The Detonator Paradox:
from Paradoxical to Inconsistent

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Abstract.-After a short review of the classical detonator paradox, this article introduces two variants of the celebrated argument that are not paradoxical arguments but true contradictions involving Fitzgerald-Lorentz contraction.

1-Introduction

The word 'paradox' in 'relativity paradoxes' is rather confusing because the paradoxes of relativity are not true paradoxes but the result of an *erroneous reasoning*. Usually the oblivion of some of the unfamiliar restrictions derived from the special theory of relativity. The consideration of the appropriate restrictions suffices to make the paradox disappear. The Detonator Paradox we will analyze in the next section is one of those (false) relativity paradoxes. As we will see, the argument is paradoxical only if we forget the relativistic behaviour of solid objects. However, the detonator argument can be slightly modified in such a way that it becomes a contradiction in which Fitzgerald-Lorentz contraction gets involved. The discussion on the Detonator paradox is purely conceptual, and then it only considers those physical details that are pertinent to the discussion. The same will apply to the variants discussed here. In my opinion, the most relevant aspect of the Detonator paradox is just that those variants have never been proposed, being as they are so immediate and conflicting. Surely, a consequence of the lack of criticism in science.

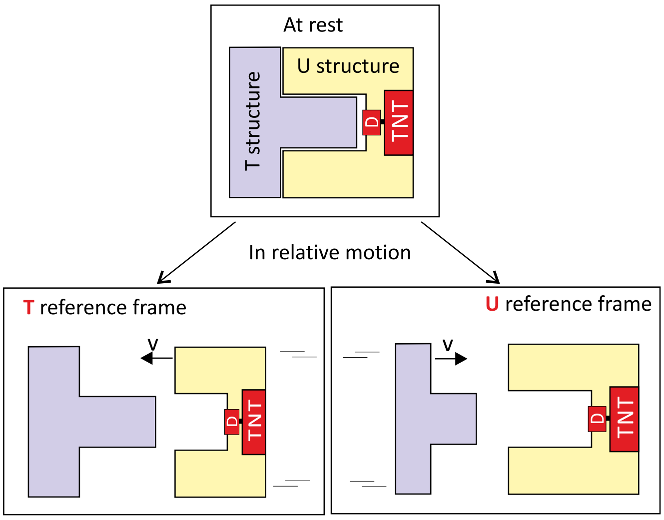

Fig. 1 – Will there be a terrible explosion, or not?

2-The Detonator Paradox

The following paradox (Detonator Paradox) is taken from [1, p. 185]: A U-shaped structure (U-hereafter) made of the strongest steel contains in its central arm a detonator switch connected by a wire to one metric ton of explosive TNT, as shown in Figure 1. A T-shaped structure (T hereafter) made of the same strong steel fits inside the U, with the long arm of the T not quite long enough to reach the detonator switch when both structures are at rest in the laboratory. Now the T structure is removed far to the left, and then accelerated towards the U structure up to reach a high velocity v. One reached v, T continues to move towards the U with the same uniform velocity v. With respect to the U structure reference frame, the long arm of the T will be Lorentz contracted. As a result it will not reach the detonator switch when the two structures collide. Therefore there will be no explosion. However, from the point of view of the T's reference frame, the long arm of the T is not contracted, while the parallel arms of the U structure are contracted in te direction of the relative motion. Therefore the long arm of the T will certainly strike the detonator switch and there will be a terrible explosion. Who are right? Will there be an explosion or not?

According to the original solution, the answer is yes, there will be a terrible explosion. The observers in the T reference frame will agree with this conclusion because from their perspective, and due to Fitzgerald-Lorentz contraction in the direction of the relative motion, the parallel arms of the U structure are shorter than the long arm of the T structure, and therefore the detonator will be hit. We have to explain, therefore, the explosion from the perspective of the U structure reference frame since from this perspective the long arm of the T structure is Lorentz contracted and then it cannot reach the detonator. From the perspective of the U reference frame we must take into account the relativistic behaviour of rigid objects (completely rigid bodies are impossible in special relativity): when the cap (C) of the T structure hits the U structure the end (E) of the T arm (Figure 2) continues to move because the notice of the C-impact lasts a certain amount of time to reach E, otherwise the Second Principle of the Special Relativity would be violated (the speed of light is the same in all reference frames, which also implies that it is the maximum speed possible for any object). Observers from the U structure frame will therefore agree in

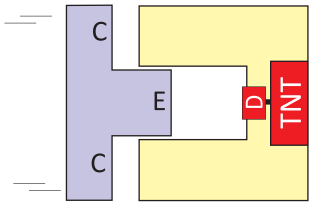

Fig. 2 – The caps (C) and the end (E) of the T structure as seen from the U frame.

that an explosion is possible: once C has impacted, E continues to move until it reaches the detonator. Although not only has to recover its original proper length, in the conditions of the discussion the long arm of the T *must also increase its proper length* enough as to impact the detonator!

