then

$$\boxed{A + \left[\begin{array}{ccccc} B_{0,0} & B_{0,1} & B_{0,2} & \cdots & B_{0,n} \\ B_{1,0} & B_{1,1} & B_{1,2} & \cdots & B_{1,n} \\ B_{2,0} & B_{2,1} & B_{2,2} & \cdots & B_{2,n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ B_{n,0} & B_{n,1} & B_{n,2} & \cdots & B_{n,n} \end{array}\right] :=}$$

$$\boxed{:= A 1_{nk} + \left[\begin{array}{ccccc} B_{0,0} & B_{0,1} & B_{0,2} & \cdots & B_{0,n} \\ B_{1,0} & B_{1,1} & B_{1,2} & \cdots & B_{1,n} \\ B_{2,0} & B_{2,1} & B_{2,2} & \cdots & B_{2,n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ B_{n,0} & B_{n,1} & B_{n,2} & \cdots & B_{n,n} \end{array}\right] =}$$

$$\boxed{= \left[\begin{array}{ccccc} B_{0,0}+A & B_{0,1} & B_{0,2} & \cdots & B_{0,n} \\ B_{1,0} & B_{1,1}+A & B_{1,2} & \cdots & B_{1,n} \\ B_{2,0} & B_{2,1} & B_{2,2}+A & \cdots & B_{2,n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ B_{n,0} & B_{n,1} & B_{n,2} & \cdots & B_{n,n}+A \end{array}\right]}. \tag{2}$$

Hence

$$1_2 := \left[\begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array}\right]; 0_2 := \left[\begin{array}{cc} 0 & 0 \\ 0 & 0 \end{array}\right]; \beta^{[0]} := -\left[\begin{array}{cc} 1_2 & 0_2 \\ 0_2 & 1_2 \end{array}\right] = -1_4.$$

The Pauli matrices:

$$\sigma_1 := \left[\begin{array}{cc} 0 & 1 \\ 1 & 0 \end{array}\right]; \sigma_2 := \left[\begin{array}{cc} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{array}\right]; \sigma_3 := -\left[\begin{array}{cc} 1 & 0 \\ 0 & -1 \end{array}\right].$$

A set $\widetilde{C}$ of complex $n \times n$ matrices is called *a Clifford set of rank n* [2, p.12] if the following conditions are fulfilled:

if $\alpha_k \in \widetilde{C}$ and $\alpha_r \in \widetilde{C}$ then $\alpha_k\alpha_r + \alpha_r\alpha_k = 2\delta_{k,r}$;

if $\alpha_k\alpha_r + \alpha_r\alpha_k = 2\delta_{k,r}$ for all elements α_r of set $\widetilde{C}$ then $\alpha_k \in \widetilde{C}$.

If $n = 4$ then a Clifford set either contains 3 matrices (*a Clifford triplet*) or contains 5 matrices (*a Clifford pentad*).

Here exist only six Clifford pentads [2, 12]: one *light pentad* β which used for a lepton descriptiom.

$$\beta^{[1]} := \left[\begin{array}{cc} \sigma_1 & 0_2 \\ 0_2 & -\sigma_1 \end{array}\right], \beta^{[2]} := \left[\begin{array}{cc} \sigma_2 & 0_2 \\ 0_2 & -\sigma_2 \end{array}\right],$$
$$\beta^{[3]} := \left[\begin{array}{cc} \sigma_3 & 0_2 \\ 0_2 & -\sigma_3 \end{array}\right], \tag{3}$$

$$\gamma^{[0]} := \begin{bmatrix} 0_2 & 1_2 \\ 1_2 & 0_2 \end{bmatrix}, \tag{4}$$

$$\beta^{[4]} := \mathrm{i} \cdot \begin{bmatrix} 0_2 & 1_2 \\ -1_2 & 0_2 \end{bmatrix}; \tag{5}$$

three *chromatic* pentads exist which used for a quark description and two *gustatory* pentads exist [1, pp.38-39]

