Breaks of Lorentz-symmetry in Optical Crystallography

(Adapted from a draft chapter of the book *Discrete Relativity*, in preparation)

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Abstract.-This paper examines some optical properties of isotropic crystals, as well as the critical reflection of light, from the perspective of different inertial reference frames in relative motion. The analysis proves several breaks of Lorentz symmetry in optical crystallography and general optics. It also demonstrates there is no other relativist alternative to Lorentz transformation able to give a solution to the problems posed by Lorentz transformation in the scenario of optics and optical crystallography. The conclusions of the arguments point to the convenience of considering a new discrete paradigm for the spacetime continuum in which this and other problems could find a solution.

1.-Conventions

All reference frames (frames hereafter) will be assumed to be inertial frames. RF_o will denote the proper frame of any object or observer at rest in that frame. RF_v will denote a frame whose axes coincide with the corresponding axes of RF_o at a certain instant, and from whose perspective RF_o moves at a uniform velocity $v = kc$, (where c is the speed of light in a vacuum, and $0 < k < 1$), being v parallel to the axis X_v. The axes in the plane XY of RF_o and RF_v will be denoted respectively by X_o, Y_o and X_v, Y_v. Lengths, times and refractive indices measured in RF_o and RF_v will be respectively sub-indexed by o and v. As usual, o will also sub-index the magnetic permeability and the electric permittivity of a vacuum, which are universal constant with the same value in all frames. Expressions like RF_o-time, RF_v-length, etc. will always indicate that the corresponding time, length, etc. have been measured respectively in RF_o or RF_v. The absolute value of a magnitude x will be denoted, as usual, by $|x|$. Lorentz transformation will be denoted by LT. The term ”velocity” will be used to refer to the module of the vector velocity, i.e. as a synonym of (physical) speed. Unless otherwise indicated, all materials will be assumed transparent and optically isotropic, and light will always be polarized and of the same wavelength.

2.-The Refractive Index

Maxwell's equations of electromagnetism lead almost immediately to the wave equation for electric fields ($\overrightarrow{E}$), and for magnetic fields ($\overrightarrow{B}$):

$$\nabla^2 \overrightarrow{E} = \mu_o \epsilon_o \frac{\partial^2 \overrightarrow{E}}{\partial^2 t}; \ \nabla^2 \overrightarrow{B} = \mu_o \epsilon_o \frac{\partial^2 \overrightarrow{B}}{\partial^2 t} \tag{1}$$

where ∇^2 is the Laplacian (a second order differential operator); μ_o is the magnetic permeability of a vacuum (magnetic constant, $\mu_o = 1.2566 \times 10^{-6}$ H/m), and ϵ_o is the electric permittivity of a vacuum (electric constant, $\epsilon_o = 8.854 \times 10^{-12}$ F/m) Both, μ_o and ϵ_o, are universal constants. The magnetic permeability is a measure of the magnetization of a medium in response to a magnetic field. The electric permeability is a measure of the electric distortion of a medium in response to an electric field. Comparing (1) with the standard form of a wave equation:

$$\nabla^2 \overrightarrow{Y} = \frac{1}{v^2} \frac{\partial^2 \overrightarrow{Y}}{\partial^2 t} \tag{2}$$

it can be immediately inferred, as Maxwell did, that:

$$v = (\mu_o \epsilon_o)^{-1/2} \tag{3}$$

$$= (4\pi \times 10^{-7} mKgC^{-2} \times 8.8541878 \times 10^{-12} C^2 s^2 Kg^{-1} m^{-3})^{-1/2} \tag{4}$$

$$= 299792.458 \, Km/s \tag{5}$$

is the speed of the electromagnetic waves, that coincides with the speed of light c in a vacuum (free space). As Maxwell suggested, and we now know well, light is a set of electromagnetic waves. Evidently, as an algebraic combination of universal constants, $(\mu_o \epsilon_o)^{-1/2}$, the speed c of light in a vacuum is also a universal constant. On the other hand, each material m has its own magnetic permeability μ_m and its own electric permittivity ϵ_m, usually greater than those of the vacuum. Light travels through a transparent medium m with a speed v less than c given by:

$$v = \frac{1}{\sqrt{\mu_m \epsilon_m}} \tag{6}$$

The relative magnetic permeability $\mu'_m = \mu_m/\mu_o$ and the relative electric permittivity $\epsilon'_m = \epsilon_m/\epsilon_o$ are frequently used in the place of μ_m and ϵ_m. They represent the extent to which the corresponding material's permeability and permittivity exceed those of free space. These relative magnitudes allow us to write:

$$v = \frac{1}{\sqrt{\mu_m \epsilon_m}} = \frac{1}{\sqrt{\mu'_m \mu_o \epsilon'_m \epsilon_o}} = \frac{1}{\sqrt{\mu'_m \epsilon'_m}} \frac{1}{\sqrt{\mu_o \epsilon_o}} = \frac{c}{\sqrt{\mu'_m \epsilon'_m}} = \frac{c}{n} \tag{7}$$

where $n = \sqrt{\mu'_m \epsilon'_m} = c/v$ is the refractive index of m. With respect to their optical properties, ordinary matter can be isotropic or anisotropic. Crystalline solids, except those of the isometric (cubic) system of symmetry, are anisotropic with respect to the index of refraction: the index of refraction varies with direction. In the case of isotropic materials the index of refraction does not change with direction. All crystals, whether isotropic or anisotropic, show what could be called *polar isotropy*: for any given direction AB through the crystal, the index of refraction is the same when light moves from A to B as when it does from B to A.

