PROVING RIEMANN HYPOTHESIS

KORN RAKPRADIT[↑]

OCTOBER 3, 2020

Abstract

In mathematics we require a proof for any hypothesis. But in the past 160 years ago a man called Bernhard Riemann refused to give a proof for his hypothesis about zeta function which he thought would be the formula for the number of primes less than a given number 'x'. He might have a very 'serious' reason for not proving this. But for me, after five years of attempt to stalk his papers and using only a high school mathematical knowledge and/or a little bit higher to prove it, I can say that all high school students can learn how to prove this hypothesis whether it is right or wrong. Let me show you the proof.

Introduction

1. Riemann Zeta Function and Riemann Hypothesis

Riemann Zeta Function is a function of a complex variable $s = (\sigma + it)$ that can be written as the summation of infinite series

$$\zeta(s) = \sum_{n=1}^{+\infty} \left(\frac{1}{n^s}\right)$$

$$= 1+\frac{1}{2^s}+\frac{1}{3^s}+\frac{1}{4^s}+\frac{1}{5^s}+\ldots;\ n = 1, 2, 3,\ldots,+\infty \text{ (all natural numbers)}$$

For $(n)^s = (e)^{s\mathrm{Log}(n)}$; $\mathrm{Log}(n)$ = natural logarithm of n

$$\zeta(s) = \frac{1}{(e)^{s\mathrm{Log}1}} + \frac{1}{(e)^{s\mathrm{Log}2}} + \frac{1}{(e)^{s\mathrm{Log}3}} + \ldots$$

$$= \frac{1}{(e)^{\sigma\mathrm{Log}1}(e)^{it\mathrm{Log}1}} + \frac{1}{(e)^{\sigma\mathrm{Log}2}(e)^{it\mathrm{Log}2}} + \frac{1}{(e)^{\sigma\mathrm{Log}3}(e)^{it\mathrm{Log}3}} + \ldots$$

$$= r_1(e)^{-it\mathrm{Log}1} + r_2(e)^{-it\mathrm{Log}2} + r_3(e)^{-it\mathrm{Log}3} + \ldots$$

$$= r_1[\cos(t\mathrm{Log}1) - i\sin(t\mathrm{Log}1)] + r_2[\cos(t\mathrm{Log}2) - i\sin(t\mathrm{Log}2)] + \ldots$$

KORN RAKPRADIT[↑] Electrical Engineer, Faculty of Engineering, Chiang Mai University, Thailand.

e-mail : kornr112@gmail.com

$= \sum_{n=1}^{+\infty} (r_n(e)^{-itLogn})$; $r_n = \frac{1}{(e)^{\sigma Logn}} = \frac{1}{(n)^{\sigma}}$ = modulus or amplitude and $(e)^{-itLogn} = [\cos(tLogn) - i\sin(tLogn)]$ = argument or phasor of each component of ζ(s) on each complex plane of each value of n in space.

1.1 Riemann Zeta Function of natural numbers (n=1,2,3,...) on the positive real line while $s = (\sigma + it)$ or any complex numbers which $\sigma > 0$.

This is the original function of $\zeta(s) = \sum_{n=1}^{+\infty} (\frac{1}{n^s})$. The value of real part, σ of complex number $s = (\sigma + it)$, which is the origin of the value of the modulus or amplitude of ζ(s), points out that ζ(s) will converge when real part of s (or σ) is more than 1 and will diverge when real part of s (or σ) is equal or less than 1but more than 0 ($0 < \sigma \leq 1$ (p-series or hyperharmonic series).

While the value of imaginary part, it (of complex number s) which is the origin of the phasor or argument (tLogn) of ζ(s) (where Log(n) stands for natural logarithm of n) shows the rotation of modulus or amplitude $\left(\frac{1}{(n)^{\sigma}}\right)$ on the complex plane of each positive value of n in space , the amplitude $\frac{1}{(n)^{\sigma}}$ of ζ(s) will lay on real line (axis) when itLogn $= 0, \pm\pi, \pm 2\pi, \ldots$ and will lay on imaginary axis when tLog n $= \pm\frac{\pi}{2}, \pm\frac{3\pi}{4}, \ldots$. With other angles or arguments (– tLogn), each of all the amplitudes, $\frac{1}{(n)^{\sigma}}$ will rotate or spread on each complex plane of each positive value of n in space.

1.1.1 Case $\sigma > 1$, t = 0

For $\sigma = 2$, t = 0 ; $\zeta(2 + i0) = \sum_{n=1}^{+\infty} (r_n(e)^{-itLogn})$

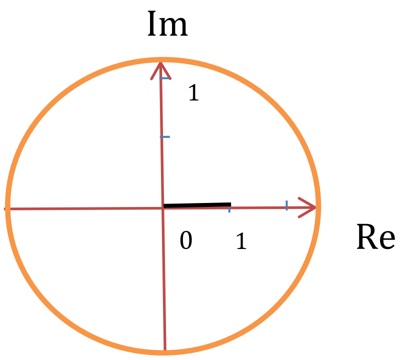
$$= r_1(e)^{-i(0)Log1} + r_2(e)^{-i(0)Log2} + r_3(e)^{-i(0)Log3} + \ldots$$

$$= \frac{1}{1^2}[\cos(0) + i\sin(0)] + \frac{1}{2^2}[\cos(0) + i\sin(0)]$$

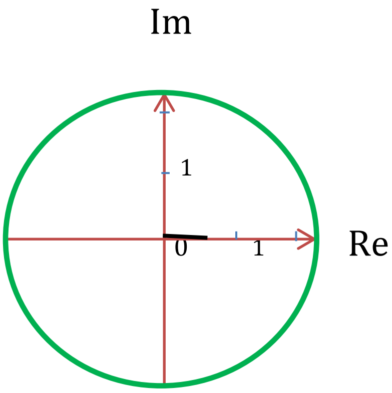
$$+ \frac{1}{3^2}[\cos(0) + i\sin(0)] + \ldots$$

$$= \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots \quad \text{(Basel Problem)}$$

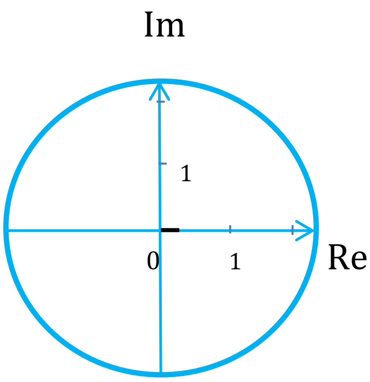
$$= \frac{(\pi)^{(2)}}{6} \quad \text{converges}$$



$r_1 = 1$

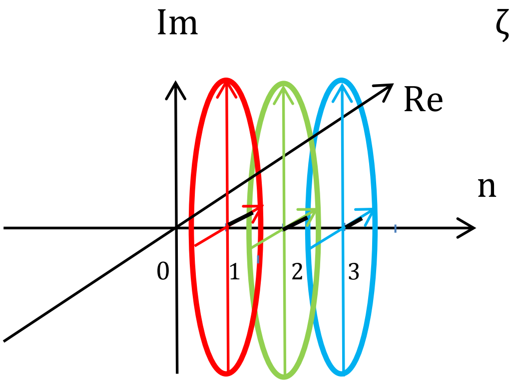


$r_2 = \frac{1}{2^2}$



$r_3 = \frac{1}{3^2}$

……….



$$\zeta(2 + i0) = \sum_{n=1}^{+\infty} \left(r_n (e)^{-it\mathrm{Log}n}\right)$$

$$= r_1 + r_2 + r_3 + \ldots$$

$$= \frac{(\pi)^{(2)}}{6}$$

≈ 1.6449 converges while each amplitude, r_n lays on the real axis of each complex plane of each positive value of n in space.

1.1.2 Case $\sigma > 1$, $t \neq 0$

For $\sigma = 2$, $t \neq 0$; $\zeta(2+it) = \sum_{n=1}^{+\infty} (r_n(e)^{-itLogn})$

$$= r_1(e)^{-itLog1} + r_2(e)^{-itLog2} + r_3(e)^{-itLog3} + \ldots$$

$$= \frac{1}{1^2}(e)^{-itLog1} + \frac{1}{2^2}(e)^{-itLog2} + \frac{1}{3^2}(e)^{-itLog3} + \ldots$$

$$= \frac{1}{1^2}[\cos(tLog1) - i\sin(tLog1)] + \frac{1}{2^2}[\cos(tLog2) - i\sin(tLog2)] + \frac{1}{3^2}[\cos(tLog3) - i\sin(tLog3)] + \ldots$$

$$= \sum_{n=1}^{+\infty} [r_n\cos(tLogn) - ir_n\sin(tLogn)]$$

Then $\zeta(2+it)$ is the summation (vector sum) of all the amplitudes (moduli) $\frac{1}{1^2}, \frac{1}{2^2}, \frac{1}{3^2}, \ldots$ which appears on each of all the complex planes of each value of n with arguments or phasors or angles of $-tLog1, -tLog2, -tLog3, \ldots$. $\zeta(2+it)$ will converge when looks over the whole complex planes of all the positive values of n in space.

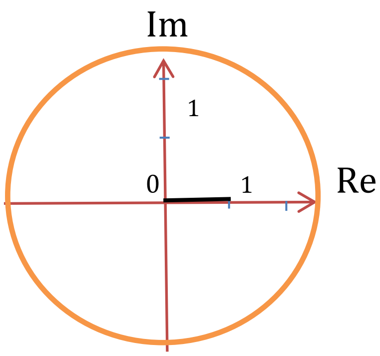
(Note that summation of scalar quantities ≥ summation of vector quantities of the same amplitudes (moduli) up to the differences of angles or phasors tLog1, tLog(2),...)

$$\frac{1}{1^2} + \frac{1}{2^2} \geq \frac{1}{1^2}(e)^{-itLog1} + \frac{1}{2^2}(e)^{-itLog2}$$

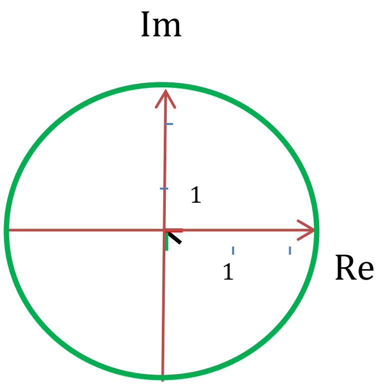
$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} \geq \frac{1}{1^2}(e)^{-itLog1} + \frac{1}{2^2}(e)^{-itLog2} + \frac{1}{3^2}(e)^{-itLog3}$$

$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots \geq \frac{1}{1^2}(e)^{-itLog1} + \frac{1}{2^2}(e)^{-itLog2} + \frac{1}{3^2}(e)^{-itLog3} + \ldots$$

Or $\zeta(2) \geq \zeta(2+it)$ and both converge



$r_1 = 1$, $(e)^{-itLog1} = [\cos tLog1 - i\sin tLog1]$

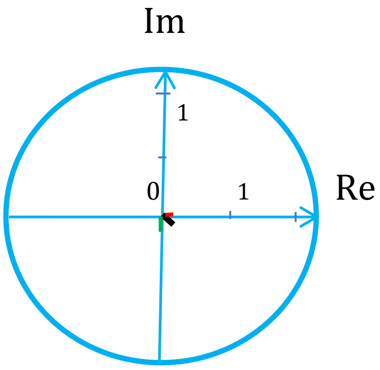


$r_2 = \frac{1}{2^2}$, $(e)^{-it\text{Log}2} = [\cos t\text{Log}2 - i\sin t\text{Log}2]$

$= r_2(e)^{-it\text{Log}2}$

$= r_2 \cos t\text{Log}2$

$= i r_2 \sin t\text{Log}2$



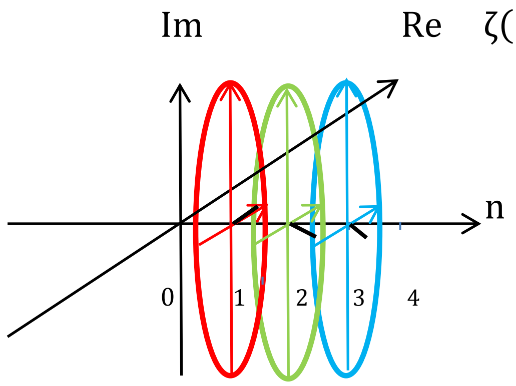
$r_3 = \frac{1}{3^2}$, $(e)^{-it\text{Log}3} = [\cos t\text{Log}3 - i\sin t\text{Log}3]$

$= r_3(e)^{-it\text{Log}3}$

$= r_3 \cos t\text{Log}3$

$= i r_3 \sin t\text{Log}3$

....



$\zeta(2+it) = \sum_{n=1}^{+\infty} (r_n(e)^{-it\text{Log}n})$

$= \frac{1}{1^2}(e)^{-it\text{Log}1} + \frac{1}{2^2}(e)^{-it\text{Log}2} + \frac{1}{3^2}(e)^{-it\text{Log}3} + \cdots$

$= \sum_{n=1}^{+\infty} [r_n \cos t\text{Log}n - i r_n \sin t\text{Log}n]$

1.1.3 Case $\sigma = 1, t = 0$

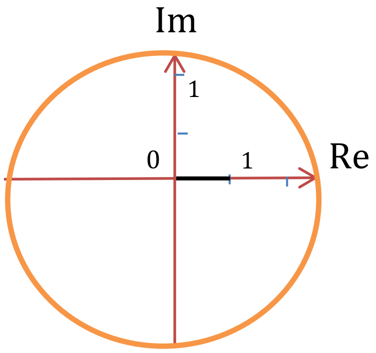
For $\sigma = 1, t = 0$; $\zeta(1 + i0) = \sum_{n=1}^{+\infty} (r_n(e)^{-it\text{Log}n})$

$= r_1(e)^{-i(0)\text{Log}1} + r_2(e)^{-i(0)\text{Log}2} + r_3(e)^{-i(0)\text{Log}3} + \ldots$

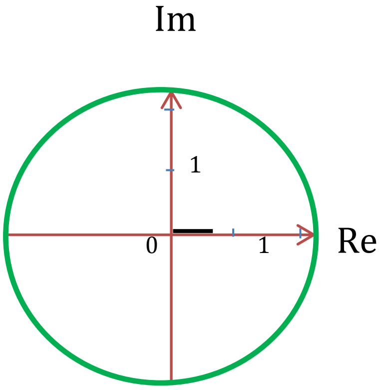
$= \frac{1}{1^1} + \frac{1}{2^1} + [\frac{1}{3^1} + \frac{1}{4^1}] + [\frac{1}{5^1} + \frac{1}{6^1} + \frac{1}{7^1} + \frac{1}{8^1}] + \ldots$ Harmonic series

$> \frac{1}{1^1} + \frac{1}{2^1} + [\frac{1}{2^1}] + [\frac{1}{2^1}] + \ldots$ which $= +\infty$

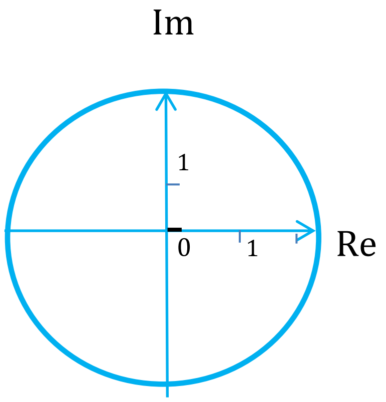
So $\zeta(1 + i0) = +\infty$ (diverges)



$r_1 = 1$

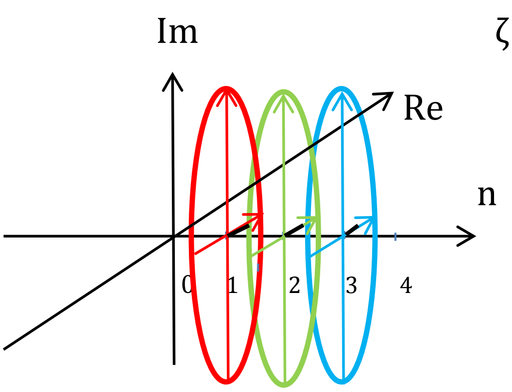


$r_2 = \frac{1}{2}$



$r_3 = \frac{1}{3}$

……….



$$\zeta(1 + i0) = \sum_{n=1}^{+\infty} \left(r_n (e)^{-itLogn}\right)$$

$$= r_1 + r_2 + r_3 + \ldots$$

$$= \frac{1}{1^1} + \frac{1}{2^1} + \frac{1}{3^1} + \ldots$$

$= +\infty$ diverges when looks over the whole complex planes of all the positive values of n in space.

1.1.4 Case $\sigma = 1$, $t \neq 0$

For $\sigma = 1, t \neq 0$; $\zeta(1 + it) = \sum_{n=1}^{+\infty} (r_n (e)^{-itLogn})$

$$= r_1 (e)^{-itLog1} + r_2 (e)^{-itLog2} + r_3 (e)^{-itLog3} + \ldots$$

$$= \frac{1}{(1)^1} (e)^{-it(1)} + \frac{1}{(2)^1} (e)^{-itLog2} + \frac{1}{(3)^1} (e)^{-itLog3} + \ldots$$

$\zeta(1 + it)$ is summation (vector sum) of all the amplitudes (moduli) which appear on each of the complex planes of each value of n with arguments or angles $of - tLog1, -tLog2, \; -tLog3, \ldots$. $\zeta(1 + it)$ may diverge or converge when looks over the whole complex planes of all the values of n.

(Note a. When $t \approx 0$; $tLog1, tLog2, \ldots \approx 0$

$$[\frac{1}{3^{(1+it)}} + \frac{1}{4^{(1+it)}}] \approx [\frac{1}{3^1} + \frac{1}{4^1}]$$

$$[\frac{1}{5^{(1+it)}} + \frac{1}{6^{(1+it)}} + \frac{1}{7^{(1+it)}} + \frac{1}{8^{(1+it)}}] \approx [\frac{1}{5^1} + \frac{1}{6^1} + \frac{1}{7^1} + \frac{1}{8^1}]$$

………

Then $\frac{1}{1^{(1+it)}} + \frac{1}{2^{(1+it)}} + [\frac{1}{3^{(1+it)}} + \frac{1}{4^{(1+it)}}] + [\frac{1}{5^{(1+it)}} + \frac{1}{6^{(1+it)}} + \frac{1}{7^{(1+it)}} + \frac{1}{8^{(1+it)}}] + \ldots\ldots$

$\approx \frac{1}{1^1} + \frac{1}{2^1} + [\frac{1}{3^1} + \frac{1}{4^1}] + [\frac{1}{5^1} + \frac{1}{6^1} + \frac{1}{7^1} + \frac{1}{8^1}] + \ldots$ which diverges ($= +\infty$).

So $\zeta(1 + it) = \frac{1}{1^{(1+it)}} + \frac{1}{2^{(1+it)}} + \frac{1}{3^{(1+it)}} + \frac{1}{4^{(1+it)}} + \frac{1}{5^{(1+it)}} + \frac{1}{6^{(1+it)}} + \frac{1}{7^{(1+it)}} + \ldots$

diverges when $t \approx 0$; $tLog1, tLog2, \ldots \approx 0$

b. When $t > 0$; $tLog2 > tLog1$; $tLog3 > tLog2; \ldots$

This may cause vector sum $[\frac{1}{3^{(1+it)}} + \frac{1}{4^{(1+it)}}]$ < or $\approx [\frac{1}{3^1} + \frac{1}{4^1}]$ scalar sum

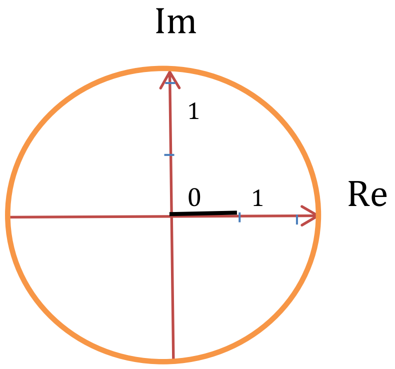
and $[\frac{1}{5^{(1+it)}} + \frac{1}{6^{(1+it)}} + \frac{1}{7^{(1+it)}} + \frac{1}{8^{(1+it)}}]$ < or $\approx [\frac{1}{5^1} + \frac{1}{6^1} + \frac{1}{7^1} + \frac{1}{8^1}]$

………..

Then $\frac{1}{1^{(1+it)}} + \frac{1}{2^{(1+it)}} + [\frac{1}{3^{(1+it)}} + \frac{1}{4^{(1+it)}}] + [\frac{1}{5^{(1+it)}} + \frac{1}{6^{(1+it)}} + \frac{1}{7^{(1+it)}} + \frac{1}{8^{(1+it)}}] + \ldots\ldots$

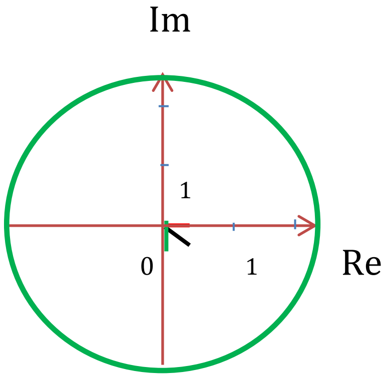
< or $\approx \frac{1}{1^1} + \frac{1}{2^1} + [\frac{1}{3^1} + \frac{1}{4^1}] + [\frac{1}{5^1} + \frac{1}{6^1} + \frac{1}{7^1} + \frac{1}{8^1}] + \ldots$

So $\zeta(1+it)$ may diverge or converge up to the differences between angles or phasors tLog1, tLog2, tLog3, ...



