The application of mathematics to warfare : the Battle of Crecy, the Battle of Carrhae, Mongol battle tactics, the Battle of Fredericksburg, the Battle of Outpost Snipe and the Battle of Medenine

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Abstract

The application of mathematics to a problem or question often leads to a deeper understanding of the problem or question and sometimes to an answer to the problem or question. The application of mathematics to warfare is possible in many situations, especially in relation to matters that involve the range, rate of fire, accuracy and effectiveness of missile weapons such as bow and arrows and firearms. This enables us to explain the results of many battles in the past and to predict the results of many battles in the future as many battles in the future may involve missile fire.

Attempts have been made in the past, particularly by Frederick Lancaster and Colonal Trevor Dupuy, to apply mathematics to the study of military history. While the idea is sound, too often they involve using statistics of dubious quality and the assignment of quite arbitrary figures, little more than guesses, to military organisations and tactics. It may be better to restrict the military analysis to factors that are more amenable to mathematics rather than to try to apply mathematics to all aspects of war. The performance of weapons, especially weapons that fire missiles, such as bow and arrows and firearms, are amenable to mathematical analysis as range, rate of fire and to a lesser extent, effectiveness are quantifiable properties of the weapons. The speed of an attacker across a firing zone is also quantifiable.

A further advantage of using figures based upon rates of fire, range and accuracy of weapons and the speed of an attacker across a firing zone is that they can be known before a battle. Many measurements of combat effectiveness in the past, such as morale and the qualities of officers and armies, can only be assessed after the results of battles. But generals need to know the mathematics involved with battles before engaging the enemy, to ensure that any attacks they make have a reasonable prospect of success.

Missile fire has been one of the most important methods used to destroy the enemy in warfare. The weapons used include the slingshot, the javelin, the crossbow and bow and arrow and more recently the arquebus, wheel-lock and flint-lock muskets, and rifle, machine gun and artillery fire. In general the slingshot and javelin have been subsidiary weapons rather than the principal weapons used by armies. The other weapons however have been the principal weapons used in many major battles. The bow was the major weapon used by the English in many battles in the Hundred Years War, by the Mongols and other horsemen occupying the vast expanse running from the area north of China to Eastern Europe, and the Parthians when defeating Crassus at Carrhae and the Saracens who fought the Crusaders. Firearms were used by Europeans, Turks, Persians, Chinese and Indians from the time they became aware of gunpowder until current times. Firearms gave victory in battles in the Hundred Years War such as at Formigny and Castillon, to the armies of Gonzalo de Cordoba, in cooperation with pikemen in the Thirty Years War, in the form of the flintlock musket to armies from the time of Marlborough to Napoleon and in the form of the rifle, machine gun and artillery since that time.

The various forms of missile fire have certain qualities in common. The fact the enemy was dealt with from a distance and the importance of accuracy and rate of fire, have resulted in certain similarities between numerous battles, in quite different times,

fought with missile type weapons. An analysis of some of these battles shows that certain common factors led to victory or defeat in these battles. Such battles involving missile fire, include those of light horsemen armed with the short bow such as Carrhae, Hattin and Mongol victories, English and French victories in the Hundred Years War, battles of the Thirty Years War, of the Napoleonic Wars, of the American Civil War, the Franco-Prussian War, Russo-Japanese War and World War I and World War II. However I will look at only a few battles as that is enough to make the point I want to make.

This paper is intended as a study of a small number of battles using missile fire. The battles are Crecy, Carrhae, Fredericksburg, the defence of Outpost Snipe during the Second Battle of El Alamein and the Battle of Medenine. The battles, other than Carrhae, all involve an attacker charging a defensive line manned by soldiers equipped with missile firing weapons. This is a quite common situation in military history, and there are many other battles that can be studied in the same way, but I have limited myself to a small number of classic examples. This paper does an analysis of each battle and explains why the battles had the outcomes they had.