As light crosses from a material m_1 of refractive index n_{o1} into another material m_2 of refractive index n_{o2} its velocity changes. As a consequence, the wavefront deviates its trajectory and the rays of light bend at the interface between both media. This phenomenon is the familiar refraction of light, the reason for which a rod partially and obliquely submerged in water seems to be bent just at the interface between air and water. The refraction of light follows Snell's Law, a simple algebraic expression that relates the angles of incidence (i) and of refraction (r) with the refractive indexes (n_{o1}, n_{o2}) of the materials through which light propagates:

$$\frac{\sin\theta_i}{\sin\theta_r} = \frac{c/n_{o1}}{c/n_{o2}} = \frac{n_{o2}}{n_{o1}} \tag{8}$$

If m_1 is a vacuum, then $n_{o1} = 1$ the refractive index of m_2 is:

$$n_{o2} = \frac{\sin\theta_i}{\sin\theta_r} \tag{9}$$

The discussion that follows analyzes the refractive index of isotropic crystals from a relativistic point of view that consider the following three alternatives:

A: The refractive index only makes sense in the proper frame of the corresponding isotropic medium, so that it can only be measured at rest, in its proper frame.

B: The refractive index depend on relative motion and can be measured in all frames.

C: The refractive index does not depend on relative motion and can be measured in all frames.

3.-Case A: the refractive index can only be measured at rest

Let AB be a transparent isotropic rod of proper length x_o placed parallel the X_o axis of its rest frame RF_o, and assume a photon ϕ moves *through* AB from its end A to its end B. Both, the length x_o of AB and the time t_o it takes ϕ to go from A to B inside AB can be measured in RF_o. If n_o is the refractive index of AB, it can be written:

$$c_o = \frac{x_o}{t_o} = \frac{c}{n_o} \tag{10}$$

$$t_o = \frac{n_o x_o}{c} \tag{11}$$

where $c_o = x_o/t_o$ is the speed of the photon ϕ through AB measured in RF_o; $n_o = c/(x_o/t_o)$ is the corresponding index of refraction determined in RF_o, and the speed of ϕ through AB is simply defined as the ratio of the length AB (or of any part of AB) to the time the photon ϕ takes to traverse it, which is the physical definition of speed [1, p. 514]. Note that c_o is also the speed of ϕ with respect to RF_o. In the frame RF_v, from which RF_o moves parallel to the X_v at the uniform velocity $v = kx, 0 < k < 1$, the length x_v of AB and the time t_{vab} it takes ϕ to go from A to B *through* AB can also be measured by means of the rulers and clocks of RF_v. And they can also be deduced from LT. As in RF_o, and according to the same definition of speed, the ratio x_v/t_v is the speed c_{vab} of light *through* AB determined in RF_v. And being c universal, the ratio c/c_{vab} is the refractive index n_v of AB in the direction from A to B and determined in RF_v. Now then, with respect to RF_v, the photon ϕ moves a distance $x_v + kct_v$ at a speed which is the relativist sum of the speeds c_o and kc, both, the speed and the distance, respectively greater than c_{vab} and x_v. So, contrarily to what happens in RF_o, the speed c_{vab} is not the speed of ϕ with respect to RF_v. And it is by this difference that this section is discussing on the possibility that $c_{vab} = x_v/t_{vab}$ were not the speed of light through AB, in spite of the fact that it is the ratio of a distance x_v through an object (AB) to the time t_{vab} it takes another object (ϕ) to traverse it, i.e a speed according to the physical definition of speed.

So, we are not dealing here with a case in which a set of measurements is being referred to a moving frame, because x_v and t_{vab} are measured and referred to RF_v by observers at rest in RF_v. What is being discussed is if the ratio of the distance

x_v to the time t_{vab}, legitimately observed and measured in RF_v, is not the speed of an object through another object simply because it is not at the same time the speed of the first object (the photon ϕ) with respect to RF_v. Now then, if the definition of *speed through an object* only holds for objects observed at rest, this restriction should be explicitly declared in both the physical definition of speed and the First Principle of relativity: the laws of physics are the same in all frames, unless the involved speeds are speeds through objects in relative motion. Evidently, according to this restriction of the First Principle of relativity, certain physical phenomena as the reflection or the refraction of light moving through two transparent media, air and water for instance, could only be examined and interpreted in physical terms in the rest frame of the corresponding transparent media. Obviously, this would make special the rest frames compared with the moving ones. Therefore, on the basis of the current physical definition of speed and the current statement of the First Principle of relativity, it can be assumed that the ratio of the distance that an object moves through another object to the time elapsed in the trip is the velocity of the first object through the second one, be this second object, or not, in relative motion. As a consequence, the index of refraction, be it or not a universal constant, could be measured in all frames. In the last section on conclusions it will be shortly examined the possibility of a discrete spacetime where the above discussion would no longer make sense.