$r_1 = \frac{1}{1^1}$, $(e)^{-itLog1} = [\cos(tLog1)$

$-i\sin(tLog1)]$



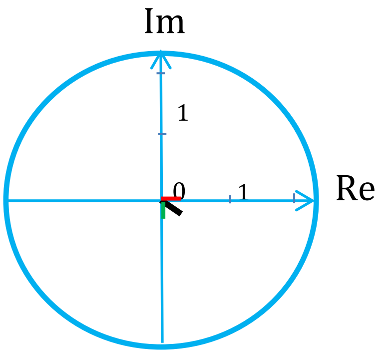
$r_2 = \frac{1}{2^1}$, $(e)^{-itLog2} = [\cos(tLog2)$

$-i\sin(tLog2)]$

\ $= r_2(e)^{-itLog2}$

– $= r_2 costLog2$

| $= ir_2 sintLog2$



$r_3 = \frac{1}{3^1}$, $(e)^{-itLog3} = [\cos(tLog3)$

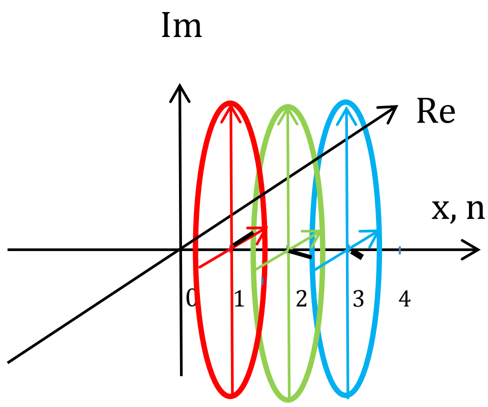
$-i\sin(tLog3)]$

\ $= r_3(e)^{-itLog3}$

– $= r_3\cos(tLog3)$

| $= ir_3\sin(tLog3)$

……….



$\zeta(1+it) = \sum_{n=1}^{+\infty} (r_n(e)^{-itLogn})$

$= \frac{1}{1^1}(e)^{-itLog1} + \frac{1}{2^1}(e)^{-itLog2}$

$+ \frac{1}{3^1}(e)^{-itLog3} + \cdots$

$= \sum_{n=1}^{+\infty} [r_n\cos(tLogn)$

$-ir_n\sin(tLogn)]$

1.1.5 Case $\sigma < 1$, t = 0

For $\sigma = \frac{1}{2}, t = 0$; $\zeta\left(\frac{1}{2} + i0\right) = \sum_{n=1}^{+\infty} \left(\frac{1}{(n)^{\left(\frac{1}{2}+i0\right)}}\right)$

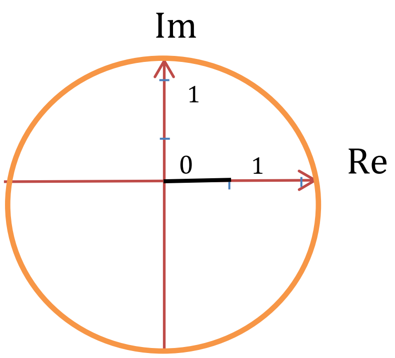
$$= \frac{1}{(1)^{\frac{1}{2}}} + \frac{1}{(2)^{\frac{1}{2}}} + \frac{1}{(3)^{\frac{1}{2}}} + \ldots$$

$$= \frac{(1)^{\frac{1}{2}}}{1} + \frac{(2)^{\frac{1}{2}}}{2} + \frac{(3)^{\frac{1}{2}}}{3} + \ldots$$

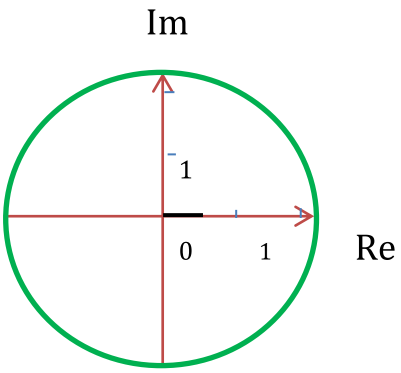
But from Harmonic series $1 + \frac{1}{2} + \frac{1}{3} + \ldots \quad = +\infty$ (diverges)

And from $1 + \frac{1.414}{2} + \frac{1.732}{3} + \ldots > 1 + \frac{1}{2} + \frac{1}{3} + \ldots$

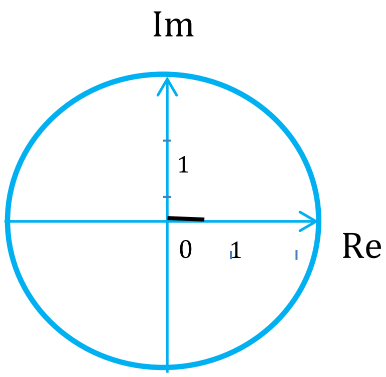
Then $\zeta\left(\frac{1}{2} + i0\right) = 1 + \frac{1.414}{2} + \frac{1.732}{3} + \ldots \quad = +\infty$ (diverges too)



$r_1 = 1$

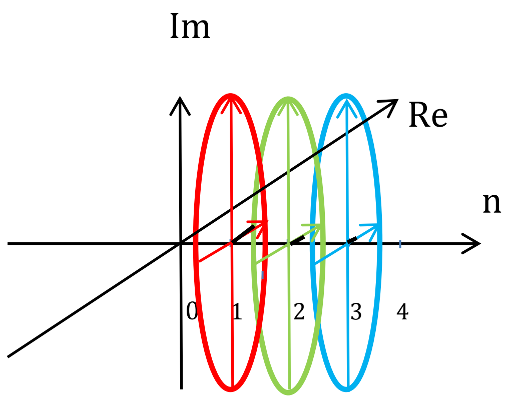


$r_2 = \frac{1}{(2)^{\frac{1}{2}}}$



$r_3 = \frac{1}{(3)^{\frac{1}{2}}}$

…………



$$\zeta\left(\frac{1}{2}+i0\right)=\sum_{n=1}^{+\infty}\left(r_n(e)^{-it\mathrm{Log}n}\right)$$

$$= r_1 + r_2 + r_3 + \dots$$

$$=\frac{1}{(1)^{\frac{1}{2}}}+\frac{1}{(2)^{\frac{1}{2}}}+\frac{1}{(3)^{\frac{1}{2}}}+\dots$$

$$= +\infty \ (\text{ diverges})$$

1.1.6 Case $\sigma < 1$, $t \neq 0$

For $\sigma = \frac{1}{2}, t \neq 0$; $\zeta\left(\frac{1}{2}+it\right)=\sum_{n=1}^{+\infty}\left(r_n(e)^{-it\mathrm{Log}n}\right)$

$$= r_1(e)^{-it\mathrm{Log}1} + r_2(e)^{-it\mathrm{Log}2} + r_3(e)^{-it\mathrm{Log}3} + \dots$$

$$= \frac{1}{(1)^{\frac{1}{2}}}(e)^{-it(0)} + \frac{1}{(2)^{\frac{1}{2}}}(e)^{-it\mathrm{Log}2} + \frac{1}{(3)^{\frac{1}{2}}}(e)^{-it\mathrm{Log}3} + \dots$$

$$= 1 + \frac{(2)^{\frac{1}{2}}(e)^{-it\mathrm{Log}2}}{2} + \frac{(3)^{\frac{1}{2}}(e)^{-it\mathrm{Log}3}}{3} + \dots$$

$\zeta\left(\frac{1}{2}+it\right)$ are summation of amplitudes (moduli) $1, \frac{1}{(2)^{\frac{1}{2}}}, \frac{1}{(3)^{\frac{1}{2}}}, \dots = (r_n)$ which appear on each of the complex planes of each value of n with arguments or angles $-t\mathrm{Log}1, -t\mathrm{Log}2, -t\mathrm{Log}3, \dots$ $\zeta\left(\frac{1}{2}+it\right)$ may diverge or converge when looks over the whole complex planes of all the values of n.

(Note a. When $t \approx 0$; $t\mathrm{Log}1, t\mathrm{Log}2 \dots \approx 0$

$$\left[\frac{1}{(3)^{\left(\frac{1}{2}+it\right)}}+\frac{1}{(4)^{\left(\frac{1}{2}+it\right)}}\right] \approx \left[\frac{1}{(3)^{\left(\frac{1}{2}\right)}}+\frac{1}{(4)^{\left(\frac{1}{2}\right)}}\right]$$

$$\left[\frac{1}{(5)^{\left(\frac{1}{2}+it\right)}}+\frac{1}{(6)^{\left(\frac{1}{2}+it\right)}}+\frac{1}{(7)^{\left(\frac{1}{2}+it\right)}}+\frac{1}{(8)^{\left(\frac{1}{2}+it\right)}}\right] \approx \left[\frac{1}{(5)^{\left(\frac{1}{2}\right)}}+\frac{1}{(6)^{\left(\frac{1}{2}\right)}}+\frac{1}{(7)^{\left(\frac{1}{2}\right)}}+\frac{1}{(8)^{\left(\frac{1}{2}\right)}}\right]$$

………

$$\frac{1}{(1)^{\left(\frac{1}{2}+it\right)}}+\frac{1}{(2)^{\left(\frac{1}{2}+it\right)}}+[\frac{1}{(3)^{\left(\frac{1}{2}+it\right)}}+\frac{1}{(4)^{\left(\frac{1}{2}+it\right)}}]+[\frac{1}{(5)^{\left(\frac{1}{2}+it\right)}}+\frac{1}{(6)^{\left(\frac{1}{2}+it\right)}}+\frac{1}{(7)^{\left(\frac{1}{2}+it\right)}}+\frac{1}{(8)^{\left(\frac{1}{2}+it\right)}}]+\ldots\ldots$$

$$\approx\frac{1}{(1)^{\left(\frac{1}{2}\right)}}+\frac{1}{(2)^{\left(\frac{1}{2}\right)}}+[\frac{1}{(3)^{\left(\frac{1}{2}\right)}}+\frac{1}{(4)^{\left(\frac{1}{2}\right)}}]+[\frac{1}{(5)^{\left(\frac{1}{2}\right)}}+\frac{1}{(6)^{\left(\frac{1}{2}\right)}}+\frac{1}{(7)^{\left(\frac{1}{2}\right)}}+\frac{1}{(8)^{\left(\frac{1}{2}\right)}}]+\ldots$$

So $\zeta\left(\frac{1}{2}+it\right)$ diverges when $t\approx 0$; $t\mathrm{Log}1, t\mathrm{Log}2, \ldots\approx 0$

b. When $t>0; t\mathrm{Log}2>t\mathrm{Log}1; t\mathrm{Log}3>t\mathrm{Log}2; \ldots$

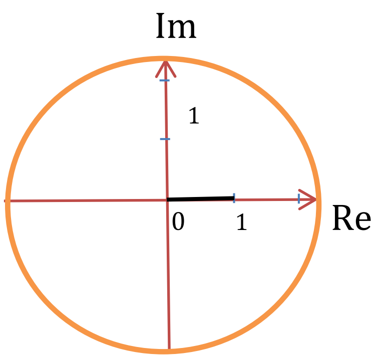
This may cause vector sum $[\frac{1}{3^{\left(\frac{1}{2}+it\right)}}+\frac{1}{4^{\left(\frac{1}{2}+it\right)}}]<$ or $\approx[\frac{1}{(3)^{\left(\frac{1}{2}\right)}}+\frac{1}{(4)^{\left(\frac{1}{2}\right)}}]$ scalar sum and $[\frac{1}{(5)^{\left(\frac{1}{2}+it\right)}}+\frac{1}{(6)^{\left(\frac{1}{2}+it\right)}}+\frac{1}{(7)^{\left(\frac{1}{2}+it\right)}}+\frac{1}{(8)^{\left(\frac{1}{2}+it\right)}}]<$ or $\approx[\frac{1}{(5)^{\left(\frac{1}{2}\right)}}+\frac{1}{(6)^{\left(\frac{1}{2}\right)}}+\frac{1}{(7)^{\left(\frac{1}{2}\right)}}+\frac{1}{(8)^{\left(\frac{1}{2}\right)}}]$

………..

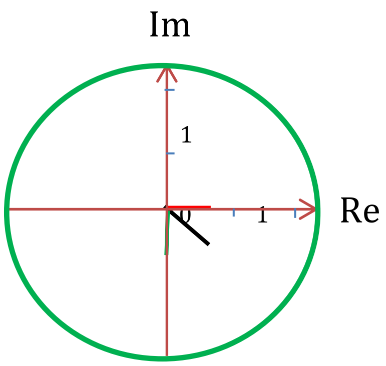
$$\frac{1}{(1)^{\left(\frac{1}{2}+it\right)}}+\frac{1}{(2)^{\left(\frac{1}{2}+it\right)}}+[\frac{1}{(3)^{\left(\frac{1}{2}+it\right)}}+\frac{1}{(4)^{\left(\frac{1}{2}+it\right)}}]+[\frac{1}{(5)^{\left(\frac{1}{2}+it\right)}}+\frac{1}{(6)^{\left(\frac{1}{2}+it\right)}}+\frac{1}{(7)^{\left(\frac{1}{2}+it\right)}}+\frac{1}{(8)^{\left(\frac{1}{2}+it\right)}}]+\ldots\ldots$$

$$<\text{ or }\approx\frac{1}{(1)^{\left(\frac{1}{2}\right)}}+\frac{1}{(2)^{\left(\frac{1}{2}\right)}}+[\frac{1}{(3)^{\left(\frac{1}{2}\right)}}+\frac{1}{(4)^{\left(\frac{1}{2}\right)}}]+[\frac{1}{(5)^{\left(\frac{1}{2}\right)}}+\frac{1}{(6)^{\left(\frac{1}{2}\right)}}+\frac{1}{(7)^{\left(\frac{1}{2}\right)}}+\frac{1}{(8)^{\left(\frac{1}{2}\right)}}]+\ldots$$

So $\zeta\left(\frac{1}{2}+it\right)$ may diverge or converge up to the differences between angles or phasors $t\mathrm{Log}1, t\mathrm{Log}2, t\mathrm{Log}3, \ldots$



$$r_1=\frac{1}{(1)^{\frac{1}{2}}},\quad (e)^{-it\mathrm{Log}1}=[\cos t\mathrm{Log}1 - i\sin t\mathrm{Log}1]$$

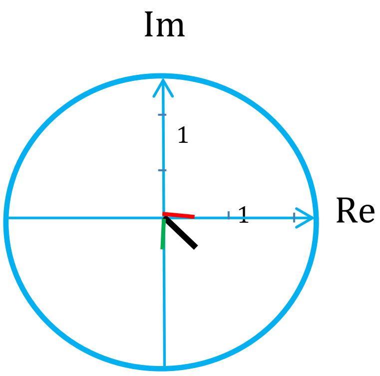


$r_2 = \frac{1}{(2)^{\frac{1}{2}}}$, $(e)^{-it\mathrm{Log}2} = [\cos t\mathrm{Log}2 - i\sin t\mathrm{Log}2]$

$= r_2(e)^{-it\mathrm{Log}2}$

$= r_2\cos t\mathrm{Log}2$

$= ir_2\sin t\mathrm{Log}2$



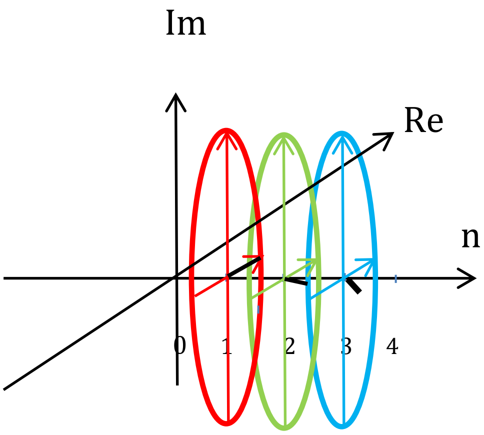
$r_3 = \frac{1}{(3)^{\frac{1}{2}}}$, $(e)^{-it\mathrm{Log}3} = [\cos t\mathrm{Log}3 - i\sin t\mathrm{Log}3]$

$= r_3(e)^{-it\mathrm{Log}3}$

$= r_3\cos(t\mathrm{Log}3)$

$= ir_3\sin(t\mathrm{Log}3)$

………



$$\zeta\left(\frac{1}{2} + it\right) = \sum_{n=1}^{+\infty} \left(r_n(e)^{-it\mathrm{Log}n}\right)$$

$$= \frac{1}{(1)^{\frac{1}{2}}}(e)^{-it\mathrm{Log}1} + \frac{1}{(2)^{\frac{1}{2}}}(e)^{-it\mathrm{Log}2} + \frac{1}{(3)^{\frac{1}{2}}}(e)^{-it\mathrm{Log}3} + \cdots$$

$$= \sum_{n=1}^{+\infty} [r_n\cos(t\mathrm{Log}n) - ir_n\sin(t\mathrm{Log}n)]$$

1.2 Riemann Zeta Function of natural numbers on the negative real Line (in addition to Riemann Zeta Function of natural numbers on the positive real line as mentioned before) while $s = (\sigma + it)$ or any complex numbers which $\sigma > 0$.

Riemann tried to find the Riemann Zeta Function of natural numbers on the negative real Line as well as on the positive real line when s

$= (\sigma + i\text{t}) =$ any complex numbers by applying analytic continuation on an equation. He started from using Pi function, $\prod(\text{s}-1) =$ Gamma function, $\Gamma(\text{s}) = \int_0^{+\infty} (e)^{(-\text{u})}(\text{u})^{(\text{s}-1)}\,\text{du}$ as source of his new derived equation $\prod(\text{s}-1)\zeta(\text{s}) = \int_0^{+\infty} \frac{(\text{x})^{(\text{s}-1)}}{(\text{e}^{\text{x}}-1)}\,\text{dx}$. Next he applied analytic continuation technique to get functional equation $2\sin(\pi s)\prod(\text{s}-1)\zeta(\text{s}) = i\int_{+\infty}^{+\infty} \frac{(-\text{x})^{(\text{s}-1)}}{(\text{e}^{\text{x}}-1)}\,\text{dx}$ of x on the whole real line and then he gave out trivial zeroes or Res $= \sigma = -2, -4, -6, \ldots$ which he thought that caused the functional equation $2\sin(\pi s)\prod(\text{s}-1)\zeta(\text{s}) = i\int_{+\infty}^{+\infty} \frac{(-\text{x})^{(\text{s}-1)}}{(\text{e}^{\text{x}}-1)}\,\text{dx} = 0$.

The interesting connection between the Riemann Zeta Function and Prime numbers was discovered by Euler as

$$\sum_{\text{n}=1}^{+\infty} \left(\frac{1}{\text{n}^{\text{s}}}\right) = \prod_{\text{p prime}} \left[\frac{1}{(1-\frac{1}{\text{p}^{\text{s}}})}\right]$$

p = 2, 3, 5, ... (all prime numbers)

While $\prod_{p\ prime} \left[\frac{1}{(1-\frac{1}{p^{\text{s}}})}\right] = \frac{1}{(1-\frac{1}{2^{\text{s}}})(1-\frac{1}{3^{\text{s}}})(1-\frac{1}{5^{\text{s}}})(1-\frac{1}{7^{\text{s}}})(1-\frac{1}{11^{\text{s}}})\ldots}$ is called the Euler Product Formula

In Riemann’s 1859 article “On the Number of Primes Less Than a Given Magnitude”, he extended the Euler Product Formula to a complex variable, presented its meromorphic continuation and functional equation, and established the relation between its zeroes (if existed) and the distribution of prime numbers.

The Riemann Zeta Function was hoped to satisfy the Riemann Functional Equation below

$$\zeta(\text{s}) = (2)^{(\text{s})}\pi^{(\text{s}-1)}\sin\left(\frac{\pi s}{2}\right)\Gamma(1-\text{s})\zeta(1-\text{s}).$$

This equation showed relation of Riemann Zeta Function at point s and (1− s).The functional equation implied that ζ(s) had zeroes at each negative even integer s =−2, −4, −6,...(which has to be proved whether it is true or false).

Riemann also defined a function

$$\prod\left(\frac{s}{2}\right)\left(\frac{s}{2}-1\right)(\pi)^{\left(-\frac{s}{2}\right)}\zeta(s) = \frac{1}{(s)(s-1)} + \int_{1}^{+\infty} \psi(x)[(x)^{\left(\frac{s}{2}-1\right)}+(x)^{-\left(\frac{1+s}{2}\right)}]dx$$

And then he set $s = \frac{1}{2} + it$

So, $\prod\left(\frac{s}{2}\right)\left(\frac{s}{2}-1\right)(\pi)^{\left(-\frac{s}{2}\right)}\zeta(s) = \frac{1}{2} + \frac{\left(tt+\frac{1}{4}\right)}{2}\int_{1}^{+\infty} \psi(x)(x)^{-\left(\frac{3}{4}\right)}\cos(\frac{1}{2}t\mathrm{Log}x)dx$

$$= \xi(t)$$

(Note that Log x = natural logarithm of x)

Riemann Hypothesis states that all nontrivial zeroes of the Riemann Zeta Function have their real parts equal to $\frac{1}{2}$ or one can say that all nontrivial zeroes are in the open strip $\{ s \in C : 0 < \mathrm{Re}s < 1\}$ which is called the critical strip, and the line $\mathrm{Re}s = \frac{1}{2}$ which all of the nontrivial zeroes lie on is called the critical line.

