Exotic Velocities in Double Relativity

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ABSTRACT: Double relativity is grounded on both, the theory of special relativity and the definition of speed as the ratio of a distance to the time it takes a body to traverse it, even if that distance is in relative motion with respect to the frame where both magnitudes are measured. This paper proves double relativity and special relativity lead to a contradiction involving the speed of light in a vacuum. It, then, proposes two alternatives to solve the inconsistency.

Abbreviations and conventions: SR, special relativity; DR, double relativity; TM, transparent medium; LD, laser device; LS, laser source; T, laser target; DSK, disk of lasers; LT, Lorentz transformation. RFo rest reference frame. RFv reference frame in relative motion with respect to RFo, whose axes coincide with the corresponding axes of RFo at a certain instant. The axes in the plane XY of RFo and RFv will be denoted respectively by Xo, Yo and Xv, Yv. Coordinates, lengths, times and refractive indices measured in RFo and RFv will be respectively sub-indexed by o and v. From the perspective of RFv, the frame RFo will be assumed to move at a uniform velocity v parallel to Xv in the sense of the increasing x, and such that $v = kc$, $0 < k < 1$, where c is the speed of light in a vacuum.

I. INTRODUCTION: DOUBLE RELATIVITY

Double relativity has nothing to do with Doubly Special Relativity [1] (also associated with the names Deformed Special Relativity and Extra Special Relativity) proposed by Amelino-Camelia as a solution to the problem posed by the incompatibility of Lorentz transformation (LT) and the universal character of Planck length and Planck time. What follows is a discussion on an aspect of the special relativity (SR) that should be termed Double Relativity (DR), because it deals with the motion of an object inside another object, in turn observed in relative motion, but focusing the attention on the relative motion of the first object with respect to second one. For instance, the motion of a photon (first object) through a transparent crystal (second object) that moves relative to a given frame RFv. The calculation of the velocity of the first object with respect to the frame RFv is a classical relativistic problem, but it is not the problem DR is interested in. DR is interested in the velocity of the first object with respect to second one, calculated by means of the rulers and the clocks of RFv, or by making an appropriate use of LT (double relative velocity).

If by means of the clocks and stick meters of a reference frame it is possible to measure the length L of an object and the time t another object, that moves inside the first object, lasts in traversing the first object (from one of its end to the other), the ratio L/t is, according to the definition of speed, the speed of the second object with respect to the first one, be this first object, or not, in relative motion with respect the observers carrying out the measurements. With respect to the rest frame RFo of the second object, the first object (for instance a photon ϕ) moves with the same speed with respect to RFo as with respect to the second object (a transparent medium TM in the case of ϕ). However, in RFv both speeds are different: although ϕ moves for the same time t_v with respect to RFv as with respect to TM, it travels a distance L_v with respect to TM less than with respect to RFv, which is the length L_v of TM plus the distance kct_v that TM moves respect to RFv while ϕ moves from one end of TM to the other. Obviously, the speed of ϕ with respect to RFv is the relativistic sum of the speed of ϕ with respect to RFo plus the speed of RFo with respect to RFv. The key question of the discussion that follows will be if the ratio L_v / t_v is, or not, the speed of ϕ trough TM measured in RFv. Being the ratio of a distance (L_v) to the time (t_v) an object (ϕ) takes in traverse it, L_v/t_v satisfies the definition of speed [2]. So, at least initially, it will be assume it is such a speed. The results of the discussion will force to rephrasing the question in new terms involving the special theory of relativity.

II. EXOTIC ANISOTROPY AND EXOTIC SPEEDS

Consider the laser device LD depicted in Figure 1, which consists of a transparent (isotropic or anisotropic) medium TM inside a transparent cylinder in whose black end a laser source LS emits a laser beam LB towards the target T placed at the other (white) end of the cylinder. Assume now that, as Figure 1 shows, twelve identical LD are radially assembled on a disk DSK, each turned 30º anticlockwise with respect to precedent one. Notice that, as Figure 1 also shows, the actual trajectory through TM of any photon ϕ of a laser beam is always the same: from A to B, where A and B are points of TM. This actual trajectory through TM will be referred to as *optical trajectory*. It can only be described relative to the geometrical and optical elements (if any) of TM, or of its container in the case of a fluid. So, it is independent from the orientation of TM relative to any reference frame: AB could be, for instance, an optical axis, in which case ϕ would always move parallel to the optical axis AB. The trajectory of ϕ with respect to a reference frame, for instance RFo or RFv, will be referred to as *relative trajectory*, and depends on the state of motion of TM with respect to that frame and on the orientation of TM in its rest frame.

