Distinguishing Between the Different Outcomes of Two Different Configurations of an EPRB Experiment.

François Barriquand
Faculté de Philosophie/Séminaire des Carmes, Institut Catholique
21 rue d'ASSAS, 75270 Paris cedex, France.

Abstract.

A variant of the well-known Einstein-Podolski-Rosen experiment is proposed, in which two protagonists may communicate in a way that constitutes a strong challenge for the most commonly accepted interpretation of the EPR experiment.

1. Introduction.

In 1935, Albert Einstein (1879–1955), Boris Podolsky (1896–1966) and Nathan Rosen (1909–1995) proposed a thought-experiment which led them to think that the theory of quantum mechanics is incomplete [1]. The so-called "EPRB experiment" refers to a variant of the original EPR experiment, due to David Bohm (1917–1992) [2], which allows one to test the implications of quantum mechanics on a discrete set of possible outcomes. In 1964, John Stewart Bell (1928–1990) noticed that the understanding of quantum mechanics advocated by many quantum physicists, including particularly Niels Bohr (1885–1962), entails the violation of an inequality which classical mechanics could not have led one to discover [3]. The experimental observation of this violation, first reported in 1972 [cf. 4], has later benefited from significant refinements, which have rendered possible the elimination of many loopholes. Many authors consider that the violation of Bell's inequalities illustrates the non-local character of quantum-mechanics [4]. It is also unanimously held that the consequences of the non-locality of quantum mechanics are too constrained to allow distant observers to communicate at a superluminal speed. The "proof" of this impossibility rests on calculating the statistical consequences of measurements performed by a first observer, traditionally called Alice (A), on the mean values of individual events observed by another individual called Bob (B). In order to calculate these mean values exactly, one needs to take into account the entire set of possible outcomes resulting from A's measurements.

Let us, for instance, suppose that Alice and Bob initially share *N* pairs of entangled spins, distinguished by the means of the index i ($1 \leq i \leq N$). All these pairs, named ψ_i , are supposed to find themselves in a singlet state :

$$\psi_i = \frac{1}{\sqrt{2}}\left(\uparrow_{i/A}\downarrow_{i/B} - \downarrow_{i/A}\uparrow_{i/B}\right) \qquad (1).$$

Since the spatial distance separating A and B can be supposed to be arbitrarily large, the time corresponding to "the" beginning of the experiment cannot be determined in a unique way. For the sake of simplicity, let us define this time from the point of view provided by A's inertial frame. Let us now try to distinguish between two different scenarios, which one may respectively name Sc_1 and Sc_2 :

(a) According to the first scenario Sc_1, Alice measures the orientation of all her spins one by one along the z axis. After this operation, each of A's spins necessarily finds itself either in the state $\uparrow_{/z}$ or in the state $\downarrow_{/z}$.

(b) According to the second scenario Sc_2, Alice measures the orientation of all her spins one by one along the *x* axis.

In all cases, we suppose that Bob later measures the orientation of all his spins along z. The statistics of Bob's measurements can be expected to correspond to what one observes when a coin is tossed N times, heads and tails both having an equal 50% chance of occurring. It is therefore quite straightforward to conclude that the statistics of Bob's results should be exactly the same in both scenarios Sc_1 and Sc_2.
It appears somewhat less trivial, however, to draw a similarly clear-cut conclusion when A and B's measurements are performed along two different axes whose respective orientations can be described by the means of an angle called θ. One might incidentally remember that some experimental observations of the violation of Bell's inequalities crucially depend on the value of a certain geometrical angle [3]. Here also, the fact that Bob cannot distinguish between Sc_1, and Sc_2 does not immediately prove that the same conclusion should hold for any value of θ, even if a well-known elementary calculation, which pays little attention to the noise generated by Alice's measurements, [5] indicates that this is the case. A slightly more involved calculation, presented in section 3 below, may serve to reinforce the idea that this *is* indeed the case. However, "the questions that we can address to nature" [6], as Werner Heisenberg (1901–1976) might have put it, can sometimes be more numerous than what one may imagine at first sight. As will now be shown, a special choice of parameters can allow Bob to distinguish between two scenarios which happen to be quite comparable to, respectively, Sc_1 and Sc_2.

2. Binomial statistics, applied to a special case of weakly probable events.

Just as in the above Eq. 1, let us suppose that Alice and Bob share a total number N of entangled pairs ψ_i ($1 \leq i \leq N$), each pair forming a singlet state. For the sake of simplicity, let us also choose N to be an even number, so that $\frac{N}{2}$ is also an integer. We further suppose that Bob can adjust the parameters of his measuring apparatus in such a way that, when submitted to this apparatus, a wave function corresponding to $\downarrow_{/z}$ (i.e., a spin state 100% oriented negatively along z) would be detected with the probability :

$$p_B = \frac{1}{N} \quad (2).$$

In what follows, we also suppose that Bob must ensure that any wave function corresponding to $\uparrow_{/z}$ would remain undetected by his apparatus. From an experimental point of view, it may seem rather unusual to demand that an experimental set-up should purposely yield a very low rate of positive results. From a theoretical point of view, however, this kind of demand does not raise any particular conceptual difficulty. Bob could for instance let his particles of spin $\downarrow_{/z}$ wander within a region where the presence of spins $\uparrow_{/z}$ would be forbidden, and use a Geiger counter to detect the presence of $\downarrow_{/z}$ states within a small volume of this region.

According to the choices performed by Alice, one may distinguish between two different scenarios *Sc*$_1$' and *Sc*$_2$', which basically correspond to the two scenarios *Sc*$_1$ and *Sc*$_2$ considered in the above introduction :

(a) According to scenario *Sc*$_1$', Alice measures the orientation of all her spins one by one along the z axis. As a result, each of her spins finds itself in the state $\uparrow_{/z}$ or in the state $\downarrow_{/z}$. One may note the number of her $\uparrow_{/z}$ measurements as being equal to $n_A^{+}=\frac{N}{2}+S$, where S is an integer comprised between $-\frac{N}{2}$ and $\frac{N}{2}$. If Bob had decided to measure all his spins along z with 100% accuracy, he would necessarily conclude that $\frac{N}{2}+S$ of his spins find themselves in the $\downarrow_{/z}$ state, whereas $\frac{N}{2}-S$ of them find themselves in the $\uparrow_{/z}$ state. Since Bob only measures $\downarrow_{/z}$ states with a probability given by Eq. 2, Bob's rate of event detection is however much lower.