Battle of Crecy

The first battle to be studied is the Battle of Crecy fought in 1346 by the English army of King Edward III composed of 16,000 men of which 7,000 were longbow men and a French army of King Phillip VI of perhaps 30,000 men composed mainly of men-at-arms and 6,000 mercenary crossbow men. The English were drawn up in 3 divisions one of which was held in the rear and the other two lay side by side and formed the front line which was situated on a ridge. Each division consisted of a center of dismounted men-at-arms and with longbow men on the flanks pushed forward. The French had their crossbow men in front and 3 lines of mounted men-at-arms behind. The longbow had twice the range and four times the rate of fire of the crossbow. When the crossbow men came within range of the longbow, the English commenced firing. The crossbow men were soon in confusion and were then ridden down by the French cavalry. The French cavalry then made a series of charges against the English line. The charges were directed against the English men-at-arms as the French could not face the rain of arrows from the longbow men but only a handful of the French reached the English line. They were quickly cut down by the English men-at-arms. Each charge was hampered by the dead and wounded and retreating remnants of previous charges. Eventually the attacks ceased with the French suffering six thousand casualties against an English loss of several hundred.

The key to this victory was the longbow with its rate of fire of 6 arrows per minute and its 300 yards maximum range. The speed at which the French cavalry crossed the 300 yard firing zone would be much reduced by the front ranks of the charging cavalry bearing the brunt of the arrow barrage, and injured men and horses hindering the cavalry behind them. A further problem for the French was they had to charge up hill and the English had dug pitfalls or *trous de loup* in front of their lines and scattered caltrops over the firing zone. The French charge across the 300 yards firing zone might have taken about 90 seconds giving each longbow man the chance to fire 9 arrows against a densely packed mass of heavy cavalry. At longer range many arrows would have bounced off the armor of the French knights yet other arrows would have brought down the unarmored or lightly armored horses. At closer range both knights

and horses would be killed or injured. Against such a dense mass of men and horses few arrows would not hit something or someone. A number of arrows would hit the same man or horse, especially those in the front or flanks of the charging mass.

It becomes a mathematical exercise to calculate the result of charges such as those made by the French at Crecy. First, one calculates the amount of time the attacker spends in the firing zone. At Crecy, the firing zone was 300 yards and this would take 90 seconds to cross. During this time the longbow men would each fire 9 arrows of which perhaps 20% would be effective, by bringing down a man or horse so that they could no longer participate in the charge. It is a fairly obvious calculation with 7,000 longbow men each releasing 9 arrows or 63,000 in total, of which 20% or 12,600 would be effective, then it is hardly surprising that a charge of 10,000 French cavalry, the number in each French line, was not going to be successful against such a rain of arrows. Even if only one out of nine arrows was effective that would be 7,000 effective arrows, more than enough to deal with a charge of 10,000 Frenchmen. The exact calculation of the percentage of arrows that were effective is a matter of surmise but given that the French cavalry would be in a close ordered mass and that at closer range arrows would kill and injure men as well as horses, one arrow out of five does not seem excessive. Many French men-at-arms would have been killed or injured, not directly from the arrows, but from falling from dying or wounded horses. A further factor assisting the longbow men was that after the first charge, subsequent charges were hindered by the dead, injured and retreating remnants of the previous charges. Also the French had to charge up hill. These factors would have the effect of causing the French to spend longer in the firing zone allowing the bowmen to fire a larger number of arrows. It should be added that while the numbers of the French are somewhat uncertain there is no question they substantially outnumbered the English which illustrates the great power of the longbow.

In order to maintain consistency between the various battles studied in this paper, the longbows of Crecy are called “weapons” and the arrows fired are called “rounds”. The expressions weapons and rounds also covers the rifled muskets and minie balls used at the Battle of Fredericksburg and also the anti-tank guns and anti-tank armored piercing rounds used in World War 2. This enables the creation of a formula consistent over all battles involving one group of soldiers charging an enemy line consisting of soldiers armed with missile firing weapons.

A calculation of the effect of the arrows on the charging French cavalry would be something like this:

r = range of weapon or firing zone (300 yards)

sa = speed of attacker (30 seconds per 100 yards or 0.3 seconds per yard)

tfz = time in firing zone (90 seconds)

rf = rate of fire (6 per minute, or 1 per 10 seconds ie time taken to fire once)

nr = number of rounds fired per weapon (9)

wf = number of weapons firing (7,000)

trf = total number of rounds fired (63,000)

e = % of rounds fired which are effective, which render opponent unable to play any further role in the battle (1 out of 5 or 20%)

nc = number of casualties (12,600)

r x sa = tfz
300 yards x 30 seconds per 100 yards or 0.3 seconds per yard = 90 seconds in firing zone

tfz ÷ rf = nr per weapon
90 seconds in firing zone ÷ by time taken to fire one round eg 10 seconds = 9 rounds fired per weapon

nr x wf = trf
9 rounds fired per weapon x number of weapons firing = total rounds fired
9 x 7,000 weapons = 63,000 rounds fired at each separate French charge

trf ÷ e = nc
total rounds fired = 63,000 ÷ effectiveness ratio = 63,000 ÷ 5 = 12,600 = number of casualties per charge