2. One-Mass State

Let form of [1, p.112, p.115] be the following:

$$\widetilde{\varphi}(t, \mathbf{x}, x_5, x_4) = \exp\left(-\mathrm{i}\frac{\mathrm{h}}{\mathrm{c}} n x_5\right) \sum_{k=1}^{4} f_k(t, \mathbf{x}, n, 0).$$

In that case the Hamiltonian has the following form (from [1, p.103]):

$$\widehat{H} = \mathrm{c}\left(\sum_{k=1}^{3} \beta^{[k]} \mathrm{i}\partial_k + \frac{\mathrm{h}}{\mathrm{c}} n \gamma^{[0]} + \widehat{G}\right)$$

with

$$\widehat{G} := \sum_{\mu=0}^{3} \beta^{[\mu]} (F_\mu + 0.5 g_1 Y B_\mu).$$

Let

$$\omega(\mathbf{k}, n) := \sqrt{\mathbf{k}^2 + n^2} = \sqrt{k_1^2 + k_2^2 + k_3^2 + n^2}$$

and

$$e_1(\mathbf{k}, n) := \frac{1}{2\sqrt{\omega(\mathbf{k}, n)(\omega(\mathbf{k}, n) + n)}} \begin{bmatrix} \omega(\mathbf{k}, n) + n + k_3 \\ k_1 + \mathrm{i}k_2 \\ \omega(\mathbf{k}, n) + n - k_3 \\ -k_1 - \mathrm{i}k_2 \end{bmatrix}. \tag{6}$$

Let

$$\widehat{H}_0 := \mathrm{c} \sum_{s=1}^{3} \beta^{[s]} \mathrm{i}\partial_s + \mathrm{h} n \gamma^{[0]}. \tag{7}$$

Let:

$$\mathrm{h} n = m \frac{\mathrm{c}^2}{\mathrm{h}}$$

then equation of moving with Hamiltonian $\widehat{H}_0$ has the following form:

$$\boxed{\frac{1}{\mathrm{c}} \mathrm{i}\partial_t \varphi = \left(\sum_{s=1}^{3} \beta^{[s]} \mathrm{i}\partial_s + m \frac{\mathrm{c}}{\mathrm{h}} \gamma^{[0]}\right) \varphi.} \tag{8}$$

This is the Dirac equation (Paul Dirac[3] formulated it in 1928).
Therefore,

$$\widehat{H}_0 e_1(\mathbf{k},n)\left(\frac{\mathrm{h}}{2\pi\mathrm{c}}\right)^{\frac{3}{2}}\exp\left(-\mathrm{i}\frac{\mathrm{h}}{\mathrm{c}}\mathbf{kx}\right)=\mathrm{h}\omega(\mathbf{k},n)\,e_1(\mathbf{k},n)\left(\frac{\mathrm{h}}{2\pi\mathrm{c}}\right)^{\frac{3}{2}}\exp\left(-\mathrm{i}\frac{\mathrm{h}}{\mathrm{c}}\mathbf{kx}\right). \tag{9}$$

Hence, function $e_1(\mathbf{k},n)\left(\frac{\mathrm{h}}{2\pi\mathrm{c}}\right)^{\frac{3}{2}}\exp\left(-\mathrm{i}\frac{\mathrm{h}}{\mathrm{c}}\mathbf{kx}\right)$ is an eigenvector of $\widehat{H}_0$ with eigenvalue

$$\mathrm{h}\omega(\mathbf{k},n)=\mathrm{h}\sqrt{\mathbf{k}^2+n^2}.$$

Similarly, function $e_2(\mathbf{k},n)\left(\frac{\mathrm{h}}{2\pi\mathrm{c}}\right)^{\frac{3}{2}}\exp\left(-\mathrm{i}\frac{\mathrm{h}}{\mathrm{c}}\mathbf{kx}\right)$ with