(b) According to the second scenario *Sc*$_2$', Alice measures the orientation of all her spins one by one along the *x* axis. As a result of this action, each of Bob's spins finds itself either in the $\frac{1}{\sqrt{2}}(\uparrow_{/z}+\downarrow_{/z})$ or the $\frac{1}{\sqrt{2}}(\uparrow_{z}-\downarrow_{/z})$ state. Each of these spins therefore enjoys a probability $\frac{p_B}{2}=\frac{1}{2N}$ of being detected by Bob.

One may now ask the following question : what are the respective probabilities $p_{\mathrm{Sc}_1'}$ and $p_{\mathrm{Sc}_2'}$ for Bob to measure *exactly one spin* (corresponding to a $\uparrow_{/z}$ state) in both scenarios *Sc*$_1$' and *Sc*$_2$' ? The answer to this question can be obtained by distinguishing between two different kinds of Bernouilli trials :

(a) In the case of *Sc*$_1$', the probability $p_{\mathrm{Sc}_1'}$ is equal to the binomial distribution $b\left(1;\frac{N}{2}+S,p_B\right)$, describing the probability of one success among $\frac{N}{2}+S$ trials, when the probability of one detection in one trial is p_B. The value $b\left(1;\frac{N}{2}+S,p_B\right)$ is not difficult to calculate exactly [7]. It is equal to :

$$b\left(1;\frac{N}{2}+S,p_B\right)=\binom{\frac{N}{2}+S}{1}.p_B.(1-p_B)^{\frac{N}{2}+S-1} \qquad (3).$$

The binomial coefficient $\binom{\frac{N}{2}+S}{1}$, which corresponds to $\frac{\left(\frac{N}{2}+S\right)!}{(1)!\left(\frac{N}{2}+S-1\right)!}$, is simply equal to $\frac{N}{2}+S$. The probability $p_{\mathrm{Sc}_1}{}'$, which has already been defined above, can therefore also be written :

$$p_{\mathrm{Sc}_1}{}' =\left(\frac{N}{2}+S\right).p_B.(1-p_B)^{\frac{N}{2}+S-1} \qquad (4).$$

(b) In the case of scenario *Sc*₂', the probability $p_{\mathrm{Sc}_2}{}'$ is equal to the binomial distribution $b\left(1;N,\frac{p_B}{2}\right)$, corresponding to one success among *N* trials, when the probability of one detection in one trial is $\frac{p_B}{2}$. The value $b\left(1;N,\frac{p_B}{2}\right)$ can be computed exactly in a similar way as above, so that $p_{\mathrm{Sc}_2}{}'$ can be written :

$$p_{\mathrm{Sc}_2}{}' = b\left(1;N,\frac{p_B}{2}\right)=N.\frac{p_B}{2}.(1-\frac{p_B}{2})^{N-1} \qquad (5).$$

Let us now take it as our task to compare attentively $p_{\mathrm{Sc}_1}{}'$ and $p_{\mathrm{Sc}_2}{}'$, using the fact that $p_B=\frac{1}{N}$. If N is sufficiently large, the order of magnitude of the absolute value $|S|$ can be expected to be comparable to $\sqrt{N}$, as implied by the central limit theorem [7]. Using the well-known Mercator series, valid for $-1<x\le 1$:

$$Log(1+x)=\sum_{k=1}^{\infty}\frac{(-1)^{k-1}}{k}x^k=x-\frac{x^2}{2}+\frac{x^3}{3}\ldots \qquad (6),$$

we may calculate the natural logarithms of $p_{\mathrm{Sc}_1}{}'$ and $p_{\mathrm{Sc}_2}{}'$, which will eventually allow us to obtain two different developments for $p_{\mathrm{Sc}_1}{}'$ and $p_{\mathrm{Sc}_2}{}'$. In the case of $p_{\mathrm{Sc}_1}{}'$, since :

$$p_{\mathrm{Sc}_1}{}' =\left(\frac{N}{2}+S\right).\frac{1}{N}.(1-\frac{1}{N})^{\frac{N}{2}+S-1}=\frac{1}{2}\left(1+\frac{2S}{N}\right).(1-\frac{1}{N})^{\frac{N}{2}+S-1} \qquad (7),$$

one obtains :

$$Log\,p_{\mathrm{Sc}_1}{}' =Log\left(\frac{1}{2}\right)+Log\left(1+\frac{2S}{N}\right)+\left(\frac{N}{2}+S-1\right).Log(1-\frac{1}{N}) \qquad (8).$$

Using Eq. 6, Eq. 8 becomes :

$$Log\, p_{\mathrm{Sc}_1{}'} = Log\left(\frac{1}{2}\right) + \frac{2S}{N} - \frac{2S^2}{N^2} + \frac{8S^3}{3N^3} - \frac{4S^4}{N^4} \dots + \left(\frac{N}{2} + S - 1\right).\left(-\frac{1}{N} - \frac{1}{2N^2} - \frac{1}{3N^3} + \cdots\right) \qquad (9).$$

Keeping only those terms whose order of magnitude is at least comparable with $\frac{1}{N^2}$, one obtains :

$$Log\, p_{\mathrm{Sc}_1{}'} = Log\left(\frac{1}{2}\right) + \frac{2S}{N} - \frac{2S^2}{N^2} + \frac{8S^3}{3N^3} - \frac{4S^4}{N^4} + \left(-\frac{1}{2} - \frac{1}{4N} - \frac{1}{6N^2} - \frac{S}{N} - \frac{S}{2N^2} + \frac{1}{N} + \frac{1}{2N^2}\right) + \cdots \quad (10),$$

which, ordaining terms according to their decreasing order of magnitude, leads to :

$$Log\, p_{\mathrm{Sc}_1{}'} = \left(Log\left(\frac{1}{2}\right) - \frac{1}{2}\right) + \left(\frac{S}{N}\right) + \left(-\frac{2S^2}{N^2} + \frac{3}{4N}\right) + \left(\frac{8S^3}{3N^3} - \frac{S}{2N^2}\right) + \left(-\frac{4S^4}{N^4} + \frac{1}{3N^2}\right) + \cdots \qquad (11).$$

