Bell's theorem refuted via elementary probability theory

Gordon Stewart Watson[1]

Abstract: Bell's theorem has been described as the most profound discovery of science. Let's see.

Introduction: Let β denote Bohm's experiment in Bell (1964); let **B**(.) denote Bell's equation (.); let $A^{\pm}$ and $B^{\pm}$ be the causally-independent same-instance results in **B**(1), pairwise correlated via λ and functions A, B. Then, reserving P for probabilities, replace Bell's expectation $P(\vec{a},\vec{b})$ in **B**(2) with its identity $E(a,b\,|\,\beta)$. So, from **B**(1), **B**(2), RHS **B**(3) and the line below it—with Λ denoting the space of λ—here's Bell's 1964 theorem (**BT**$_1$) in our notation:

$$\textbf{BT}_1\text{: } E(a,b\,|\,\beta) = \textstyle\int_{\Lambda} d\lambda\, \rho(\lambda) A(a,\lambda) B(b,\lambda) \neq -a\cdot b \text{ [sic]:} \tag{1}$$

$$\text{with } A(a,\lambda) = \pm 1 \equiv A^{\pm},\ B(b,\lambda) = \mp 1 \equiv B^{\mp},\ A(a,\lambda)B(b,\lambda) = \pm 1. \tag{2}$$

Refutation: LHS (1) is a standard definition of an expectation. So, under relativistic causality and functions (A,B) satisfying (2) and LHS (1): let Λ^{+} be the sub-space that delivers $A(a,\lambda)B(b,\lambda) = 1$; then the remainder Λ^{-} delivers $A(a,\lambda)B(b,\lambda) = -1$. So, from (1):

$$E(a,b|\beta) = \int_{\Lambda^+} d\lambda\, \rho(\lambda) A(a,\lambda) B(b,\lambda) + \int_{\Lambda^-} d\lambda\, \rho(\lambda) A(a,\lambda) B(b,\lambda) \tag{3}$$

$$= P\left(AB = 1\,|\,a,b,\Lambda^{+}\right) - P\left(AB = -1\,|\,a,b,\Lambda^{-}\right) : \text{the weighted-sum of } AB \text{ results.} \tag{4}$$

$$= \left[P\left(A^{+}B^{+}\right) + P\left(A^{-}B^{-}\right)\right] - \left[P\left(A^{+}B^{-}\right) + P\left(A^{-}B^{+}\right)\right] : \text{with conditions suppressed,}$$
$$\text{the weighted-sum of the same-instance results } (\pm 1) \text{ that deliver each } AB \text{ result.} \tag{5}$$

$$= P(A^{+})P(B^{+}\,|\,A^{+}) + P(A^{-})P(B^{-}\,|\,A^{-}) - P(A^{+})P(B^{-}\,|\,A^{+}) - P(A^{-})P(B^{+}\,|\,A^{-}) :$$
$$\text{via the product rule for the paired (same-instance) results correlated as in (2).} \tag{6}$$

$$= \tfrac{1}{2}\left[P\left(B^{+}\,|\,A^{+}\right) + P\left(B^{-}\,|\,A^{-}\right) - P\left(B^{-}\,|\,A^{+}\right) - P\left(B^{+}\,|\,A^{-}\right)\right] : \text{for, with}$$
$$\lambda \text{ a random latent variable, the marginal probabilities } \left[\text{like } P\left(A^{+}\right)\right] = \tfrac{1}{2}. \tag{7}$$

$$= \tfrac{1}{2}\left[\sin^2\tfrac{1}{2}(a,b) + \sin^2\tfrac{1}{2}(a,b) - \cos^2\tfrac{1}{2}(a,b) - \cos^2\tfrac{1}{2}(a,b)\right] : \text{equating the probability}$$
$$\text{functions in (7) to } \beta\text{-based laws (akin to Malus' Law for light-beams).} \tag{8}$$

$$= -\cos(a,b) = -a\cdot b. \text{ So RHS (1) is refuted. QED. [See also: } \textbf{BT}_2 \text{ at (20).]} \tag{9}$$

[1] eprb@me.com [Ex: 1989.v0, 2019R.v1, 2020H.v4c] Ref: 2020H.v5 20201024

Further: Bell uses **B**(15), Bell's inequality (**BI**), as proof of his theorem: so we now refute it.

$$\textbf{BI:}\ |E(a,b)-E(a,c)|-1 \leq E(b,c)\ \text{[sic]: ie, } \mathbf{B}(15) \text{ in our notation,} \tag{10}$$

$$\text{where } -1 \leq E(a,b) \leq 1, -1 \leq E(a,c) \leq 1, -1 \leq E(b,c) \leq 1. \text{ However:} \tag{11}$$

$$E(a,b)[1+E(a,c)] \leq 1+E(a,c); \text{ for, if } V \leq 1, \text{ and } 0 \leq W, \text{ then } VW \leq W. \tag{12}$$

$$\therefore E(a,b)-E(a,c)-1 \leq -E(a,b)E(a,c). \text{ Similarly:} \tag{13}$$

$$E(a,c)-E(a,b)-1 \leq -E(a,b)E(a,c). \text{ Hence our irrefutable} \tag{14}$$

$$\text{counter-inequality, } \textbf{WI:}\ |E(a,b)-E(a,c)|-1 \leq -E(a,b)E(a,c). \tag{15}$$

$$\text{So, with test-settings } 0 < (a,c) < \pi;\ (a,b) = (b,c) = \frac{(a,c)}{2} = \frac{x}{2}: \text{ and, via (9),} \tag{16}$$

$$\text{using test-functions } E(a,b) = E(b,c) = -\cos\left(\frac{x}{2}\right), E(a,c) = -\cos(x): \text{ please} \tag{17}$$

$$\text{copy and test this next expression in WolframAlph}a^{\circledR}\text{; free-online, see References.} \tag{18}$$

$$\text{plot}|cos(x)-cos(x/2)|-1\&\&-cos(x/2)\&\&-cos(x/2)cos(x), 0 \leq x \leq \pi \tag{19}$$