There is, however, a little detail that neither the U-observers nor the T-observers have considered. According to their respective explanations the U structure (for T-observers) and the T structure (for U-observers) suffer a real FitzGerald-Lorentz contraction. It is not an apparent deformation, as in the case of a rod partially submerged in water, but a real deformation as in the case of a mechanically deformed rod (see A). This is particularly true for the T-observers since from their perspective there is only one impact: the impact of the central arm of the U structure with the end of the T's long arm, and this collision is only possible if the U's long arms are really, not apparently, contracted. Therefore, the deformation would be real for all observers, as is real the deformation of the mechanically bent rod. Does this conclusion resolve the problem of the apparent or real nature of Fitzgerald-Lorentz contraction? Or is it another relativistic infelicity related to Lorentz transformation?

3.-The Detonator Inconsistency

We will now modify the detonator argument so that its conclusions become much more conflicting. Indeed, what would happen if the long arm of the T were clearly shorter than the parallel arms of the U when both structures are at rest in the lab? As we will immediately see, for the observers in the U's reference frame the explosion will always be impossible unless the T's long arm is sufficiently enlarged immediately after the collision, which is an inadmissible *ad hoc* mechanical assumption. In the conditions of the previous section, let L_{oT} be the proper length of the T's long arm, L_{oU} the proper length of the U's

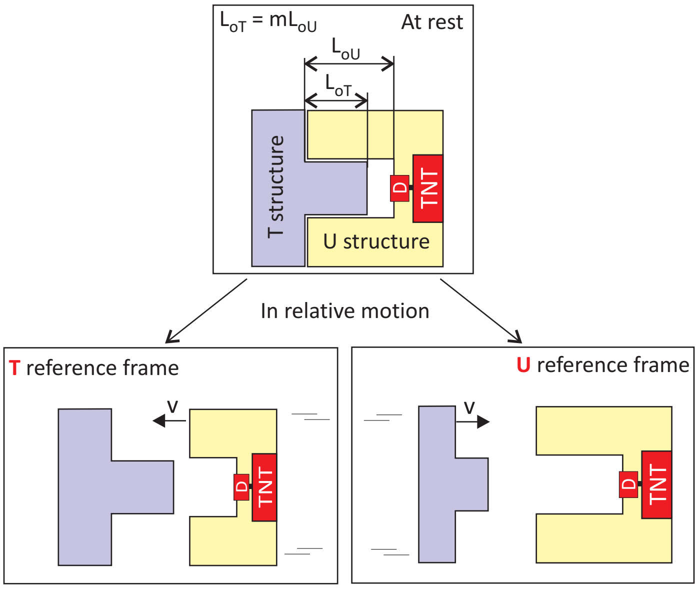

Fig. 3 – The detonator argument becomes inconsistent if the T's long arm is clearly shorter than the parallel arms of the U

parallel arms, and assume it holds:

$$L_{oT} = mL_{oU};\ 0 < m < 1 \tag{1}$$

Assume also the relative velocity between the T and the U structures is $v = kc$, being $0 < k < 1$. In those conditions the relativistic factor γ will be given by:

$$\gamma = \frac{1}{\sqrt{1 - \frac{k^2c^2}{c^2}}} \tag{2}$$

$$= \frac{1}{\sqrt{1-k^2}} \tag{3}$$

And the proper lengths L_{oT} and L_{oU} will be seen contracted by a factor:

$$\gamma^{-1} = \sqrt{1-k^2} < 1 \tag{4}$$

when observed in relative motion (Figure 3). In the next discussion we will make use of the concept of *kf-solid*: a solid that

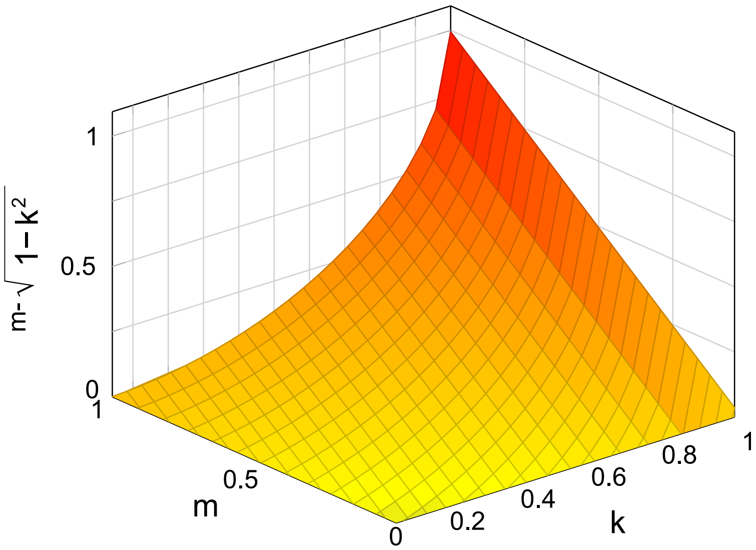

Fig. 4 – Each point above the represented surface corresponds to a pair of values of m and k for which the explosion occurs from the perspective of the T reference frame. An explosion that is impossible from the perspective of the U reference frames, unless the long arm of the T structure enlarges arbitrarily by a certain factor.