Using the well-known development of the natural exponential :

$$e^x = 1 + x + \frac{x^2}{2} + \cdots \qquad (12),$$

one then obtains, to order $\frac{1}{N^2}$:

$$p_{\mathrm{Sc}_2{}'} p_{\mathrm{Sc}_1{}'} = \frac{1}{2\sqrt{e}}.\{1 + \frac{S}{N} + \left(-\frac{2S^2}{N^2} + \frac{3}{4N}\right) + \left(\frac{8S^3}{3N^3} - \frac{S}{2N^2}\right) + \left(-\frac{4S^4}{N^4} + \frac{1}{3N^2}\right) + \\ \frac{1}{2}\left(\frac{S}{N} - \frac{2S^2}{N^2} + \frac{3}{4N} + \frac{8S^3}{3N^3} - \frac{S}{2N^2}\right)^2 + \cdots\} \qquad (13).$$

The last parenthesis present in Eq. 13 may be rewritten, to order $\frac{1}{N^2}$:

$$\begin{aligned}\frac{1}{2}\left(\frac{S}{N} - \frac{2S^2}{N^2} + \frac{3}{4N} + \frac{8S^3}{3N^3} - \frac{S}{2N^2}\right)^2 &= \frac{S^2}{2N^2} + \frac{S}{N}\left(-\frac{2S^2}{N^2} + \frac{3}{4N} + \frac{8S^3}{3N^3} - \frac{S}{2N^2}\right) + \frac{1}{2}\left(-\frac{2S^2}{N^2} + \frac{3}{4N} + \cdots\right)^2 \\ &= \frac{S^2}{2N^2} - \frac{2S^3}{N^3} + \frac{3S}{4N^2} + \frac{8S^4}{3N^4} - \frac{S^2}{2N^3} + \frac{2S^4}{N^4} - \frac{3S^2}{2N^3} + \frac{9}{32N^2} + \cdots \\ &= \frac{S^2}{2N^2} + \left(-\frac{2S^3}{N^3} + \frac{3S}{4N^2}\right) + \left(\frac{14S^4}{3N^4} - \frac{2S^2}{N^3} + \frac{9}{32\ {}^2}\right) + \cdots \qquad (14),\end{aligned}$$

so that, ordaining terms according to their decreasing order of magnitude,

$$p_{\mathrm{Sc}_1{}'} = \frac{1}{2\sqrt{e}}.\{1 + \frac{S}{N} + \left(-\frac{3S^2}{2N^2} + \frac{3}{4N}\right) + \left(\frac{2S^3}{3N^3} + \frac{S}{4N^2}\right) + \left(\frac{2S^4}{3N^4} - \frac{2S^2}{N^3} + \frac{59}{96N^2}\right) + \cdots \} \qquad (15).$$

The mean value of $p_{\mathrm{Sc}_1{}'}$, which may be noted $< p_{\mathrm{Sc}_1{}'} >$, can be obtained by calculating the average of Bob's results over a large number of series of exactly N measurements. Such a large number, quite independent from *N*, may be called *N'*. The contribution of odd powers of *S* terms, such as $\frac{1}{2\sqrt{e}}\frac{S}{N}$ or $\frac{1}{2\sqrt{e}}\left(\frac{2S^3}{3N^3}+\frac{S}{4N^2}\right)$, to $< p_{\mathrm{Sc}_1'} >$, may therefore be described by a kind of random walk comprising *N'* steps, whose evolution can be represented by the means of the sum $X_1 + \cdots + X_{\mathrm{N'}}$, where all X_is ($1 \leq i \leq N'$) are independent variables whose mean value is zero, so that $< (X_1 + \cdots + X_{\mathrm{N'}})^2 >$ is only equal to $N'.< {X_1}^2 >$. In other words, the addition of all the contributions of the *odd powers* of *S* terms present in Eq. 15 to Bob's statistical estimation of the mean value of $p_{\mathrm{Sc}_1'}$, obtained after *N'* series of *N* measurements, can be expected to grow in proportion with $\sqrt{N'}$. In contrast, even the smallest *even power* of *S* term present in Eq. 13 can be expected to contribute to the sum of Bob's measurements in proportion with *N'*. Since *N'* may be chosen to be much larger than *N*, the contribution of all the *odd powers* of *S* terms may ultimately be neglected in the calculation of $< p_{\mathrm{Sc}_1'} >$. The condition for neglecting the first of these terms in Eq. 15, which is of order $\frac{S}{N} \sim \frac{1}{\sqrt{N}}$, in front of the term $\left(\frac{2S^4}{3N^4} - \frac{2S^2}{N^3} + \frac{59}{96N^2}\right)$, which is of order $\frac{1}{N^2}$, is simply $\frac{\sqrt{N'}}{\sqrt{N}} \ll \frac{N'}{N^2}$, which is equivalent to :

$$N' \gg N^3 \qquad (16).$$

Our next step should consist in calculating the mean values $< S^2 >$ and $< S^4 >$. Alice's measurement of $n_A^+ = \frac{N}{2} + S$ spins may also be described as a kind of random walk, for which one may write :

$$n_A^+ - n_A^- = \frac{N}{2} + S - \left(\frac{N}{2} - S\right) = 2S = X_1 + X_2 \ldots + X_N \qquad (17),$$

where all X_js ($1 \leq j \leq N$) are independent variables taking either the value $X_j = +1$ or $X_j = -1$, with a probability equal to $\frac{1}{2}$ for each alternative, as indicated by the application of Born's rules to Alice's measurements. Since :