2. Gamma function

Gamma function is an extension of the factorial function n! (which is the product of all positive integers less than or equal to n) to real line and complex plane with its argument shifted by 1 .

$\Gamma(n) = (n-1)!$, for n = 1, 2, 3, … (positive integers)

$\Gamma(s+1) = (s)! = s\,\Gamma(s)$

And $\Gamma(s) = \int_{0}^{+\infty}(e)^{(-x)}(x)^{(s-1)}dx$, for s = complex numbers $(\sigma + it)$ with a positive real part, $\mathrm{Re}s = \sigma = 1, 2, 3, \ldots$

This improper integral can be extended by analytic continuation technique to all real and complex numbers except the non-positive integers (where $\Gamma(s)$ has simple poles) yielding the meromorphic function.

3. Gamma function on the whole complex plane

Gamma function, which defines in positive half- complex plane, has a unique analytic continuation to the negative half- complex plane.

$$\Gamma(s) = \frac{\Gamma(s+k)}{s(s+1)(s+2)\ldots(s+k-1)}$$

$k = 0, 1, 2, 3, \ldots$ and $s+k > 0$, or $s > -k$, $s \neq 0, -1, -2, \ldots, -(k-1)$.

The product of the denominator is zero when s equals any integer 0, $-1, -2, \ldots$ Then the gamma function must be undefined at those points. It is a meromorphic function with simple poles at those non-positive integers.

The gamma function is non-zero everywhere along the real line, although it comes nearly close to zero as $s \to \infty$. There is no complex number s for which $\Gamma(s) = 0$ and hence the reciprocal gamma function $\frac{1}{\Gamma(s)}$ is an entire function with zeroes at $s = 0, -1, -2, -3, \ldots$

4. Analytic function and analytic continuation technique

Analytic function is a function that is locally given by a convergent power series. There are both real and complex analytic functions. Analytic functions of each type are infinitely differentiable. A function is analytic if and only if its Taylor series about x_0 converges to the function in some neighborhood for every x_0 in its domain.

Any real analytic function on some open set on the real line can be extended to a complex analytic function on some open set of the complex plane. However, not every real analytic function defined on the whole real line can be extended to a complex analytic function defined on the whole complex plane. For example, the function $f(x) = \frac{1}{x^2+1}$ is not defined for $x = \pm i$.

Analytic continuation is a technique to extend the domain of a given analytic function. Analytic continuation often succeeds in defining further values of a function, for example in a new region where an infinite series represent in terms of which it is initially defined to be divergent.

Suppose f is an analytic function defined on a non-empty open subset U of the complex plane C. If V is a larger open subset than U of C (U is contained in V), and F is an analytic function defined on V such that $F(z) = f(z)$, then F is called an analytic continuation of f. The restriction of F to U is the function f we started with.

5. Pi function

Pi function is an alternative notation which was originally introduced by Gauss which in term of the gamma function is

$$\prod(s) = \Gamma(s + 1)$$

$$= s\Gamma(s)$$

$$= \int_0^{+\infty} (e)^{(-x)}(x)^{(s)} dx$$

Or $\prod(n) = n!$, for n = 1, 2, 3,... (positive integers)

6. Functional equation

A functional equation is any equation that cannot be reduced to algebraic equation easily. The equation relates the values of a function (or functions) at some points with its values at the other points, for example

$$f(x + y) = f(x) + f(y)$$

A main method to solve elementary functional equation is substitution.

7. Cauchy's Integral Theorem

Cauchy's Integral Theorem implies that the line integral of every holomorphic function along a loop vanishes.

$$\oint_\gamma f(z)\, dz = 0$$

Here γ is a rectifiable path in a simply connected open subset U of the complex plane C whose starting point is equal to its end point, and f: U→ C is a holomorphic function.

<u>Disproof of Riemann Zeta Function's derivation and Riemann Hypothesis</u>

1. Proof that $\zeta(s)\prod(s - 1) \neq \int_0^{+\infty} \frac{(x)^{(s-1)}}{(e^x - 1)} dx$, $s = \sigma + it$, Res or $\sigma > 1$, x = zero and positive real numbers

From $\prod(s-1) = \Gamma(s)$

$$= \int_0^{+\infty} (e)^{(-x)} (x)^{(s-1)} dx \quad , s = \sigma + it \quad , \text{Res or } \sigma > 0$$

When defined $\Gamma(s)$ in term of an improper integral, s = all complex numbers $(\sigma + it)$ with positive real part, and x = zero and positive real numbers, or in term of a limit

$$\prod(s-1) = \lim_{a\to 0, b\to +\infty} \int_a^b (e)^{(-x)} (x)^{(s-1)} dx \text{ , } s = \sigma + it \text{ , Res or } \sigma > 0$$

Riemann tried to substitute nx for x as the starting variable in the integrands, he derived his equation as

$$\prod(s-1) = \int_0^{+\infty} (e)^{(-nx)} (nx)^{(s-1)} dnx \text{ , } s = \sigma + it \text{ , Res or } \sigma > 0,$$

n = 1, 2, 3,..., $+\infty$

Or in term of a limit,

$$\prod(s-1) = \lim_{a\to 0, b\to +\infty} \int_a^b (e)^{(-nx)} (nx)^{(s-1)} dnx$$

But he forgot to change the boundaries of the integral (or the value of nx) after he separated out the variable of the integrands from nx to n and x, and moved term $(n)^{(s)}$ to the left hand side as he might have done below

$$\prod(s-1) = \lim_{a\to 0, b\to +\infty} \int_a^b (e)^{(-nx)} (n)^{(s)} (x)^{(s-1)} dx$$

Multiply by $\left(\frac{1}{n^s}\right)$ both sides

$$\left(\frac{1}{n^s}\right) \prod(s-1) = \lim_{a\to 0, b\to +\infty} \int_a^b (e)^{(-x)(n)} (x)^{(s-1)} dx$$

$$\left(\frac{1}{1^s}\right) \prod(s-1) = \lim_{a\to 0, b\to +\infty} \int_a^b (e)^{(-x)(1)} (x)^{(s-1)} dx$$

$$\left(\frac{1}{2^s}\right) \prod(s-1) = \lim_{a\to 0, b\to +\infty} \int_a^b (e)^{(-x)(2)} (x)^{(s-1)} dx$$

...

$$\left(\frac{1}{\infty^s}\right) \prod(s-1) = \lim_{a\to 0, b\to +\infty} \int_a^b (e)^{(-x)(b)} (x)^{(s-1)} dx$$

Then Riemann took summation of the above equations from $\left(\frac{1}{1^s}\right) \prod(s-1)$ to $\left(\frac{1}{\infty^s}\right) \prod(s-1)$ and he thought that terms $(e)^{(-x)(b)}$ of all the right hand side integrals could be sum to infinite or $= \lim_{b\to+\infty} \sum_{n=1}^{b} (e)^{(-x)(n)}$. So he made the summation.

$$= \lim_{a\to0,b\to+\infty} \int_a^b [(e)^{(-x)(1)} + (e)^{(-x)(2)} + \cdots + (e)^{(-x)(b)}]\,(x)^{(s-1)}\,dx$$

$$\sum_{n=1}^{+\infty} (\tfrac{1}{n^s}) \prod(s-1) = \lim_{a\to0,b\to+\infty} \int_a^b [\sum_{n=1}^{b} (e)^{(-x)(n)}]\,(x)^{(s-1)}\,dx$$

From Geometric series $\sum_{n=1}^{+\infty} (e)^{(-x)(n)} = (\frac{(e)^{(-x)}}{1-(e)^{(-x)}}) = (\frac{1}{(e)^{(x)}-1})$

Then $\zeta(s)\prod(s-1) = \lim_{a\to0,b\to+\infty} \int_a^b \frac{(x)^{(s-1)}}{(e)^{(x)}-1}\,dx$

Or $\zeta(s)\prod(s-1) = \int_0^{+\infty} \frac{(x)^{(s-1)}}{(e)^{(x)}-1}\,dx \quad , s = \sigma + it \; , \text{Res or } \sigma > 1 \; \ldots \text{(A)}$

As you can see, the above equation (integral) is wrong because the boundaries of the new integral were still from a=0 to b=+∞. Actually the boundaries had to be changed after separating variable of the integrands from nx_n to x_n and n, preparing for moving $(n)^{(s)}$ to the left hand side.

From $\prod(s-1) = \lim_{a\to0,b\to+\infty} \int_a^b (e)^{(-x)}\,(x)^{(s-1)}\,dx \; , s = \sigma + it$

, Res or $\sigma > 0$

$n = 1, 2, 3, \ldots, +\infty$; $x = nx_n$ and $nx_n = b, x_n = b/n$; $nx_n = a, x_n = a/n$

Then $\prod(s-1) = \lim_{a\to0,b\to+\infty} \int_a^b (e)^{(-x_n)(n)}\,(nx_n)^{(s-1)}\,dx_n \; , s = \sigma + it$

, Res or $\sigma > 0$

$$\prod(s-1) = \lim_{a\to0,b\to+\infty} \int_{a/n}^{b/n} (e)^{(-x_n)(n)}\,(n)^{(s)}(x_n)^{(s-1)}\,dx_n \; , s = \sigma + it$$

, Res or $\sigma > 0$

Term $(n)^{(s)}$ is separated out from the integral and moved to the left hand side of the equation and then n is increased in value from 1 to +∞ in each equation and finally combined to become $\sum_{n=1}^{+\infty} (\frac{1}{n^s})$ or $\zeta(s)$ at the left hand side. The values of nx_n change from 0 to +∞ and x_n is also changing upto increasing value of n of those integrals at the right hand side. The lower boundaries of those new integrals vary from a/1 to a/+∞, (a/n, n = 1, 2, 3, … , +∞) and the upper boundaries vary from b/1 to b/+∞,(b/n, n = 1, 2, 3, … , +∞).

Multiply by $\frac{1}{n^s}$ both sides

$$\left(\frac{1}{n^s}\right)\prod(s-1) = \lim_{a\to 0, b\to +\infty} \int_{a/n}^{b/n} (e)^{(-x_n)(n)} (x_n)^{(s-1)}\, dx_n$$

Substitute $s = (\sigma + it)$ and $(x_n)^{(s-1)} = (e)^{(\sigma+it-1)\mathrm{Log}(x_n)}$

$$\left(\frac{1}{n^s}\right)\prod(s-1) = \left(\frac{1}{(e)^{(\sigma+it)\mathrm{Log}(n)}}\right)\prod(\sigma + it-1)$$

$$= \lim_{a\to 0, b\to +\infty} \int_{a/n}^{b/n} (e)^{(-x_n)(n)} (e)^{(\sigma+it-1)\mathrm{Log}(x_n)}\, dx_n$$

$$= \lim_{a\to 0, b\to +\infty} \int_{a/n}^{b/n} (e)^{(-x_n)(n)+(\sigma+it-1)\mathrm{Log}(x_n)}\, dx_n$$

From $\frac{d}{dx_n}\ (e)^{(-x_n)(n)}.\ (e)^{(\sigma+it-1)\mathrm{Log}(x_n)}$

$$=[(e)^{(-x_n)(n)} \frac{d}{dx_n} (e)^{(\sigma+it-1)\mathrm{Log}(x_n)}]+[(e)^{(\sigma+it-1)\mathrm{Log}(x_n)} \frac{d}{dx_n} (e)^{(-x_n)(n)}]$$

$$=(e)^{(-x_n)(n)}.(e)^{(\sigma+it-1)\mathrm{Log}(x_n)}.\frac{(\sigma+it-1)}{(x_n)} + (e)^{(\sigma+it-1)\mathrm{Log}(x_n)}.(e)^{(-x_n)(n)}.(-n)$$

$$= [\,-(n) + \frac{(\sigma-1)}{(x_n)} + \frac{(it)}{(x_n)}\,]\,[(e)^{(-x_n)(n)+(\sigma+it-1)\mathrm{Log}(x_n)}\,]$$

Then $\left(\frac{1}{n^s}\right)\prod(s-1) = \lim_{a\to 0, b\to +\infty} \int_{a/n}^{b/n} (e)^{(-x_n)(n)+(\sigma+it-1)\mathrm{Log}(x_n)}\, dx_n$

$$= \lim_{a\to 0, b\to +\infty} [\frac{1}{-(n)+\frac{(\sigma-1)}{(x_n)}+\frac{(it)}{(x_n)}}][(e)^{(-x_n)(n)+(\sigma+it-1)\mathrm{Log}(x_n)}]\,]_{a/n}^{b/n}$$

$$= \lim_{a\to 0, b\to +\infty} [\frac{(x_n)^{(\sigma-1)}}{(e)^{(x_n)(n)}}][\frac{(x_n)^{(it)}}{[-(n)+\frac{(\sigma-1)}{(x_n)}+\frac{(it)}{(x_n)}]}\,]\,]_{a/n}^{b/n}$$

$$= \lim_{a\to 0, b\to +\infty} [\frac{(b/n)^{(\sigma-1)}(b/n)^{(it)}}{e^{(b/n)n}[-(n)+\frac{n(\sigma-1)}{(b)}+\frac{n(it)}{(b)}]}] - [\frac{(a/n)^{(\sigma-1)}(a/n)^{(it)}}{e^{(a/n)n}[-(n)+\frac{n(\sigma-1)}{(a)}+\frac{n(it)}{(a)}]}]$$

$$= \lim_{a\to 0, b\to +\infty} [\frac{(b)^{(\sigma+it-1)}}{(n)^{(\sigma+it-1)}(e)^{(b)}[-(n)+\frac{n(\sigma-1)}{(b)}+\frac{n(it)}{(b)}]}]$$

$$-[\frac{(a)^{(\sigma+it-1)}}{(n)^{(\sigma+i-1)}(e)^{(a)}[-(n)+\frac{n(\sigma-1)}{(a)}+\frac{n(it)}{(a)}]}]$$

$$=[\frac{1}{(n)^{(\sigma+it-1)}}]\,[\frac{\approx(+\infty)^{(\sigma+it-1)}}{(e)^{\approx(+\infty)}[-(n)+\frac{n(\sigma-1)}{\approx(+\infty)}+\frac{n(it)}{\approx(+\infty)}]}]$$

$$-\,[\frac{1}{(n)^{(\sigma+it-1)}}]\,[\frac{\approx(0)^{(\sigma+it-1)}}{(e)^{\approx(0)}[-(n)+\frac{n(\sigma-1)}{\approx(0)}+\frac{n(it)}{\approx(0)}]}]$$

$$=[\frac{1}{(n)^{(\sigma+it-1)}}]\,[\frac{\approx(+\infty)}{\approx(+\infty)}]\;-\,[\frac{1}{(n)^{(\sigma+it-1)}}]\,[\frac{\approx(0)}{\approx(+\infty)}]$$

$$=[\frac{+\infty}{+\infty}]\text{ indeterminate form}$$

And from $\frac{d}{dx_n}\,(x_n)^{(\sigma-1)}(x_n)^{(it)}$

$$=\frac{d}{dx_n}\,(e)^{(\sigma-1)\mathrm{Log}(x_n)}(e)^{(it)\mathrm{Log}(x_n)}$$

$$=(e)^{(\sigma-1)\mathrm{Log}(x_n)}.\frac{d}{dx_n}\,(e)^{(it)\mathrm{Log}(x_n)}+(e)^{(it)\mathrm{Log}(x_n)}.\frac{d}{dx_n}\,(e)^{(\sigma-1)\mathrm{Log}(x_n)}$$

$$=(e)^{(\sigma-1)\mathrm{Log}(x_n)}.(e)^{(it)\mathrm{Log}(x_n)}.\frac{(it)}{x_n}+(e)^{(it)\mathrm{Log}(x_n)}.(e)^{(\sigma-1)\mathrm{Log}(x_n)}.\frac{(\sigma-1)}{x_n}$$

$$=(e)^{(\sigma-1)\mathrm{Log}(x_n)}(e)^{(it)\mathrm{Log}(x_n)}(e)^{-\mathrm{Log}(x_n)}(it)$$

$$+(e)^{(it)\mathrm{Log}(x_n)}(e)^{(\sigma-1)\mathrm{Log}(x_n)}(e)^{-\mathrm{Log}(x_n)}(\sigma-1)$$

$$=[(it)+(\sigma-1)][\,(e)^{(\sigma-1)\mathrm{Log}(x_n)}(e)^{-\mathrm{Log}(x_n)}(e)^{(it)\mathrm{Log}(x_n)}]$$

$$=(\sigma+it-1)\,[\,(e)^{\,[(\sigma-1)-1]\mathrm{Log}(x_n)}(e)^{(it)\mathrm{Log}(x_n)}]$$

$$=(\sigma+it-1)\,[\,(x_n)^{[(\sigma-2)]}(x_n)^{(it)}]$$

Then $\frac{d}{dx_n}\,(\sigma+it-1)(x_n)^{(\sigma-2)}(x_n)^{(it)}$

$$=(\sigma+it-1)\frac{d}{dx_n}[(e)^{(\sigma-2)\mathrm{Log}(x_n)}(e)^{(it)\mathrm{Log}(x_n)}]$$

$$=(\sigma+it-1)[(e)^{(\sigma-2)\mathrm{Log}(x_n)}\frac{d}{dx_n}(e)^{(it)\mathrm{Log}(x_n)}$$

$$+\;(e)^{(it)\mathrm{Log}(x_n)}\frac{d}{dx_n}(e)^{(\sigma-2)\mathrm{Log}(x_n)}]$$

$$= (\sigma + it - 1)[(e)^{(\sigma-2)\mathrm{Log}(x_n)}(e)^{(it)\mathrm{Log}(x_n)} \frac{(it)}{x_n}$$

$$+(e)^{(it)\mathrm{Log}(x_n)}(e)^{(\sigma-2)\mathrm{Log}(x_n)} \frac{(\sigma-2)}{x_n}]$$

$$= (\sigma + it - 1)[(e)^{(\sigma-2)\mathrm{Log}(x_n)}(e)^{(it)\mathrm{Log}(x_n)}(e)^{-\mathrm{Log}(x_n)}(it)$$

$$+(e)^{(it)\mathrm{Log}(x_n)}(e)^{(\sigma-2)\mathrm{Log}(x_n)}(e)^{-\mathrm{Log}(x_n)}(\sigma - 2)]$$

$$= (\sigma + it - 1)[(it)+(\sigma - 2)][\,(e)^{(\sigma-2)\mathrm{Log}(x_n)}(e)^{-\mathrm{Log}(x_n)}(e)^{(it)\mathrm{Log}(x_n)}]$$

$$= (\sigma + it - 1)(\sigma + it - 2)\ [\,(e)^{[(\sigma-2)-1]\mathrm{Log}(x_n)}(e)^{(it)\mathrm{Log}(x_n)}]$$

$$= (\sigma + it - 1)(\sigma + it - 2)\ [\,(x_n)^{[(\sigma-3)]}(x_n)^{(it)}]$$

Or $\frac{(d)^2}{(dx_n)^2}\ [(x_n)^{(\sigma-1)}(x_n)^{(it)}]=(\ \sigma + it - 1)\ (\sigma + it - 2)\ [\,(x_n)^{[(\sigma-3)]}(x_n)^{(it)}]$

Hence $\frac{d^k}{(dx_n)^k}[(x_n)^{(\sigma-1)}(x_n)^{(it)}]$

$$= [(\sigma + it - 1)\ (\sigma + it - 2) \ldots (\sigma + it - k)\ ][\,(x_n)^{(\sigma-1-k)}(x_n)^{(it)}]$$

And so $\frac{d^{(\sigma-1)}}{(dx_n)^{(\sigma-1)}}[(x_n)^{(\sigma-1)}(x_n)^{(it)}]$

$$= [(it + \sigma - 1)(it + \sigma - 2) \ldots (it + 1)\ [\,(x_n)^{(0)}(x_n)^{(it)}]$$

Also $\frac{d^{(\sigma)}}{(dx_n)^{(\sigma)}}[(x_n)^{(\sigma)}(x_n)^{(it)}]$

$$= [(it + \sigma)(it + \sigma - 1) \ldots (it + 1)\ [\,(x_n)^{(0)}(x_n)^{(it)}]$$

And from $\frac{d}{dx_n}[(e)^{(x_n)(n)}[(-n) + \frac{(\sigma+it-1)}{x_n}]]$

Or $\frac{d}{dx_n}[-(n)(e)^{(x_n)(n)} + [(\sigma + it - 1)(e)^{(-\mathrm{Log}x_n)}(e)^{(x_n)(n)}]\,]$

$$=-(n)^{(2)}(e)^{(x_n)(n)} + (\sigma + it - 1)[(e)^{(-\mathrm{Log}x_n)}(n)(e)^{(x_n)(n)} +$$

$$(e)^{(x_n)(n)}(\frac{1}{-x_n})(e)^{(-\mathrm{Log}x_n)}\ ]$$

$$=-(n)^{(2)}(e)^{(x_n)(n)} + (\sigma + it - 1)(e)^{(x_n)(n)}\left[(n)(e)^{(-\mathrm{Log}x_n)} - (e)^{(-2\mathrm{Log}x_n)}\right] \ldots H$$