$$e_2(\mathbf{k},n):=\frac{1}{2\sqrt{\omega(\mathbf{k},n)(\omega(\mathbf{k},n)+n)}}\begin{bmatrix}k_1-\mathrm{i}k_2\\ \omega(\mathbf{k},n)+n-k_3\\ -k_1+\mathrm{i}k_2\\ \omega(\mathbf{k},n)+n+k_3\end{bmatrix} \tag{10}$$

is eigenvector of $\widehat{H}_0$ with eigenvalue $\mathrm{h}\omega(\mathbf{k})=\mathrm{h}\sqrt{\mathbf{k}^2+n^2}$, too, and functions

$$e_3(\mathbf{k},n)\left(\frac{\mathrm{h}}{2\pi\mathrm{c}}\right)^{\frac{3}{2}}\exp\left(-\mathrm{i}\frac{\mathrm{h}}{\mathrm{c}}\mathbf{kx}\right) \text{ and } e_4(\mathbf{k},n)\left(\frac{\mathrm{h}}{2\pi\mathrm{c}}\right)^{\frac{3}{2}}\exp\left(-\mathrm{i}\frac{\mathrm{h}}{\mathrm{c}}\mathbf{kx}\right)$$

with

$$e_3(\mathbf{k},n):=\frac{1}{2\sqrt{\omega(\mathbf{k},n)(\omega(\mathbf{k},n)+n)}}\begin{bmatrix}-\omega(\mathbf{k},n)-n+k_3\\ k_1+\mathrm{i}k_2\\ \omega(\mathbf{k},n)+n+k_3\\ k_1+\mathrm{i}k_2\end{bmatrix} \tag{11}$$

and

$$e_4(\mathbf{k},n):=\frac{1}{2\sqrt{\omega(\mathbf{k},n)(\omega(\mathbf{k},n)+n)}}\begin{bmatrix}k_1-\mathrm{i}k_2\\ -\omega(\mathbf{k},n)-n-k_3\\ k_1-\mathrm{i}k_2\\ \omega(\mathbf{k},n)+n-k_3\end{bmatrix} \tag{12}$$

are eigenvectors of $\widehat{H}_0$ with eigenvalue $-\mathrm{h}\omega(\mathbf{k},n)$.

3. Bi-mass state

Let U be a 8×8 matrix such that for every $\widetilde{\varphi}$: [1, p.107] :

$$\begin{aligned}(U\widetilde{\varphi},U\widetilde{\varphi}) &= \rho, \\ \left(U\widetilde{\varphi},\beta^{[s]}U\widetilde{\varphi}\right) &= -\frac{j_s}{\mathrm{c}}\end{aligned} \tag{13}$$

[3] Paul Adrien Maurice Dirac (1902 – 1984) was an English theoretical physicist who made fundamental contributions to the early development of both quantum mechanics and quantum electrodynamics.

here (1)

$$\widetilde{\varphi}=\widetilde{\varphi}1_8;\ \beta^{[s]}=\beta^{[s]}1_8.$$

In that case:

$$U^{\dagger}\beta^{[\mu]}U=\beta^{[\mu]}$$

for $\mu\in\{0,1,2,3\}$.

Such transformation has a matrix of the following shape [1, p.126]:

$$U^{(-)}:=\left[\begin{array}{cccc}(a+\mathrm{i}b)\,1_2 & 0_2 & (c+\mathrm{i}g)\,1_2 & 0_2\\ 0_2 & 1_2 & 0_2 & 0_2\\ (-c+\mathrm{i}g)\,1_2 & 0_2 & (a-\mathrm{i}b)\,1_2 & 0_2\\ 0_2 & 0_2 & 0_2 & 1_2\end{array}\right] \tag{14}$$

where:

$$a^2+b^2+c^2+g^2=1,$$

4. Neutrino

Let:

$$\widehat{H}_{0,4}\overset{Def.}{=}\sum_{r=1}^{3}\beta^{[r]}\mathrm{i}\partial_r+h\left(n_0\gamma^{[0]}+s_0\beta^{[4]}\right).$$

$$\underline{u}_1(\mathbf{k},n)\overset{Def}{=}\frac{1}{2\sqrt{\omega(\mathbf{k},n)\left(\omega(\mathbf{k},n)+n\right)}}\left[\begin{array}{c}0\\0\\0\\0\\ \omega(\mathbf{k},n)+n+k_3\\ k_1+\mathrm{i}k_2\\ \omega(\mathbf{k},n)+n-k_3\\ -k_1-\mathrm{i}k_2\end{array}\right]$$

and

$$\underline{u}_2(\mathbf{k},n)\overset{Def}{=}\frac{1}{2\sqrt{\omega(\mathbf{k},n)\left(\omega(\mathbf{k},n)+n\right)}}\left[\begin{array}{c}0\\0\\0\\0\\ k_1-\mathrm{i}k_2\\ \omega(\mathbf{k},n)+n-k_3\\ -k_1+\mathrm{i}k_2\\ \omega(\mathbf{k},n)+n+k_3\end{array}\right]$$

correspond to eigenvectors of $\widehat{H}_{0,4}$ with eigenvalue

$$\omega(\mathbf{k},n)=\sqrt{\mathbf{k}^2+n^2}$$

and 8-vectors

$$\underline{u}_3(\mathbf{k},n) \overset{Def}{=} \frac{1}{2\sqrt{\omega(\mathbf{k},n)(\omega(\mathbf{k},n)+n)}} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -\omega(\mathbf{k},n)-n+k_3 \\ k_1+\mathrm{i}k_2 \\ \omega(\mathbf{k},n)+n+k_3 \\ k_1+\mathrm{i}k_2 \end{bmatrix}$$

and

$$\underline{u}_4(\mathbf{k},n) \overset{Def}{=} \frac{1}{2\sqrt{\omega(\mathbf{k},n)(\omega(\mathbf{k},n)+n)}} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ k_1-\mathrm{i}k_2 \\ -\omega(\mathbf{k},n)-n-k_3 \\ k_1-\mathrm{i}k_2 \\ \omega(\mathbf{k},n)+n-k_3 \end{bmatrix}$$

correspond to eigenvectors of $\widehat{H}_{0,4}$ with eigenvalue $-\omega(\mathbf{k}.n)$.
Let

$$\widehat{H}'_{0,4} \overset{Def}{=} U^{(-)}\widehat{H}_{0,4}U^{(-)\dagger},$$
$$\underline{u}'_\mu(\mathbf{k},n) \overset{Def}{=} U^{(-)}\underline{u}_\mu(\mathbf{k},n).$$

That is

$$\underline{u}'_1(\mathbf{k},n) = \frac{1}{2\sqrt{\omega(\mathbf{k},n)(\omega(\mathbf{k},n)+n)}} \begin{bmatrix} (c+\mathrm{i}q)(\omega(\mathbf{k},n)+n+k_3) \\ (c+\mathrm{i}q)(k_1+\mathrm{i}k_2) \\ 0 \\ 0 \\ (a-\mathrm{i}b)(\omega(\mathbf{k},n)+n+k_3) \\ (a-\mathrm{i}b)(k_1+\mathrm{i}k_2) \\ \omega(\mathbf{k},n)+n-k_3 \\ -k_1-\mathrm{i}k_2 \end{bmatrix},$$