$$(X_1 + \cdots + X_N)^2 = \sum_{j=1}^{N} {X_j}^2 + \sum_{j,k/_{j\neq k}} X_j X_k \qquad (18),$$

and since X_j and X_k, which verify $< X_j > = < X_k > = 0$, are independent when $i \neq j$, the mean value $< \sum_{j,k/_{j\neq k}} X_j X_k >$ is also equal to zero. Since, furthermore, ${X_j}^2 = 1$ for all values of the index *j* ($1 \leq j \leq N$), Eq. 18 ensures us that :

$$< (X_1 + \cdots + X_N)^2 > = N \qquad (19).$$

In a similar way, we may also calculate :

$$< (X_1 + \cdots + X_N)^4 > = < \left(\sum_{j=1}^{N} X_j{}^2 + \sum_{j,k/_{j\neq k}} X_j X_k \right) . \left(\sum_{j'=1}^{N} X_{j'}{}^2 + \sum_{j',k'/_{j'\neq k'}} X_{j'} X_{k'} \right) >$$
$$= < N^2 + 2 \sum_{j,k/_{j\neq k}} X_j{}^2 X_k{}^2 >$$
$$= N^2 + 2N(N-1) = 3N^2 - 2N \qquad (20).$$

Since $2S = X_1 + X_2 \ldots + X_N$ (cf. Eq. 17), one deduces from equations 19 and 20 :

$$< S^2 > = \frac{N}{4} \qquad (21).$$

$$< S^4 > = \frac{3N^2}{16} - \frac{N}{8} \qquad (22).$$

Inserting these values of $< S^2 >$ and $< S^4 >$ into Eq. 15, one obtains for $< p_{\mathrm{Sc}_1{}'} >$ the following equation, valid to order $\frac{1}{N^2}$:

$$< p_{\mathrm{Sc}_1{}'} > \sim \frac{1}{2\sqrt{e}} . \{1 + \left(-\frac{3S^2}{2N^2} + \frac{3}{4N} \right) + \left(\frac{2S^4}{3N^4} - \frac{2S^2}{N^3} + \frac{59}{96\ ^2} \right) + \cdots\}$$

$$\sim \frac{1}{2\sqrt{e}} . \{1 + \frac{3}{8N} + \frac{23}{96N^2} \ldots\} \qquad (23).$$

Let us now turn towards the calculation of $p_{\mathrm{Sc}_2{}'}$. Inserting Eq. 2 in Eq. 5, one obtains :

$$p_{\mathrm{Sc}_2{}'} = N . \frac{1}{2N} . (1 - \frac{1}{2N})^{N-1} \qquad (24).$$

The natural logarithm of $p_{\mathrm{Sc}_2{}'}$ is therefore :

$$Log\, p_{\mathrm{Sc}_2{}'} = Log\left(\frac{1}{2}\right) + (N-1) Log(1 - \frac{1}{2N}) \qquad (25).$$

Using again Mercator's formula (Eq. 6), one obtains :

$$Log\, p_{\mathrm{Sc}_2{}'} = Log\left(\frac{1}{2}\right) + (N-1)(-\frac{1}{2N} - \frac{1}{8N^2} - \frac{1}{24\ ^3} \ldots) \qquad (26).$$

Keeping only the lowest order terms of this series, this leads to :

$$Log\, p_{\mathrm{Sc}_2{}'} \approx Log\left(\frac{1}{2}\right) - \frac{1}{2} + \frac{3}{8N} + \frac{1}{12N^2} + \cdots \qquad (27).$$

Using Eq. 12, this last equation implies that :

$$p_{\mathrm{Sc}_2{}'} = \frac{1}{2\sqrt{e}}.\left\{1+\frac{3}{8N}+\frac{1}{12N^2}+\frac{9}{128N^2}\dots\right\} = \frac{1}{2\sqrt{e}}.\left\{1+\frac{3}{8N}+\frac{59}{384N^2}\dots\right\} \qquad (28).$$

Since, in the case of Eq. 28, no difference can be made between $p_{\mathrm{Sc}_2{}'}$ and $< p_{\mathrm{Sc}_2{}'} >$, the contents of Eq. 23 and Eq. 28 may be more conveniently compared by writing :

$$< p_{\mathrm{Sc}_1{}'} > \sim \frac{1}{2\sqrt{e}}.\left\{1+\frac{3}{8N}+\frac{23}{96N^2}+\cdots\right\} \qquad (29),$$

and

$$< p_{\mathrm{Sc}_2{}'} > = \frac{1}{2\sqrt{e}}.\left\{1+\frac{3}{8N}+\frac{59}{384N^2}+\cdots\right\} \qquad (30).$$

The appearance of the factor $e^{-1/2}$ in both Eq. 29 and Eq. 30 is reminiscent of the Poisson's distribution. In order to illustrate the order of magnitude of the discrepancy that distinguishes $< p_{\mathrm{Sc}_1{}'} >$ from $< p_{\mathrm{Sc}_2{}'} >$, let us choose, for instance, $N \sim 100$. In order to allow the difference between equations 29 and 30 to start to exert an influence on the measurements of $< p_{\mathrm{Sc}_1{}'} >$ and $< p_{\mathrm{Sc}_2{}'} >$, *N'* should at least amount, in a rough approximation, to :

$$N' \sim \frac{1}{|< p_{\mathrm{Sc}_1{}'} > - < p_{\mathrm{Sc}_2{}'} >|} = \frac{2\sqrt{e}}{\left|\frac{23}{96\;{}^{2}} - \frac{59}{384N^2}\right|} \qquad (31).$$