4.-Case B: the refractive index as a relative constant

The refractive index of a crystal depends very closely on its internal structure, which in turns is a consequence of the physicochemical laws driving the nucleation and growing of crystals according to their corresponding ionic (atomic or molecular) composition. As is well known for more than a century, and exhaustively confirmed by X-ray diffraction and other experimental analysis, the internal structure of a crystalline material is essentially periodic in any spacial direction within the crystal: in any of such directions there is the same density and types of particles and chemical bonds separated by its own repetition period (structural anisotropy), which is the same in each of the two senses of each considered direction within the crystal. Contrarily to what happens in anisotropic crystals, when a polarized beam of light strikes an isotropic crystal, the beam does not split into two polarized (ordinary and extraordinary) beams, nor there is an alteration of its direction of polarization. The polarized beam passes through the crystal at a speed that depends on the refractive index of the crystal, which is a universal property for each isotropic mineral species. The trajectory of the polarized beam is then deviated according to Snell's Law, because isotropic crystals obey Snell's Law. In isotropic crystals (minerals of the isometric system), the refractive index is always the same for each mineral species and wavelength, whatsoever be the angle of incidence.

That said, let A and B be two points within an isotropic crystal ICR of refractive index n_o in its rest frame RF_o. Whatsoever be the direction of the straight line AB joining A and B, light travels through ICR from A to B at the same speed $c_o = c/n_o$, because, being an isotropic crystal, ICR has the same refractive index n_o in all directions. Assume AB is placed parallel to X_o, being its proper length x_o. In RF_o a photon ϕ will travel from A to B, and from B to A, in the same time t_o given by:

$$\frac{c}{n_o} = \frac{x_o}{t_o} \tag{12}$$

From the perspective of the frame RF_v, the crystal ICR moves at a uniform velocity $v = kc, 0 < k < 1$, parallel to X_v. We are assuming in this section that it is possible to determine the refractive index of an isotropic crystal in relative motion, i.e. that it is possible to determine the speed of light *through* a crystal in relative motion. This speed can only be the ratio of the distance a photon travels *within* the crystal, for instance the length of AB or of any part of AB, to the time it takes in completing the trip. In our case, the photon ϕ travels a horizontal distance $\gamma^{-1}x_o$ (the length of AB at the relative velocity kc) for a time t_{vab}. This time t_{vab} will be calculated in two different ways to test each other. In the first way t_{ab} will be calculated by simply application of LT to t_o, which is the RF_o-interval of time between two events (ϕ starts moving at A, and ϕ ends moving at B) separated by a proper distance x_o in the direction of the relative motion. In the second way, it will be calculated as the time elapsed while ϕ traverses the RF_v-distance $\gamma^{-1}x_o + kct_{vab}$ at the speed resulting from the relativistic sum of c/n_o and kc, which is the speed of ϕ with respect to RF_v. In the first case, and according to LT it holds:

$$t_{vab} = \gamma t_o + \frac{\gamma x_o kc}{c^2} \tag{13}$$

$$= \gamma\left(t_o + \frac{x_o k}{c}\right) \tag{14}$$

As noted above, t_{vab} is also the time ϕ takes to traverse the RF_v-distance $\gamma^{-1}x_o + kct_{vab}$ at the RF_v-velocity c_v resulting from the relativistic sum of c/n_o and kc, which is given by:

$$c_v = \frac{c/n_o + kc}{1 + \dfrac{kcc/n_o}{c^2}} = \frac{(c + n_o kc)/n_o}{(n_o + k)/n_o} = \frac{c(1 + n_o k)}{n_o + k} \tag{15}$$

In consequence, it can be written:

$$t_{vab} = \frac{\gamma^{-1}x_o + kct_{vab}}{\frac{c(1+n_o k)}{n_o + k}} = \frac{(\gamma^{-1}x_o + kct_{vab})(n_o + k)}{c(1+n_o k)} \tag{16}$$

$$ct_{vab}(1+n_o k) = (\gamma^{-1}x_o + kct_{vab})(n_o + k) \tag{17}$$

$$ct_{vab}(1+n_o k) = \gamma^{-1}x_o(n_o + k) + kct_{vab}(n_o + k) \tag{18}$$

$$ct_{vab}(1+n_o k - n_o k - k^2) = \gamma^{-1}x_o(n_o + k) \tag{19}$$

$$ct_{vab}(1-k^2) = \gamma^{-1}x_o(n_o + k) \tag{20}$$

$$ct_{vab}\gamma^{-2} = \gamma^{-1}x_o(n_o + k) \tag{21}$$

$$t_{vab} = \gamma\left(\frac{n_o x_o}{c} + \frac{k x_o}{c}\right) \tag{22}$$

$$= \gamma\left(t_o + \frac{x_o k}{c}\right) \tag{23}$$

which coincides with (14). Therefore, if c_{vab} denotes the speed of light when going from A to B *through ICR*, and measured in RF_v, it can be written:

$$c_{vab} = \frac{\gamma^{-1}x_o}{\gamma\left(t_o + \frac{x_o k}{c}\right)} = \frac{\gamma^{-2}}{\frac{t_o}{x_o} + \frac{k}{c}} = \frac{\gamma^{-2}}{\frac{n_o}{c} + \frac{k}{c}} = \frac{c(1-k^2)}{n_o + k} \tag{24}$$