Number of casualties per charge = 12,600. This is what would have happened if the French had reached the English line, but they did not suffer this number of casualties per charge, as in many cases, the French cavalry were able to retreat outside the range of the arrows. Total French casualties for the battle were probably about 6,000, but this is a very uncertain figure. Wikipedia, as at 5 October 2020, suggests at least 4,000 were killed from the French army.

The English army was organised in three separate divisions so the individual French charges may have been against a single English division so the number of arrows fired at an individual French charge may only have been a third of the 63,000 arrows calculated above, so 21,000 arrows with an effectiveness ratio of 20% would mean 4,200 casualties per charge. Even with these figures it is very clear why the individual French charges were unsuccessful. Alternatively, if the French charges were against the whole English front line, which consisted of two divisions, as the third division was in reserve, the number of arrows fired during a French charge would be 42,000 and with an effectiveness rate of 20%, that would cause 8,400 casualties per charge. Again it is quite clear why the French charges failed.

Linking it all together, the final formula, is:

r x sa = tfz ÷ rf = nr x wf = trf ÷ e = nc

or to spell it out

range x speed of attacker = time in firing zone ÷ rate of fire = number of rounds fired per weapon x number of weapons firing = total number of rounds fired ÷ effectiveness = number of casualties.

All figures used in these calculations are necessarily approximations, but reasonably close approximations, to what actually happened in the battles studied. Many of the figures could be slightly altered and still reflect what happened in the battles, and still show how and why the battles were won and lost.

The English victory at Crecy was repeated at Poitier in 1356 and at Agincourt in 1415. At Poitier the French attacked on foot which may have reduced the percentage of arrows which would be effective as there would be no unarmored or lightly armored horses brought down by the arrows. However it would have greatly increased the time the French would have taken to cross the firing zone so allowing the longbow men to fire a considerably greater number of arrows. At Agincourt, the French men-at-arms, although vastly out-numbering the English, were delayed in the firing zone as mud slowed their advance.

Battle of Carrhae

The English army at Crecy, Poitier and Agincourt fought dismounted. However, the bow was used by mounted archers particularly in the Middle East and in the area stretching from the Pacific Ocean, North of China, to Eastern Europe. One great victory won by mounted archers was at Carrhae in 53 BCE when the Parthian mounted archers destroyed a Roman army commanded by Crassus. The Roman army consisted of 35,000 infantry and a few cavalry and the Parthian army had 11,000 cavalry of whom 10,000 were horse archers. The Parthian horse archers used the recurved composite bow made of a mixture of a wooden core, animal horn and animal sinew glued together which produced a short bow convenient for use on horseback, but which also had considerable power.

When the two armies met the Parthian mounted archers unleashed a hail of arrows at the Romans and tried to get around the Romans flanks. The Romans attempted to counter this by forming a great hollow square. The rain of arrows continued and the Romans found they were unable to come to close quarters with the enemy as the Parthians would retreat before the Romans while still maintaining the barrage of arrows. The Parthians were skilled at riding away from their enemies while turning in the saddle and firing backwards; a technique that became known as the Parthian shot. The Roman hope that the Parthians would run out of arrows evaporated when they saw the large number of camels carrying spare arrows. Eventually the Romans decided they would have to retreat during the night into the town of Carrhae. The next night the retreat continued but the Romans were betrayed by their guide and Crassus was killed. Only a small number of Romans eventually reached safety. Carrhae can be analyzed in the same way as Crecy even though the battles are in some ways quite different.