In the rest frame RFo of DSK, the optical trajectories of all laser beams have the same length L_o. Therefore, each photon of each laser beam travels through TM the same distance L_o for the same time t_o. In these conditions, the speed c_o through TM of each photon ϕ of each laser beam is given by:

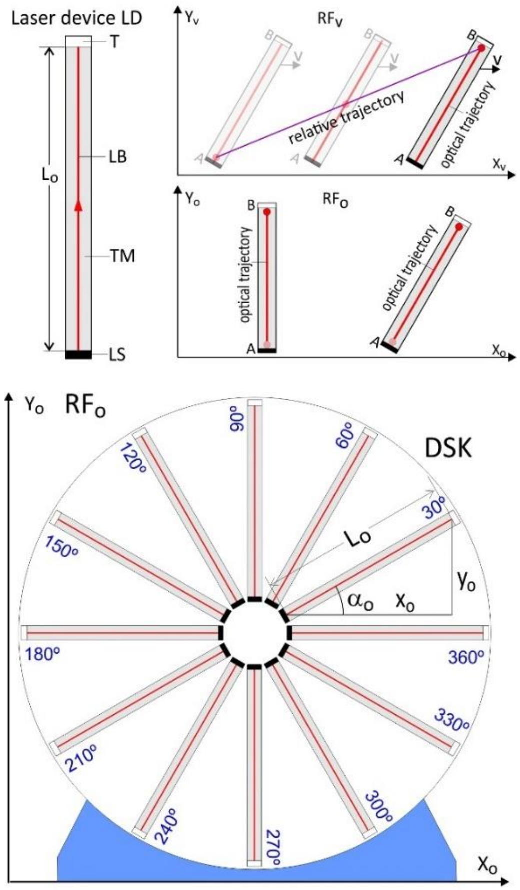

Figure 1. Laser Device LD (top) and Laser disk DSK (bottom). LS: laser source; LB: laser beam; TM: transparent medium; T: target; L_O: length of the optical trajectory.

$$c_o = \frac{L_o}{t_o} = \frac{c}{n_o};\ n_o > 1 \tag{1}$$

where n_o is the refractive index of the transparent medium TM (measured in RF_o). Evidently, it holds:

$$\left(c_o = \frac{c}{n_o}\right) \wedge (n_o > 1) \Rightarrow c_o < c \tag{2}$$

Being DSK at rest in RF_o, the speed c_o is also the speed of ϕ with respect to RFo. Therefore, in the rest frame RFo of DSK, equations (1)-(2) hold for each photon ϕ of each of the twelve laser beams, independently of its orientation. Things are quite different from the perspective of the frame RFv, defined in accordance with the above conventions. In this frame, DSK moves at a velocity v = kc; 0 < k < 1 parallel to its axis Xv, and the distance d_v a photon ϕ moves throw TM from its source LS to its target T (i.e. the length of its optical trajectory) has a horizontal component $x_v = \gamma^{-1} x_o \cos\alpha_o$ and a vertical component $y_v = y_o$, where α_o is the angle the corresponding laser beam of ϕ makes with Xo, i.e. the angle that its relative trajectory in RFo makes with the axis Xo of RFo.. In consequence:

$$d_v = \sqrt{x_v^2 + y_v^2} = \sqrt{\gamma^{-2} L_o^2 \cos^2\alpha_o + L_o^2 \sin^2\alpha_o} \tag{3}$$

$$= L_o\sqrt{(1-k^2)\cos^2\alpha_o + \sin^2\alpha_o} \tag{4}$$

$$= L_o\sqrt{\cos^2\alpha_o - k^2\cos^2\alpha_o + \sin^2\alpha_o} \tag{5}$$

$$= L_o\sqrt{1 - k^2\cos^2\alpha_o} \tag{6}$$

Therefore, in RFv the optical trajectory of each photon of each laser beam in DSK depends on the relative trajectory of that laser beam in the rest frame RFo of its source. The instant at which ϕ begins to move through its optical trajectory is the same as the instant it begins to move through its relative trajectory; and the same applies to the instant it ends both trajectories. So, the time t_v it lasts in completing its optical trajectory is the same as the time its lasts in complete its relative trajectory, which according to LT, is given by:

$$t_v = \gamma\left(t_o + \frac{kcL_o\cos\alpha_o}{c^2}\right) = \gamma\left(t_o + \frac{kL_o\cos\alpha_o}{c}\right) \tag{7}$$