In RF_v the refractive index n_{vab} of *ICR* in the direction from A to B is, therefore:

$$n_{vab} = \frac{c}{\frac{c(1-k^2)}{n_o + k}} \tag{25}$$

$$= \frac{n_o + k}{1 - k^2} \tag{26}$$

Let us now assume the photon ϕ moves in the same direction AB parallel to X_v but in the opposite sense, i.e. from B to A. From the perspective of RF_o it can be written:

$$\frac{c}{n_o} = \frac{x_o}{t_o} \tag{27}$$

Denoting by t_{vba} the RF_v-time ϕ takes to go from B to A through AB, it can be written for RF_v:

$$t_{vba} = \gamma t_o - \frac{\gamma x_o k c}{c^2} \tag{28}$$

$$= \gamma\left(t_o - \frac{x_o k}{c}\right) \tag{29}$$

The time t_{vba} is also the time ϕ takes to traverse the RF_v-distance $\gamma^{-1}x_o - kct_{vba}$ at the RF_v-velocity c'_v resulting from the relativistic sum of c/n_o and kc, now in the same direction but in opposite senses, which is given by:

$$c'_v = \frac{c/n_o - kc}{1 - \frac{kcc/n_o}{c^2}} = \frac{(c - n_o kc)/n_o}{(n_o - k)/n_o} = \frac{c(1 - n_o k)}{n_o - k} \tag{30}$$

In consequence, it can be written:

$$t_{vba} = \frac{\gamma^{-1}x_o - kct_{vba}}{\frac{c(1-n_o k)}{n_o - k}} = \frac{(\gamma^{-1}x_o - kct_{vba})(n_o - k)}{c(1-n_o k)} \tag{31}$$

$$ct_{vba}(1-n_o k) = (\gamma^{-1}x_o - kct_{vba})(n_o - k) = \gamma^{-1}x_o(n_o - k) - kct_{vba}(n_o - k) \tag{32}$$

$$ct_{vba}(1 - n_o k + k(n_o - k)) = \gamma^{-1}x_o(n_o - k) \tag{33}$$

$$ct_{vba}(1 - n_o k + n_o k - k^2) = \gamma^{-1} x_o (n_o - k) \tag{34}$$

$$ct_{vba}(1 - k^2) = \gamma^{-1} x_o (n_o - k) \tag{35}$$

$$ct_{vba}\gamma^{-2} = \gamma^{-1} x_o (n_o - k) \tag{36}$$

$$t_{vba} = \gamma\left(\frac{n_o x_o}{c} + \frac{k x_o}{c}\right) \tag{37}$$

$$= \gamma\left(t_o + \frac{k x_o}{c}\right) \tag{38}$$

which coincides with (29). Therefore, if c_{vba} denotes the speed of light when going from B to A *through ICR*, and measured in RF_v, it can be written:

$$c_{vba} = \frac{\gamma^{-1} x_o}{\gamma\left(t_o - \frac{x_o k}{c}\right)} = \frac{\gamma^{-2}}{\frac{t_o}{x_o} - \frac{k}{c}} = \frac{\gamma^{-2}}{\frac{n_o}{c} - \frac{k}{c}} = \frac{c(1-k^2)}{n_o - k} \tag{39}$$

In RF_v the refractive index n_{vba} of ICR in the direction from B to A is, therefore:

$$n_{vba} = \frac{c}{\frac{c(1-k^2)}{n_o - k}} = \frac{n_o - k}{1 - k^2} \tag{40}$$

which is different from n_{vab}, and the difference increases with k (Figure 1). So then, according to LT, light travels through

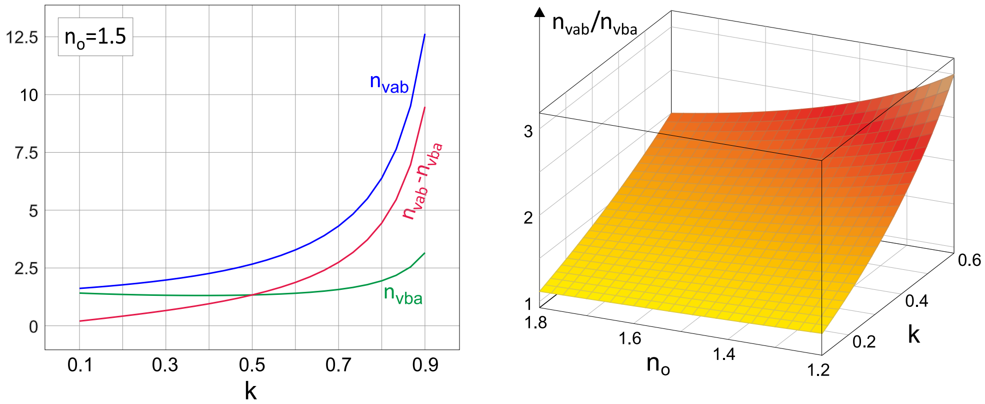

Fig. 1 – Left: the relative refractive indices n_{vab} and n_{vba} and the ratio between them in terms of the relative velocity coefficient k and the refractive index at rest $n_o = 1.5$ (note that n_{vab} can be several times greater than n_{vba}) Note that polar anisotropy increases exponentially with relative velocity. Right: Surface of n_{vab} in terms of n_o and k.

an isotropic crystal at different speeds in each sense (from A to B, and from B to A) of the same direction AB. Let us call *polar anisotropy* to this relativistic anisotropy of the refractive index. As Figure 1 shows, this polar anisotropy is far from being infinitesimal: the refractive index in one of the senses of the same direction can be several times greater than the one in the other sense. The problem is that all of our empirical and theoretical knowledge in the field of optical crystallography indicates such a polar anisotropy does not exist in the rest frame of crystals, whether isotropic or anisotropic, not even at the infinitesimal scale compatible with experimental detection.