And

$$\frac{d}{dx_n}[H]=-(n)^{(3)}(e)^{(x_n)(n)}+(\sigma+it-1)[[(n)^{(2)}(e)^{(-\mathrm{Log}x_n)}$$

$$+(n)(e)^{(x_n)(n)}\left(\frac{1}{-x_n}\right)(e)^{(-\mathrm{Log}x_n)}]-[(e)^{(-2\mathrm{Log}x_n)}(n)(e)^{(x_n)(n)}$$

$$+(e)^{(x_n)(n)}\left(\frac{2}{-x_n}\right)(e)^{(-\mathrm{Log}x_n)}]]$$

$$=-(n)^{(3)}(e)^{(x_n)(n)}+(\sigma+it-1)(e)^{(x_n)(n)}[(n)^{(2)}(e)^{(-\mathrm{Log}x_n)}-$$

$$(2n)(e)^{(-2\mathrm{Log}x_n)}+(2)(e)^{(-3\mathrm{Log}x_n)}] \qquad \ldots I$$

$$\frac{d}{dx_n}[I]=-(n)^{(4)}(e)^{(x_n)(n)}+(\sigma+it-1)(e)^{(x_n)(n)}[(n)^{(3)}(e)^{(-\mathrm{Log}x_n)}$$

$$-3(n)^{(2)}(e)^{(-2\mathrm{Log}x_n)}+(6n)(e)^{(-3\mathrm{Log}x_n)}-(6)(e)^{(-4\mathrm{Log}x_n)}] \qquad \ldots J$$

$$\frac{d}{dx_n}[J]=-(n)^{(5)}(e)^{(x_n)(n)}+(\sigma+it-1)(e)^{(x_n)(n)}[(n)^{(4)}(e)^{(-\mathrm{Log}x_n)}$$

$$-4(n)^{(3)}(e)^{(-2\mathrm{Log}x_n)}+12(n)^{(2)}(e)^{(-3\mathrm{Log}x_n)}-(24n)(e)^{(-4\mathrm{Log}x_n)}$$

$$+(24)(e)^{(-5\mathrm{Log}x_n)}] \qquad \ldots K$$

$$\frac{d}{dx_n}[K]=-(n)^{(6)}(e)^{(x_n)(n)}+(\sigma+it-1)(e)^{(x_n)(n)}[(n)^{(5)}(e)^{(-\mathrm{Log}x_n)}$$

$$-5(n)^{(4)}(e)^{(-2\mathrm{Log}x_n)}+20(n)^{(3)}(e)^{(-3\mathrm{Log}x_n)}-60(n)^{(2)}(e)^{(-4\mathrm{Log}x_n)}$$

$$+(120n)(e)^{(-5\mathrm{Log}x_n)}-(120)(e)^{(-6\mathrm{Log}x_n)}]$$

So $\frac{d^{(\sigma-1)}}{(dx_n)^{(\sigma-1)}}\ [(e)^{(x_n)(n)}[(-n)+\frac{(\sigma+it-1)}{x_n}]]$

$$=-(n)^{(\sigma)}(e)^{(x_n)(n)}+(\sigma+it-1)(e)^{(x_n)(n)}[(n)^{(\sigma-1)}(e)^{(-\mathrm{Log}x_n)}$$

$$-(\sigma-1)(n)^{(\sigma-2)}(e)^{(-2\mathrm{Log}x_n)}+(\sigma-1)(\sigma-2)(n)^{(\sigma-3)}(e)^{(-3\mathrm{Log}x_n)}$$

$$-(\sigma-1)(\sigma-2)(\sigma-3)(n)^{(\sigma-4)}(e)^{(-4\mathrm{Log}x_n)}+\ldots]$$

Now apply L' Hospital's Rule to

$$\lim_{a\to 0,b\to+\infty}\left[\frac{(x_n)^{(\sigma-1)}(x_n)^{(it)}}{-(n)(e)^{(x_n)(n)}+\frac{(\sigma-1)}{(x_n)}(e)^{(x_n)(n)}+\frac{(it)}{(x_n)}(e)^{(x_n)(n)}}\right]\Big]_{a/n}^{b/n}$$

Repeat L' Hospital's $(\sigma-1)$ times until $(x_n)^{(\sigma-1)}=\ (x_n)^{(0)}=1$

Then $\lim_{a\to 0,b\to +\infty}\left[\frac{(x_n)^{(\sigma-1)}(x_n)^{(it)}}{-(n)(e)^{(x_n)(n)}+\frac{(\sigma-1)}{(x_n)}(e)^{(x_n)(n)}+\frac{(it)}{(x_n)}(e)^{(x_n)(n)}}\right]\Big]_{a/n}^{b/n}$

$$=\lim_{a\to 0,b\to +\infty}\left[\frac{(it+\sigma-1)(it+\sigma-2)\dots(it+1)(x_n)^{(0)}(x_n)^{(it)}}{(e)^{(x_n)(n)}\,[-(n)^{(\sigma)}+(\sigma+it-1)][(n)^{(\sigma-1)}(e)^{(-Logx_n)}-(\sigma-1)(n)^{(\sigma-2)}\dots]}\right]\Big]_{a/n}^{b/n}$$

$$=\lim_{a\to 0,b\to +\infty}\left[\frac{(it+\sigma-1)(it+\sigma-2)\dots(it+1)(b)^{(it)}(n)^{(-it)}}{(e)^{(b)}[-(n)^{(\sigma)}+(\sigma+it-1)[(n)^{(\sigma)}(e)^{(-Logb)}-(\sigma-1)(n)^{(\sigma)}(e)^{(-2Logb)}\dots]}\right]$$

$$-\left[\frac{(it+\sigma-1)(it+\sigma-2)\dots(it+1)(a)^{(it)}(n)^{(-it)}}{(e)^{(a)}[-(n)^{(\sigma)}+(\sigma+it-1)[(n)^{(\sigma)}(e)^{(-Loga)}-(\sigma-1)(n)^{(\sigma)}(e)^{(-2Loga)}\dots]}\right]$$

$$=\lim_{a\to 0,b\to +\infty}\left[\frac{(it+\sigma-1)(it+\sigma-2)\dots(it+1)(e)^{(itLogb)}}{(e)^{(b)}(n)^{(\sigma+it)}[-1+(\sigma+it-1)[(e)^{(-Logb)}-(\sigma-1)(e)^{(-2Logb)}\dots]}\right]$$

$$-\left[\frac{(it+\sigma-1)(it+\sigma-2)\dots(it+1)(e)^{(itLoga)}}{(e)^{(a)}(n)^{(\sigma+it)}[-1+(\sigma+it-1)[(e)^{(-Log(a)}-(\sigma-1)\,(e)^{(-2Loga)}\dots]}\right]$$

[Let $\lim_{a\to 0}$Log (a) = z, or Log(≈ 0) = z, then (e) $^{(z)}\approx 0$,but $(e)^{(-\infty)}=\frac{1}{(e)^{(\infty)}}$ ≈ 0 So $\lim_{a\to 0}$ Log (a) or $\lim_{a\to 0}$ Log (≈ 0) $\approx -\infty$]

$$=\lim_{a\to 0,b\to +\infty}\left[\frac{(it+\sigma-1)(it+\sigma-2)\dots(it+1)[\cos\approx(+\infty t)+i\sin\approx(+\infty t)]}{(n)^{(\sigma+it)}(e)^{\approx(+\infty)}[-1+(\sigma+it-1)[(e)^{(-Log\approx(+\infty))}-(\sigma-1)(e)^{(-2Log\approx(+\infty))}\dots]}\right]$$

$$-\left[\frac{(it+\sigma-1)(it+\sigma-2)\dots(it+1)[\cos\approx(-\infty t)+i\sin\approx(-\infty t)]}{(n)^{(\sigma+it)}(e)^{\approx(0)}[-1+(\sigma+it-1)[(e)^{(-Log\approx(0))}-(\sigma-1)\,(e)^{(-2Log\approx(0))}\dots]}\right]$$

$$=\lim_{a\to 0,b\to +\infty}\left[\frac{(it+\sigma-1)(it+\sigma-2)\dots(it+1)}{(n)^{(\sigma+it)}}\right]\left[\frac{\approx(1)+\approx(0)}{(e)^{\approx(+\infty)}[\approx(-1)+\approx(0)-\approx(0)+\cdots]}\right]$$

$$-\left[\frac{(it+\sigma-1)(it+\sigma-2)\dots(it+1)}{(n)^{(\sigma+it)}}\right]\left[\frac{\approx(1)+\approx(0)}{(e)^{\approx(0)}[\approx(-1)+\approx(0)-\approx(0)+\cdots]}\right]$$

$$=\lim_{a\to 0,b\to +\infty}\left[\frac{(it+\sigma-1)(it+\sigma-2)\dots(it+1)}{(n)^{(\sigma+it)}}\right][\approx(0)]$$

$$-\left[\frac{(it+\sigma-1)(it+\sigma-2)\dots(it+1)}{(n)^{(\sigma+it)}}\right]\left[\frac{\approx(1)}{\approx(-1)}\right]$$

$$=\lim_{a\to 0,b\to +\infty}\left[\frac{(it+\sigma-1)(it+\sigma-2)\dots(it+1\,)}{(n)^{(\sigma+it)}}\right]$$

$$=\frac{(\sigma+it-1)!}{(n)^{(\sigma+it)}}$$

$$=\frac{(s-1)!}{(n)^{(s)}}$$

For $n = 1$ $\quad \frac{1}{(1)^s}\prod(s-1) = \lim_{a\to 0, b\to +\infty} \int_{a/1}^{b/1} (e)^{(-x_1)(1)}(x_1)^{(s-1)}dx_1$

$$= \frac{(s-1)!}{(1)^{(s)}}$$

For $n = 2$ $\quad \frac{1}{(2)^s}\prod(s-1) = \lim_{a\to 0, b\to +\infty} \int_{a/2}^{b/2} (e)^{(-x_2)(2)}(x_2)^{(s-1)}dx_2$

$$= \frac{(s-1)!}{(2)^{(s)}}$$

...

For $n = \infty$ $\quad \frac{1}{(+\infty)^s}\prod(s-1) = \lim_{a\to 0, b\to +\infty} \int_{a/+\infty}^{b/+\infty} (e)^{(-x_\infty)(+\infty)}(x_\infty)^{(s-1)}dx_\infty$

$$= \frac{(s-1)!}{(+\infty)^{(s)}}$$

Actually the correct method of integration of the above equations should be.

$$[\left(\frac{1}{1^s}\right) + \left(\frac{1}{2^s}\right) + \ldots + \left(\frac{1}{\infty^s}\right)] \prod(s-1)$$

$$= \lim_{a\to 0, b\to +\infty} [\int_{a/(1)}^{b/(1)} (e)^{(-x_1)(1)}(x_1)^{(s-1)}dx_1$$

$$+ \int_{a/(2)}^{b/(2)} (e)^{(-x_2)(2)}(x_2)^{(s-1)}dx_2$$

$$+ \ldots$$

$$+ \int_{a/(+\infty)}^{b/(+\infty)} (e)^{(-x_\infty)(+\infty)}(x_\infty)^{(s-1)}dx_\infty \,]$$

$$\sum_{n=1}^{+\infty} \left(\frac{1}{n^s}\right) \prod(s-1) = \frac{(s-1)!}{(1)^{(s)}}$$

$$+ \frac{(s-1)!}{(2)^{(s)}}$$

$$+ \ldots$$

$$+ \frac{(s-1)!}{(+\infty)^{(s)}}$$

Or $\zeta(s) \prod(s-1) = \frac{(s-1)!}{(1)^{(s)}} + \frac{(s-1)!}{(2)^{(s)}} + \ \ldots \ + \frac{(s-1)!}{(+\infty)^{(s)}}$

$$= \sum_{n=1}^{+\infty} \left(\frac{1}{n^s}\right)(s-1)!$$

$\zeta(s)\prod(s-1) \neq \int_0^{+\infty} \frac{(x)^{(s-1)}}{(e)^{(x)}-1}dx,$ because infinite summation (Geometric series), $\sum_{n=1}^{+\infty} (e)^{(-x)n} = \frac{1}{(e)^{(x)}-1}$ of one of the integrand, $(e)^{(-x)n}$ can be taken only when value of (x) does not change while n changes. But in this case the value of x changes to $x_1, x_2, \ldots, x_\infty$ while n changes, and

$$\int_{a/(1)}^{b/(1)} (x_1)^{(s-1)} dx_1 \neq \int_{a/(2)}^{b/(2)} (x_2)^{(s-1)} dx_2$$

$$\neq \cdots$$

$$\neq \int_{a/+(\infty)}^{b/+(\infty)} (x_\infty)^{(s-1)} dx_\infty$$

so infinite summation of the integrands $(e)^{(-x)n}$ of the above integrals can not be done as Riemann did.

2. Proof that $2\sin(\pi s)\prod(s-1)\zeta(s) \neq i\int_{+\infty}^{+\infty} \frac{(-x)^{(s-1)}}{(e^x-1)}dx$, $s = \sigma + it$, Res $= \sigma > 1$. And ζ(s) of this functional equation has no trivial zeroes (−2, −4, −6,...).

Start from finding ζ(s) for x on the whole real line (negative and positive integers) especially for x = negative integers using analytic continuation while s = complex number $(\sigma + it)$

From Riemann's equation

$$\zeta(s)\prod(s-1) = \int_0^{+\infty} \frac{(x)^{(s-1)}}{(e)^{(x)}-1}dx \quad , s = \sigma + it, \text{ Res or } \sigma > 1 \qquad \ldots (A)$$

$$= \lim_{a\to 0, b\to +\infty} \int_a^b \frac{(x)^{(s-1)}}{(e)^{(x)}-1}dx$$

Apply analytic continuation technique (by stalking Riemann's work, and to do this we have to assume that equation ... A is correct) then

$$\zeta(s)\prod(s-1) = \lim_{b\to +\infty} \int_{-b}^{b} \frac{(-1)^{(s-1)}(-x)^{(s-1)}}{(e)^{(x)}-1}dx \qquad \ldots (B)$$

$$= (-1)^{(s-1)} \lim_{b\to+\infty} \int_{-b}^{b} \frac{(-x)^{(s-1)}}{(e)^{(x)}-1} dx$$

From $(\cos\pi - i\sin\pi) = -1$; $(\cos\pi = -1, \sin\pi = 0)$

$$\zeta(s)\prod(s-1) = \frac{(\cos\pi - i\sin\pi)^{(s)}}{(-1)} \lim_{b\to+\infty} \int_{-b}^{b} \frac{(-x)^{(s-1)}}{(e)^{(x)}-1} dx$$

From $(\cos\pi - i\sin\pi) = (e)^{(-\pi i)}$

$$\zeta(s)\prod(s-1) = -(e)^{(-\pi is)} \lim_{b\to+\infty} \int_{-b}^{b} \frac{(-x)^{(s-1)}}{(e)^{(x)}-1} dx$$

Multiply by $-(e)^{(\pi is)}$ both sides

$$-(e)^{(\pi is)}\zeta(s)\prod(s-1) = \lim_{b\to+\infty} \int_{-b}^{b} \frac{(-x)^{(s-1)}}{(e)^{(x)}-1} dx \qquad \text{... (C)}$$

From $(\cos\pi + i\sin\pi) = -1$; $(\cos\pi = -1, \sin\pi = 0)$

$$\zeta(s)\prod(s-1) = \frac{(\cos\pi + i\sin\pi)^{(s)}}{(-1)} \lim_{b\to+\infty} \int_{-b}^{b} \frac{(-x)^{(s-1)}}{(e)^{(x)}-1} dx$$

$$= -(e)^{(\pi is)} \lim_{b\to+\infty} \int_{-b}^{b} \frac{(-x)^{(s-1)}}{(e)^{(x)}-1} dx$$

Multiply by $-(e)^{(-\pi is)}$ both sides

$$-(e)^{(-\pi is)}\zeta(s)\prod(s-1) = \lim_{b\to+\infty} \int_{-b}^{b} \frac{(-x)^{(s-1)}}{(e)^{(x)}-1} dx \qquad \text{... (D)}$$

(C) $-$(D) ; $[-(e)^{(\pi is)} + (e)^{(-\pi is)}]\zeta(s)\prod(s-1)$

$$= \lim_{b\to+\infty} \int_{-b}^{b} \frac{(-x)^{(s-1)}}{(e)^{(x)}-1} dx - \lim_{b\to+\infty} \int_{-b}^{b} \frac{(-x)^{(s-1)}}{(e)^{(x)}-1} dx$$

$$[-\cos\pi s - i\sin\pi s + \cos\pi s - i\sin\pi s]\zeta(s)\prod(s-1)$$

$$= \lim_{b\to+\infty} \int_{-b}^{b} \frac{(-x)^{(s-1)}}{(e)^{(x)}-1} dx - \lim_{b\to+\infty} \int_{-b}^{b} \frac{(-x)^{(s-1)}}{(e)^{(x)}-1} dx$$

$$-2i\sin\pi s\zeta(s)\prod(s-1) = \lim_{b\to+\infty} \int_{-b}^{b} \frac{(-x)^{(s-1)}}{(e)^{(x)}-1} dx + \lim_{b\to+\infty} \int_{b}^{-b} \frac{(-x)^{(s-1)}}{(e)^{(x)}-1} dx$$

$$= \lim_{b \to +\infty} \int_{b}^{b} \frac{(-x)^{(s-1)}}{(e)^{(x)}-1} dx$$

$$= \int_{+\infty}^{+\infty} \frac{(-x)^{(s-1)}}{(e)^{(x)}-1} dx$$

Multiply by i both sides

$$2\sin\pi s\zeta(s)\prod(s-1) \quad = i \int_{+\infty}^{+\infty} \frac{(-x)^{(s-1)}}{(e)^{(x)}-1} dx \qquad \ldots (E)$$

$= 0$ (by Cauchy's Integral Formula and by method which was used to derive the equation above)

But the above derivation (all red characters) is wrong because it starts from wrong equation

$$\zeta(s)\prod(s-1) = \int_0^{+\infty} \frac{(x)^{(s-1)}}{(e^x-1)} dx \qquad \ldots (A)$$

The right derivation in this case should start from equation

$$\prod(s-1) = \lim_{a \to 0, b \to +\infty} \int_a^b (e)^{(-nx_n)} (nx_n)^{(s-1)} dnx_n \qquad , s = \sigma + it$$

, Res or $\sigma > 0$

Then change the boundaries in accordance with separation of the variable from nx_n to n and x_n, preparing for moving $(n)^{(s)}$ to the left hand side.

$$\prod(s-1) = \lim_{a \to 0, b \to +\infty} \int_{a/n}^{b/n} (e)^{(-x_n)(n)} (n)^{(s)} (x_n)^{(s-1)} dx_n$$

Multiply by $\frac{1}{(n)^s}$ both sides

$$\frac{1}{(n)^s} \prod(s-1) = \lim_{a \to 0, b \to +\infty} \int_{a/n}^{b/n} (e)^{(-x_n)(n)} (x_n)^{(s-1)} dx_n$$

Set $n = 1$ to $+\infty$ and take summation of all the equations as above, then

$$\zeta(s)\prod(s-1) = \sum_{n=1}^{+\infty} \frac{1}{(n)^s} \prod(s-1)$$

$$= \sum_{n=1}^{+\infty} \lim_{a\to 0, b\to +\infty} \int_{a/n}^{b/n} (e)^{(-x_n)(n)} (x_n)^{(s-1)} dx_n$$

Apply analytic continuation technique for x on the whole real line especially x = negative integers then

$$\zeta(s)\prod(s-1) = \lim_{a\to -\infty, b\to +\infty} [\int_{a/(1)}^{b/(1)} (e)^{(-x_1)(1)} (-1)^{(s-1)} (-x_1)^{(s-1)} dx_1$$

$$+ \int_{a/(2)}^{b/(2)} (e)^{(-x_2)(2)} (-1)^{(s-1)} (-x_2)^{(s-1)} dx_2$$

$$+ \ldots$$

$$+ \int_{a/(+\infty)}^{b/(+\infty)} (e)^{(-x_\infty)(+\infty)} (-1)^{(s-1)} (-x_\infty)^{(s-1)} dx_\infty]$$

$$= (-1)^{(s-1)} \lim_{b\to +\infty} [\int_{-b/(1)}^{b/(1)} (e)^{(-x_1)(1)} (-x_1)^{(s-1)} dx_1$$

$$+ \int_{-b/(2)}^{b/(2)} (e)^{(-x_2)(2)} (-x_2)^{(s-1)} dx_2$$

$$+ \ldots$$

$$+ \int_{-b/(+\infty)}^{b/(+\infty)} (e)^{(-x_\infty)(+\infty)} (-x_\infty)^{(s-1)} dx_\infty]$$

From $(\cos\pi - i\sin\pi) = -1$; $(\cos\pi = -1, \sin\pi = 0)$

$$\zeta(s)\prod(s-1) = \frac{(\cos\pi - i\sin\pi)^{(s)}}{(-1)} \lim_{b\to +\infty} [\int_{-b/(1)}^{b/(1)} (e)^{(-x_1)(1)} (-x_1)^{(s-1)} dx_1$$

$$+ \int_{-b/(2)}^{b/(2)} (e)^{(-x_2)(2)} (-x_2)^{(s-1)} dx_2$$

$$+ \ldots$$

$$+ \int_{-b/(+\infty)}^{b/(+\infty)} (e)^{(-x_\infty)(+\infty)} (-x_\infty)^{(s-1)} dx_\infty]$$