In the same time, neglecting the influence of the $\frac{S}{N}$ term of Eq. 15 in Eq. 29 also requires one to ensure that Eq. 16 is verified, which imposes an even stronger condition on *N'* than Eq. 31: if *N*=100, Eq. 16 imposes the condition $N' \gg 10^6$, which may start to be verified in a reliable way for, let us say, $N' \sim 10^8$. The total number *N***N'* of entangled pairs needed by Bob to start distinguishing between Alice's choice of Sc_1' or Sc_2' should therefore approximately amount to 10^{10}. If Bob wanted to obtain a more reliable result, defined by a degree of confidence of several *sigmas*, an even much larger number of entangled states would be required. This number could fortunately be significantly reduced if Bob, being a good mathematician, were able to add up the information derived simultaneously from different values of *N*. For instance, Bob's measuring $N' = 10^8$ series of *N*=100 spins is experimentally equivalent to his measuring *N'*=101010101 series of *N*=99 spins, provided that Bob can be willing to ignore one among his 10^8 measurements, since 101010101*99=(10^{10}- 1). Bob may also decompose 10^{10} of his measurements into 1020408 series of 98 measurements, while ignoring 32 of his measurements, since 102040816*98=(10^{10}- 32), *etc*. Bob may furthermore benefit from the statistical information contained in his detections of *exactly 2* spins for *N* particles, as well as *exactly 3* spins, *etc*. In the end, Bob could dispose of a wealth of meaningful information that would largely exceed what a succinct comparison between Eq. 29 and Eq. 30 may suggest.

Let us also signal the fact that the value chosen for p_B in Eq. 2 (which is : $p_B = \frac{1}{N}$), whose principal mathematical merit consists in being easy to manipulate by hand, does not correspond to the value that would best optimize the difference $|< p_{Sc_1'} > - < p_{Sc_2'} >|$. By studying the more general situation defined by :

$$p_B = \frac{2x}{N} \qquad (32),$$

where x is an arbitrary real number (verifying $x \ll N$), one may follow exactly the same steps as those reported above, from Eq. 3 to Eq. 30, in order to obtain, in the end, the following developments, valid to order $\frac{1}{N^2}$:

$$< p_{Sc_1'} > = xe^{-x}.\left\{1 + \left(x - \frac{x^2}{2}\right)\frac{1}{N} + \left(\frac{5}{8} - 2x + \frac{7}{2}x^2 - \frac{7}{3}x^3 + \frac{1}{2}x^4\right)\frac{1}{N^2} + \cdots\right\} \quad (33),$$

and

$$< p_{Sc_2'} > = xe^{-x}.\left\{1 + \left(x - \frac{x^2}{2}\right)\frac{1}{N} + \left(x^2 - \frac{5}{6}x^3 + \frac{1}{8}x^4\right)\frac{1}{N^2} + \cdots\right\} \quad (34).$$

According to equations 33 and 34, the difference $|< p_{Sc_1'} > - < p_{Sc_2'} >|$ happens to be zero for x=0 and x=1 only. The question of its maximization, as a function of x, lies outside of the scope of the present study. Another interesting question may be related to the experimental consequences of the presence of the term proportional to $\frac{S}{N}$ in Eq. 15. Such a term occurs nowhere in the calculation of $p_{Sc_2'}$. It only affects the measurement of $p_{Sc_1'}$, in scenario *Sc*1'. The strategy followed in the present article has consisted in focusing on the analysis of the mean values $< p_{Sc_1'} >$ and $< p_{Sc_2'} >$, which both remain unaffected by any contribution attributable to $\frac{S}{N}$. It might be interesting, however, to ask whether the impact of the "noise" associated with $\frac{S}{N}$ could also be taken into account by Bob in order to improve the efficiency of his discrimination between *Sc*1' and *Sc*2'. This question will be left for further studies.

Although, as just indicated, Eq. 16 provides a very conservative estimate of the number of measurements that might be necessary for enabling Bob to distinguish between *Sc*1' and *Sc*2', one may also note, on a more pessimistic tone, that since all the information gathered by Bob can only be obtained by the means of a single large set of data, one single experimental error among *N*N'* measurements may potentially jeopardize the entire scheme upon which Bob is supposed to base his analysis. If Bob wishes to exclude the slightest possibility of experimental error in his measurements, even an accurate result obtained with 10^3 spins may already seem extremely challenging. The small difference that separates $< p_{Sc_1'} >$ and $< p_{Sc_2'} >$ in Eq. 33 and Eq. 34 may therefore encourage us to come back to the original question raised in section 1 above, in order to verify whether, by any chance, the difference of angles chosen by A and B in Sc_1 and Sc_2 may enable A and B to communicate in a significantly more efficient way than what Bob's distinction between Eq. 33 and Eq. 34 already seems to allow.

3. First order calculation of the impact exerted by the angular dependence of A's measurements on the statistics of B's measurements.

The calculation contained in the present section 3 contains a negative result that may be safely ignored by most readers. Such a calculation may nevertheless serve as a time-saving reference for statisticians who might wish to venture beyond the level of approximation that will be used here below.

Let us again suppose, as indicated in Eq. 1, that two observers, named Alice and Bob, share *N* pairs of entangled spins $1/2$, each pair initially forming a singlet state. Let us again also suppose that Bob will measure all his spins along the direction z. We further suppose that if Bob disposed of one single spin $\downarrow_{/z}$, he would obtain a positive result indicating him that his spin is indeed in the state $\downarrow_{/z}$ with a probability p_B, with $0 \leq p_B \leq 1$. For the sake of simplicity, one could have supposed that Bob's capacity of detection is perfect, so that $p_B = 1$, but it may seem more instructive, in the light of the results obtained in section 2 above, to take into account a slightly more general picture.

According to the experimental protocol that has already been briefly mentioned in the introduction here above, Alice is supposed to measure her spins along an axis differing from z by a constant angle θ, whose value she is free to choose (Alice may also change the value of θ after *N* measurements). In contrast with Bob's measurements, the efficiency of Alice's measurements is supposed to be perfect. In reference to her own axis of measurement, the number of her "spin up" measurements may be noted :

$$n_A^+ = \frac{N}{2} + S \qquad (35),$$

where the integer *S*, with $-\frac{N}{2} \leq S \leq \frac{N}{2}$, plays the same role as the integer *S* present in Eq. 3. The number of "spin down" states measured by Alice is therefore :

$$n_A^- = \frac{N}{2} - S \qquad (36),$$

Alice's n_A^+ "spin up" states lead B to measure a number of $\downarrow_{/z}$ spins equal to :

$$n_{B_{/n_A^+}}^- = \left(\frac{N}{2} + S\right) p_B cos^2\frac{\theta}{2} + S' \qquad (37),$$

where *S*' is an integer comprised between $-\left(\frac{N}{2}+S\right)p_B cos^2\frac{\theta}{2}$ and $-\left(\frac{N}{2}+S\right)p_B cos^2\frac{\theta}{2}+\frac{N}{2}+S$, which corresponds to the statistical quantum "noise" of Bob's measurements of $n^-_{B_{/n^+_A}}$.

