

Electron propagation in a lattice

Jean Louis Van Belle, *Drs, MAEc, BAEC, BPhil*

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Email: jeanlouisvanbelle@outlook.com

Abstract

This paper applies our realist interpretation of quantum mechanics to an analysis of the mechanics of electron propagation through a crystal. We derive a formula for the *effective* mass of an electron which differs by a factor 2 from Feynman's. We think this solves his rather weird remark on the relation between the effective and free-space mass of an electron, which says the effective mass turns out to be 2 to 20 times the free-space mass of the electron. Our calculations imply the effective mass equals the free-space mass in the absence of a potential barrier between successive atoms in an lattice, which is what is to be expected.

In the next version of this paper, we will comment on Feynman's development of the Schrödinger equation in the context of simple linear motion.

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Electron propagation in a lattice

Introduction

Feynman's lectures on how an electron – or an excitation or some other irregularity¹ – might move through a one-, two- or three-dimensional array of atoms (a *lattice*, in other words²) yield the right formulas but – even after multiple readings – leave me completely bewildered because of the imprecision in concepts and the chaotic heuristic arguments. Let us try to do better by exploring possible definitions of the various concepts that are used, and also by injecting some *realism* in the description: what is that we are trying to talk about, *exactly*?³

The results of Feynman's analysis can be summarized in one or two formulas, and so let us try to find out how we get these. The formula we want to understand is the formula for the (kinetic) energy of an electron as it is moving through the lattice:

$$E = E_v = \frac{1}{2} m_{\text{eff}} v^2 = \frac{p v}{2}$$
$$\text{with } m_{\text{eff}} = \frac{p}{v} = \frac{\hbar k}{v} = \frac{\hbar^2}{2A b^2} \Leftrightarrow k = \frac{\hbar v}{2A b^2}$$

The m_{eff} is the *effective* mass of the electron as it moves through the lattice. Feynman tells us that " m_{eff} has nothing to do with the real mass of an electron", although in "commonly used metals and semiconductors it often happens to turn out to be the same general order of magnitude, about 0.1 to 30 times the free-space mass of the electron."⁴ We do not agree with that. There must be *some* relation between the two mass concepts because, otherwise, the concept of a momentum ($p = m_{\text{eff}} v$) and a wavenumber $k = p/\hbar$ would not make any sense and should, therefore, not be used. We will come back to this later.

We know the value of this energy is constrained to a band which is usually referred to as the conduction band. This band is defined by a minimum and a maximum energy level which Feynman writes as a function of some (average) energy level E_0 and some energy *difference* A (or $4A$: we will discuss this 4-factor in a minute). The graph below (Figure 1) plots E as a function of the mentioned wave number $k = p/\hbar = m_{\text{eff}} v/\hbar = \hbar v/2A b^2$. Needless to say, v is the (classical) velocity of the electron in material: quantum physicists think of it as a group velocity of a wave *packet*, but we think it is the actual drift velocity of the pointlike electron.⁵ As for b and A , b is the linear distance between two atoms (or molecules) in the array and A is... A requires some more explaining, so let us start.

¹ We deliberately avoid the use of terms like excitons or phonons for the time being – until we are ready to define them more precisely.

² The term *crystal* is usual added (crystal lattice) to emphasize we are talking a *regular* or repetitive atomic or molecular structure, as opposed to say, a nuclear lattice. We think the context is clear enough.

³ We refer to Feynman's Lectures, Volume III, Chapter 13 and – to a much lesser extent – Chapter 14. We assume the reader is more or less familiar with them, if only because many modern textbooks basically copy Feynman's treatment of the subject – even if it was written in the early 1960s.

⁴ [Feynman's Lectures, Vol. III, p. 13-7](#). The original 1963 print edition talks about "real crystals" (instead of commonly used metals and semiconductors) and gives a range of "2 to 20 times the (free) electron mass."

⁵ See [our paper on de Broglie's matter-wave](#).

