Bell's theorem refuted via elementary probability theory

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Abstract: Bell's theorem has been described as the most profound discovery of science. Let's see.

Introduction: Let β denote the thought-experiment in Bell (1964). Let **B**(.) denote his equations (.). Let the causally-independent same-instance results in **B**(1) be $A^{\pm}$ and $B^{\pm}$, pairwise correlated via the functions A & B and the variable λ. Then, reserving P for probabilities, let's replace Bell's expectation $P(\vec{a},\vec{b})$ in **B**(2) with its identity $E(a,b|\beta)$. So, from **B**(1), **B**(2), RHS **B**(3) and the line below **B**(3)—with Λ denoting the space of λ—here's Bell's theorem (**BT**) in our notation:

$$\textbf{BT}\text{: } E(a,b|\beta) = \textstyle\int_{\Lambda} d\lambda\, \rho(\lambda) A(a,\lambda)B(b,\lambda) \neq -a\cdot b \text{ [sic];} \tag{1}$$

$$\text{with } A(a,\lambda) = \pm 1 \equiv A^{\pm},\, B(b,\lambda) = \pm 1 \equiv B^{\pm},\, A(a,\lambda)B(b,\lambda) = \pm 1. \tag{2}$$

Refutation: Via RHS (2), and independent of the functions A and B, we divide Λ into two subsets: Λ^{+} is the space that delivers $A(a,\lambda)B(b,\lambda) = 1$, Λ^{-} delivers $A(a,\lambda)B(b,\lambda) = -1$. Thus, from (1):

$$E(a,b|\beta) = \int_{\Lambda^{+}} d\lambda\, \rho(\lambda)A(a,\lambda)B(b,\lambda) - \int_{\Lambda^{-}} d\lambda\, \rho(\lambda)A(a,\lambda)B(b,\lambda) \tag{3}$$

$$= P\left(AB = 1\,|\,a,b,\Lambda^{+}\right) - P\left(AB = -1\,|\,a,b,\Lambda^{-}\right), \text{ the weighted-sum of } AB \text{ results.} \tag{4}$$

$$= \left[P\left(A^{+}B^{+}\right) + P\left(A^{-}B^{-}\right)\right] - \left[P\left(A^{+}B^{-}\right) + P\left(A^{-}B^{+}\right)\right], \text{ with conditions suppressed,}$$
$$\text{the weighted-sum of the same-instance results } (\pm 1) \text{ that deliver each } AB \text{ result.} \tag{5}$$

$$= P(A^{+})P(B^{+}|A^{+}) + P(A^{-})P(B^{-}|A^{-}) - P(A^{+})P(B^{-}|A^{+}) - P(A^{-})P(B^{+}|A^{-})$$
$$\text{via the product rule for the paired (same-instance) results correlated as in (2).} \tag{6}$$

$$= \tfrac{1}{2}\left[P\left(B^{+}\,|\,A^{+}\right) + P\left(B^{-}\,|\,A^{-}\right) - P\left(B^{-}\,|\,A^{+}\right) - P\left(B^{+}\,|\,A^{-}\right)\right] \text{ for, with}$$
$$\lambda \text{ a random latent variable, the marginal probabilities } \left[\text{like } P\left(A^{+}\right)\right] = \tfrac{1}{2}. \tag{7}$$

$$= \tfrac{1}{2}\left[\sin^2\tfrac{1}{2}(a,b) + \sin^2\tfrac{1}{2}(a,b) - \cos^2\tfrac{1}{2}(a,b) - \cos^2\tfrac{1}{2}(a,b)\right]\text{: replacing the probability}$$
$$\text{functions in (7) with our } \beta\text{-based laws (akin to Malus' Law for light-beams).} \tag{8}$$

$$= \sin^2\tfrac{1}{2}(a,b) - \cos^2\tfrac{1}{2}(a,b) = -\cos(a,b) = -a\cdot b. \text{ So RHS (1) is refuted: QED.} \tag{9}$$

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Confirmation: We now refute **B**(15), Bell's inequality (**BI**), offered by Bell as proof of his theorem.

$$\textbf{BI:}\ |E(a,b)-E(a,c)|-1 \le E(b,c) \text{ [sic]: ie, } \textbf{B}(15) \text{ in our notation,} \tag{10}$$

$$\text{where } -1 \le E(a,b) \le 1,\ -1 \le E(a,c) \le 1,\ -1 \le E(b,c) \le 1. \tag{11}$$

$$\text{However: } E(a,b)[1+E(a,c)] \le 1+E(a,c); \text{ for, if } V \le 1, \text{ and } 0 \le W, \text{ then } VW \le W. \tag{12}$$

$$\therefore E(a,b)-E(a,c)-1 \le -E(a,b)E(a,c). \tag{13}$$

$$\text{Similarly: } E(a,c)-E(a,b)-1 \le -E(a,b)E(a,c). \text{ Hence our irrefutable inequality} \tag{14}$$

$$\textbf{WI:}\ |E(a,b)-E(a,c)|-1 \le -E(a,b)E(a,c). \tag{15}$$

$$\text{So, with test-settings } 0<(a,c)<\pi; (a,b)=(b,c)=\frac{(a,c)}{2}=\frac{x}{2}, \text{ and, via (9),} \tag{16}$$

$$\text{with test-functions } E(a,b)=E(b,c)=-\cos\left(\frac{x}{2}\right), E(a,c)=-\cos(x):\ \text{please} \tag{17}$$

copy, paste and test this next expression in WolframAlpha®; free-online, see References. (18)

$$\text{plot}|cos(x)-cos(x/2)|-1\&\&-cos(x/2)\&\&-cos(x)cos(x/2), 0 \le x \le \pi \tag{19}$$

Thus, under the generality of (16)-(17):[2] (i) For $0<x<\pi$, (10) is everywhere false, (15) is everywhere true. (ii) For $x=0$ and $x=\pi$, (10) and (15) are true. (iii) Let the unnumbered relations between **B**(14) and **B**(15) be **B**(14a)-**B**(14c). (iv) Then Bell's error is his move from true **B**(14a) to false **B**(14b): for **B**(14b) leads to false **B**(15). (v) In other words, given the common LHS in (10) and (15): Bell's error equates irrefutable $-E(a,b)E(a,c)$ from (15) to false $E(b,c)$ from (10); hence, as above, Bell's equality only holds at $x=0$ and $x=\pi$. That is: when Bell's $-\cos\left(\frac{x}{2}\right)=-\cos\left(\frac{x}{2}\right)\cos(x)$.

Conclusions: (i) Bell's theorem (1) and Bell's inequality (10) are refuted. (ii) In (8), via our heuristic debt to Malus, we provide the first of a family of laws that refute Bell's theorem in other settings.

References:

1. Bell, J. S. (1964). "On the Einstein Podolsky Rosen paradox." Physics 1, 195-200. http://cds.cern.ch/record/111654/files/vol1p195-200_001.pdf
2. WolframAlpha®. "WolframAlpha: computational intelligence." https://www.wolframalpha.com

[2] Under β, the plane of the coplanar angles need not be orthogonal to the line-of-light axis: just not parallel to it.