Alice's n^-_A "spin down" states also lead B to measure a supplementary number of $\downarrow_{/z}$ spins, which is equal to :

$$n^-_{B_{/n^-_A}} = \left(\frac{N}{2}-S\right)p_B sin^2\frac{\theta}{2}+S'' \qquad (38),$$

where *S*″ is an integer comprised between $-\left(\frac{N}{2}-S\right)p_B sin^2\frac{\theta}{2}$ and $-\left(\frac{N}{2}-S\right)p_B sin^2\frac{\theta}{2}+\frac{N}{2}-S$, which corresponds to the statistical quantum "noise" of Bob's measurements of $n^-_{B_{/n^-_A}}$. The total number of $\downarrow_{/z}$ spins measured by Bob is therefore :

$$n^-_B = \frac{Np_B}{2} + Sp_B cos\theta + S' + S'' \qquad (39).$$

Let us define $P_{S,S',S''}$ as being the probability that Alice and Bob's measurements can be described by the triplet (*S*, *S*', *S*″). What Bob can measure, independently of Alice, corresponds to n^-_B. In other words, if the same experiment, performed on *N* pairs of spins, is repeated a sufficient number of times, Bob can determine the probability for the occurrence of any value of n^-_B, which may be noted $P(n^-_B)$. This probability corresponds to the sum of all $P_{S,S',S''}$ for which Bob obtains the same result n^-_B. It can therefore be written :

$$P(n^-_B) = \sum_{S,S',S''\big/\frac{Np_B}{2}+Sp_B cos\theta+\ '+S''=n^-_B} P_{S,S',S''} \qquad (40)$$

Since Eq. 39 may also be rewritten as :

$$S' = n^-_B - \frac{Np_B}{2} - Sp_B cos\theta - S'' \qquad (41),$$

and since one may define a certain constant quantity (*i.e.* independent of θ), designed by *C*, as being equal to :

$$C = n^-_B - \frac{Np_B}{2} \qquad (42),$$

one may write, instead of Eq. 40 :

$$P(n^-_B) = \sum_{S,S''\big/S'=C-Sp_B cos\theta-S''} P_{S,S',S''} \qquad (43).$$

In virtue of the central limit theorem [7], one may estimate that, in the limit of large *N*, the probability $P_{S,S',S''}$ is provided by the product of three Gaussians, each respectively corresponding to the probability of occurrence of *S*, *S*', and *S''*. Before writing down the expression of this product, one may introduce the following simplifying notations, where the letters f_c and f_s may be understood to mean, respectively, "cosine factor" and "sine factor" :

$$f_c = p_B cos^2\frac{\theta}{2}(1 - p_B cos^2\frac{\theta}{2}) \quad (44)$$

$$f_s = p_B sin^2\frac{\theta}{2}(1 - p_B sin^2\frac{\theta}{2}) \quad (45).$$

Using these notations, one may write :

$$P_{S,S',S''} \sim \frac{1}{\sqrt{\frac{\pi N}{2}}} e^{-\frac{2S^2}{N}} . \frac{1}{\sqrt{2\pi(\frac{N}{2}+S)f_c}} e^{-\frac{S'^2}{2(\frac{N}{2}+S)f_c}} . \frac{1}{\sqrt{2\pi(\frac{N}{2}-S)f_s}} e^{-\frac{S''^2}{2(\frac{N}{2}-S)f_s}} \quad (46),$$

so that :

$$P(n_B^-) \sim \int_{-\propto}^{+\propto} dS \int_{-\propto}^{+\propto} dS'' \frac{1}{\sqrt{\frac{\pi N}{2}}} e^{-\frac{2S^2}{N}} . \frac{1}{\sqrt{2\pi(\frac{N}{2}+S)f_c}} e^{-\frac{(C - S p_B cos\theta - S'')^2}{2(\frac{N}{2}+S)f_c}} . \frac{1}{\sqrt{2\pi(\frac{N}{2}-S)f_s}} e^{-\frac{S''^2}{2(\frac{N}{2}-S)f_s}} \quad (47).$$

Adventurous mathematicians might wish to obtain from Eq. 47 an approximation for $P(n_B^-)$ valid to increasing orders of $\frac{1}{N}$, in analogy with what we have written in Eq. 33 and Eq. 34. This could indeed be done by noting that :

$$\frac{1}{\sqrt{\left(\frac{N}{2}+S\right)}\sqrt{\left(\frac{N}{2}-S\right)}} = \frac{2}{N} + \frac{4S^2}{N^3} + \cdots \quad (48);$$

$$\frac{1}{\frac{N}{2}+S} = \frac{2}{N} - \frac{4S}{N^2} + \cdots \quad (49);$$

$$\frac{1}{\frac{N}{2}-S} = \frac{2}{N} + \frac{4S}{N^2} + \cdots \quad (50).$$

After having inserted equations 48, 49 and 50 in Eq. 47, a fastidious calculation would need to be done in order to inquire whether $P(n_B^-)$ can remain independent of θ at any order of $\frac{1}{N}$. However, even if the result of this calculation indicated that the approximation of $P(n_B^-)$ provided by Eq. 47 does indeed depend on θ, such a result would not suffice to prove that Bob enjoys the capacity to detect any θ variation in his results, since the Gaussian formula introduced in Eq. 46 only corresponds to an approximation valid for large N, whose speed of convergence is not taken into account by Eq. 46 itself. The only physical information that one may immediately extract with confidence from Eq. 47 can be obtained after three further simplifications, which consist in approximating $\sqrt{\left(\frac{N}{2}+S\right)}\sqrt{\left(\frac{N}{2}-S\right)}$, as well as $\left(\frac{N}{2}+S\right)$ and $\left(\frac{N}{2}-S\right)$, by $\frac{N}{2}$. These three simplifications lead us to the following equation :

$$P(n_B^-) \sim \int_{-\propto}^{+\propto} dS \int_{-\propto}^{+\propto} dS'' \frac{1}{\sqrt{\frac{\pi N}{2}}} e^{-\frac{2S^2}{N}} . \frac{1}{\sqrt{\pi N}\ c} e^{-\frac{(C - S p_B cos\theta - S'')^2}{Nfc}} . \frac{1}{\sqrt{\pi N f_s}} e^{-\frac{S''^2}{N f_s}} \qquad (51)$$