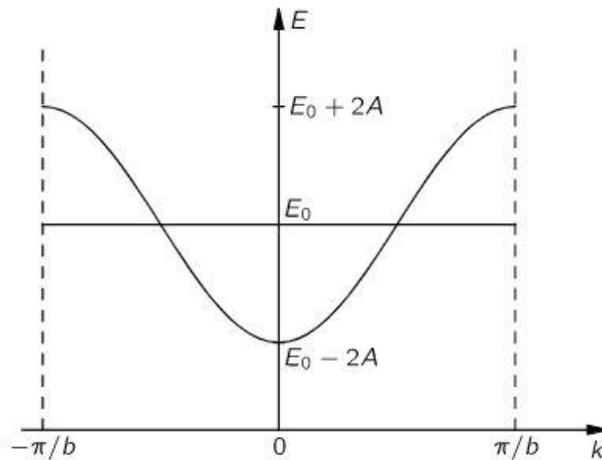


Figure 1: The energy band $E_0 - 2A \leq E \leq E_0 + 2A$ as a function of $k = p/\hbar = \hbar v/2Ab^2$

The amplitude A

We should first note that A and b (and \hbar too, of course) are *given* constants, so k varies with the drift velocity v . We should probably expect this drift velocity to vary with the voltage (potential difference) that is being applied, but let us think about this later. Also note we have a rather annoying factor 2 in the $k = \hbar v/2Ab^2$ formula: the energy difference in a simple two-state system is usually defined as the difference between an $E_0 + A$ and $E_0 - A$ level. Hence, the energy *difference* is usually defined as *twice* A, whatever A might represent. So here we have an energy difference that is equal to *four* times that ubiquitous factor A. We are not sure why right now but we might find out later.

So what *is* that factor A? Feynman does not say all that much about it. He *defines* it as part of iA/\hbar , which is “the amplitude for the electron to jump from one atom to the next (per unit time)”, and then simply states that:

“There is not much more to be said about the various coefficients, such as the amplitude A, which appear in the theory. Generally they are very hard to calculate, so in practical cases very little is known theoretically about these parameters and for any particular real situation we can only take values determined experimentally.”⁶

The factor that gives us the actual *probability* of the electron going from here to there is A, of course. The division by \hbar just *scales* the energy and gives us a value expressed per unit time. Because we cannot assume the reader read our other papers on these matters, it is useful to rewrite the Planck-Einstein relation ($E = h \cdot f = \hbar \cdot \omega$) as $\omega = E/\hbar$. In this particular case, this is to be written as $\omega = A/\hbar$, which reminds us of the *cycle time* between two quantum-mechanical states: $T = 1/f = 2\pi/\omega = 2\pi\hbar/A = h/A$. We remind the reader of Feynman’s illustration of two probabilities ‘*sloshing back and forth*’, as he famously phrased it, as a function of time:

⁶ Feynman’s *Lectures*, Vol. III, p. 13-2 and 13-9. It is customary to include the imaginary unit (i) in the amplitude, but we think this is not justified: the K_{ij} coefficients in the original Hamiltonian equations are real-valued numbers. See our paper on [Feynman’s Time Machine](#), June 2020.

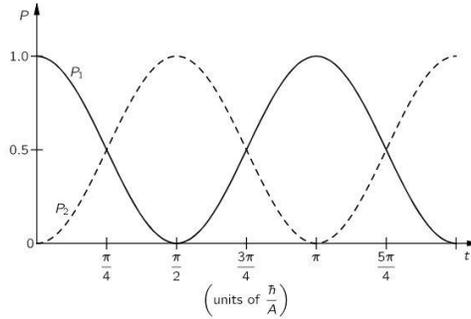


Figure 2: Probabilities sloshing back and forth⁷

The illustration triggers an obvious question: how do we *know* we are in state $|1\rangle$ at $t = 0, \pi, 2\pi, \dots$ as measured in the relevant unit here, which is \hbar/A ? Or – what amounts to the same – state $|2\rangle$ at $t = \pi/2, 3\pi/2, \dots$

The answer is: we do not. That is the point of quantum mechanics: *we do not know* – not exactly, at least. Quantum mechanics models statistical indeterminacy: we do not know the initial conditions and, therefore, we do not know when or how our electron is going to move from one electron to another. However, we should be able to say some more about that value A – more than what the quantum physicists want to give us, in any case.