after impacting at a speed kc its not impacting parts does enlarge in the direction of motion by a factor less than f, being f a real number. From the perspective of the T reference frame, the explosion will occur if:

$$L_{oT} \geq L_{vU} = \sqrt{1-k^2}L_{oU} \tag{5}$$

That is to say, if, according to (1):

$$mL_{oU} \geq \sqrt{1-k^2}L_{oU} \tag{6}$$

And then if:

$$m \geq \sqrt{1-k^2} \tag{7}$$

For example if $k = 0.5$ and $m = 0.87$ (see Figure 4). From the perspective of the U reference frame the condition for the explosion to occur will be:

$$L_{oU} \leq L_{vT} = \sqrt{1-k^2}L_{oT} \tag{8}$$

That is to say:

$$L_{oU} \leq \sqrt{1-k^2}mL_{oU} \tag{9}$$

And then if:

$$1 \leq m\sqrt{1-k^2} \tag{10}$$

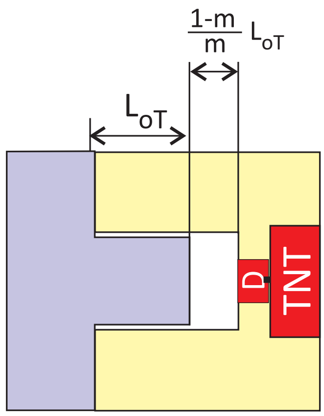

Fig. 5 – After the collision the long arm of the T structure would have to be enlarged by a factor of $(1-m)/m$ in order to make the explosion possible.

which is impossible since m and $\sqrt{1-k^2}$ are both less than 1. From the perspective of the U reference frame, after the collision the explosion will not occur unless the long arm of the T enlarges at least by a length λ such that:

$$\lambda = L_{oU} - L_{oT} \tag{11}$$

$$= \frac{1}{m}L_{oT} - L_{oT} \tag{12}$$

$$= L_{oT}\left(\frac{1}{m} - 1\right) \tag{13}$$

$$= L_{oT}\frac{1-m}{m} \tag{14}$$

Thus, our enlargement factor f will be $(1-m)/m$, as Figure 5 shows. In consequence each point of the shadowed area drawn in Figure 6 represents an explosion from the perspective of the T reference frame that is impossible from the perspective of the U reference frame if the long arm of the T is not enlarged by a factor equal or greater than $(1-m)/m$. According to T's

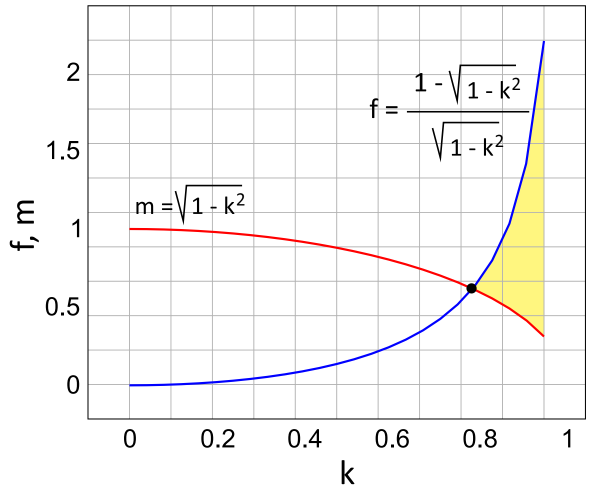

Fig. 6 – The conflicting area within which the explosion occurs (U's observers) and does not occur (T's observers) if the mechanical properties of kf-solids are not violated.

observers the explosion occurs if:

$$m \geq \sqrt{1-k^2} \tag{15}$$

According to U's observers the explosion does not occur if the long arm of the T enlarges by a factor less than f:

$$f = \frac{1-m}{m} \leq \frac{1-\sqrt{1-k^2}}{\sqrt{1-k^2}} \tag{16}$$

Therefore, the functions:

$$m = \sqrt{1-k^2} \tag{17}$$

$$f = \frac{1-\sqrt{1-k^2}}{\sqrt{1-k^2}} \tag{18}$$

define a conflicting area in which the explosion occurs and does not occur, as Figure 6 shows. According to T's observers the explosion occurs in all points above the graph of (17). According to U's observers the explosion does not occur in all points below the graph of (18). The intersection of both areas is the conflicting zone where the explosion occurs and does not occur if the mechanical properties of kf-solids are not violated.