From $(\cos\pi - i\sin\pi) = (e)^{(-\pi i)} = -1$

$$\zeta(s)\prod(s-1) = -(e)^{(-\pi i)s} \lim_{b\to +\infty} [\int_{-b/(1)}^{b/(1)} (e)^{(-x_1)(1)} (-x_1)^{(s-1)} dx_1$$

$$+ \int_{-b/(2)}^{b/(2)} (e)^{(-x_2)(2)} (-x_2)^{(s-1)} dx_2$$

$$+\ldots$$

$$+\int_{-b/(\infty)}^{b/(\infty)}(e)^{(-x_\infty)(+\infty)}(-x_\infty)^{(s-1)}dx_\infty\,]$$

Multiply by $-(e)^{(\pi is)}$ both sides

$$-(e)^{(\pi is)}\zeta(s)\prod(s-1)=\lim_{b\to+\infty}[\int_{-b/(1)}^{b/(1)}(e)^{(-x_1)(1)}(-x_1)^{(s-1)}dx_1$$

$$+\int_{-b/(2)}^{b/(2)}(e)^{(-x_2)(2)}(-x_2)^{(s-1)}dx_2$$

$$+\ldots$$

$$+\int_{-b/(+\infty)}^{b/(+\infty)}(e)^{(-x_\infty)(+\infty)}(-x_\infty)^{(s-1)}dx_\infty \quad \ldots\ (C')$$

From $(\cos\pi+i\sin\pi)=-1$; $(\cos\pi=-1\,,\sin\pi=0)$

$$\zeta(s)\prod(s-1)=\frac{(\cos\pi+i\sin\pi)^{(s)}}{(-1)}\lim_{b\to+\infty}[\int_{-b/(1)}^{b/(1)}(e)^{(-x_1)(1)}(-x_1)^{(s-1)}dx_1$$

$$+\int_{-b/(2)}^{b/(2)}(e)^{(-x_2)(2)}(-x_2)^{(s-1)}dx_2$$

$$+\ldots$$

$$+\int_{-b/(+\infty)}^{b/(+\infty)}(e)^{(-x_\infty)(+\infty)}(-x_\infty)^{(s-1)}dx_\infty\,]$$

From $(\cos\pi+i\sin\pi)=(e)^{(\pi i)}=-1$

$$\zeta(s)\prod(s-1)=-(e)^{(\pi i)s}\lim_{b\to+\infty}[\int_{-b/(1)}^{b/(1)}(e)^{(-x_1)(1)}(-x_1)^{(s-1)}dx_1$$

$$+\int_{-b/(2)}^{b/(2)}(e)^{(-x_2)(2)}(-x_2)^{(s-1)}dx_2$$

$$+\ldots$$

$$+\int_{-b/(+\infty)}^{b/(+\infty)}(e)^{(-x_\infty)(+\infty)}(-x_\infty)^{(s-1)}dx_\infty\,]$$

Multiply by $-(e)^{(-\pi is)}$ both sides

$$-(e)^{(-\pi is)}\zeta(s)\prod(s-1)=\lim_{b\to+\infty}[\int_{-b/(1)}^{b/(1)}(e)^{(-x_1)(1)}(-x_1)^{(s-1)}dx_1$$

$$+\int_{-b/(2)}^{b/(2)}(e)^{(-x_2)(2)}(-x_2)^{(s-1)}dx_2$$

$$+\ldots$$

$$+\int_{-b/(+\infty)}^{b/(+\infty)}(e)^{(-x_\infty)(+\infty)}(-x_\infty)^{(s-1)}dx_\infty\,] \quad \ldots (D')$$

(C') −(D'); $[-(e)^{(\pi is)}+(e)^{(-\pi is)}]\zeta(s)\prod(s-1)$

$$=\lim_{b\to+\infty}[\int_{-b/(1)}^{b/(1)}(e)^{(-x_1)(1)}(-x_1)^{(s-1)}dx_1$$

$$+\int_{-b/(2)}^{b/(2)}(e)^{(-x_2)(2)}(-x_2)^{(s-1)}dx_2$$

$$+\ldots$$

$$+\int_{-b/(+\infty)}^{b/(+\infty)}(e)^{(-x_\infty)(+\infty)}(-x_\infty)^{(s-1)}dx_\infty\,]$$

$$-\lim_{b\to+\infty}[\int_{-b/(1)}^{b/(1)}(e)^{(-x_1)(1)}(-x_1)^{(s-1)}dx_1$$

$$+\int_{-b/(2)}^{b/(2)}(e)^{(-x_2)(2)}(-x_2)^{(s-1)}dx_2$$

$$+\ldots$$

$$+\int_{-b/(+\infty)}^{b/(+\infty)}(e)^{(-x_\infty)(+\infty)}(-x_\infty)^{(s-1)}dx_\infty\,]$$

$[-\cos\pi s - i\sin\pi s + \cos\pi s - i\sin\pi s]\zeta(s)\prod(s-1)$

$$=\lim_{b\to+\infty}[\int_{-b/(1)}^{b/(1)}(e)^{(-x_1)(1)}(-x_1)^{(s-1)}dx_1$$

$$+\int_{-b/(2)}^{b/(2)}(e)^{(-x_2)(2)}(-x_2)^{(s-1)}dx_2$$

$$+\ldots$$

$$+\int_{-b/(+\infty)}^{b/(+\infty)}(e)^{(-x_\infty)(+\infty)}(-x_\infty)^{(s-1)}dx_\infty\,]$$

$$-\lim_{b\to+\infty}[\int_{-b/(1)}^{b/(1)}(e)^{(-x_1)(1)}(-x_1)^{(s-1)}dx_1$$

$$+\int_{-b/(2)}^{b/(2)}(e)^{(-x_2)(2)}(-x_2)^{(s-1)}dx_2$$

$$+\dots$$

$$+\int_{-b/(+\infty)}^{b/(+\infty)}(e)^{(-x_\infty)(+\infty)}(-x_\infty)^{(s-1)}dx_\infty\ ]$$

$$-2i\sin\pi s\zeta(s)\prod(s-1)\ =\lim_{b\to+\infty}[\int_{-b/(1)}^{b/(1)}(e)^{(-x_1)(1)}(-x_1)^{(s-1)}dx_1$$

$$+\int_{-b/(2)}^{b/(2)}(e)^{(-x_2)(2)}(-x_2)^{(s-1)}dx_2$$

$$+\dots$$

$$+\int_{-b/(+\infty)}^{b/(+\infty)}(e)^{(-x_\infty)(+\infty)}(-x_\infty)^{(s-1)}dx_\infty\ ]$$

$$+\lim_{b\to+\infty}[\int_{b/(1)}^{-b/(1)}(e)^{(-x_1)(1)}(-x_1)^{(s-1)}dx_1$$

$$+\int_{b/(2)}^{-b/(2)}(e)^{(-x_2)(2)}(-x_2)^{(s-1)}dx_2$$

$$+\dots$$

$$+\int_{b/(+\infty)}^{-b/(+\infty)}(e)^{(-x_\infty)(+\infty)}(-x_\infty)^{(s-1)}dx_\infty\ ]$$

$$-2i\sin\pi s\zeta(s)\prod(s-1)\quad=\lim_{b\to+\infty}[\int_{b/(1)}^{b/(1)}(e)^{(-x_1)(1)}(-x_1)^{(s-1)}dx_1$$

$$+\int_{b/(2)}^{b/(2)}(e)^{(-x_2)(2)}(-x_2)^{(s-1)}dx_2$$

$$+\dots$$

$$+\int_{b/(+\infty)}^{b/(+\infty)}(e)^{(-x_\infty)(+\infty)}(-x_\infty)^{(s-1)}dx_\infty\ ]$$

$$=\ [\int_{+\infty/(1)}^{+\infty/(1)}(e)^{(-x_1)(1)}(-x_1)^{(s-1)}dx_1$$

$$+\int_{+\infty/(2)}^{+\infty/(2)}(e)^{(-x_2)(2)}(-x_2)^{(s-1)}dx_2$$

$$+\dots$$

$$+\int_{+\infty/+(\infty)}^{+\infty/+(\infty)}(\mathrm{e})^{(-\mathrm{x}_\infty)(+\infty)}(-\mathrm{x}_\infty)^{(\mathrm{s}-1)}\mathrm{dx}_\infty\ ]$$

Multiply by i both sides

$$2\sin\pi s\zeta(\mathrm{s})\prod(\mathrm{s}-1) = i[\int_{+\infty/(1)}^{+\infty/(1)}(\mathrm{e})^{(-\mathrm{x}_1)(1)}(-\mathrm{x}_1)^{(\mathrm{s}-1)}\mathrm{dx}_1$$

$$+\int_{+\infty/(2)}^{+\infty/(2)}(\mathrm{e})^{(-\mathrm{x}_2)(2)}(-\mathrm{x}_2)^{(\mathrm{s}-1)}\mathrm{dx}_2$$

$$+\ldots$$

$$+\int_{+\infty/+(\infty)}^{+\infty/+(\infty)}(\mathrm{e})^{(-\mathrm{x}_\infty)(+\infty)}(-\mathrm{x}_\infty)^{(\mathrm{s}-1)}\mathrm{dx}_\infty\ ]$$

$$= i\sum_{n=1}^{+\infty}\int_{+\infty/(\mathrm{n})}^{+\infty/(\mathrm{n})}(\mathrm{e})^{(-\mathrm{x}_\mathrm{n})(\mathrm{n})}(-\mathrm{x}_\mathrm{n})^{(\mathrm{s}-1)}\mathrm{dx}_\mathrm{n}$$

$= 0$ (by Cauchy's Integral Formula and the method which was used to derive the equation above)

Consider $\prod(\mathrm{s}-1) = \Gamma(\mathrm{s}) = \frac{\Gamma(\mathrm{s}+\mathrm{k})}{\mathrm{s}(\mathrm{s}+1)(\mathrm{s}+2)\ldots(\mathrm{s}+\mathrm{k}-1)}$

k =0, 1, 2, 3,... and $\mathrm{s} + \mathrm{k} > 0$, or s – k , $\mathrm{s} \neq 0, -1, -2, \ldots, -(\mathrm{k}-1)$

$\prod(\mathrm{s}-1) = \Gamma(\mathrm{s})$ has poles at $\mathrm{s} = 0, -1, -2, -3, \ldots$ (or undefined) and $\prod(\mathrm{s}-1) = \Gamma(\mathrm{s})$ never becomes zero for any values of s.

$\mathrm{Sin}\pi s = 0$ all the time by the method that was used to derive $2\sin\pi s\ \zeta(\mathrm{s})\prod(\mathrm{s}-1)$ as mentioned above or πs must be $+\pi, +3\pi, +5\pi, \ldots$ while $\sin\pi s$ must be 0 , $\cos\pi s$ must be -1, and s must be $+1, +3, +5, \ldots$ $\mathrm{s} \neq 0, -1, -2, \ldots$ (negative integers) which will cause $\prod(\mathrm{s}-1)$ to has poles.

Note that $0\times\mathrm{a} = 0$, a is not 0, but can be any numbers and/or $\mathrm{a} = \frac{0}{0}$ (undefined).In this case $\mathrm{a} = \zeta(\mathrm{s})\prod(\mathrm{s}-1)$ while $\sin\pi s$ must $= 0$, $\prod(\mathrm{s}-1) \neq 0$ and $\zeta(\mathrm{s})\prod(\mathrm{s}-1) = (\mathrm{s}-1)!\ \sum_{\mathbf{n=1}}^{+\infty}\left(\frac{1}{\mathrm{n}^{\mathrm{s}}}\right)$.

So $2\sin\pi s\ \zeta(\mathrm{s})\prod(\mathrm{s}-1) = i\sum_{n=1}^{+\infty}\int_{+\infty/(\mathrm{n})}^{+\infty/(\mathrm{n})}(\mathrm{e})^{(-\mathrm{x}_\mathrm{n})(\mathrm{n})}(-\mathrm{x}_\mathrm{n})^{(\mathrm{s}-1)}\mathrm{dx}_\mathrm{n} = 0$

Or $0\times\ \zeta(\mathrm{s})\prod(\mathrm{s}-1) = i\sum_{n=1}^{+\infty}\int_{+\infty/(\mathrm{n})}^{+\infty/(\mathrm{n})}(\mathrm{e})^{(-\mathrm{x}_\mathrm{n})(\mathrm{n})}(-\mathrm{x}_\mathrm{n})^{(\mathrm{s}-1)}\mathrm{dx}_\mathrm{n} = 0$

That means $\zeta(s)\prod(s-1)$ can be any numbers and/or $= \frac{0}{0}$ (undefined). In this case, nothing supports that $\zeta(s) = 0$ and $\sin\pi s = 0$, $s = -2, -4, -6$, ... (which are called trivial zeroes, and cause $\zeta(s) = 0$ as Riemann misunderstood). On the contrary s must not be negative integers which will cause $\prod(s-1)$ to has poles and make conflict to the above derivation of the function which we have $(-1)^{(s-1)} = \frac{(\cos\pi + i\sin\pi)^{(s)}}{(-1)} = \frac{(\cos\pi s + i\sin\pi s)}{(-1)}$

where $\cos\pi s$ must$=-1$ and $\sin\pi s$ must$=0$.

The above derivation gives out that there are no trivial zeroes of Riemann Zeta Function $2\sin\pi s\,\zeta(s)\prod(s-1) = 0$ for $x =$ negative integers (or for x on the whole real line).

3. Proof that $\zeta(s)\pi^{\left(-\frac{s}{2}\right)}\prod\left(\frac{s}{2}-1\right) \neq \frac{1}{s(s-1)} + \int_{1}^{+\infty} \psi(x)[(x)^{\left(\frac{s}{2}-1\right)} + (x)^{-\left(\frac{1+s}{2}\right)}]\,dx$

From $\prod(s-1) = \Gamma(s)$

$= \int_{0}^{+\infty}(e)^{(-x)}(x)^{(s-1)}dx$, $s = \sigma + it$, Res or $\sigma > 0$, $x =$ zero and positive real numbers, or in limit form

$$\prod(s-1) = \lim_{a\to 0, b\to +\infty} \int_{a}^{b}(e)^{(-x)}(x)^{(s-1)}\,dx$$

Riemann tried to start from denoted $s = \frac{s}{2}$ or $\frac{(\sigma+it)}{2}$ and $x = nn\pi x$ to the above equation and then

$$\prod\left(\frac{s}{2}-1\right) = \lim_{a\to 0, b\to +\infty} \int_{a}^{b}(e)^{(-nn\pi x)}\,(nn\pi x)^{\left(\frac{s}{2}-1\right)}\,dnn\pi x$$

But he forgot (again) to change the boundaries (upper and lower) of the integral after separating the variable from $nn\pi x$ to nn , π, and x, as shown below

$$\prod\left(\frac{s}{2}-1\right) = \lim_{a\to 0, b\to +\infty} \int_{a}^{b}(e)^{(-nn\pi x)}(n)^{(s)}\,(\pi)^{\left(\frac{s}{2}\right)}(x)^{\left(\frac{s}{2}-1\right)}\,dx$$

Multiplied by $\frac{1}{(n)^{s}}\,(\pi)^{\left(-\frac{s}{2}\right)}$ both sides.

$$\frac{1}{(n)^{s}}\,(\pi)^{\left(-\frac{s}{2}\right)}\prod\left(\frac{s}{2}-1\right) = \lim_{a\to 0, b\to +\infty} \int_{a}^{b}(e)^{(-nn\pi x)}\,(x)^{\left(\frac{s}{2}-1\right)}\,dx$$

$$\frac{1}{(1)^s}(\pi)^{\left(-\frac{s}{2}\right)}\prod\left(\frac{s}{2}-1\right)=\lim_{a\to 0,b\to+\infty}\int_a^b(e)^{-(1)(1)(\pi)(x)}\,(x)^{\left(\frac{s}{2}-1\right)}\,dx$$

$$\frac{1}{(2)^s}(\pi)^{\left(-\frac{s}{2}\right)}\prod\left(\frac{s}{2}-1\right)=\lim_{a\to 0,b\to+\infty}\int_a^b(e)^{-(2)(2)(\pi)(x)}\,(x)^{\left(\frac{s}{2}-1\right)}\,dx$$

...

$$\frac{1}{(\infty)^s}(\pi)^{\left(-\frac{s}{2}\right)}\prod\left(\frac{s}{2}-1\right)=\lim_{a\to 0,b\to+\infty}\int_a^b(e)^{(-bb\pi x)}\,(x)^{\left(\frac{s}{2}-1\right)}\,dx$$

Then Riemann took summation of the above equations from $\left(\frac{1}{1^s}\right)(\pi)^{\left(-\frac{s}{2}\right)}\prod\left(\frac{s}{2}-1\right)$ to $\left(\frac{1}{\infty^s}\right)(\pi)^{\left(-\frac{s}{2}\right)}\prod\left(\frac{s}{2}-1\right)$ and he thought that term $(e)^{(-x)(b)}$ of all the right hand sides could be sum to infinite or $=\sum_{n=1}^{\infty}(e)^{(-nn\pi x)}$.

$$\left[\left(\frac{1}{1^s}\right)+\left(\frac{1}{2^s}\right)+\ldots+\left(\frac{1}{\infty^s}\right)\right](\pi)^{\left(-\frac{s}{2}\right)}\prod\left(\frac{s}{2}-1\right)$$

$$=\lim_{a\to 0,b\to+\infty}\int_a^b\left[(e)^{-(1)(1)(\pi)(x)}+(e)^{-(2)(2)(\pi)(x)}+\ldots\right.$$

$$\left.+(e)^{(-bb\pi x)}\right](x)^{\left(\frac{s}{2}-1\right)}dx$$

$$=\lim_{a\to 0,b\to+\infty}\int_a^b\left[\sum_{n=1}^{b}(e)^{(-nn\pi x)}\right](x)^{\left(\frac{s}{2}-1\right)}dx$$

Or $\sum_{n=1}^{+\infty}\left(\frac{1}{n^s}\right)(\pi)^{-\left(\frac{s}{2}\right)}\prod\left(\frac{s}{2}-1\right)=\lim_{a\to 0,b\to+\infty}\int_a^b\left[\sum_{n=1}^{b}(e)^{(-nn\pi x)}\right](x)^{\left(\frac{s}{2}-1\right)}dx$

Or $\zeta(s)(\pi)^{-\left(\frac{s}{2}\right)}\prod\left(\frac{s}{2}-1\right)=\lim_{a\to 0,b\to+\infty}\int_a^b\left[\sum_{n=1}^{b}(e)^{(-nn\pi x)}\right](x)^{\left(\frac{s}{2}-1\right)}dx$

$$=\int_0^{+\infty}\left[\sum_{n=1}^{+\infty}(e)^{(-nn\pi x)}\right](x)^{\left(\frac{s}{2}-1\right)}dx$$

Then Riemann denoted $\sum_{n=1}^{+\infty}(e)^{(-nn\pi x)}=\psi(x)$

So $\zeta(s)(\pi)^{\left(-\frac{s}{2}\right)}\prod\left(\frac{s}{2}-1\right)=\lim_{a\to 0,b\to+\infty}\int_a^b\psi(x)\,(x)^{\left(\frac{s}{2}-1\right)}dx$

From $(2\psi(x)+1)=(x)^{\left(-\frac{1}{2}\right)}\left(2\psi\left(\frac{1}{x}\right)+1\right)$ (Jacobi, Fund S.184)

$$\zeta(s)(\pi)^{\left(-\frac{s}{2}\right)}\prod\left(\frac{s}{2}-1\right)=\lim_{a\to 0,b\to+\infty}\int_a^1\psi(x)(x)^{\left(\frac{s}{2}-1\right)}\,dx+\int_1^b\psi(x)(x)^{\left(\frac{s}{2}-1\right)}\,dx$$

$$= \lim_{a\to 0, b\to +\infty} \int_a^1 \psi\left(\frac{1}{x}\right)(x)^{\left(\frac{s-3}{2}\right)} dx$$

$$+ \frac{1}{2}\int_a^1 [(x)^{\left(\frac{s-3}{2}\right)} - (x)^{\left(\frac{s}{2}-1\right)}]\, dx + \int_1^b \psi(x)(x)^{\left(\frac{s}{2}-1\right)} dx$$

$$\lim_{a\to 0} \frac{1}{2}\int_a^1 [(x)^{\left(\frac{s-3}{2}\right)} - (x)^{\left(\frac{s}{2}-1\right)}]\, dx = \lim_{a\to 0} [(\tfrac{1}{2})\frac{(x)^{\left(\frac{s-1}{2}\right)}}{\left(\frac{s-1}{2}\right)}]_a^1 - (\tfrac{1}{2})\frac{(x)^{\left(\frac{s}{2}\right)}}{\left(\frac{s}{2}\right)}]]_a^1$$

$$= +\frac{1}{2}[\frac{(1-0)}{\left(\frac{s-1}{2}\right)}] - \frac{1}{2}[\frac{(1-0)}{\left(\frac{s}{2}\right)}]$$

$$= \frac{1}{(s)(s-1)}$$

Considered $\lim_{a\to 0} \int_a^1 \psi\left(\frac{1}{x}\right)(x)^{\left(\frac{s-3}{2}\right)} dx$

Riemann might let $u = \frac{1}{x}$ then $du = (-1)\ (x)^{(-2)}dx$ (or something liked this)

And $dx = (-1)\ (u)^{(-2)}du$

Then $\lim_{a\to 0} \int_a^1 \psi\left(\frac{1}{x}\right)(x)^{\left(\frac{s-3}{2}\right)} dx = \lim_{a\to 0} \int_{1/a}^{1/1} \psi(u)\left(\frac{1}{u}\right)^{\left(\frac{s-3}{2}\right)} (-1)\ (u)^{(-2)}du$

$$= -\lim_{b\to +\infty} \int_b^1 \psi(u)(u)^{-\left(\frac{1+s}{2}\right)} du$$

$$= \lim_{b\to +\infty} \int_1^b \psi(u)(u)^{-\left(\frac{1+s}{2}\right)} du$$

Then Riemann set back $\lim_{b\to +\infty} \int_1^b \psi(u)(u)^{-\left(\frac{1+s}{2}\right)} du = \lim_{b\to +\infty} \int_1^b \psi(x)(x)^{-\left(\frac{1+s}{2}\right)} dx$

(which was wrong because he had let $u=\frac{1}{x}$, so u should not go back to = x again that would make confusion between the value of x of all the integrals involved.)