Let us now consider the general case in which ϕ travels through the isotropic crystal ICR in any direction DE that makes an angle $\alpha_o, 0^\circ \leq \alpha_o \leq 360^\circ$, with the axis X_o of RF_o (hereafter direction α_o). Since ICR is isotropic, the refractive index in the direction DE will continue to be n_o. Assume ϕ moves a distance h_o through ICR whose respective horizontal and vertical components are x_o and y_o respectively parallel to X_o and Y_o. In RF_o it holds:

$$\frac{c}{n_o} = \frac{h_o}{t_o} = \frac{x_o}{t_o|\cos(\alpha_o)|} \tag{41}$$

$$\frac{t_o}{x_o} = \frac{n_o}{c|\cos(\alpha_o)|} \tag{42}$$

From the perspective of the frame RF_v, ϕ moves through ICR a distance h_v for a time t_v, being $\gamma^{-1}x_o$ and y_o respectively the horizontal (parallel to X_v) and vertical (parallel to Y_v) components of h_v. The double sign $\pm$ is used because the LT term

$\gamma x_o kc/c^2$ (difference in phase synchronization) is positive in the direction of the relative motion (increasing x) and negative in the opposite one. Since the sign of $\cos(\alpha_o)$ changes in the same way, $\pm$ will opportunely replaced with $+$. Thus, from the perspective of RF_v it can be written:

$$\frac{h_v}{t_v} = \frac{\sqrt{\gamma^{-2}x_o^2 + y_o^2}}{\gamma t_o \pm \dfrac{\gamma x_o kc}{c^2}} = \frac{\sqrt{\gamma^{-2} + \tan^2(\alpha_o)}}{\gamma \dfrac{t_o}{x_o} \pm \dfrac{\gamma k}{c}} = \frac{\sqrt{\gamma^{-2} + \tan^2(\alpha_o)}}{\gamma \dfrac{t_o}{h_o|\cos(\alpha_o)|} \pm \dfrac{\gamma k}{c}} \tag{43}$$

$$= \frac{\sqrt{\gamma^{-2} + \tan(\alpha_o)}}{\gamma \dfrac{n_o}{c|\cos(\alpha_o)|} \pm \gamma \dfrac{k}{c}} = \frac{\sqrt{\gamma^{-2} + \tan^2(\alpha_o)}}{\dfrac{\gamma}{c}\left(\dfrac{n_o}{|\cos(\alpha_o)|} \pm k\right)} \tag{44}$$

$$= \frac{c\sqrt{\gamma^{-2} + \tan^2(\alpha_o)}}{\gamma\left(\dfrac{n_o + k\cos(\alpha_o)}{|\cos(\alpha_o)|}\right)} = \frac{c|\cos(\alpha_o)|\sqrt{1 - k^2 + \tan^2(\alpha_o)}}{\gamma(n_o + k\cos(\alpha_o))} \tag{45}$$

$$= \frac{c|\cos(\alpha_o)|\sqrt{\sec^2(\alpha_o) - k^2}}{\gamma(n_o + k\cos(\alpha_o))} = \frac{c|\cos(\alpha_o)|\sqrt{1/\cos^2(\alpha_o) - k^2}}{\gamma(n_o + \cos(\alpha_o))} \tag{46}$$

$$= \frac{c\sqrt{1 - k^2\cos^2(\alpha_o)}}{\gamma(n_o + k\cos(\alpha_o))} \tag{47}$$

Therefore, if $c_{v\alpha_v}$ and $n_{v\alpha_v}$ denote respectively the speed of ϕ through ICR and the refractive index of ICR, both in the RF_v-direction α_v, which is related to the RF_o-direction α_o through $\tan(\alpha_v) = \gamma\tan(\alpha_o)$, it can be written:

$$\frac{c}{n_{v\alpha_v}} = c_{v\alpha_v} = \frac{h_v}{t_v} = \frac{c\sqrt{1 - k^2\cos^2(\alpha_o)}}{\gamma(n_o + k\cos(\alpha_o))} \tag{48}$$

$$n_{v\alpha_v} = \frac{\gamma(n_o + k\cos(\alpha_o))}{\sqrt{1 - k^2\cos^2(\alpha_o)}} = \frac{n_o + k\cos(\alpha_o)}{\sqrt{(1-k^2)(1 - k^2\cos^2(\alpha_o))}} \tag{49}$$

For the particular values $\alpha_o = 90^\circ$ and $\alpha_o = 270^\circ$ it holds:

$$n_{v90^\circ} = n_{v270^\circ} = \frac{n_o}{\sqrt{1-k^2}} = \gamma n_o \tag{50}$$

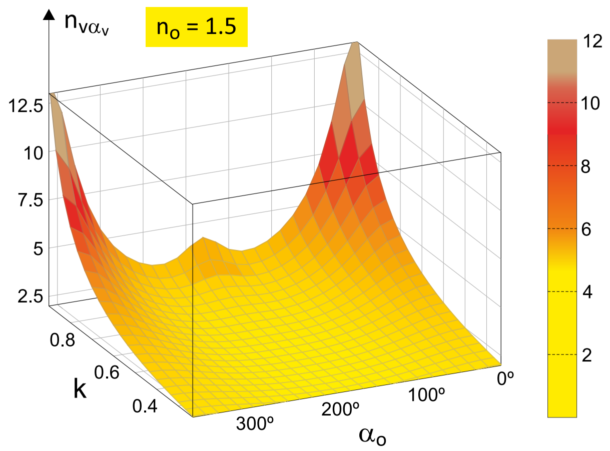

Fig. 2 – The anisotropy of an isotropic crystal in relative motion according to LT (for an isotropic crystal of refractive index at rest $n_o = 1.5$). Note that some of them can be more than six times greater than other, which is hard to explain in terms of structural and optical crystallography.