In conclusion, while for the T's observers the explosion occurs without violating the mechanical properties of kf-solids, for the U's observers the explosion would imply that violation. In this case the T frame and the U frame would not be totally equivalent, which goes against the First Principle of Relativity. In a world where at least a kf-solid exists, being f defined by the conditions of the above discussion Fitzgerald-Lorentz contraction leads either to a contradictory result (the explosion occurs and does not occur) or to a violation of the First Principle of Relativity.

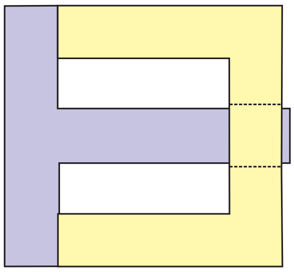

Fig. 7 – A variant of the Detonator Paradox.

4.-A variant of the Detonator Paradox

Consider again the above scenario of the Detonator Paradox and assume that, in the place of the detonator, the U structure has a central hole of a diameter greater than the diameter of the long arm of the T structure and in such a way that when both structures are at rest in the lab they fit as Figure 7 illustrates. Assume also the information of a mechanical collision travels through the material of both structures at a velocity v:

$$v = kc, \ 0 < k < 1 \tag{19}$$

The discussion that follows will be carried out from the perspective of the U reference frame, that will also referred to as RF_o. As in the case of the previous sections, assume the T structure is removed far to the left and accelerated towards the U up to reach a velocity that now is just kc, being k the same as in (19). Assume the caps (C) of the T hits the U structure at instant t_{o1}. The end E of the T's long arm continues to move until the notice of the collision reaches it. At t_{o1} the long arm of the T has a length L_{v1}. So, the notice of the collision lasts a time Δt_{o1} to reach E:

$$\Delta t_{o1} = \frac{L_{v1}}{kc} \tag{20}$$

Now then, during Δt_{o1} the end E of the T structure moves a distance d_{o1}:

$$d_{o1} = kc\Delta t_{o1} \tag{21}$$

$$= kc\frac{L_{v1}}{kc} \tag{22}$$

$$= L_{v1} \tag{23}$$

Therefore, when the notice of the impact reaches the position of E at t_{o1}, E is no longer in this position but at a distance L_{v1} from it, which is the same distance the notice of the impact has just traversed. This notice must traverse again the same distance L_{v1} to reach E, which takes it a time Δt_{o2}. Since the notice of the impact must again traverse the same distance L_{v1}

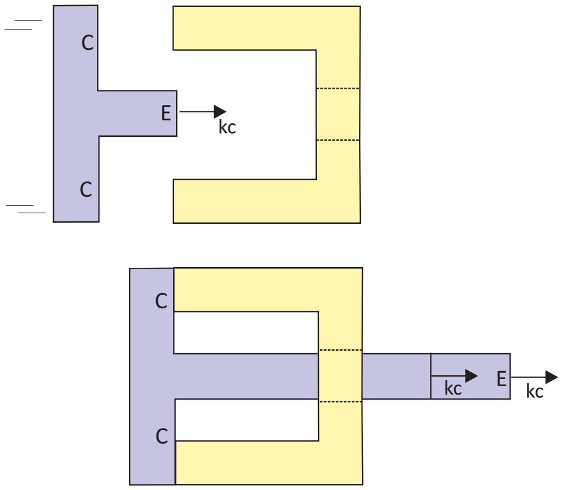

Fig. 8 – For U's observers, the long arm of the T should enlarge indefinitely.

at the same velocity kc as in the case of t_{o1}, it is evident that $\Delta t_{o2} = \Delta t_{o1}$. It is clear, then, this argument leads to the absurdity that, from the U's reference frame perspective, the long arm of the T will enlarge indefinitely. The reader could easily find other absurdities derived from this variant of the Detonator Paradox. The only way to avoid the above absurdity is by a new ad hoc mechanical assumption: it is impossible for a body to move with a speed equal or greater than the speed at which the notice of a mechanical impact travels through the material the body is made of. The observers in the T and the observers in the U would have to conclude their observations on objects in relative motion could not be appropriate in order to get conclusions on what really happen in the corresponding proper frames of the observed objects.

References

[1] Edwin F. Taylor and John Archibald Wheeler, *Spacetime physics. Introduction to special relativity*, W. H. Freeman and Company, New York, 1997.