So $\zeta(s)(\pi)^{\left(-\frac{s}{2}\right)} \prod\left(\frac{s}{2}-1\right) = \lim_{a\to 0, b\to +\infty} \frac{1}{2}\int_a^1 [(x)^{\left(\frac{s-3}{2}\right)} - (x)^{\left(\frac{s}{2}-1\right)}]\, dx$

$$+ \int_1^b \psi(x)(x)^{\left(\frac{s}{2}-1\right)} dx + \int_1^b \psi(x)(x)^{-\left(\frac{1+s}{2}\right)} dx$$

$$= \frac{1}{(s)(s-1)} + \lim_{b\to +\infty} \int_1^b \psi(x)[(x)^{\left(\frac{s}{2}-1\right)} + (x)^{-\left(\frac{1+s}{2}\right)}]dx$$

$$= \frac{1}{(s)(s-1)} + \int_1^{+\infty} \psi(x)[(x)^{(\frac{s}{2}-1)} + (x)^{-(\frac{1+s}{2})}]dx \quad \text{... (F)}$$

The above equation (integration)… (F) was wrong because of his trick to change $\lim_{a\to 0, b\to +\infty} \int_a^1 \psi\left(\frac{1}{x}\right)(x)^{(\frac{s-3}{2})} dx$ to $\lim_{b\to +\infty} \int_1^b \psi(x)(x)^{-(\frac{1+s}{2})} dx$ which would make confusion about the value of x of all the integrals involved, and was wrong because the boundaries of the integral were still from a=0 to b=+∞. Actually, the boundaries had to be changed after separating variable of the integrands from $nn\pi x$ to nn, π, and x, as below

From $\prod\left(\frac{s}{2}-1\right) = \lim_{a\to 0, b\to +\infty} \int_a^b (e)^{(-nn\pi x_n)} (nn\pi x_n)^{(\frac{s}{2}-1)} dnn\pi x_n$, s= $(\sigma + it)$, Res or $\sigma > 0$, x = zero and positive real numbers

Then $\prod\left(\frac{s}{2}-1\right) = \lim_{a\to 0, b\to +\infty} \int_{a/nn\pi}^{b/nn\pi} (e)^{(-nn\pi x_n)} (n)^{(s)} (\pi)^{(\frac{s}{2})} (x_n)^{(\frac{s}{2}-1)} dx_n$

Term $(n)^{(s)}$ is separated out from the integral and moved to the left hand side of the equation and then n is increased in value from 1 to +∞ in each equation and finally combined to become $\sum_{n=1}^{+\infty} \left(\frac{1}{n^s}\right)$ or ζ(s) at the left hand side. The values of x_n at each n of each integral at the right hand side are changed too (the lower boundaries vary from $a/(1)(1)\pi$ to $a/(\infty)(\infty)\pi$ and the upper boundaries vary from $b/(1)(1)\pi$ to $b/(\infty)(\infty)\pi$. (Remember that $nn\pi x_n$ had boundaries from 0 to +∞ in the original integral).

Multiply by$(\pi)^{-(\frac{s}{2})} \left(\frac{1}{n^s}\right)$ both sides

$$(\pi)^{-(\frac{s}{2})} \left(\frac{1}{n^s}\right) \prod\left(\frac{s}{2}-1\right) = \lim_{a\to 0, b\to +\infty} \int_{a/nn\pi}^{b/nn\pi} (e)^{(-nn\pi x_n)} (x_n)^{(\frac{s}{2}-1)} dx_n$$

Substitute $s = (\sigma + it)$ and $(x_n)^{(\frac{s}{2}-1)} = (e)^{\left(\frac{(\sigma+it)}{2}-1\right)\mathrm{Log}(x_n)}$

$$(\pi)^{-(\frac{s}{2})} \left(\frac{1}{n^s}\right) \prod\left(\frac{s}{2}-1\right) = (\pi)^{-\frac{(\sigma+it)}{2}} \left(\frac{1}{(e)^{\frac{(\sigma+it)}{2}\mathrm{Log}(n)}}\right) \prod\left(\frac{(\sigma+it)}{2}-1\right)$$

$$= \lim_{a\to 0, b\to +\infty} \int_{a/nn\pi}^{b/nn\pi} (e)^{(-x_n)(nn\pi)} (e)^{\left(\frac{(\sigma+it)}{2}-1\right)\mathrm{Log}(x_n)} dx_n$$

$$= \lim_{a\to 0, b\to +\infty} \int_{a/nn\pi}^{b/nn\pi} (e)^{(-x_n)(nn\pi)+\left(\frac{(\sigma+it)}{2}-1\right)\mathrm{Log}(x_n)}\, dx_n$$

$$= \lim_{a\to 0, b\to +\infty} \left[\frac{1}{-(nn\pi)+\frac{\left(\frac{(\sigma+it)}{2}-1\right)}{(x_n)}}\right]\left[(e)^{\left[(-x_n)(nn\pi)+\left(\frac{(\sigma+it)}{2}-1\right)\mathrm{Log}(x_n)\right]}\right]_{a/nn\pi}^{b/nn\pi}$$

$$= \lim_{a\to 0, b\to +\infty} \left[\frac{(x_n)^{\left(\frac{\sigma}{2}-1\right)}}{(e)^{(x_n)(nn\pi)}}\right]\left[\frac{(x_n)^{\left(\frac{it}{2}\right)}}{\left[-(nn\pi)+\frac{\left(\frac{\sigma}{2}-1\right)}{(x_n)}+\frac{\left(\frac{it}{2}\right)}{(x_n)}\right]}\right]_{a/nn\pi}^{b/nn\pi}$$

$$= \lim_{a\to 0, b\to +\infty} \left[\frac{(x_n)^{\left(\frac{\sigma}{2}-1\right)}(x_n)^{\left(\frac{it}{2}\right)}}{(e)^{(x_n)(nn\pi)}\left[-(nn\pi)+\frac{\left(\frac{\sigma}{2}-1\right)}{(x_n)}+\frac{\left(\frac{it}{2}\right)}{(x_n)}\right]}\right]_{a/nn\pi}^{b/nn\pi}$$

$$= \lim_{a\to 0, b\to +\infty} \left(\left[\frac{(b/nn\pi)^{\left(\frac{\sigma}{2}-1\right)}(b/nn\pi)^{\left(\frac{it}{2}\right)}}{-(e)^{(b/nn\pi)(nn\pi)}\left[-(nn\pi)+\frac{\left(\frac{\sigma}{2}-1\right)}{(b/nn\pi)}+\frac{\left(\frac{it}{2}\right)}{(b/nn\pi)}\right]}\right] - \left[\frac{(a/nn\pi)^{\left(\frac{\sigma}{2}-1\right)}(a/nn\pi)^{\left(\frac{it}{2}\right)}}{(e)^{(a/nn\pi)(nn\pi)}\left[-(nn\pi)+\frac{\left(\frac{\sigma}{2}-1\right)}{(a/nn\pi)}+\frac{\left(\frac{it}{2}\right)}{(a/nn\pi)}\right]}\right] \right)$$

$$= \lim_{a\to 0, b\to +\infty} \left(\left[\frac{(b)^{\left(\frac{\sigma}{2}-1\right)}(b)^{\left(\frac{it}{2}\right)}(nn\pi)^{\left(-\frac{it}{2}\right)}}{(e)^{(b)}\left[-(nn\pi)^{\left(\frac{\sigma}{2}\right)}+\frac{(nn\pi)^{\left(\frac{\sigma}{2}\right)}.\left(\frac{\sigma}{2}-1\right)}{(b)}+\frac{(nn\pi)^{\left(\frac{\sigma}{2}\right)}.\left(\frac{it}{2}\right)}{(b)}\right]}\right] - \left[\frac{(a)^{\left(\frac{\sigma}{2}-1\right)}(a)^{\left(\frac{it}{2}\right)}(nn\pi)^{\left(-\frac{it}{2}\right)}}{(e)^{(a)}\left[-(nn\pi)^{\left(\frac{\sigma}{2}\right)}+\frac{(nn\pi)^{\left(\frac{\sigma}{2}\right)}.\left(\frac{\sigma}{2}-1\right)}{(a)}+\frac{(nn\pi)^{\left(\frac{\sigma}{2}\right)}.\left(\frac{it}{2}\right)}{(a)}\right]}\right] \right)$$

$$= \lim_{a\to 0, b\to +\infty} \left(\left[\frac{(b)^{\left(\frac{\sigma}{2}-1\right)}(e)^{\left(\frac{it\mathrm{Log}b}{2}\right)}(nn\pi)^{\left(-\frac{it}{2}\right)}}{(e)^{(b)}\left[-(nn\pi)^{\left(\frac{\sigma}{2}\right)}+\frac{(nn\pi)^{\left(\frac{\sigma}{2}\right)}.\left(\frac{\sigma}{2}-1\right)}{(b)}+\frac{(nn\pi)^{\left(\frac{\sigma}{2}\right)}.\left(\frac{it}{2}\right)}{(b)}\right]}\right] - \left[\frac{(a)^{\left(\frac{\sigma}{2}-1\right)}(e)^{\left(\frac{it\mathrm{Log}a}{2}\right)}(nn\pi)^{\left(-\frac{it}{2}\right)}}{(e)^{(a)}\left[-(nn\pi)^{\left(\frac{\sigma}{2}\right)}+\frac{(nn\pi)^{\left(\frac{\sigma}{2}\right)}.\left(\frac{\sigma}{2}-1\right)}{(a)}+\frac{(nn\pi)^{\left(\frac{\sigma}{2}\right)}.\left(\frac{it}{2}\right)}{(a)}\right]}\right] \right)$$

$$= \lim_{a\to 0, b\to +\infty} \left(\left[\frac{(b)^{\left(\frac{\sigma}{2}-1\right)}[\cos(t\mathrm{Log}b)+i\sin(t\mathrm{Log}b)](nn\pi)^{\left(-\frac{it}{2}\right)}}{(e)^{(b)}\left[-(nn\pi)^{\left(\frac{\sigma}{2}\right)}+\frac{(nn\pi)^{\left(\frac{\sigma}{2}\right)}\left(\frac{\sigma}{2}-1\right)}{(b)}+\frac{(nn\pi)^{\left(\frac{\sigma}{2}\right)}\left(\frac{it}{2}\right)}{(b)}\right]}\right]\right.$$

$$-\left[\frac{(a)^{\left(\frac{\sigma}{2}-1\right)}[\cos(tLoga)+i\sin(tLoga)](nn\pi)^{\left(-\frac{it}{2}\right)}}{(e)^{(a)}\left[-(nn\pi)^{\left(\frac{\sigma}{2}\right)}+\frac{(nn\pi)^{\left(\frac{\sigma}{2}\right)}.\left(\frac{\sigma}{2}-1\right)}{(a)}+\frac{(nn\pi)^{\left(\frac{\sigma}{2}\right)}.\left(\frac{it}{2}\right)}{(a)}\right]}\right]\Bigg)$$

[$\lim_{a\to 0} \text{Log}\,(a) = z$, or Log $(\approx 0) = z$,

But $(e)^{(z)} \approx 0$, or $(e)^{(-\infty)} = \frac{1}{(e)^{(\infty)}} \approx 0$,

So $\lim_{a\to 0} \text{Log}\,(a)$ or Log $(\approx 0) \approx -\infty$]

$$= \Bigg(\left[\frac{\approx(+\infty)^{\left(\frac{\sigma}{2}-1\right)}[\cos\approx(+\infty)+i\sin\approx(+\infty)](nn\pi)^{\left(-\frac{it}{2}\right)}}{(e)^{\approx(+\infty)}\left[-(nn\pi)^{\left(\frac{\sigma}{2}\right)}+\frac{(nn\pi)^{\left(\frac{\sigma}{2}\right)}\left(\frac{\sigma}{2}-1\right)}{\approx(+\infty)}+\frac{(nn\pi)^{\left(\frac{\sigma}{2}\right)}\left(\frac{it}{2}\right)}{\approx(+\infty)}\right]}\right]$$

$$-\left[\frac{\approx(0)^{\left(\frac{\sigma}{2}-1\right)}[\cos\approx(-\infty)+i\sin\approx(-\infty)](nn\pi)^{\left(-\frac{it}{2}\right)}}{(e)^{\approx(0)}\left[-(nn\pi)^{\left(\frac{\sigma}{2}\right)}+\frac{(nn\pi)^{\left(\frac{\sigma}{2}\right)}.\left(\frac{\sigma}{2}-1\right)}{\approx(0)}+\frac{(nn\pi)^{\left(\frac{\sigma}{2}\right)}.\left(\frac{it}{2}\right)}{\approx(0)}\right]}\right]\Bigg)$$

$$= \Bigg(\left[\frac{\approx(+\infty)^{\left(\frac{\sigma}{2}-1\right)}[\approx(1)+i\approx(0)](nn\pi)^{\left(-\frac{it}{2}\right)}}{(e)^{\approx(+\infty)}\left[-(nn\pi)^{\left(\frac{\sigma}{2}\right)}+\frac{(nn\pi)^{\left(\frac{\sigma}{2}\right)}.\left(\frac{\sigma}{2}-1\right)}{\approx(+\infty)}+\frac{(nn\pi)^{\left(\frac{\sigma}{2}\right)}.\left(\frac{it}{2}\right)}{\approx(+\infty)}\right]}\right]$$

$$-\left[\frac{\approx(0)^{\left(\frac{\sigma}{2}-1\right)}[\approx(1)+i\approx(0)](nn\pi)^{\left(-\frac{it}{2}\right)}}{(e)^{\approx(0)}\left[-(nn\pi)^{\left(\frac{\sigma}{2}\right)}+\frac{(nn\pi)^{\left(\frac{\sigma}{2}\right)}.\left(\frac{\sigma}{2}-1\right)}{\approx(0)}+\frac{(nn\pi)^{\left(\frac{\sigma}{2}\right)}.\left(\frac{it}{2}\right)}{\approx(0)}\right]}\right]\Bigg)$$

$$= \Bigg(\left[\frac{\approx(+\infty).\approx(1)(nn\pi)^{\left(-\frac{it}{2}\right)}}{(e)^{\approx(+\infty)}\left[-(nn\pi)^{\left(\frac{\sigma}{2}\right)}+\approx(0)+\approx(0)\right]}\right]$$

$$-\left[\frac{\approx(0).\approx(1)(nn\pi)^{\left(-\frac{it}{2}\right)}}{(e)^{\approx(0)}\left[-(nn\pi)^{\left(\frac{\sigma}{2}\right)}+\approx(+\infty)+\approx(+\infty)\right]}\right]\Bigg)$$

$$= -\left(\frac{+\infty}{+\infty} - 0\right) \text{ (indeterminate form)}$$

And from $\frac{d}{dx_n}(x_n)^{\left(\frac{\sigma}{2}-1\right)}(x_n)^{\left(\frac{it}{2}\right)}$

$$= \frac{d}{dx_n}\,(e)^{\left(\frac{\sigma}{2}-1\right)\text{Log}(x_n)}(e)^{\left(\frac{it}{2}\right)\text{Log}(x_n)}$$

$$= (e)^{\left(\frac{\sigma}{2}-1\right)\mathrm{Log}(x_n)} \frac{d}{dx_n}(e)^{\left(\frac{it}{2}\right)\mathrm{Log}(x_n)} + (e)^{\left(\frac{it}{2}\right)\mathrm{Log}(x_n)} \frac{d}{dx_n}(e)^{\left(\frac{\sigma}{2}-1\right)\mathrm{Log}(x_n)}$$

$$= \frac{\left(\frac{it}{2}\right)(e)^{\left(\frac{it}{2}\right)\mathrm{Log}(x_n)}}{x_n}(e)^{\left(\frac{\sigma}{2}-1\right)\mathrm{Log}(x_n)} + \frac{\left(\frac{\sigma}{2}-1\right)(e)^{\left(\frac{\sigma}{2}-1\right)\mathrm{Log}(x_n)}}{x_n}(e)^{\left(\frac{it}{2}\right)\mathrm{Log}(x_n)}$$

$$= [(e)^{\left(\frac{\sigma}{2}-1\right)\mathrm{Log}(x_n)}\left(\frac{it}{2}\right)(e)^{\left(\frac{it}{2}\right)\mathrm{Log}(x_n)}(e)^{-\mathrm{Log}(x_n)}$$

$$+(e)^{\left(\frac{it}{2}\right)\mathrm{Log}(x_n)}\left(\frac{\sigma}{2}-1\right)(e)^{\left(\frac{\sigma}{2}-1\right)\mathrm{Log}(x_n)}(e)^{-\mathrm{Log}(x_n)}]$$

$$= [\left(\frac{it}{2}\right)+\left(\frac{\sigma}{2}-1\right)][\,(e)^{\left(\frac{\sigma}{2}-1\right)\mathrm{Log}(x_n)}(e)^{-\mathrm{Log}(x_n)}(e)^{\left(\frac{it}{2}\right)\mathrm{Log}(x_n)}]$$

$$= [\left(\frac{\sigma}{2}+\frac{it}{2}-1\right)][\,(e)^{[\left(\frac{\sigma}{2}-1\right)-1]\mathrm{Log}(x_n)}(e)^{\left(\frac{it}{2}\right)\mathrm{Log}(x_n)}]$$

$$= [\left(\frac{\sigma}{2}+\frac{it}{2}-1\right)][\,(x_n)^{[\left(\frac{\sigma}{2}-1\right)-1]}(x_n)^{\left(\frac{it}{2}\right)}]$$

$$= [\left(\frac{\sigma}{2}+\frac{it}{2}-1\right)][\,(x_n)^{\left(\frac{\sigma}{2}-2\right)}(x_n)^{\left(\frac{it}{2}\right)}]$$

And $\frac{d}{dx_n}[\left(\frac{\sigma}{2}+\frac{it}{2}-1\right)][\,(x_n)^{\left(\frac{\sigma}{2}-2\right)}(x_n)^{\left(\frac{it}{2}\right)}]$

$$= \left(\frac{\sigma}{2}+\frac{it}{2}-1\right)\frac{d}{dx_n}(e)^{\left(\frac{\sigma}{2}-2\right)\mathrm{Log}(x_n)}(e)^{\left(\frac{it}{2}\right)\mathrm{Log}(x_n)}$$

$$= \left(\frac{\sigma}{2}+\frac{it}{2}-1\right)[(e)^{\left(\frac{\sigma}{2}-2\right)\mathrm{Log}(x_n)}\frac{d}{dx_n}(e)^{\left(\frac{it}{2}\right)\mathrm{Log}(x_n)}$$

$$+ (e)^{\left(\frac{it}{2}\right)\mathrm{Log}(x_n)}\frac{d}{dx_n}(e)^{\left(\frac{\sigma}{2}-2\right)\mathrm{Log}(x_n)}]$$

$$= \left(\frac{\sigma}{2}+\frac{it}{2}-1\right)[(e)^{\left(\frac{\sigma}{2}-2\right)\mathrm{Log}(x_n)}\frac{\left(\frac{it}{2}\right)}{x_n}(e)^{\left(\frac{it}{2}\right)\mathrm{Log}(x_n)}$$

$$+(e)^{\left(\frac{it}{2}\right)\mathrm{Log}(x_n)}\frac{\left(\frac{\sigma}{2}-2\right)}{x_n}(e)^{\left(\frac{\sigma}{2}-2\right)\mathrm{Log}(x_n)}\;]$$

$$= \left(\frac{\sigma}{2}+\frac{it}{2}-1\right)[\left(\frac{it}{2}\right)(e)^{\left(\frac{\sigma}{2}-2\right)\mathrm{Log}(x_n)}(e)^{\left(\frac{it}{2}\right)\mathrm{Log}(x_n)}(e)^{-\mathrm{Log}(x_n)}$$

$$+\left(\frac{\sigma}{2}-2\right)(e)^{\left(\frac{it}{2}\right)\mathrm{Log}(x_n)}(e)^{\left(\frac{\sigma}{2}-2\right)\mathrm{Log}(x_n)}(e)^{-\mathrm{Log}(x_n)}]$$

$$= \left(\frac{\sigma}{2} + \frac{it}{2} - 1\right)\left(\frac{\sigma}{2} + \frac{it}{2} - 2\right)[\,(e)^{\left(\frac{\sigma}{2}-2\right)\mathrm{Log}(x_n)}(e)^{-\mathrm{Log}(x_n)}(e)^{\left(\frac{it}{2}\right)\mathrm{Log}(x_n)}]$$

$$= \left(\frac{\sigma}{2} + \frac{it}{2} - 1\right)[\left(\frac{\sigma}{2} + \frac{it}{2} - 2\right)][\,(e)^{[\left(\frac{\sigma}{2}-2\right)-1]\mathrm{Log}(x_n)}(e)^{\left(\frac{it}{2}\right)\mathrm{Log}(x_n)}]$$

$$=\left(\frac{\sigma}{2} + \frac{it}{2} - 1\right)[\left(\frac{\sigma}{2} + \frac{it}{2} - 2\right)][\,(x_n)^{\left(\frac{\sigma}{2}-3\right)}(x_n)^{\left(\frac{it}{2}\right)}]$$