Hence, in the vertical direction parallel to Y_v, and only in it, there is no polar anisotropy. On the other hand, and as Figure 2 illustrates, LT transforms any isotropic crystal into an anisotropic one whose birefringence (the difference between the maximum and the minimum refractive index of a transparent material) can be (depending on the relative velocity kc) several times greater than the natural birefringence of anisotropic minerals: the birefringence of a highly birefringent mineral as rutile is 0.287, while Figure 2 shows birefringences thirty times greater than that of the rutile. There is no way to explain such a difference in crystallographic terms. We must conclude that LT transforms the proper world at rest in a world in which isotropic minerals cannot exist and all minerals, whether isotropic or anisotropic, exhibit a type of anisotropy until now unknown in the physical world: polar anisotropy. We have proved that the refractive index n_{vab} is greater than the refractive index n_{vba}, which means light travels through ICR faster in the direction from B to A than in the direction from A to B. If that unexpected polar anisotropy were real, related to the periodic internal structure of ICR, it would also be present but in the opposite direction after rotating ICR 180º. But this is not what happens. What happen is that, according to LT, after rotating ICR light travel faster now from A to B than from B to A. Polar anisotropy can only be, therefore, a mathematical artifact unrelated to the physical electromagnetic interactions between light an the internal periodical structure of crystals driving the speed of light through them. And for the same reasons, the relativistic impossibility of isotropic crystals, of which we have the highest empirical and theoretical evidence, can only be another mathematical artifact unrelated to the true electromagnetic interactions between light and isotropic minerals.

We could accept that a ruler may have different lengths, one for each observer that observes it a different relative velocity (though it is hard to believe that a ruler could have simultaneously an indefinite number of different lengths). But it seems unacceptable that each mineral species has simultaneously an indefinite number of internal structures, one for each possible relativistic observer, simply because a mineral can only belong to one of the 230 spacial groups of symmetry (or to a few number of them in the case of polymorphic minerals) defining its internal structure, which are the consequences of the physicochemical laws driving crystallogenesis. In its turn, the internal structure of crystals are the responsible for their electromagnetic interactions with light, which are universal attributes of each mineral species. And recall that in the derivation of the anisotropic refractive index $n_{v\alpha_v}$ and its consequences have intervened not only Fitzgerald-Lorentz contraction, but also time dilation and, particularly important in the deduction of polar anisotropy, the difference in phase synchronization. LT seems to transform the actual world into a deformed reality that can only be apparent. And what is worse, an apparent world whose appearance disagrees with the known laws of structural and optical crystallography.

The above results have been deduced from the initial hypothesis that the refractive indexes of transparent media are not universal constants, as is the case of the refractive index of free space, but relative constants whose relative values can be determined in relative motion. Under this hypothesis, LT has been proved to be inappropriate. So, to end this section, let us ask for another relativistic transformation different form LT that could be compatible with the laws of optical crystallography. As we know, LT includes one functional factor for converting between relative lengths in the direction of relative motion, and two functional factors for converting between relative times, including the differences in phase synchronization in both senses of the direction of relative motion, the three of them depending exclusively on the relative velocity factor k. The length functional factor $L(k)$ immediately follows from:

$$\forall x_o : x_v = \gamma^{-1} x_o = \sqrt{1-k^2}\,x_o \tag{51}$$

$$L(k) = \sqrt{1-k^2} \tag{52}$$

In our case, the functional factors $T_{ab}(k)$ and $T_{ba}(k)$ are immediately derived from LT. Indeed, consider a photon moving through free space and so that in RF_o it travels from the end A to the end B of a rest ruler AB of proper length x_o and parallel to the axis X_o. In RF_o the photon lasts the same time t_o in going from A to B as in going from B to A, t_o. According to LT, things are not that way in RF_v:

$$t_{vab} = \gamma t_o + \frac{\gamma k c x_o}{c^2} = \gamma t_o + \frac{\gamma k c c t_o}{c^2} = \gamma(1+k)t_o \tag{53}$$

$$t_{vba} = \gamma t_o - \frac{\gamma k c x_o}{c^2} = \gamma t_o - \frac{\gamma k c c t_o}{c^2} = \gamma(1-k)t_o \tag{54}$$

So that:

$$T_{ab}(k) = \frac{1+k}{\sqrt{1-k^2}} \tag{55}$$

$$T_{ba}(k) = \frac{1-k}{\sqrt{1-k^2}} \tag{56}$$

As could not be otherwise, $T_{ab}(k) \neq T_{ba}(k)$. And it cannot be otherwise because from the perspective of RF_v the photon travels different distances when going from A to B ($L(k)x_o + kct_v$) and when going from B to A ($L(k)x_o - kct_v$), while the speed of light c is the same in both cases (Second Principle of Relativity). As in the case of LT, the new transformation we are asking for, must have at least one functional factor $L^*(k)$ for converting between relative lengths, and two functional factors $T^*_{ab}(k)$ and $T^*_{ba}(k)$ for converting between relative times. In the case of the above photon moving through ICR from A to B we would have:

$$c_{vab} = \frac{L^*(k)x_o}{T^*_{ab}(k)t_o} = \frac{L^*(k)}{T^*_{ab}(k)t_o/x_o} \tag{57}$$