$$\underline{u}'_2(\mathbf{k},n) = \frac{1}{2\sqrt{\omega(\mathbf{k},n)(\omega(\mathbf{k},n)+n)}} \begin{bmatrix} (c+\mathrm{i}q)(k_1-\mathrm{i}k_2) \\ (c+\mathrm{i}q)(\omega(\mathbf{k},n)+n-k_3) \\ 0 \\ 0 \\ (a-\mathrm{i}b)(k_1-\mathrm{i}k_2) \\ (a-\mathrm{i}b)(\omega(\mathbf{k},n)+n-k_3) \\ -k_1+\mathrm{i}k_2 \\ \omega(\mathbf{k},n)+n+k_3 \end{bmatrix},$$

$$\underline{u}_3'(\mathbf{k},n) = \frac{1}{2\sqrt{\omega(\mathbf{k},n)(\omega(\mathbf{k},n)+n)}} \begin{bmatrix} -(c+\mathrm{i}q)(\omega(\mathbf{k},n)+n-k_3) \\ (c+\mathrm{i}q)(k_1+\mathrm{i}k_2) \\ 0 \\ 0 \\ -(a-\mathrm{i}b)(\omega(\mathbf{k},n)+n-k_3) \\ (a-\mathrm{i}b)(k_1+\mathrm{i}k_2) \\ \omega(\mathbf{k},n)+n+k_3 \\ k_1+\mathrm{i}k_2 \end{bmatrix},$$

$$\underline{u}_4'(\mathbf{k},n) = \frac{1}{2\sqrt{\omega(\mathbf{k},n)(\omega(\mathbf{k},n)+n)}} \begin{bmatrix} (c+\mathrm{i}q)(k_1-\mathrm{i}k_2) \\ -(c+\mathrm{i}q)(\omega(\mathbf{k},n)+n+k_3) \\ 0 \\ 0 \\ (a-\mathrm{i}b)(k_1-\mathrm{i}k_2) \\ -(a-\mathrm{i}b)(\omega(\mathbf{k},n)+n+k_3) \\ k_1-\mathrm{i}k_2 \\ \omega(\mathbf{k},n)+n-k_3 \end{bmatrix}.$$

Here $\underline{u}_1'(\mathbf{k},n)$ and $\underline{u}_2'(\mathbf{k},n)$ correspond to eigenvectors of $\widehat{H}_{0,4}'$ with eigenvalue $\omega(\mathbf{k}) = \sqrt{\mathbf{k}^2+n^2}$, and $\underline{u}_3'(\mathbf{k},n)$ and $\underline{u}_4'(\mathbf{k})$ correspond to eigenvectors of $\widehat{H}_{0,4}'$ with eigenvalue $-\omega(\mathbf{k},n)$.

Vectors

$$\nu_{n,(1)}(\mathbf{k},n) = \begin{bmatrix} \omega(\mathbf{k},n)+n+k_3 \\ k_1+\mathrm{i}k_2 \\ 0 \\ 0 \end{bmatrix} \text{ and } \nu_{n,(2)}(\mathbf{k},n) = \begin{bmatrix} k_1-\mathrm{i}k_2 \\ \omega(\mathbf{k},n)+n-k_3 \\ 0 \\ 0 \end{bmatrix}$$

$$\nu_{n,(3)}(\mathbf{k},n) = \begin{bmatrix} -(n+\omega(\mathbf{k},n)-k_3) \\ (k_1+ik_2) \\ 0 \\ 0 \end{bmatrix}, \nu_{n,(4)}(\mathbf{k},n) = \begin{bmatrix} (k_1-ik_2) \\ -(n+\omega(\mathbf{k},n)+k_3) \\ 0 \\ 0 \end{bmatrix}$$

are denoted as *neutrinno components* of bi-n-leptonn basic vectors. These vectors form a linear space of functions of the type

$$\varphi := \begin{bmatrix} \varphi_1 \\ \varphi_2 \\ 0 \\ 0 \end{bmatrix}.$$

From [1, p.84]

$$\rho = \sum_{s=1}^{4} \varphi_s^* \varphi_s, \tag{15}$$

$$\frac{j_\alpha}{\mathrm{c}} = -\sum_{k=1}^{4}\sum_{s=1}^{4} \varphi_s^* \beta_{s,k}^{[\alpha]} \varphi_k$$