Mongol battle tactics

The Mongols used the same battle tactics the Parthians used at Carrhae. The great majority of their army consisted of horse archers, although they had some heavy cavalry. The horse archers would shower the enemy with arrows while keeping out of range of the enemy's weapons. If charged by enemy cavalry they would retreat to avoid close contact with the enemy cavalry, while continuing to fire at the enemy during the retreat. Marco Polo described Mongol battle tactics as follows:

"*When they come to an engagement with the enemy, they will gain the victory in this fashion. [They never let themselves get into a regular medley, but keep perpetually riding round and shooting into the enemy. And] as they do not count it any shame to run away in battle, they will [sometimes pretend to] do so, and in running away they turn in the saddle and shoot hard and strong at the foe, and in this way make great havoc.*"

And

"*Thus they fight to as good purpose in running away as if they stood and faced the enemy, because of the vast volleys of arrows that they shoot in this way, turning round upon their pursuers, who are fancying that they have won the battle. But when the Tartars see that they have killed and wounded a good many horses and men, they wheel round bodily and return to the charge in perfect order and with loud cries; and in a very short time the enemy are routed.*"

The Mongols simply ensured that the firing zone was never completely crossed by the enemy, by retreating before an advancing enemy, and by pursuing a retreating enemy, they kept the enemy in the firing zone for as long as they wanted. When the enemy were sufficiently weakened, they would unleash a charge of their heavy cavalry to finish them off.

The vital difference between English longbow victories and battles won by mounted archers such as Carrhae, Manzikert, Hattin and the Mongol victories is that the mobility of the mounted archers ensures that the if the opponents are infantry they are never able to get through the firing zone to close with the mounted archers to use their weapons. The mounted archers would simply ride away, while maintaining the rain of arrows on the enemy. Even when the opposition were cavalry like the European knights during the Crusades they were often not able to close with the Saracen horse archers due to the greater mobility of the lightly armored horse archers. It was simply a case of keeping the enemy in the firing zone, and facing the shower of arrows, but never allowing them to get close enough to engage in close order combat.

Battle of Fredericksburg

The Battle of Fredericksburg was fought in December 1862 between Union General Ambrose E Burnside's Army of the Potomac and Confederate General Robert E Lee's Army of Northern Virginia. A major part of the battle involved an assault by Union troops on the Confederates strongly entrenched behind a four foot high stone wall at the bottom of a low ridge called Marye's Heights. There were further Confederate troops and artillery entrenched on Marye's Heights itself. The Union troops had to cross two bridges over a ditch while under Confederate artillery fire and then cross 400 hundred yards of open ground to reach the Confederate's line. One Confederate artillery officer had told Confederate General James Longstreet just before the battle "Once we open fire, not even a chicken could live on the ground out there". When Lee saw the Union troops form up, he had a touch of doubt the Confederate line would hold and Longstreet told him that if every man in the Union army assaulted Marye's Heights "I will kill them all."

There were initially 2,000 Confederate troops behind the stone wall, with 7,000 men in reserve. During a pause in the fighting the Confederate troops were reinforced so they were able to stand in four lines and maintain almost continuous fire against the advancing Union troops, with one line firing while the other three lines were reloading.

Assault after assault were repelled by Confederate rifle and artillery fire. Union brigades were sent forward one at a time, so the Confederates never had to beat off a single massive attack. Union brigades were typically 1,000 to 1,500 men in strength and as each brigade attacked separately, the defending Confederate troops never faced an attack by more than 1,500 men at a time. Fourteen separate attacks were made, but not a

single Union soldier got within 100 feet of the stone wall. The time taken for troops carrying full equipment to cross the 400 yards to the Confederate line was greatly increased by the Union practice of marching in formation, firing and then stopping to reload which greatly increased the time they spent in the firing zone. The Union fire would have had a limited effect on the Confederate troops protected by the stone wall, while Union troops were very exposed to the Confederate fire. At one stage a Union officer, Colonel Nelson Miles, suggested a bayonet charge might take the Confederate position but permission was refused. Survivors from the assaults were pinned to the ground by the continuous Confederate fire. Only when night fell were the Union troops able to withdraw to safety out of range of the Confederate rifle fire and artillery fire. Seven thousand Union soldiers lay dead on the ground between Fredericksburg and Marye's Heights. Confederate losses in the attack on Marye's Heights were about one thousand two hundred men.