According to (12), $t_o/x_o = n_o/c$, so that:

$$c_{vab} = \frac{L^*(k)}{T^*_{ab}(k)n_o/c} = \frac{cL^*(k)}{n_o T^*_{ab}(k)} \tag{58}$$

$$c_{vab} n_o T^*_{ab}(k) = cL^*(k) \tag{59}$$

$$T^*_{ab}(k) = \frac{cL^*(k)}{c_{vab} n_o} \tag{60}$$

$$T^*_{ab}(k) = \frac{n_{vab}}{n_o} L^*(k) \tag{61}$$

In the case of the above photon moving through *ICR* from *B* to *A* we would have:

$$c_{vba} = \frac{L^*(k)x_o}{T^*_{ba}(k)t_o} = \frac{L^*(k)}{T^*_{ba}(k)t_o/x_o} = \frac{L^*(k)}{T^*_{ba}(k)n_o/c} = \frac{cL^*(k)}{n_oT^*_{ba}(k)} \tag{62}$$

$$c_{vba}n_oT^*_{ba}(k) = cL^*(k) \tag{63}$$

$$T^*_{ba}(k) = \frac{cL^*(k)}{c_{vba}n_o} \tag{64}$$

$$T^*_{ba}(k) = \frac{n_{vba}}{n_o}L^*(k) \tag{65}$$

According to (61)-(65), and being polar anisotropy theoretically and experimentally impossible, we would get $T^*_{ab}(k) = T^*_{ba}(k), \forall k$, which goes against the Second Principle of relativity, as noted above.

Let us end this section by summarizing its main results. To assume the refractive index is a relative constant in isotropic minerals that can be measured in relative motion has the following unacceptable consequences:

1.-Isotropic crystals show polar anisotropy, which is incompatible with all of our theoretical and experimental knowledge in optical and structural crystallography.

2.-Isotropic crystals, of whose existence we have the highest theoretical and empirical evidence, cannot exist in relative motion.

3.-Some universal properties of isotropic crystals, as their respective refractive indexes, become non-universal thanks to LT.

4-.-There is no relativistic alternative to LT able to resolve the above optical conflicts.

5.-Case C: the refractive index as a universal constant

This section assumes that the magnetic permeability μ_m and the electric permittivity ϵ_m of any transparent medium *m* are universal constants, as is the case of the magnetic permeability μ_o and electric permittivity ϵ_o of a vacuum (free space). In these conditions, and considering that any algebraic combination of universal constants is also a universal constant, it can be written:

$$\left.\begin{aligned} c &= 1/\sqrt{\mu_o\epsilon_o} \\ \mu'_m &= \mu_m/\mu_o \\ \epsilon'_m &= \epsilon_m/\epsilon_o \\ n &= \sqrt{\mu'_m\epsilon'_m} \end{aligned}\right\} \text{are universal constants}$$

Thus, to assume the refractive index of an isotropic medium is a universal constant is a logical consequence of assuming its magnetic permeability and its electric permittivity are universal constants. Assuming that this is the case, the next discussion analyzes, from a relativistic point of view, the critical angle of reflection, the minimal angle of incidence at which the refraction of light between two isotropic media ceases to occur and all light is internally reflected.

When light crosses from a isotropic material of refractive index n_{o1} into another isotropic material of a less refractive index n_{o2}, the angle of incidence is less than the angle of refraction. As the angle of incidence increases the angle of refraction approaches to 90º. There is a critical angle of incidence θ_c ($0^o < \theta < 90^o$) for which the refracted angle is just 90º. Over this critical angle no refraction occurs and all incident rays are reflected. Let us call *critical reflection* to the reflection of light when the incident angle is just the critical angle. We will examine the critical reflection of light from the perspective of two inertial reference frames that move relative to each other. Let RF_o be the proper frame of m_1 and m_2, two isotropic transparent materials whose refractive indices are respectively n_{o1} and n_{o2}, being $n_{o1} > n_{o2}$. The critical angle of incidence is immediately deduced from Snell's Law: it is the angle of incidence θ_c for which the refracted angle is 90º:

$$\frac{\sin\theta_c}{\sin 90} = \frac{n_{o2}}{n_{o1}} \tag{66}$$

$$\sin\theta_c = n_{o2}/n_{o1} \tag{67}$$

$$\theta_c = \arcsin n_{o2}/n_{o1} \tag{68}$$

Assuming the refractive indices of isotropic transparent media are universal constants, any algebraic combination of them, as n_{o2}/n_{o1}, is also a universal constant. In consequence, the critical angle θ_c ($0^o < \theta < 90^o$) is a universal constant for any two given isotropic transparent materials. Assume that in m_1 a laser beam is emitted inclined to the Normal by the critical angle θ_c. The critical reflection occurs and a visible light is emitted by a critical sensor placed at the appropriate vertical distance

y_{o1} (Figure 3). Assume also a photon of the critical incident ray lasts a time t_o in traversing the distance h_o at its velocity $c_1 = c/n_{o1}$ through m_1. We can write:

