An elementary derivative analysis of Lorentzian time dilation and related complexifications to time travel

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Abstract;
This paper comprises a series of relatively trivial, but nonetheless intriguing propositions with regards to Lorentz transformations and complex-valued time travel. They follow from the postulations that allow for the emergence of special relativity, and their consequent implications through differentiation. A complement to them, and another constituent of this paper, is a theoretical exposition to the conclusions they draw.

Firstly, consider the fundamental Lorentz transformation for time dilation across two proper time intervals, one of which embodies a moving frame of reference.

$$t' = t\sqrt{1-\frac{v^2}{c^2}}$$

wherein the left hand side of the expression is representative of the proper time interval experienced across a moving frame of reference.

It may be concluded from the above that in order for the expression to be entirely real,

$\sqrt{1-\frac{v^2}{c^2}}$ must necessarily be real. Consequently, its internal expression must equal a number that exceeds, or matches zero (in avoiding complex solutions).

$$1-\frac{v^2}{c^2} \geq 0$$

$$-\frac{v^2}{c^2} \geq -1$$

$$\frac{v^2}{c^2} \leq 1$$

$$v^2 \leq c^2$$

$$|v| \leq |c|$$

Carrying out the mathematical reasoning delineated above, invariably obtains the well-known impossibility of faster-than-light travel.

With regards to the same function, nonetheless, one may opt to re-verify this result in the construction of its derivative (with respect to velocity).

$$\frac{dt'}{dv} = \frac{d}{dv}\left[t\sqrt{1-\frac{v^2}{c^2}}\right] = t\frac{d}{dv}\left[\sqrt{1-\frac{v^2}{c^2}}\right]$$

$$= t\frac{d}{dv}\left(1-\frac{v^2}{c^2}\right)^{1/2}$$

$$= t\frac{1}{2\left[\sqrt{1-\frac{v^2}{c^2}}\right]}\frac{-2v}{c^2}$$

$$= t\frac{-2v}{2c^2\left[\sqrt{1-\frac{v^2}{c^2}}\right]} = \frac{-tv}{c^2\left[\sqrt{1-\frac{v^2}{c^2}}\right]}$$

$$= \frac{-tv}{\sqrt{c^4 - v^2c^2}}$$

A brief deconstruction, such as the one to the left, reveals the same boundary conditions as previously unearthed.

In order for the effects of special relativity to be falsified, or reversed; time dilation must necessarily possess a positive derivative with respect to velocity I.e. time speeding up, as opposed to slowing down with an advantage in velocity. For this to be met, the denominator associated with the derivative (to the bottom of the derivation to the left) must comprise a negative number. Whilst there exist mathematical solutions for negative roots to positive integers; herein they may be declared inexistent.

Additionally, for the derivative to remain real, or for it not to collapse onto the imaginary plane;

$$\sqrt{c^4 - v^2c^2} \in R$$
$$c^4 \geq v^2c^2$$
$$c^2 \geq v^2$$
$$|v| \leq |c|$$

Consequently, a Lorentzian structure does indeed necessitate a maximal bound on the speed of an object with non-zero mass.

Perhaps more significantly, one may opt to pursue a complex solution to the paradigm entirely. In order to positively reverse the effects of special relativity (in theory, naturally), it is of interest to construct a complex solution (for the velocity of an entity) that satisfies a positive derivative [in correspondence to dilated time and associated velocity].

In order to do so, one must attempt to reconstruct the original Lorentzian expression (placed at the beginning of this paper) for time dilation, and reformulate it so as to yield a function whose image is an entity's velocity.

$$t' = t\sqrt{1 - \frac{v^2}{c^2}}$$
$$\frac{t'}{t} = \sqrt{1 - \frac{v^2}{c^2}}$$
$$\left[\frac{t'}{t}\right]^2 = 1 - \frac{v^2}{c^2}$$
$$-\frac{v^2}{c^2} = \left[\frac{t'}{t}\right]^2 - 1$$
$$\frac{v^2}{c^2} = 1 - \left[\frac{t'}{t}\right]^2$$
$$v^2 = c^2 - c^2\left[\frac{t'}{t}\right]^2$$
$$v = \sqrt{c^2 - c^2\left[\frac{t'}{t}\right]^2} = c\sqrt{1 - \left[\frac{t'}{t}\right]^2}$$

In a manner commensurate to the rearrangement to the left, it is entirely plausible a proposition that solutions for an unorthodox, inverted movement across a closed time-like curve, exist perhaps even without a faster-than-light velocity. Naturally, in order for this condition to be satisfied, a body's velocity must be entirely complex; or for that matter, not philosophically determinate or unattainable.

In order to ascertain this very idea, nonetheless, we may proceed with this line of thinking.

If one were to travel to the 'past' in an accelerated reference frame, instead of to the 'future', $\frac{t'}{t}$ must necessarily encapsulate a value greater than one [having experienced a greater time differential in comparison to a stationary reference frame].

In that event,

$\left[\frac{t'}{t}\right]^2$ must also necessarily equal a number that exceeds 1.

As a direct result, the radical component in an entity's velocity-centered trajectory

$\sqrt{1-\frac{v^2}{c^2}}$ will be manifestly complex.

Noting that v equals

$c\sqrt{1-\left[\frac{t'}{t}\right]^2}$; one may set out to find a complex valued solution.

It happens to be a known fact that $\sqrt{-A}=\sqrt{A}i=\sqrt{|A|}i$ for any positive integer A.

In an inferential understanding, one may mathematically posit that any purely complex solutions for a physical entity's velocity may so be represented in the form;

$$v = ic\sqrt{\left|1-\left[\frac{t'}{t}\right]^2\right|}$$

wherein the related expression inside the root is an absolute conversion, and i denotes the imaginary unit.

In another intuitive exploitation, one may recognize that if

$$\left[\frac{t'}{t}\right]^2 \geq 1$$

$$\left|1-\left[\frac{t'}{t}\right]^2\right| = \left[\frac{t'}{t}\right]^2 - 1$$

; given that either is interchangeable with the other, and is representative of the absolute difference between the two terms.

As such, reformulating the expression of a quintessential complex velocity;

$$v = ic\sqrt{\left[\frac{t'}{t}\right]^2 - 1}$$

It may be noted profoundly, that the theoretical derivation of an idyllic solution such as this is by no means acknowledging of or emblematic of the philosophical indeterminacy that encapsulates it. In any event, an enthusing corollary of such a solution is a theoretical proposition long conceived of in both general relativity and quantum mechanics: negative energy.

As is abundantly understood, the existence of a purely imaginary velocity is the sole prerequisite for a body to externally demonstrate a kinetic energy of a sub-zero state.

$$KE = \frac{1}{2}mv^2$$

$$v = ic\sqrt{\left[\frac{t'}{t}\right]^2 - 1}$$

$$v^2 = i^2c^2\left[\left[\frac{t'}{t}\right]^2 - 1\right]$$

$$v^2 = -c^2\left[\left[\frac{t'}{t}\right]^2 - 1\right]$$

$$v^2 = \left[-c^2\left[\frac{t'}{t}\right]^2 + c^2\right] = \left[c^2 - c^2\left[\frac{t'}{t}\right]^2\right]$$

$$\left[\frac{t'}{t}\right]^2 \geq 1;$$

$$v^2 \leq 0$$

$$KE \leq 0$$

Discerning the causal association between a sub-zero classical energy state, and the constriction of a velocity onto a complex plane is theoretically redundant. Both, for all intents and purposes, may engender one another.

In evaluation, nevertheless, building complex-valued solutions for alternative Lorentzian transformations may follow from the hitherto illustrated logical schematic.