Confederate troops were armed with a variety of muzzle loading rifled muskets. The most common were the Springfield Model 1861 and the Pattern 1853 Enfield both of which fired the same .58 caliber minie ball. The Springfield could fire 3 rounds per minute with an effective range of 400 yards. The Pattern Enfield could fire 3 rounds per minute and had a maximum range of 2,000 yards. The distance from the Union start line to the Confederate line was 400 yards so the firing zone was 400 yards. The speed of the attacker is hard to assess as there is no known timing of the Union assaults but given the practice of firing and reloading during the attack and obstacles on the ground including the dead and wounded from prior attacks it may well have been 60 seconds per 100 yards or 240 seconds in the 400 yard firing zone. The effectiveness ratio is the most difficult figure to assess but given the accuracy of the rifled musket and the close order of the Union troops who stood shoulder to shoulder, it would be quite high. A Union regiment of 500 men might attack in two lines each of 250 men, with a front of 200 yards and 13 inches between the front line and the back line. An effectiveness rate of 25% may even be a conservative figure especially at closer ranges. This would cause 6,000 potential casualties except there were only about 1,500 men in each attack so clearly the attacks were going to fail.

Confederate artillery consisted of a variety of guns, many captured from the Union in previous battles, of varying ranges and rates of fire. Artillery typically had a range of 1,500 yards but this was often limited by the distance to the nearest ridge or other cover for enemy troops. Artillery could fire a variety of ammunition such as solid shot and spherical case rounds at longer ranges, and the more lethal canister shot for ranges of less than 400 yards. The rate of fire of artillery was about one round per minute. The Confederate artillery was placed on the top of Marye's Heights and their effect was described by one Confederate officer as "We could see our shells bursting in their ranks, making great gaps; but on they came as though they would go straight through and over us. Now we gave them canister and that staggered them." But as the day advanced, accurate Union sharpshooting and a shortage of ammunition reduced the effect of the Confederate artillery. But by that time the Confederate troops behind the stone wall had been reinforced so it did not get any easier for the attacking Union troops.

A calculation of the effect of the Confederate rifle fire on the Union troops would be something like this:

r = range of weapon or firing zone (400 yards)

sa = speed of attacker (60 seconds per 100 yards)

tfz = time in firing zone (240 seconds)

rf = rate of fire (3 per minute, or 1 per 20 seconds ie time taken to fire once)

nr = number of rounds fired per weapon (12)

wf = number of weapons firing (2,000)

trf = total number of rounds fired (24,000)

e = % of shots fired which are effective, which render opponent unable to take any further part in the battle (1 out of 4 or 25%)

nc = number of casualties (6,000)

r x sa = tfz
400 yards x 60 seconds per 100 yards or 0.6 seconds per yard = 240 seconds in firing zone

tfz ÷ rf = nr per weapon
240 seconds in firing zone ÷ by time taken to fire one round eg 20 seconds = 12 rounds fired per weapon

nr x wf = total rounds fired
12 rounds per weapon x number of weapons firing = total rounds fired
12 x 2,000 weapons = 24,000 rounds fired at each separate Union attack

Rounds fired = 24,000 ÷ effectiveness ratio (25%) = 24,000 ÷ 4 = 6,000 = number of casualties per charge

Number of casualties per charge = 6,000.

The above calculation does not even take into account the effect of the Confederate artillery, which of course would have different figures for range, rate of fire and effectiveness, so it can easily be seen that the Union attacks on Marye's Heights had no chance of success. Total Union casualties in front of Marye's Heights were 7,000 which is only 500 per charge but the above calculation shows how little chance there was of success for the Union attacks. It also shows that a lot of Union troops hit the ground to save their lives when under fire from the Confederate positions. The later Union attacks were hindered by survivors of earlier attacks lying on the ground and clutching at the legs of soldiers involved in the later attacks, in an attempt to stop the pointless waste of lives, in attacks which were clearly not going to succeed.

Defence of Outpost Snipe

The defence of Outpost Snipe occurred during the Second Battle of El Alamein. The 2nd Battalion of the Rifle Brigade equipped with 13 six-pounder anti-tank guns and 239th Battery of the 76th Anti-Tank Regiment equipped with 6 six-pounder anti-tank

guns were ordered to occupy a position known as Snipe. The units moved forward on the night of 26 October 1942 and as sometimes happens when moving at night they ended up in the wrong place. The place they occupied however was an excellent defensive position being a shallow depression with a three to four foot high edge and with scruffy bushes that were ideal for hiding anti-tank guns. There was a problem however in that it was right in the middle of the 15th Panzer Division and the Italian Littorio Armoured Division and was a thousand yards behind the Axis front line. There was also a problem with soft sand which made it difficult to move the guns.