So $\frac{d^{(\frac{\sigma}{2}-1)}}{d(x_n)^{(\frac{\sigma}{2}-1)}}\ [(x_n)^{(\frac{\sigma}{2}-1)}(x_n)^{\left(\frac{it}{2}\right)}\,]$

$$= [\left(\frac{\sigma}{2} + \frac{it}{2} - 1\right)\left(\frac{\sigma}{2} + \frac{it}{2} - 2\right)\ldots\left(\frac{it}{2} + 1\right)][\,(x_n)^{(0)}(x_n)^{\left(\frac{it}{2}\right)}\,]$$

$$=\left(\frac{s}{2} - 1\right)!\ [\,(x_n)^{(0)}(x_n)^{\left(\frac{it}{2}\right)}]$$

And $\frac{d^{(\frac{\sigma}{2})}}{d(x_n)^{(\sigma)}}[(x_n)^{(\frac{\sigma}{2})}(x_n)^{\left(\frac{it}{2}\right)}]$

$$=[\left(\frac{\sigma}{2} + \frac{it}{2}\right)\left(\frac{\sigma}{2} + \frac{it}{2} - 1\right)\ldots\left(\frac{it}{2} + 1\right)][(x_n)^{(0)}(x_n)^{\left(\frac{it}{2}\right)}]$$

$$=(\frac{s}{2})!\,[\,(x_n)^{(0)}(x_n)^{\left(\frac{it}{2}\right)}]$$

And from $\frac{d}{dx_n}[(e)^{(x_n)(nn\pi)}[(-nn\pi) + \frac{\left(\frac{\sigma}{2}+\frac{it}{2}-1\right)}{x_n}]]$

Or $\frac{d}{dx_n}[-(nn\pi)(e)^{(x_n)(nn\pi)} + \left(\frac{\sigma}{2} + \frac{it}{2} - 1\right)[(e)^{(-\mathrm{Log}x_n)}(e)^{(x_n)(nn\pi)}]\,]$

$$=-(nn\pi)^{(2)}(e)^{(x_n)(nn\pi)} + \left(\frac{\sigma}{2} + \frac{it}{2} - 1\right)[(e)^{(-\mathrm{Log}x_n)}(nn\pi)(e)^{(x_n)(nn\pi)} + (e)^{(x_n)(nn\pi)}(\frac{1}{-x_n})(e)^{(-\mathrm{Log}x_n)}\,]$$

$$=-(nn\pi)^{(2)}(e)^{(x_n)(nn\pi)} + \left(\frac{\sigma}{2} + \frac{it}{2} - 1\right)(e)^{(x_n)(nn\pi)}\left[(nn\pi)(e)^{(-\mathrm{Log}x_n)} - (e)^{(-2\mathrm{Log}x_n)}\right] \quad \ldots P$$

And

$$\frac{d}{dx_n}[P] =-(nn\pi)^{(3)}(e)^{(x_n)(nn\pi)} + \left(\frac{\sigma}{2} + \frac{it}{2} - 1\right)[\,[(nn\pi)^{(2)}(e)^{(-\mathrm{Log}x_n)}$$

$+(nn\pi)(e)^{(x_n)(nn\pi)}\left(\frac{1}{-x_n}\right)(e)^{(-\mathrm{Log}x_n)}] - [(e)^{(-2\mathrm{Log}x_n)}(nn\pi)(e)^{(x_n)(nn\pi)}$

$+(e)^{(x_n)(nn\pi)}\left(\frac{2}{-x_n}\right)(e)^{(-\mathrm{Log}x_n)}]\,]$

$=-(nn\pi)^{(3)}(e)^{(x_n)(nn\pi)} + [\left(\frac{\sigma}{2}+\frac{it}{2}-1\right)(e)^{(x_n)(nn\pi)}][(nn\pi)^{(2)}(e)^{(-\mathrm{Log}x_n)}$
$\quad -(2nn\pi)(e)^{(-2\mathrm{Log}x_n)}+(2)(e)^{(-3\mathrm{Log}x_n)}]$ ...Q

$\frac{d}{dx_n}[Q] = -(nn\pi)^{(4)}(e)^{(x_n)(nn\pi)} + \left(\frac{\sigma}{2}+\frac{it}{2}-1\right)(e)^{(x_n)(nn\pi)}[(nn\pi)^{(3)}(e)^{(-\mathrm{Log}x_n)}$

$\quad -3(nn\pi)^{(2)}(e)^{(-2\mathrm{Log}x_n)}+(6nn\pi)(e)^{(-3\mathrm{Log}x_n)} - (6)(e)^{(-4\mathrm{Log}x_n)}]$ ...R

$\frac{d}{dx_n}[R] = -(nn\pi)^{(5)}(e)^{(x_n)(nn\pi)} + \left(\frac{\sigma}{2}+\frac{it}{2}-1\right)(e)^{(x_n)(nn\pi)}[(nn\pi)^{(4)}(e)^{(-\mathrm{Log}x_n)}$

$\quad -\ 4(nn\pi)^{(3)}(e)^{(-2\mathrm{Log}x_n)} + 12(nn\pi)^{(2)}(e)^{(-3\mathrm{Log}x_n)} -$

$\quad -\ (24nn\pi)(e)^{(-4\mathrm{Log}x_n)}+(24)(e)^{(-5\mathrm{Log}x_n)}]$ ...S

$\frac{d}{dx_n}[S] = -(nn\pi)^{(6)}(e)^{(x_n)(nn\pi)} + [\left(\frac{\sigma}{2}+\frac{it}{2}-1\right)(e)^{(x_n)(nn\pi)}]$
$[(nn\pi)^{(5)}(e)^{(-\mathrm{Log}x_n)} -5(nn\pi)^{(4)}(e)^{(-2\mathrm{Log}x_n)} +20(nn\pi)^{(3)}(e)^{(-3\mathrm{Log}x_n)}$

$-60(nn\pi)^{(2)}(e)^{(-4\mathrm{Log}x_n)} +(120nn\pi)(e)^{(-5\mathrm{Log}x_n)} -(120)(e)^{(-6\mathrm{Log}x_n)}]$

So $\frac{d^{(\frac{\sigma}{2}-1)}}{(dx_n)^{(\frac{\sigma}{2}-1)}}\ [(e)^{(x_n)(nn\pi)}[(-nn\pi)+\frac{\left(\frac{\sigma}{2}+\frac{it}{2}-1\right)}{x_n}]\,]$

$= -(nn\pi)^{\left(\frac{\sigma}{2}\right)}(e)^{(x_n)(nn\pi)} + [\left(\frac{\sigma}{2}+\frac{it}{2}-1\right)(e)^{(x_n)(nn\pi)}][(nn\pi)^{\left(\frac{\sigma}{2}-1\right)}(e)^{(-\mathrm{Log}x_n)}$

$-\left(\frac{\sigma}{2}-1\right)(nn\pi)^{\left(\frac{\sigma}{2}-2\right)}(e)^{(-2\mathrm{Log}x_n)} + \left(\frac{\sigma}{2}-1\right)\left(\frac{\sigma}{2}-2\right)(nn\pi)^{\left(\frac{\sigma}{2}-3\right)}(e)^{(-3\mathrm{Log}x_n)}$

$-\left(\frac{\sigma}{2}-1\right)\left(\frac{\sigma}{2}-2\right)\left(\frac{\sigma}{2}-3\right)(nn\pi)^{\left(\frac{\sigma}{2}-4\right)}(e)^{(-4\mathrm{Log}x_n)}+\ldots]$

Apply L' Hospital's Rule $\left(\frac{\sigma}{2}-1\right)$ times to

$$\lim_{a\to 0, b\to +\infty}\left[\frac{(x_n)^{(\frac{\sigma}{2}-1)}(x_n)^{\left(\frac{it}{2}\right)}}{(e)^{(x_n)(nn\pi)}[-(nn\pi)+\frac{\left(\frac{\sigma}{2}-1\right)}{(x_n)}+\frac{\left(\frac{it}{2}\right)}{(x_n)}]}\right]\Bigg]_{a/nn\pi}^{b/nn\pi}$$

until $(x_n)^{(\frac{\sigma}{2}-1)} = (x_n)^{(0)} = 1$

Then $\lim_{a\to 0,b\to+\infty}\left[\frac{(x_n)^{(\frac{\sigma}{2}-1)}(x_n)^{(\frac{it}{2})}}{(e)^{(x_n)(nn\pi)}[-(nn\pi)+\frac{(\frac{\sigma}{2}-1)}{(x_n)}+\frac{(\frac{it}{2})}{(x_n)}]}\right]\Big]_{a/nn\pi}^{b/nn\pi}$

$$=\lim_{a\to 0,b\to+\infty}\left[\frac{\left(\frac{it}{2}+\frac{\sigma}{2}-1\right)\left(\frac{it}{2}+\frac{\sigma}{2}-2\right)\ldots\left(\frac{it}{2}+1\right)(x_n)^{(0)}(x_n)^{\left(\frac{it}{2}\right)}}{(e)^{(x_n)(nn\pi)}[-(nn\pi)^{\left(\frac{\sigma}{2}\right)}+\left(\frac{\sigma}{2}+\frac{it}{2}-1\right)][(nn\pi)^{\left(\frac{\sigma}{2}-1\right)}(e)^{(-\mathrm{Log}x_n)}-\left(\frac{\sigma}{2}-1\right)\ldots]}\right]\Big]_{a/nn\pi}^{b/nn\pi}$$

$$=\lim_{a\to 0,b\to+\infty}\left[\frac{\left(\frac{it}{2}+\frac{\sigma}{2}-1\right)\left(\frac{it}{2}+\frac{\sigma}{2}-2\right)\ldots\left(\frac{it}{2}+1\right)(b)^{\left(\frac{it}{2}\right)}(nn\pi)^{\left(-\frac{it}{2}\right)}}{(e)^{(b)}\left[-(nn\pi)^{\left(\frac{\sigma}{2}\right)}+\left(\frac{\sigma}{2}+\frac{it}{2}-1\right)[(nn\pi)^{\left(\frac{\sigma}{2}-1\right)}\frac{(e)^{(-\mathrm{Log}b)}}{(e)^{(-\mathrm{Log}nn\pi)}}-\left(\frac{\sigma}{2}-1\right)(nn\pi)^{\left(\frac{\sigma}{2}-2\right)}\ldots]\right]}\right]$$

$$-\left[\frac{\left(\frac{it}{2}+\frac{\sigma}{2}-1\right)\left(\frac{it}{2}+\frac{\sigma}{2}-2\right)\ldots\left(\frac{it}{2}+1\right)(a)^{\left(\frac{it}{2}\right)}(nn\pi)^{\left(-\frac{it}{2}\right)}}{(e)^{(a)}\left[-(nn\pi)^{\left(\frac{\sigma}{2}\right)}+\left(\frac{\sigma}{2}+\frac{it}{2}-1\right)[(nn\pi)^{\left(\frac{\sigma}{2}-1\right)}\frac{(e)^{(-\mathrm{Log}a)}}{(e)^{(-\mathrm{Log}nn\pi)}}-\left(\frac{\sigma}{2}-1\right)(nn\pi)^{\left(\frac{\sigma}{2}-2\right)}\ldots]\right]}\right]$$

$$=\lim_{a\to 0,b\to+\infty}\left[\frac{\left(\frac{it}{2}+\frac{\sigma}{2}-1\right)\left(\frac{it}{2}+\frac{\sigma}{2}-2\right)\ldots\left(\frac{it}{2}+1\right)(b)^{\left(\frac{it}{2}\right)}(nn\pi)^{\left(-\frac{it}{2}\right)}}{(e)^{(b)}\left[-(nn\pi)^{\left(\frac{\sigma}{2}\right)}+\left(\frac{\sigma}{2}+\frac{it}{2}-1\right)[(nn\pi)^{\left(\frac{\sigma}{2}\right)}(e)^{(-\mathrm{Log}b)}-\left(\frac{\sigma}{2}-1\right)(nn\pi)^{\left(\frac{\sigma}{2}\right)}(e)^{(-2\mathrm{Log}b)}\ldots]\right]}\right]$$

$$-\left[\frac{\left(\frac{it}{2}+\frac{\sigma}{2}-1\right)\left(\frac{it}{2}+\frac{\sigma}{2}-2\right)\ldots\left(\frac{it}{2}+1\right)(a)^{\left(\frac{it}{2}\right)}(nn\pi)^{\left(-\frac{it}{2}\right)}}{(e)^{(a)}\left[-(nn\pi)^{\left(\frac{\sigma}{2}\right)}+\left(\frac{\sigma}{2}+\frac{it}{2}-1\right)[(nn\pi)^{\left(\frac{\sigma}{2}\right)}(e)^{(-\mathrm{Log}a)}-\left(\frac{\sigma}{2}-1\right)(nn\pi)^{\left(\frac{\sigma}{2}\right)}(e)^{(-2\mathrm{Log}a)}\ldots]\right]}\right]$$

$$=\lim_{a\to 0,b\to+\infty}\left[\frac{\left(\frac{it}{2}+\frac{\sigma}{2}-1\right)\left(\frac{it}{2}+\frac{\sigma}{2}-2\right)\ldots\left(\frac{it}{2}+1\right)(e)^{\left(\frac{it}{2}\mathrm{Log}b\right)}}{(e)^{(b)}(nn\pi)^{\left(\frac{\sigma}{2}+\frac{it}{2}\right)}\left[-1+\left(\frac{\sigma}{2}+\frac{it}{2}-1\right)[(e)^{(-\mathrm{Log}b)}-\left(\frac{\sigma}{2}-1\right)(e)^{(-2\mathrm{Log}b)}\ldots]\right]}\right]$$

$$-\left[\frac{\left(\frac{it}{2}+\frac{\sigma}{2}-1\right)\left(\frac{it}{2}+\frac{\sigma}{2}-2\right)\ldots\left(\frac{it}{2}+1\right)(e)^{(it\mathrm{Log}a)}}{(e)^{(a)}(nn\pi)^{\left(\frac{\sigma}{2}+\frac{it}{2}\right)}\left[-1+\left(\frac{it}{2}+\frac{\sigma}{2}-1\right)[(e)^{(-\mathrm{Log}(a)}-\left(\frac{\sigma}{2}-1\right)(e)^{(-2\mathrm{Log}a)}\ldots]\right]}\right]$$

[Let $\lim_{a\to 0}\mathrm{Log}\,(a)=z$, or $\mathrm{Log}(\approx 0)=z$, then $(e)^{(z)}\approx 0$,but $(e)^{(-\infty)}=\frac{1}{(e)^{(\infty)}}$ ≈ 0 So $\lim_{a\to 0}\mathrm{Log}\,(a)$ or $\lim_{a\to 0}\mathrm{Log}\,(\approx 0)\approx -\infty$]

$$=\lim_{a\to 0,b\to+\infty}\left[\frac{\left(\frac{it}{2}+\frac{\sigma}{2}-1\right)\left(\frac{it}{2}+\frac{\sigma}{2}-2\right)\ldots\left(\frac{it}{2}+1\right)[\cos\approx(+\infty t)+i\sin\approx(+\infty t)]}{(nn\pi)^{\left(\frac{it}{2}+\frac{\sigma}{2}\right)}(e)^{\approx(+\infty)}\left[-1+\left(\frac{it}{2}+\frac{\sigma}{2}-1\right)[(e)^{(-\mathrm{Log}\approx(+\infty))}-\left(\frac{\sigma}{2}-1\right)(e)^{(-2\mathrm{Log}\approx(+\infty))}\ldots]\right]}\right]$$

$$-\left[\frac{\left(\frac{it}{2}+\frac{\sigma}{2}-1\right)\left(\frac{it}{2}+\frac{\sigma}{2}-2\right)\ldots\left(\frac{it}{2}+1\right)[\cos\approx(-\infty t)+i\sin\approx(-\infty t)]}{(nn\pi)^{\left(\frac{it}{2}+\frac{\sigma}{2}\right)}(e)^{\approx(0)}\left[-1+\left(\frac{it}{2}+\frac{\sigma}{2}-1\right)[(e)^{(-\mathrm{Log}\approx(0))}-\left(\frac{\sigma}{2}-1\right)(e)^{(-2\mathrm{Log}\approx(0))}\ldots]\right]}\right]$$

$$=\lim_{a\to 0,b\to+\infty}\left[\frac{\left(\frac{it}{2}+\frac{\sigma}{2}-1\right)\left(\frac{it}{2}+\frac{\sigma}{2}-2\right)\ldots\left(\frac{it}{2}+1\right)}{(nn\pi)^{\left(\frac{it}{2}+\frac{\sigma}{2}\right)}}\right]\left[\frac{\approx(1)+\approx(0)}{(e)^{\approx(+\infty)}[\approx(-1)+\approx(0)-\approx(0)+\cdots]}\right]$$

$$-\left[\frac{\left(\frac{it}{2}+\frac{\sigma}{2}-1\right)\left(\frac{it}{2}+\frac{\sigma}{2}-2\right)\ldots\left(\frac{it}{2}+1\right)}{(nn\pi)^{\left(\frac{it}{2}+\frac{\sigma}{2}\right)}}\right]\left[\frac{\approx(1)+\approx(0)}{(e)^{\approx(0)}[\approx(-1)+\approx(0)-\approx(0)+\cdots]}\right]$$

$$=\lim_{a\to 0,b\to+\infty}\left[\frac{\left(\frac{it}{2}+\frac{\sigma}{2}-1\right)\left(\frac{it}{2}+\frac{\sigma}{2}-2\right)\ldots\left(\frac{it}{2}+1\right)}{(nn\pi)^{\left(\frac{it}{2}+\frac{\sigma}{2}\right)}}\right][\approx(0)]$$

$$-\left[\frac{(it+\sigma-1)\left(\frac{it}{2}+\frac{\sigma}{2}-2\right)\ldots\left(\frac{it}{2}+1\right)}{(nn\pi)^{\left(\frac{it}{2}+\frac{\sigma}{2}\right)}}\right]\left[\frac{\approx(1)}{\approx(-1)}\right]$$

$$=\lim_{a\to 0,b\to+\infty}\left[\frac{\left(\frac{it}{2}+\frac{\sigma}{2}-1\right)\left(\frac{it}{2}+\frac{\sigma}{2}-2\right)\ldots\left(\frac{it}{2}+1\right)}{(nn\pi)^{\left(\frac{it}{2}+\frac{\sigma}{2}\right)}}\right]$$

$$=\frac{\left(\frac{it}{2}+\frac{\sigma}{2}-1\right)!}{(nn\pi)^{\left(\frac{it}{2}+\frac{\sigma}{2}\right)}}$$

$$=\frac{(\frac{s}{2}-1)!}{(nn\pi)^{\left(\frac{s}{2}\right)}}$$

For $n=1$ $\quad \frac{1}{(1)^s}\prod\left(\frac{s}{2}-1\right)(\pi)^{-\left(\frac{s}{2}\right)}$

$$=\lim_{a\to 0,b\to+\infty}\int_{a/(1)(1)\pi}^{b/(1)(1)\pi}(e)^{(-x_1)(1)(1)\pi}(x_1)^{(\frac{s}{2}-1)}dx_1$$

$$=\frac{(\frac{s}{2}-1)!}{((1)(1)\pi)^{\left(\frac{s}{2}\right)}}$$

For $n=2$ $\quad \frac{1}{(2)^s}\prod\left(\frac{s}{2}-1\right)(\pi)^{-\left(\frac{s}{2}\right)}$

$$=\lim_{a\to 0,b\to+\infty}\int_{a/(2)(2)\pi}^{b/(2)(2)\pi}(e)^{(-x_2)(2)(2)\pi}(x_2)^{(\frac{s}{2}-1)}dx_2$$

$$=\frac{(\frac{s}{2}-1)!}{((2)(2)\pi)^{\left(\frac{s}{2}\right)}}$$

For $n=+\infty$ $\quad \frac{1}{(+\infty)^s}\prod\left(\frac{s}{2}-1\right)(\pi)^{-\left(\frac{s}{2}\right)}$

$$=\lim_{a\to 0,b\to+\infty}\int_{a/+(\infty)(+\infty)\pi}^{b/(+\infty)(+\infty)\pi}(e)^{(-x_\infty)(+\infty)}(x_\infty)^{\left(\frac{s}{2}-1\right)}dx_\infty$$

$$= \frac{(\frac{s}{2}-1)!}{((+\infty)(+\infty)\pi)^{\left(\frac{s}{2}\right)}}$$