Using (twice) the well-known formula :

$$\int_{-\propto}^{+\propto} e^{-(ax^2+b\ )} = \sqrt{\frac{\pi}{a}} e^{\frac{b^2}{a}} \qquad (52),$$

it then becomes a matter of straightforward, albeit tedious exercise to verify that :

$$P(n_B^-) \sim \frac{1}{\sqrt{2\pi N \frac{p_B}{2}(1-\frac{p_B}{2})}} e^{-\frac{(n_B^- - \frac{N p_B}{2})^2}{2N\frac{p_B}{2}(1-\frac{p_B}{2})}} \qquad (53).$$

Eq. 53 does not depend on θ in any way. It also corresponds to the statistics that Bob could be expected to verify in case Alice did not perform any measurement at all.

The main lesson that can be derived from Eq. 53 is that there does not seem to exist any obvious way that could allow Bob to distinguish between Alice's choices in a much more efficient way than Eq. 33 and Eq. 34 allow him to do in the case of Sc_1' and Sc_2'. Even if Eq. 51 seems, at first sight, more sophisticated than the series of equations that have enabled us to obtain Eq. 33 and Eq. 34, the physical relevance of Eq. 51 appears, in fact, much weaker. *A posteriori*, this conclusion may not seem too surprising, since the Gaussian approximation introduced in Eq. 46 is known to perform poorly in the case of low rates of detection, which can be more satisfactorily described by a Poissonian approximation. What is more, as has already been noted, Eq. 46 does not directly instruct us about the speed according to which an increasing number of measurements may lead Bob to obtain an accurate knowledge of the convergence of his measurements. One may also note that Eq. 53 does not directly provide us with the kind of "2-level" information considered in section 2, obtained after taking into account *N'* series of *N* measurements.

3. Physical analysis.

In a French textbook devoted to the foundations of quantum mechanics, Franck Laloë has written that "although the notion of *events*, viewed as isolated processes in space-time, or the notion of *causality*, remain very fundamental in relativity, Bell's theorem indicates that they are not as universal as one might have presumed. Quantum mechanics compel us to adopt these notions with a slight *grain of salt*." [8] Although a large number of physicists would presumably agree with Franck Laloë's assessment, it has also often been considered that this kind of philosophical considerations, interesting as they may seem from an epistemological point of view, is deprived of any useful observable consequence.

Now, the situation seems to have changed. An interesting task might consist in examining the implications of the above Eq. 33 and Eq. 34 from the points of views of different approaches of quantum mechanics, such as the so-called "standard interpretation" (often called "Copenhagen's interpretation", in reference to the Danish physicist Niels Bohr), or the so-called "pilot's wave interpretation" originally studied by Louis de Broglie (1892–1987), or "decoherence scenarios", or Hugh Everett (1930–1982)'s so-called "many-worlds" interpretation, *etc*. Since such a task could easily require the redaction of an entire book, I shall limit myself, in a first step, to distinguish briefly between a non-exhaustive number of highly different conjectures.

3.1 Conjecture n°1.

One might reasonably wish to ask whether the reasonings that have been followed in order to show that scenarios Sc_1' and Sc_2' can lead Bob to measure two slightly different values for $< p_{\mathrm{Sc}_1{}'} >$ and $< p_{\mathrm{Sc}_2{}'} >$, as shown in Eq. 33 and Eq. 34, contain an error. If this hypothesis turned out to be correct, I would wish to be able to present my excuses to all the potential readers I might have misled [9]. On the other hand, since I have been unable to find any incoherence in the above reasonings, I feel myself compelled to question either : (a) the validity of the postulate according to which superluminal communication between two distant observers is impossible (cf. conjectures 2 and 3 below), or (b) the validity of the interpretation of quantum mechanics which is usually presented as most "standard" in contemporary textbooks (cf. conjectures 4 and 5 below), or perhaps even both postulates (a)+(b), since it seems difficult to abandon the first of these two without also abandoning the second.

3.2 Conjecture n°2.

In view of the fact that the violation of Bell's inequalities has already been experimentally observed many times, one may boldly suppose that equations 33 and 34 indicate that superluminal communication between two observers is, eventually, allowed by quantum mechanics. Such an hypothesis would immediately raise serious theoretical difficulties, however, since it might inevitably lead one to suppose that two observers A and B could find themselves in a situation wherein B would be able to communicate to A the results of A's own measurements before A would have started to perform them. Rejecting this possibility on purely logical grounds might be premature. But accepting it might dangerously weaken the notion of *randomness*, as it is now used in quantum mechanics – among other things. In order to escape from this difficult situation, one might imagine that some patterns of quantum measurements, hitherto considered as possible, should eventually be considered as forbidden, on purely logical grounds, so as to preserve the rational consistency of our observations of the natural world.