When the sun rose the next day the British troops found they were surrounded by German and Italian tanks, self-propelled guns and other vehicles and guns. Opening fire with their six-pounder guns they quickly destroyed 14 tanks, 2 self-propelled guns, some trucks, an 88 mm gun and a staff car. The enemy returned fire and the British lost 3 guns and 1 gun that sunk into the sand.

British troops sent to relieve the troops at Outpost Snipe got lost and then bombarded Snipe with "friendly fire" understandable as the troops in Outpost Snipe were in the wrong place. The Axis forces turned to face the relieving troops but lost some more tanks to the six-pounder guns in Outpost Snipe. The Axis forces however drove off the relieving British troops mocking them on the British radio frequency as they retreated.

The British troops in Outpost Snipe were now alone and Axis artillery fire reduced the British force to 13 six-pounders. They were then attacked by 9 Italian M14/41 tanks and some Semoverte self propelled guns at a point where only one of the British guns could fire back. This was due to the limited traverse of the six-pounder guns and because the soft sands in Outpost Snipe made it difficult for gun crews to change the gun's positions. The one gun that could fire destroyed all 9 of the Italian tanks. The troops at Snipe then faced some more "friendly fire".

The Germans then prepared 70 tanks to attack the British 1st Armoured Division in the British front line and some of the German forces, due to communication failures, were unaware of the British anti-tank guns in Outpost Snipe. Their advance went within 200 yards of the hidden British guns and exposed the thin side armour of the German tanks. Four of the British guns were able to knock out 9 panzers. The German tanks then turned to face Snipe but this exposed their flanks to fire from other British forces and they eventually withdrew.

The Germans then attacked Snipe again with 15 Panzer Mark III tanks against which only 2 of the British six-pounders were able to bring fire to bear against the German tanks. One of these British guns was put out of action but 4 German tanks were knocked out, but with the odds greatly against the British troops it was decided to withdraw to the British lines, a process that was completed under cover of darkness.

The British defence of Snipe had frustrated Rommel's attempt to attack the British 1st Armoured Division and had inflicted significant losses on German and Italian armoured divisions. It was conservatively estimated the defenders of Snipe had destroyed or disabled 32 Axis tanks, 5 self propelled guns, 2 artillery pieces, some trucks and a staff car and probably a further 20 tanks had been damaged and probably were never repaired and brought back into action. The British losses were 18 six-pounder guns, as a number of guns had to be abandoned due to a lack of towing vehicles, some Bren gun carriers and 72 troops. Axis troop losses were unknown but certainly exceeded the British losses.

Battle of Medenine

The Battle of Medenine was fought on 6 March 1943 in North Africa between the British 8th Army commanded by General Montgomery and the German-Italian army commanded by Field Marshal Erwin Rommel. Following his defeat at El Alamein Rommel had retreated to the Libyan Tunisian border and occupied old French fortifications known as the Mareth line. Montgomery's 8th Army had cautiously followed Rommel to the Mareth line and Rommel decided on a spoiling attack to disrupt British preparations for an attack on the Mareth line. The British were warned of the planned German attack by Ultra code breakers reading German communications relating to the planned attack. The British rushed reinforcements, consisting of the 2nd New Zealand Division, the 51st Highland Division and the 7th Armoured Division to the area so they had 470 anti-tank guns, 350 field guns and 400 tanks to block the German advance.

The British forces were deployed with anti-tank guns and infantry holding the front line with armor kept in reserve for a possible counter attack. The British positions were well dug in and camouflaged so they could not be seen from more than 200 to 300 yards. The Germans advanced cautiously and the British waited until the Germans had come within close range when the British artillery and anti-tank guns opened fire. The 10th Panzer Division was within 400 yards of the New Zealand anti-tank guns when they opened fire, destroying 5 tanks. The 21st Panzer ran up against the 201st Guards Brigade and quickly lost 3 tanks and 12 more a little later. The 15th Panzer came up against the 131st Brigade and lost 20 tanks. The tank attack having failed the Germans sent forward infantry which were hit by a heavy artillery barrage and forced to retreat. The Germans withdrew early the next day leaving 52 destroyed tanks on the battlefield while the British lost no tanks and few guns and men. The vast majority of German tank losses were caused by fire from the British and New Zealand anti-tanks guns.