$$c_o = \frac{c}{n_{o1}} = \frac{h_o}{t_o} \tag{69}$$

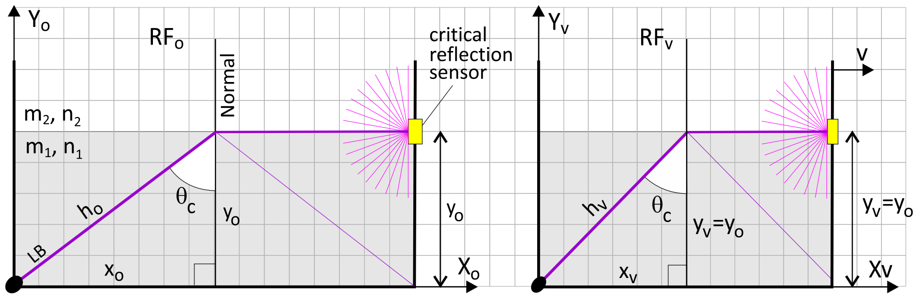

Fig. 3 – Reflection of light at the critical angle for the isotropic transparent materials m_1 and m_2 in RF_o (top) and in RF_v (bottom). In both cases a critical sensor emits a visible light if, and only if, the critical reflection takes place.

From the perspective of RF_v it can be written:

$$\left.\begin{array}{r} n_{v1} = n_{o1} \\ c \text{ universal constant} \end{array}\right\} \Rightarrow c_v = \frac{c}{n_{v1}} = \frac{c}{n_{o1}} = c_o \tag{70}$$

$$\left.\begin{array}{r} y_v = y_o \\ \theta_c \text{ universal constant} \end{array}\right\} \Rightarrow x_v = x_o \tag{71}$$

$$\left.\begin{array}{r} y_v = y_o \\ x_v = x_o \end{array}\right\} \Rightarrow h_v = h_o \tag{72}$$

$$\left.\begin{array}{r} h_v = h_o \\ c_v = c_o \end{array}\right\} \Rightarrow t_v = t_o \tag{73}$$

In consequence, and according to (71) and (73), LT functional factors for lengths $L(k)$ and LT functional factor for times $T(k)$ (52)-(55) must hold:

$$\forall k \in (0,1):\ L(k) = \sqrt{1-k^2} = 1;\ T(k) = \frac{1+k}{\sqrt{1-k^2}} = 1 \tag{74}$$

which is impossible except if $k = 0$, i.e. at rest with respect to RF_o. And for the same reasons, the corresponding length and time factors $L^*(k)$ and $T^*(k)$ of any other relativistic transformation alternative to LT, must also verify:

$$\forall k \in (0,1):\ L^*(k) = T^*(k) = 1 \tag{75}$$

which means that, under the assumption that the refractive indexes of transparent media are universal constants, there cannot be changes neither in lengths nor in times when measured from different frames in relative motion, if the corresponding transformation is compatible with the critical reflection of light.

6.-Conclusions

By assuming the index of refraction is a relative constant depending on relative motion, it has been proved that LT implies the relativistic existence of an impossible polar anisotropy as well as the impossibility of transparent isotropic materials, which also goes against our theoretical and empirical evidence regarding the optical behaviour of isometric minerals. We have then assumed the alternative assumption, according to which the refractive index is a universal constant for each transparent crystalline material. The corresponding discussion has proved that LT cannot account for the critical reflection of light when observed in relative uniform motion, which is another evident break of Lorentz symmetry. The results of both discussions go far beyond the breaking of Lorentz symmetry. They prove there is no other relativistic alternative to explain the considered optical properties, of which we have the highest theoretical and experimental support. We would have to conclude that, at least with respect to these optical properties, the proper (rest) frame of the transparent materials interacting with light is not equivalent to other frames in relative uniform motion. A conclusion that put to the test the Principle of Relativity in its current version.

The special theory of relativity is a theory on the spacetime continuum, but space and time could be discontinuous, discrete, made of indivisible minima, as is the case of matter and energy. It is convenient to recall at this point that the *continuum* is a set theoretical object built on one of the most controverted hypothesis in history, the hypothesis of the actual infinity (subsumed into the Axiom of Infinity founding modern set theories), according to which, and to put it into colloquial terms, the list of the natural numbers exists as a complete totality in spite of the fact that no last number complete the list. The alternative hypothesis of the potential infinity -it is always possible to count a number greater than any given number, but the complete list of numbers does not actually exist- does not deserve the attention of contemporary mathematics, whose main stream is infinitist even in the more pure teoplatonic sense. Physics should not depend on the consistency of a set theoretical axiom that could be inconsistent (for its shortness, I recommend to take a glance at this proof).

Other aspects of our knowledge of reality also point to the discrete nature of both space and time. In this regard, it is worth noting that the factor for converting between continuous and discrete geometries has the algebraic form of the relativistic Lorentz's factor γ, of capital importance in LT. So, ironic as it may be, the theory of special relativity could be a theory on the inconsistent spacetime continuum, whose experimental support comes from an unexpected algebraic relation between discrete and continuous geometries. The discrete interpretation of nature, including space and time, has a significant number of advantages (see A). One of the most relevant is that, contrarily to the points of the spacetime continuum, which are primitive concepts devoid of physical meaning, the indivisible units of space and time would be physical elements of the physical world plenty of physical meaning. This physical fabric of indivisible units of space and time would be the actual scenario where all physical phenomena takes place, being all of them subjected to the same physical laws. Although these physical phenomena could be also be referred to abstract reference frames, these abstract frames could only be defined in agreement with the absolute character of the physical world, including its discrete fabric of space and time.

References

[1] Database Oxford University Press, *Dictionary of Physics,* Oxford University Press, New York, 2009.