In consequence, the speed c_v through the transparent medium TM of a photon ϕ of a laser beam of DSK, i.e. the ratio of the distance d_v to the time t_v (both measured in RFv) will be:

$$c_v = \frac{d_v}{t_v} = \frac{L_o\sqrt{1-k^2\cos^2\alpha_o}}{\gamma\left(t_o + \frac{kL_o\cos\alpha_o}{c}\right)} \tag{8}$$

$$= \frac{\sqrt{1-k^2\cos^2\alpha_o}}{\gamma\left(\frac{t_o}{L_o} + \frac{k\cos\alpha_o}{c}\right)} \tag{9}$$

$$= \frac{\sqrt{1-k^2\cos^2\alpha_o}}{\gamma\left(\frac{n_o}{c} + \frac{k\cos\alpha_o}{c}\right)} \tag{10}$$

$$= \frac{c\sqrt{1-k^2}\sqrt{1-k^2\cos^2\alpha_o}}{n_o + k\cos\alpha_o} \tag{11}$$

$$= \frac{c\sqrt{(1-k^2)(1-k^2\cos^2\alpha_o)}}{n_o + k\cos\alpha_o} \tag{12}$$

which also depends upon the orientation α_o of the laser beam with respect to RFo. But whatsoever be that orientation, light always travels through TM in the same direction and the same sense: from its source at the black end of each LD to its target at the white end of each LD, i.e. through its optical trajectory. Therefore, light should always move at the same speed through TM, be TM a fluid or a crystalline material, isotropic or anisotropic.

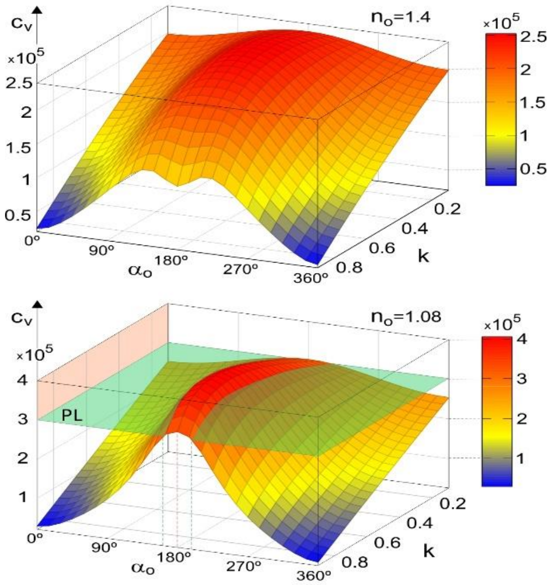

Figure 2. Top: exotic optical anisotropy of a transparent medium of refractive index 1.4. Bottom: for small refractive indexes, the speed of light through the medium can be greater than the speed of light through a vacuum (red points over the green plane PL).

As Figure 2 (top) shows, the anomalous anisotropy (12) is really extreme, with differences of up to one hundred thousand kilometers per second. Thus, while in RFo the speed of light through a transparent medium is always the same for the same optical trajectory irrespective of the orientation of the relative trajectory, in RFv the speed of light through the same optical trajectory in a transparent medium strongly depends on the orientation of the relative trajectory with respect to the rest frame RFo of TM. And things can get worse because for certain refractive indexes of the transparent media, c_v can be greater than the speed of light in a vacuum. For instance, if the transparent medium is oriented at an angle of 180º in RFo, in RFv the speed of light through that transparent medium would be 387931.034 Km/s if its refractive index is 1.08, and 432692.307 Km/s if it is 1.02. So then, in these cases it holds:

$$c_v > c \tag{13}$$

which goes against the Second Principle of SR

III. CONCLUSIONS

According to the above contradiction between the Second Principle of SR and (13), SR and DR are not compatible. We will have to decide which aspects of which of them would have to be changed. SR is founded on two principles, the Principle of Relativity (the laws of physics are the same in all inertial frames) and the Principle of the Speed of Light (the speed of light is the same in all inertial frames), and its key operator for converting between measurements performed at rest and in relative motion is LT. DR is grounded on SR and on the definition of speed as the ratio of the distance traversed by a body to the time taken. A first alternative to solve the incompatibility between SR and DR could be to consider the world transformed by LT as an apparent world, as is apparent, for example, the deformation of a rigid rod partially and obliquely submerged in water. And for the same reasons we cannot use the apparent deformation of the submerged rod to get conclusions on what happen in the internal structure of the rod, we should not use the appearances of the apparent world resulting from LT to get conclusion on what actually happens in that world, under penalty of inconsistencies as the above one. A second alternative could be to restrict the definition of speed so that it does not apply to bodies moving inside other objects when these second objects are observed in relative motion. In these case, the Principle of Relativity should also be modified: the laws of physics are the same in all inertial frames except for the moving parts of bodies. In such a case, the moving parts of bodies could only be examined in the corresponding rest frames of the bodies. At least for this reason, rest frames would be different from frames observed at relative motion.

REFERENCES

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[2]. Daintith, J. (2009). *Dictionary of Physics.* (J. Daintith, Ed.) New York: Oxford University Press.