The British and New Zealand troops at Medenine were predominantly using the six-pounder 57mm anti-tank gun, although there were also some two-pounder and seventeen-pounder anti-tank guns as well. The only anti-tank guns used by the troops defending Outpost Snipe were six-pounders. The six-pounder guns had an effective range of 1,650 yards, but often dug in and camouflaged anti-tank guns would hold their fire until enemy tanks were much closer, for example 400 yards at Medenine. The rate of fire was 15 rounds per minute, but would be slower if the target was changed between shots. A reasonable assessment of the rate of fire, when time taken for changes of target is included, would be 10 per minute.

The speed of the attacking tanks over rough terrain would be about 16 kilometres per hour or about 10 miles per hour. The top road speed of a Panzer III or Panzer IV was about 40 kph or 25 mph, but tanks were much slower off the road. At a speed of 10 mph a tank would take about 82 seconds to cross a 400 yard firing zone. 400 yards is 22.72% of a mile. A tank does 10 miles in an hour or in 3,600 seconds. This is 360 seconds for one mile and 22.72% of 360 seconds is around 82 seconds.

Unlike the massed French cavalry at Crecy, tanks in World War 2 were "point targets" and usually moving point targets, so the way to hit them for anti-tank guns was with direct fire. This meant the six-pounder had a sight or sighted telescope with crosshairs and the six-pounder had a 96% chance of hitting a 6 foot by 6 foot target at 1,000 yards and a 55% chance of hitting the target at 2,000 yards. Obviously this would be reduced in conditions of poor visibility or if the gun crew were under fire.

Hitting a tank is of little value if the shot bounces off the tank, due to the thickness or slope of the tank's armor. Penetration of tank armor by six-pounder armor

piercing shot will vary from tank to tank but increases at closer range. Often well camouflaged and dug in anti-tank units would delay fire until enemy tanks were quite close which would ensure high accuracy and good penetration of tank armor and limit the wastage of ammunition. This was exactly the tactics employed in the Battle of Medenine and due to this the firing zone could be considered to be 400 yards.

Given the accuracy and penetration of tank armor at close range, the effectiveness of anti-tank fire would be quite high, possibly as high as one tank put out of action for every two shots from close range and one put out of action for every 3 or 4 shots for longer ranges. It would be possible to assign a particular effectiveness rate for particular ranges, but battles are dynamic and ranges change as tanks move forwards or backwards, so an overall effectiveness rate is simpler and more realistic for any given battle. A reasonable overall effectiveness rate of one tank put out of action for every three shots fired could be assessed. It is likely a similar or better effectiveness rate will apply to German anti-tank guns, especially the 88mm gun. The 88mm had a higher muzzle velocity than the six-pounder although its higher profile made it more difficult to conceal.

This effectiveness rate would explain the considerable success of anti-tank guns in World War 2. British successes such as Outpost Snipe and the Battle of Medenine and Alam Halfa, were matched by German successes such as the defeat of the British Operation Battleaxe and in the Battle of Gazala. The Americans also stopped the German Mortain offensive in early August 1944 in Normandy with their 57mm M1 anti-tank gun which was very similar to the British six-pounder gun.

A calculation of the effect of the anti-tank guns on the Axis tanks at Snipe and Medenine would be something like this:

r = range of weapon or firing zone (1,650 yards, but in reality 400 yards due to the tactics of holding fire until the enemy was at close range)

sa = speed of attacker (10 miles per hour or 10 miles in 3,600 seconds, 360 seconds per mile and 82 seconds for 400 yards)

tfz = time in firing zone (82 seconds)

rf = rate of fire (10 per minute, or 1 per 6 seconds ie time taken to fire once)

nr = number of rounds fired per weapon (82 ÷ 6 = 14 rounds approx)

wf = number of weapons firing (will vary, at Snipe soft sand made it difficult to bring guns to bear but that was unusual, say 10 guns)

trf = total rounds fired (14 x 10 = 140)

e = % of rounds fired which are effective, which render opponent unable to take any further role in the battle (1 out of 3 or 33%)

nc = number of casualties (140 ÷ 3 = 47 approx)

r x sa = tfz
400 yards x 20.449 seconds per 100 yards or 0.20449 seconds per yard = 82 seconds in firing zone

tfz ÷ rf = nr per weapon
82 seconds in firing zone ÷ by time taken to fire one round eg 6 seconds = 14 rounds fired per weapon

nr x wf = total rounds fired
14 rounds per weapon x number of weapons firing = rounds fired
14 x 10 weapons = 140 rounds fired at each separate tank attack

Rounds fired = 140 ÷ effectiveness ratio (33%) = 140 ÷ 3 = 47 = number of casualties per tank attack

Number of casualties per tank attack = 47.