Thus, under the generality of (16)-(17): (i) For $0 < x < \pi$, Bell's (10) is everywhere false, our (15) is everywhere true. (ii) For $x = 0$ and $x = \pi$, (10) and (15) are true. (iii) Let the relations between **B**(14) and **B**(15) be **B**(14a)-**B**(14c). (iv) Then Bell's error is his move from *true* **B**(14a) to *false* **B**(14b): for **B**(14b) leads to *false* **B**(15). (v) In other words, given the common LHS in (10) and (15): Bell's error equates his false $E(b,c)$ in (10) to our irrefutable $-E(a,b)E(a,c)$ in (15); hence, as above, Bell's equality only holds at $x = 0$ and $x = \pi$. That is: when Bell's $-\cos\left(\frac{x}{2}\right) = -\cos\left(\frac{x}{2}\right)\cos(x)$.

Conclusions: Under relativistic causality (no influence propagates superluminally) and true (non-naive) realism (some existents change interactively): (i) Bell's theorem (1) and Bell's inequality (10) are refuted; his error identified. (ii) In (8), via our heuristic debt to Malus, we provide the first of a family of laws that refute Bell's theorem in other settings. (iii) A variation of (1), from Bell (1975), is similarly refuted: see Appendix. (iv) Thus, with an improved notation, we confirm a result in Watson 2017D: ie, our detector $\partial_a^{\pm}$ detects the equivalence classes to which each pre-test $p(\lambda)$ and $p(-\lambda)$ belong. That is, on the elements of $\partial_a^{\pm}$'s domain, let $\overset{\partial_a^{\pm}}{\sim}$ denote the equivalence relation *has the same output under* $\partial_a^{\pm}$; $\partial_b^{\pm}$ similarly. Then these clearly-local classes, under the laws in (8), also refute Bell's theorem: to thus expose and dismiss *nonlocality* in an irrefutable relativistically-causal way.

Appendix: Bell (1975) varies his first theorem to propose, in our terms, a second theorem: $\mathbf{BT}_2$.

$$\mathbf{BT}_2\text{: } E(a,b\,|\,\beta) \neq \int_\Lambda d\lambda\, \rho(\lambda) A(a,\lambda) B(b,\lambda) \neq -a\cdot b = E(a,b\,|\,\beta) \text{ [sic]. For, after}$$

Bell (1975:3): with these *local* forms $A(a,\lambda), B(b,\lambda)$, it is *not* possible to find functions

A and B and a probability distribution ρ which give the correlation $E(a,b\,|\,\beta) = -a\cdot b$. (20)

With $\mathbf{BT}_2$ sandwiched between results proven in (3)-(9), we refute (20) via two physically-significant A and B functions under $\int_\Lambda d\lambda\, \rho(\lambda) = 1$ and every β-relevant existent. So: source $[S_\beta]$ emits particle-pairs $p(\lambda)$ and $p(-\lambda)$; their properties (.) pairwise-correlated via $\lambda + (-\lambda) = 0$. $p(\lambda)$ interacts with detector $\partial_a^\pm$, a 2-channel polarizer-analyzer with principal-axis a and output channels $a^\pm \equiv \pm a$. Within $\partial_a^\pm$, polarizer $\Phi_a^\pm$ transforms $p(\lambda)$ to $p(\varphi = a^\pm)$, where φ denotes the post-interaction spin-axis. $p(\varphi = a^\pm)$ then interacts with analyzer $a\cdot\varphi$ to deliver the result $A^\pm = \pm 1$; etc. In shorthand: $A(a,\lambda) = \partial_a^\pm(\lambda) = \pm 1$, $B(b,\lambda) = \partial_b^\pm(-\lambda) = \pm 1$, and $\mathbf{BT}_2$ is refuted as in (3)-(9). Thus:

$$\pm 1 = A^\pm \leftarrow \partial_a^\pm \leftarrow p(\lambda) \leftarrow [S_\beta] \rightarrow p(-\lambda) \rightarrow \partial_b^\pm \rightarrow B^\mp = \mp 1\text{; ie,} \tag{21}$$

$$\pm 1 = [a\cdot\varphi \leftarrow p(\varphi = a^\pm) \leftarrow \Phi_a^\pm] \leftarrow p(\lambda) \leftarrow [S_\beta] \rightarrow p(-\lambda) \rightarrow \left[\Phi_b^\pm \rightarrow p(\varphi = b^\mp) \rightarrow b\cdot\varphi\right] = \mp 1\text{:} \tag{22}$$

$$\text{thus, via } \partial_a^\pm\text{: } p(\lambda) \rightarrow [\Phi_a^\pm \rightarrow p(\varphi = a^\pm) \rightarrow a\cdot\varphi] = \pm 1 = A^\pm\text{: in short, } \partial_a^\pm(\lambda) = \pm 1\text{; etc.} \tag{23}$$

$$\text{Thus, as in (3)-(9), } \mathbf{BT}_2 \text{ is refuted: } E(a,b|\beta) = \int_\Lambda d\lambda\, \rho(\lambda) \partial_a^\pm(\lambda) \partial_b^\pm(-\lambda) = -a\cdot b\text{. QED.} \tag{24}$$

For some proposed consequences of the results here: see Watson (2020E). (25)

References:

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