3.3 Conjecture n°3.

In order to avoid the kind of logical difficulties mentioned in the preceding conjecture, one might prefer to suppose that one single privileged space-time reference frame allows "instantaneous" communication between two observers, whereas other reference frames forbid the same phenomenon to happen. Supposing, as many cosmologists are inclined to do, that the expansion of the universe is partially governed by a so-called "dark energy" term that may vary in "time", or that has varied in "time" during a previous stage of the history of our universe, the existence of a privileged reference frame enabling one to describe the "temporal" evolution of this dark energy term becomes a rather natural hypothesis. As far as the interpretation of the discrepancy between Eq. 33 and Eq. 34 is concerned, however, the existence of one privileged frame within which "instantaneous" communication between distant observers would be possible should also be accompanied by the fact that within other frames, especially fast receding ones, "immediate communication" would not be allowed. This, in turn, should forbid one to apply the standard rules of quantum mechanics to the description of phenomena studied by observers remaining immobile in these fast receding frames. In a way, therefore, the present conjecture n°3 would also lead one to abandon the "standard" rules of quantum mechanics as we know them today.

3.4 Conjecture n°4.

Perhaps the easiest way to rid oneself from the possibility of superluminal communication might consist in adding to the "standard" description of quantum mechanics a supplementary mechanism of decorrelation, hitherto unknown. Since such a mechanism has never been observed, at least as of today, in any of the variants of the EPR experiment that have been put to the test during the last decades, the plausibility of the present conjecture n°4 may seem rather low. On the other hand, the high degree of experimental precision needed to check the validity of both Eq. 33 and Eq. 34 does not allow one to exclude it completely. It the existence of a decorrelation mechanism capable of suppressing all observable consequences of the discrepancy that distinguishes Eq. 33 from Eq. 34 were proven, physicists should naturally attempt to investigate its spatio-temporal/dynamical properties, which seem very difficult to conjecture.

3.5 Conjecture n°5.

Instead of imagining the existence of a kind of "decorrelation" mechanism capable of suppressing the effects of the discrepancy that distinguishes Eq. 33 from Eq. 34, one may suppose, on the contrary, that a supplementary "correlation" mechanism could forbid Bob to distinguish between the above scenarios Sc_1' and Sc_2'. The description of such a "correlation" might perhaps require one to take into account a rather sophisticated set of "global" parameters, which might share some common features with the famously hypothesized "hidden variables" that have sometimes been thought to govern the evolution of quantum mechanics.

4. Conclusion.

Werner Heisenberg, in the philosophical essay already quoted in the introduction of the present article, asserted that, from the point of view of a 20th century observer of nature, "the predictable course of phenomena in space and time is no longer the firm skeleton of the world but only one nexus among others that becomes separated from the web of relations that we call the world by the way we examine it, by the questions we address to nature" [6]. The successes encountered by quantum mechanics during more than a century have been so impressive that the kind of philosophical reflection developed by W. Heisenberg in his essay, written between 1942 and 1943, has been largely forgotten. The present article may possibly contribute to vindicate W. Heisenberg's rather bold epistemological point of view in a stronger way than W. Heisenberg had himself anticipated. The analysis of the variant of the EPRB experiment considered in section 2 above suggests that, if the "standard" rules of quantum mechanics are flawless, superluminal communication between two observers might be possible. The experimental consequences of this conclusion are, however, difficult to anticipate with certainty. The logical "skeleton" upon which the frame of quantum mechanics has been built appears more puzzling than ever. Precise experimental studies will be needed in order to discriminate between widely diverging conjectures, suggested by the present theoretical situation.

Notes and References

[1] Einstein A., Podolsky B. and Rosen N., *Can quantum-mechanical description of physical reality be considered complete ?* Physical Review, **47**, p. 777–780 (1935).

[2] Bohm D., *Quantum Theory*. Prentice Hall, Englewoodcliffs (1951).

[3] Bell J.-S., *On the Einstein Podolsky Rosen Paradox*. Physics, **1**, p. 195–200 (1964).

[4] Laloë F., *Comprenons-nous vraiment la mécanique quantique ?* 2^e édition révisée et augmentée, CNRS éditions, Paris, 2017. Cf. particularly the list of historical experiments reported in § **IV.A.4** and Laloë's own theoretical analysis, developed in § **IV.C.2** : *La mécanique quantique est-elle non locale ? Contrafactualité*.

[5] Laloë F., *ibid*. Appendice **F** : *Impossibilité d'une transmission superluminale de messages*, p. 481–485.

[6] Heisenberg W., *Reality and its Order*. Edited by Konrad Kleinknecht, Springer, 2019, p. 23. The exact German phrase written by W. Heisenberg was "Für uns ist der gesetzmäßige Ablauf im Raum und Zeit nicht mehr das feste Skelett der Welt, sondern eher nur ein Zusammenhang unter anderen, der durch die Art, wie wir ihn untersuchen, durch die Fragen, die wir an die Natur richten, aus dem Gewebe von Zusammenhängen herausgelöst wird, das wir die Welt nennen".

[7] Krishnan V., *Probability and Random Processes*, with contributions from K. Chandra. Wiley, 2016. Cf. particularly § **4.2** : *Binomial Distribution* ; § **6.4** : *Normal or Gaussian distribution* ; § **10.3** : *Means and Variances of Some Distributions*.

[8] Laloë F., *ibid*. p. 112–113.

[9] In 2005, I wrote an erroneous article for the *Revue des Questions Scientifiques* entitled *Teleportation and Information Decoding*, which basically amounted to suppose that the here-above Eq. 53 does depend on θ in a measurable way. I later recognized my error in an article entitled *Retrospective Examination of Three Articles Published in the Revue des Questions Scientifiques in 2005 and 2006* [cf. http://vixra.org/abs/1006.0057 (2010)]. While recognizing this error, I proposed a scheme in which gravity was supposed to introduce a new challenging ingredient to the original EPR experiment. Unfortunately, such an idea was also erroneous. The reason for this failure is simple : the way I took gravity into account would have allowed one to replace it by electromagnetism. Fortunately, the major part of my 2010 *Retrospective Examination* is not devoted to the question of the relationship between gravity and quantum mechanics. Its main content deals with the second principle of thermodynamics in a way that seems to me fundamentally correct. I have developed my understanding of the limits of this "principle" in two later articles, also accessible on the internet site vixra.org. The content of the last of these articles, entitled *Motion of an Object Due to the Adjusted rate of Modifications Performed on its Environment*, seems to me particularly recommendable for its clarity.