This calculation shows that camouflaged and dug-in six-pounder guns could knock out tanks at the rate 10 guns knocking out 47 tanks or a ratio of 4.7 to one anti-tank gun. This might reduce a bit if the tanks were firing back, but usually in World War 2 tanks did not fire their main armament when on the move as it affects accuracy and if they stopped to fire to get more accuracy they would be spending more time in the firing zone. The tanks' machine guns could fire on the move, but anti-tank guns have shields and such fire would be unlikely to be very effective against dug-in anti-tank guns in camouflaged positions. A ratio of 4.7 tanks knocked out for each anti-tank gun shows why German tank ace Michael Wittman reputedly hated anti-tank guns.

Conclusion

The final equation for calculating the likely result of an attack on an enemy position defended by troops armed with missile firing weapons is:

r x sa = tfz ÷ rf = nr x wf = trf ÷ e = nc

or to spell it out

range x speed of attacker = time in firing zone ÷ rate of fire = number of rounds fired per weapon x number of weapons firing = total number of rounds fired ÷ effectiveness = number of casualties.

This formula applies to both historical battles fought with missile firing weapons and to modern battles fought between armies armed with missile firing weapons. It may have to have minor alterations for particular battles, but it is a general formula for many battles involving missile firing weapons.

The aim of this article is not to show the defensive is superior to the offensive which is not always the case. For example in medieval times a heavy cavalry change could scatter infantry not armed with the longbow or pike and modern blitzkrieg attacks with armor concentrated against a weak point in the enemy’s line have been shown to be successful in World War 2 and subsequently. The aim is to show what prospects an attack has for success can be rationally calculated before the attack takes place. In some cases the calculation will show an attack should succeed, on other occasions that it will fail. The analysis still has relevance to today’s armies. After all modern armies are built

around the missile fire of small arms, machine guns, artillery, tanks and anti-tank weapons. In a modern style attack of tanks and armored infantry with artillery and air support, the aim is to break the enemy's line, which will be defended with modern missile firing weapons. There is a certain amount of asymmetrical warfare in the modern world, but current wars in Syria, Yemen, Afghanistan and Libya will still involve soldiers firing missiles against each other in the form of bullets, artillery shells and armor piercing rounds.

Nor is the aim of this article to reduce warfare to mathematics. There are many other factors involved in battle, most obviously morale, how troops and weapons are deployed etc. However ignorance of the analysis of the likely results of combat could cause attacks to be made which are likely to be defeated. Equally knowledge of the analysis of the likely results of an attack may have avoided such disasters as Fredericksburg, Cold Harbor, Crecy, Poitier, Agincourt, Alam Halfa, Medenine, the Cauldron and Operation Battle Axe.

Generals will know the figures for the performance of their own weapons, and from experience and captured weapons, the performance of the enemy's weapons. The figures they will be less aware of is the number of enemy troops facing them and the total number of weapons aimed at them. This highlights the importance of reconnaissance, whether by drone, small numbers of troops scouting forward, or simple means such as binoculars or telescopes. What generals should not do is to throw troops forward, hoping for the best or to see what happens. Before they engage the enemy they should do their maths.

Rates of fire, range and accuracy of weapons are reasonably easily tested to produce accurate numbers for the mathematical analysis of battles. What cannot be easily examined are these matters when under fire from the enemy. Speed of cavalry, infantry and tanks when traversing the firing zone can also be reasonably accurately calculated, but speed across the firing zone when under fire and when movement is hindered by dead or wounded men and horses is much more difficult to calculate. But if we are to avoid disastrous attacks, such as occurred in the battles examined in this paper, we do need to do these calculations and even calculations with approximate figures for such variables are better than no calculations.

It is not surprising that at the time of the Battle of Crecy no one had done a mathematical analysis of a cavalry charge against longbow men. This is because, apart from the longbow being a relatively new weapon in Europe, mathematics itself was quite unsophisticated at that time. The Hindu-Arabic numeral system was only just arriving in Europe and the Battle of Crecy was fought before the development of modern science. However Fredericksburg, Snipe and Medenine were fought long after the development of modern mathematics and science and yet there is little indication in those battles of any mathematical analysis of the prospects for a successful attack.

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