Hence,

$$\begin{aligned}
\rho &= \varphi^{\dagger}\varphi = \varphi_1\varphi_1^* + \varphi_2\varphi_2^*, \\
\frac{j_1}{c} &= -\varphi^{\dagger}\beta^{[1]}\varphi = -\left(\varphi_1\varphi_2^* + \varphi_2\varphi_1^*\right), \\
\frac{j_2}{c} &= -\varphi^{\dagger}\beta^{[2]}\varphi = -i\left(\varphi_1\varphi_2^* - \varphi_2\varphi_1^*\right), \\
\frac{j_3}{c} &= -\varphi^{\dagger}\beta^{[3]}\varphi == -\left(\varphi_1\varphi_1^* - \varphi_2\varphi_2^*\right).
\end{aligned}$$

From [1, p.82] velocities:

$$\begin{aligned}
u_1 &= \frac{j_1}{\rho} = \frac{-c\left(\varphi_1\varphi_2^* + \varphi_2\varphi_1^*\right)}{\varphi_1\varphi_1^* + \varphi_2\varphi_2^*}, \\
u_2 &= \frac{j_2}{\rho} = \frac{-ci\left(\varphi_1\varphi_2^* - \varphi_2\varphi_1^*\right)}{\varphi_1\varphi_1^* + \varphi_2\varphi_2^*}, \\
u_3 &= \frac{j_3}{\rho} = \frac{-c\left(\varphi_1\varphi_1^* - \varphi_2\varphi_2^*\right)}{\varphi_1\varphi_1^* + \varphi_2\varphi_2^*}.
\end{aligned}$$

That is

$$u_1^2 + u_1^2 + u_1^2 = c^2.$$

Hence, the neutrimo velocity eqaual to the light velocity.

From [1, p.108] the neutrino mass:

$$m = \varphi^{\dagger}\left(\beta^{[4]}n_0 + \gamma^{[0]}s_0\right)\varphi.$$

But

$$\varphi^{\dagger}\beta^{[4]}\varphi = 0 \text{ and } \varphi^{\dagger}\gamma^{[0]}\varphi = 0.$$

Hence, $m = 0$.
Hence, a neutrino has ZERO mass.

The electromagnetic potencial [1, 145]:

$$\widehat{A} = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 9 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}$$

Therefore, :

$$\begin{aligned}\widehat{A}\underline{u}'_1(\mathbf{k},n) &= \underline{u}_1(\mathbf{k},n),\\ \widehat{A}\underline{u}'_2(\mathbf{k},n) &= \underline{u}_2(\mathbf{k},n),\\ \widehat{A}\underline{u}'_3(\mathbf{k},n) &= \underline{u}_3(\mathbf{k},n),\\ \widehat{A}\underline{u}'_4(\mathbf{k},n) &= \underline{u}_4(\mathbf{k},n).\end{aligned}$$

Hence, the neutrino does not interact with the electomagnetic field.

Linear space of vectors $\nu_{n,(s)}(\mathbf{k},n)$ can be representated as linear space of vectors $\nu_{m,(s)}(\mathbf{k},m)$. Therefore, neitrinos of lepton with mass n oscillirues on neutrinos of lepton with mass m.

Conclusion

A netrino is derivative from a lepton under some unitar transformation.

the neutrino does not interact with the electomagnetic field,

a neutrino has ZERO mass,

the neutrimo velocity eqaual to the light velocity,

neitrinos of lepton with mass n oscillirues on neutrinos of lepton with mass m.

References

[1] Gunn Quznetsov, 2020, *Time-Space, Probability and Physics*, PRESS

[2] For instance, Madelung, E., *Die Mathematischen Hilfsmittel des Physikers.* Springer Verlag, (1957) p.29