Actually the correct method of integration of the above equations should be.

$$[\left(\frac{1}{1^s}\right) + \left(\frac{1}{2^s}\right) + \ldots + \left(\frac{1}{\infty^s}\right)] \prod\left(\frac{s}{2} - 1\right) (\pi)^{-\left(\frac{s}{2}\right)}$$

$$= \lim_{a\to 0, b\to +\infty} [\int_{a/(1)(1)\pi}^{b/(1)(1)\pi} (e)^{(-x_1)(1)(1)\pi} (x_1)^{\left(\frac{s}{2}-1\right)} dx_1$$

$$+ \int_{a/(2)(2)\pi}^{b/(2)(2)\pi} (e)^{(-x_2)(2)(2)\pi} (x_2)^{\left(\frac{s}{2}-1\right)} dx_2$$

$$+ \ldots$$

$$+ \int_{a/(+\infty)(+\infty)\pi}^{b/(+\infty)(+\infty)\pi} (e)^{(-x_\infty)(+\infty)(+\infty)\pi} (x_\infty)^{\left(\frac{s}{2}-1\right)} dx_\infty$$

$$\sum_{n=1}^{+\infty} \left(\frac{1}{n^s}\right) \prod\left(\frac{s}{2} - 1\right) (\pi)^{-\left(\frac{s}{2}\right)} = \frac{(\frac{s}{2}-1)!}{[(1)(1)\pi]^{\left(\frac{s}{2}\right)}}$$

$$+ \frac{(\frac{s}{2}-1)!}{[(2)(2)\pi]^{\left(\frac{s}{2}\right)}}$$

$$+ \ldots$$

$$+ \frac{(\frac{s}{2}-1)!}{[(+\infty)(+\infty)\pi]^{\left(\frac{s}{2}\right)}}$$

Or $\zeta(s) \prod\left(\frac{s}{2} - 1\right) (\pi)^{-\left(\frac{s}{2}\right)} = \sum_{n=1}^{+\infty} \frac{(\frac{s}{2}-1)!}{[nn\pi]^{\left(\frac{s}{2}\right)}}$

$$\zeta(s)(\pi)^{\left(-\frac{s}{2}\right)} \prod\left(\frac{s}{2}-1\right) \neq \frac{1}{(s)(s-1)} + \int_1^{+\infty} \psi(x)[(x)^{\left(\frac{s}{2}-1\right)} + (x)^{-\left(\frac{1+s}{2}\right)}]dx$$

because the boundaries are not the same as Riemann did before. In this case the different boundaries tell us that $x_1, x_2, \ldots, x_\infty$ are not the same,

so $\int_{a/(1)(1)(\pi)}^{b/(1)(1)(\pi)} (x_1)^{\left(\frac{s}{2}-1\right)} dx_1 \neq \int_{a/(2)(2)(\pi)}^{b/(2)(2)(\pi)} (x_2)^{\left(\frac{s}{2}-1\right)} dx_2$

$$\neq \cdots$$

$$\neq \int_{a/(+\infty)(+\infty)(\pi)}^{b/(+\infty)(+\infty)(\pi)} (x_\infty)^{\left(\frac{s}{2}-1\right)}\, dx_\infty$$

, and infinite summation of the integrands , $(e)^{(-nn\pi x)}$ of all the above integrals cannot be done as Riemann did because infinite summation, $\sum_{n=1}^{+\infty} (e)^{(-nn\pi x)}$ of one of the integrand can be done only when value of another integrand $(x_n)^{\left(\frac{s}{2}-1\right)}$ does not change when n changes.

4. Proof that $\prod\left(\frac{s}{2}\right)\left(\frac{s}{2}-1\right)(\pi)^{\left(-\frac{s}{2}\right)}\zeta(s)$, for $s=\frac{1}{2}+it$

$$\neq \frac{1}{2}+\frac{\left(tt+\frac{1}{4}\right)}{2}\int_1^{+\infty}\psi(x)(x)^{-\left(\frac{3}{4}\right)}\cos\left(\frac{1}{2}tLogx\right)dx$$

$$\neq \xi(t)$$

From Riemann's equation (suppose that it is correct)

$$\zeta(s)(\pi)^{\left(-\frac{s}{2}\right)}\prod\left(\frac{s}{2}-1\right) = \frac{1}{(s)(s-1)}+\int_1^{+\infty}\psi(x)\left[(x)^{\left(\frac{s}{2}-1\right)}+(x)^{-\left(\frac{1+s}{2}\right)}\right]dx \quad \text{... (F)}$$

Multiply $\left(\frac{s}{2}\right)(s-1)$ both sides and set $s=\frac{1}{2}+it$

$$\zeta(s)(\pi)^{\left(-\frac{s}{2}\right)}\prod\left(\frac{s}{2}-1\right)\left(\frac{s}{2}\right)(s-1)$$

$$= \frac{\left(\frac{s}{2}\right)(s-1)}{(s)(s-1)}+\left(\frac{s}{2}\right)(s-1)\int_1^{+\infty}\psi(x)\left[(x)^{\left(\frac{s}{2}-1\right)}+(x)^{-\left(\frac{1+s}{2}\right)}\right]dx$$

$$= \frac{1}{2}+\left(\frac{\frac{1}{2}+it}{2}\right)\left(\frac{1}{2}+it-1\right)\int_1^{+\infty}\psi(x)\left[(x)^{\left(\frac{1}{4}+\frac{it}{2}-1\right)}+(x)^{-\left(\frac{1+\frac{1}{2}+it}{2}\right)}\right]dx$$

$$= \frac{1}{2}+\left(\frac{tt+\frac{1}{4}}{2}\right)\int_1^{+\infty}\psi(x)\left[(x)^{\left(-\frac{3}{4}\right)}(x)^{\left(\frac{it}{2}\right)}+(x)^{\left(-\frac{3}{4}\right)}(x)^{\left(-\frac{it}{2}\right)}\right]dx$$

$$= \frac{1}{2}+\left(\frac{tt+\frac{1}{4}}{2}\right)\int_1^{+\infty}\psi(x)(x)^{\left(-\frac{3}{4}\right)}\left[(e)^{\left(\frac{1}{2}tLogx\right)}+(e)^{\left(-\frac{1}{2}tLogx\right)}\right]dx$$

$$= \frac{1}{2}+\left(\frac{tt+\frac{1}{4}}{2}\right)\int_1^{+\infty}\psi(x)(x)^{\left(-\frac{3}{4}\right)}\left[\cos\left(\frac{1}{2}tLogx\right)+i\sin\left(\frac{1}{2}tLogx\right)\right.$$

$$\left.\cos\left(\frac{1}{2}tLogx\right)-i\sin\left(\frac{1}{2}tLogx\right)\right]dx$$

$$= \frac{1}{2} + \left(\frac{tt+\frac{1}{4}}{2}\right) \int_1^{+\infty} \psi(x)(x)^{\left(-\frac{3}{4}\right)} 2\cos(\tfrac{1}{2}tLogx)dx$$

$$= \frac{1}{2} + \left(tt + \frac{1}{4}\right) \int_1^{+\infty} \psi(x)(x)^{\left(-\frac{3}{4}\right)}\cos(\tfrac{1}{2}tLogx)dx$$

$$= \xi(t)$$

Consider $\zeta(s)(\pi)^{\left(-\frac{s}{2}\right)} \prod\left(\frac{s}{2}-1\right)\left(\frac{s}{2}\right)(s-1)$

From $\Gamma(s) = \frac{\Gamma(s+1)}{s}$ for $s \neq 0$

And $\Gamma(s) = \frac{\Gamma(s+k)}{s(s+1)(s+2)\ldots(s+k-1)}$ for $k = 0, 1, 2, 3, \ldots$ and $s+k > 0$, or $s > -k$

$, s \neq 0, -1, -2, \ldots, -(k-1)$

And $\prod(s) = \Gamma(s+1) = s\Gamma(s) = s\prod(s-1)$

Then $\prod(\frac{s}{2}) = \Gamma(\frac{s}{2}+1) = \frac{s}{2}\Gamma(\frac{s}{2}) = \frac{s}{2}\prod\left(\frac{s}{2}-1\right)$

Thus $\zeta(s)(\pi)^{\left(-\frac{s}{2}\right)}\left(\frac{s}{2}\right)\prod\left(\frac{s}{2}-1\right)(s-1) = \zeta(s)(\pi)^{\left(-\frac{s}{2}\right)}\prod\left(\frac{s}{2}\right)(s-1)$

So $\zeta(s)(\pi)^{\left(-\frac{s}{2}\right)}\prod\left(\frac{s}{2}\right)(s-1) = \frac{1}{2} + \left(tt + \frac{1}{4}\right)\int_1^{+\infty} \psi(x)(x)^{\left(-\frac{3}{4}\right)}\cos(\tfrac{1}{2}tLogx)\,dx$

$$= \xi(t)$$

However, the above equation is not true because it was derived from the wrong equation… (F) as proof above

$$\zeta(s)(\pi)^{\left(-\frac{s}{2}\right)}\prod\left(\frac{s}{2}-1\right) = \frac{1}{(s)(s-1)} + \int_1^{+\infty} \psi(x)[(x)^{\left(\frac{s}{2}-1\right)} + (x)^{-\left(\frac{1+s}{2}\right)}]dx \quad \ldots (F)$$

While the correct one is

$$\zeta(s)(\pi)^{\left(-\frac{s}{2}\right)}\prod\left(\frac{s}{2}-1\right) = \sum_{n=1}^{+\infty} \lim_{a\to 0, b\to +\infty} \int_{a/nn\pi}^{b/nn\pi} (e)^{(-nn\pi x_n)}(x_n)^{\left(\frac{s}{2}-1\right)}dx_n$$

$$= \sum_{n=1}^{+\infty} \frac{(\frac{s}{2}-1)!}{[nn\pi]^{\left(\frac{s}{2}\right)}}$$

Multiply by $\left(\frac{s}{2}\right)$ both sides

Then $\zeta(s)(\pi)^{\left(-\frac{s}{2}\right)}\left(\frac{s}{2}\right)\prod\left(\frac{s}{2}-1\right)$

$$= \left(\frac{s}{2}\right) \sum_{\mathbf{n=1}}^{+\infty} \lim_{a\to 0, b\to +\infty} \int_{a/nn\pi}^{b/nn\pi} (e)^{(-nn\pi x_n)} (x_n)^{\left(\frac{s}{2}-1\right)} dx_n$$

$$= \left(\frac{s}{2}\right) \sum_{n=1}^{+\infty} \frac{\left(\frac{s}{2}-1\right)!}{[nn\pi]^{\left(\frac{s}{2}\right)}}$$

From $\prod\left(\frac{s}{2}\right) = \Gamma\left(\frac{s}{2}+1\right) = \frac{s}{2}\Gamma\left(\frac{s}{2}\right) = \frac{s}{2}\prod\left(\frac{s}{2}-1\right)$

So $\zeta(s)(\pi)^{\left(-\frac{s}{2}\right)}\left(\frac{s}{2}\right)\prod\left(\frac{s}{2}-1\right) = \zeta(s)(\pi)^{\left(-\frac{s}{2}\right)}\prod\left(\frac{s}{2}\right)$

Hence $\zeta(s)(\pi)^{\left(-\frac{s}{2}\right)}\prod\left(\frac{s}{2}\right) = \left(\frac{s}{2}\right)\sum_{n=1}^{+\infty} \frac{\left(\frac{s}{2}-1\right)!}{[nn\pi]^{\left(\frac{s}{2}\right)}}$

And if one set $s = \frac{1}{2} + it = \sigma + it$

Then $\zeta\left(\frac{1}{2}+it\right)(\pi)^{\left(-\frac{\left(\frac{1}{2}+it\right)}{2}\right)}\prod\left(\frac{\frac{1}{2}+it}{2}\right)$

$$= \xi(t)$$

$$= \left(\frac{\left(\frac{1}{2}+it\right)}{2}\right) \sum_{\mathbf{n=1}}^{+\infty} \lim_{a\to 0, b\to +\infty} \int_{a/nn\pi}^{b/nn\pi} (e)^{(-nn\pi x_n)} (x_n)^{\left(\frac{\left(\frac{1}{2}+it\right)}{2}-1\right)} dx_n$$

$$= \left(\frac{1}{4}+\frac{it}{2}\right) \sum_{n=1}^{+\infty} \frac{\left(\frac{1}{4}+\frac{it}{2}-1\right)!}{[nn\pi]^{\left(\frac{1}{4}+\frac{it}{2}\right)}}$$

Hence $\xi(t) \neq \frac{1}{2} + \left(tt+\frac{1}{4}\right)\int_1^{+\infty} \psi(x)(x)^{\left(-\frac{3}{4}\right)}\cos\left(\frac{1}{2}tLogx\right)dx$ and $\xi(t)$ will not vanish $(= 0)$ for any value of t. So there are no nontrivial zeroes of $\zeta(s)$ on the critical line, $Res = \sigma = \frac{1}{2}$ within the critical strip $\{s \in C: 0 < Res < 1\}$.

5. Proof that $\zeta(s) = (2)^{(s)}(\pi)^{(s-1)}\sin\left(\frac{\pi s}{2}\right)\Gamma(1-s)\,\zeta(1-s)$ only when $s = \sigma + it = \frac{1}{2} + i0 = \frac{1}{2}$.

Or in the other word, there are no nontrivial zeroes that can be found from this functional equation because $\zeta(s) = \zeta\left(\frac{1}{2}\right)$ will diverge $(= +\infty)$ not vanish $(= 0)$.

From $(\pi)^{\left(-\frac{s}{2}\right)}\prod\left(\frac{s}{2}-1\right)\zeta(s)$

Or $(\pi)^{\left(-\frac{s}{2}\right)}\Gamma\left(\frac{s}{2}\right)\zeta(s)$

Let $s = (1-s)$, and this will be true if and only if

$s = (\sigma + it)$

$= \left(\frac{1}{2} + it\right)$

$= (1-s)$

$= 1 - \left(\frac{1}{2} + it\right)$

$= \left(\frac{1}{2} - it\right)$

or $\left(\frac{1}{2} + it\right) = \left(\frac{1}{2} - it\right)$ which will be true only when t =0, $s = \frac{1}{2}$

From $(\pi)^{\left(-\frac{s}{2}\right)}\Gamma\left(\frac{s}{2}\right)\zeta(s) = (\pi)^{-\left(\frac{1-s}{2}\right)}\Gamma\left(\frac{1-s}{2}\right)\zeta(1-s)$, $s = (1-s) = \frac{1}{2}$

Multiply by $\Gamma\left(\frac{1+s}{2}\right)$ both sides

$$(\pi)^{\left(-\frac{s}{2}\right)}\Gamma\left(\frac{s}{2}\right)\zeta(s)\Gamma\left(\frac{1+s}{2}\right) = (\pi)^{-\left(\frac{1-s}{2}\right)}\Gamma\left(\frac{1-s}{2}\right)\zeta(1-s)\Gamma\left(\frac{1+s}{2}\right)$$

From Duplication Formula

$$\Gamma(z)\,\Gamma\left(z+\frac{1}{2}\right) = (2)^{(1-2z)}\sqrt{\pi}\,\Gamma(2z)$$

If $z = \frac{s}{2}$

Then $\Gamma\left(\frac{s}{2}\right)\Gamma\left(\frac{1+s}{2}\right) = (2)^{(1-s)}\sqrt{\pi}\,\Gamma(s)$

Thus $(\pi)^{\left(-\frac{s}{2}\right)}(2)^{(1-s)}\sqrt{\pi}\Gamma(s)\zeta(s) = (\pi)^{-\left(\frac{1-s}{2}\right)}\Gamma\left(\frac{1-s}{2}\right)\zeta(1-s)\Gamma\left(\frac{1+s}{2}\right)$

From Euler's Reflection Formula

$$\Gamma(1-z)\,\Gamma(z) = \frac{\pi}{\sin(\pi z)}$$

Then $(\pi)^{\left(-\frac{s}{2}\right)}(2)^{(1-s)}\sqrt{\pi}\frac{\pi}{\sin(\pi s)\Gamma(1-s)}\zeta(s) = (\pi)^{-\left(\frac{1-s}{2}\right)}\Gamma\left(\frac{1-s}{2}\right)\zeta(1-s)\Gamma\left(\frac{1+s}{2}\right)$

Or $\quad \zeta(s) = \frac{(2)^{(s)}}{(2)}(\pi)^{(s)}\sin(\pi s)\Gamma(1-s)\zeta(1-s)\Gamma(\frac{1-s}{2})\Gamma(\frac{1+s}{2})$

From Euler's Reflection Formula again,

$$\Gamma(1-z)\,\Gamma(z) = \frac{\pi}{\sin(\pi z)}$$

$$Z = \left(\frac{1-s}{2}\right)$$

$$(1-z) = \left(1 - \frac{(1-s)}{2}\right) = \left(\frac{1+s}{2}\right)$$

Then $\quad \Gamma(\frac{1-s}{2})\Gamma(\frac{1+s}{2}) = \frac{\pi}{\sin(\pi\frac{(1-s)}{2})} = \frac{\pi}{\sin(\frac{\pi}{2}-\frac{\pi s}{2})} = \frac{\pi}{\cos(\frac{\pi s}{2})}$

And $\quad \sin(\pi s) = 2\sin(\frac{\pi s}{2})\cos(\frac{\pi s}{2})$

Hence $\quad \zeta(s) = \frac{(2)^{(s)}}{(2)}(\pi)^{(s)}\sin(\pi s)\Gamma(1-s)\zeta(1-s)\Gamma(\frac{1-s}{2})\Gamma(\frac{1+s}{2})$

$$= \frac{(2)^{(s)}}{(2)}(\pi)^{(s)}2\sin(\frac{\pi s}{2})\cos(\frac{\pi s}{2})\Gamma(1-s)\zeta(1-s)\frac{\pi}{\cos(\frac{\pi s}{2})}$$

$$=(2)^{(s)}(\pi)^{(s-1)}\sin(\frac{\pi s}{2})\Gamma(1-s)\zeta(1-s)\text{ , for } s = (1-s) = \frac{1}{2}$$

So, there are no nontrivial zeroes that can be found from this functional equation $\zeta(s)=(2)^{(s)}(\pi)^{(s-1)}\sin\left(\frac{\pi s}{2}\right)\Gamma(1-s)\zeta(1-s)$, because the condition of deriving the equation was $s = (1-s)$ or $(\frac{1}{2}+it) = (1-(\frac{1}{2}+it))$, or $(\frac{1}{2}+it) = (\frac{1}{2}-it)$ which will be true only when $t = 0$, $s = \frac{1}{2}$. In this case $\zeta(s) \neq 0$ but $\zeta(s) = \zeta\left(\frac{1}{2}\right)$ which will diverge $(=+\infty)$.

[Note that $\zeta\left(\frac{1}{2}\right) = \sum_{n=1}^{+\infty}\left(\frac{1}{(n)^{(\frac{1}{2})}}\right)$

$$= 1 + \frac{1}{(2)^{(\frac{1}{2})}} + \frac{1}{(3)^{(\frac{1}{2})}} + \frac{1}{(4)^{(\frac{1}{2})}} + \frac{1}{(5)^{(\frac{1}{2})}} + \ldots$$

$$= 1 + \frac{(2)^{(\frac{1}{2})}}{2} + \frac{(3)^{(\frac{1}{2})}}{3} + \frac{(4)^{(\frac{1}{2})}}{4} + \frac{(5)^{(\frac{1}{2})}}{5} + \ldots$$

But $\quad 1 + \frac{(2)^{(\frac{1}{2})}}{2} + \frac{(3)^{(\frac{1}{2})}}{3} + \frac{(4)^{(\frac{1}{2})}}{4} + \frac{(5)^{(\frac{1}{2})}}{5} + \ldots > 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \ldots$

And because $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\ldots+\frac{1}{+\infty}=+\infty$ (harmonic series)

So $\zeta\left(\frac{1}{2}\right)=1+\frac{(2)^{\left(\frac{1}{2}\right)}}{2}+\frac{(3)^{\left(\frac{1}{2}\right)}}{3}+\frac{(4)^{\left(\frac{1}{2}\right)}}{4}+\frac{(5)^{\left(\frac{1}{2}\right)}}{5}+\ldots=+\infty$]

Summary

I think it is nonsense to go on proving the rest of Riemann's paper. I hope that my paper is clear enough to point out all mistakes and give disproof of the original Riemann Zeta Function and Riemann Hypothesis. I feel good if my paper can give warning to people who are trying to apply the Riemann Hypothesis to explain any phenomena which may be very dangerous in some cases especially in experimental high energy physics.

In my opinion, mathematics is nature. It is everywhere around us, flowers, leaves, water flow, wind blow, sunshine, rain, earth and the whole universe. Humans especially one who called himself mathematician should not play tricks with mathematics just for honour without ashamedness.

References

1. Riemann, Bernhard (1859). "Ueber die Anzahl der Primzahlen unter einer gegebenen Grösse".

2. William F. Trench. " Introduction to Real Analysis". Professor Emeritus. Trinity University. San Antonio, Tx, USA. ISBN 0-13-045786-8.

3. E. Bomberri, "Problems of the millennium: The Riemann Hypothesis," CLAY, (2000).

4. John Derbyshire, Prime Obsession: Bernhard Riemann and The Greatest Unsolved Problem in Mathematics, Joseph Henry Press, 2003, ISBN 9780309085496.