We may assume the electron lowers its energy when joining an atom (or a molecule⁸) so as to create an ion.⁹ To move from one atom to another, it must, therefore, go through a potential barrier. This is nothing mysterious: potential barriers – or their corollary: potential wells – should not be thought of as static fields: they vary in time. They result from various charges moving around and creating some joint or superposed field which varies in time. Hence, a particle breaking through a ‘potential wall’ or coming out of a potential ‘well’ just uses a temporary opening corresponding to a very classical trajectory in space and in time: think of it being in the right place at the right time – with one addition, perhaps: the *direction* of the (linear or angular) momentum needs to be right too.

Because we are modeling *regular* motion – something with a cycle time (period) or, what amounts to the same, a frequency – we know the electron *must* move at some point in time: otherwise we would not be able to calculate probabilities. Now that we are mentioning the calculation of probabilities, how does that work, *exactly*? Let us continue the example of the two-state system before going back to our semiconductor. We said we could not know for sure that the electron is in state $|1\rangle$ at $t = 0, \hbar/2A, \hbar/A, \dots$ ¹⁰ but the graph obviously assumes we knew at point $t = 0$, at least! Perhaps we had switched on some strong electric field for a while so as to polarize the ammonia molecule—and then we switched it

⁷ We gratefully acknowledge the online edition of Feynman’s Lectures for [this illustration](#) too. The graph measures time in radians, as opposed to seconds. Indeed, cycle times – in seconds, that is – are measured in units of \hbar/A . It is a deep philosophical point which we could – but do not want to – enter into at this point of the narrative.

⁸ It is usually an atom but molecular orbitals are theoretically possible too.

⁹ As you know, we may also have electron *holes*, in which case the ion is positive instead of negative.

¹⁰ We are measuring time in proper seconds now, rather than in units of \hbar/A . We do so by effectively multiplying $0, \pi, 2\pi, \dots$ by \hbar/A . For example, 2π becomes $2\pi\hbar/A = \hbar/A$. It is always useful to recheck the physical dimension: \hbar (or \hbar) is expressed in $\text{N}\cdot\text{m}\cdot\text{s}$, while A is an energy and, hence, is expressed in $\text{N}\cdot\text{m}$. Hence, the division of \hbar by A does yield seconds.

off at $t = 0$. But then we do not know. All we know is that the cycle time is equal to $\pi\hbar/A = h/2A$. Hence, we may expect it will have *swapped* from state $|1\rangle$ to state $|2\rangle$ and then back again in the time interval $h/2A$. That is a full cycle. If we are interested only in it swapping from one state to the other, we may consider that to be the cycle, perhaps, and then the cycle time will be $\pi\hbar/2A = h/4A$. The *frequency* of swapping is just the inverse of the cycle time, of course: $f = 1/T = 4A/h$. We will usually write that as an angular frequency – $\omega = 2\pi f = 4A/\hbar$ – and, just to confuse the uninformed reader and introduce mysterious quantum-mechanical language, we will call that the quantum-mechanical probability *amplitude* for the electron to swap states.

Let us go back to the conduction band of our semiconductor: that factor 4 is no longer so annoying because, yes, an amplitude that is equal to $\omega = A/\hbar$ reads nicer than an amplitude that is equal to $\omega = 4A/\hbar$. So the factor 4 is there – it refers to a quarter (1/4) of a cycle – but you can choose where you put it: in your definition of the potential barrier (which is the conduction band, in this particular case) or in the definition of the amplitude. Thanks for *not* telling us these things, Mr. Feynman: I guess it is part of you challenging your students by not telling them everything, right? :-/

What about normalization? We may think this is normalized already, because it is *real*, right? An amplitude is something *per unit time*, so its *value* is equal the angular frequency but it is actually a *fraction*: the amplitude is A/\hbar seconds *per second*. Hence, this quantum-mechanical amplitude is, effectively, a dimensionless number. How can we relate this to the concept of probabilities or – to be precise – to probability *densities*? We need to take an absolute square somewhere, right? To be precise: we need to square the absolute value of a complex-valued amplitude or wavefunction, right? Not here. It is, again, part of the utterly confusing language quantum physicists like to use to mystify things. The concept of an amplitude may refer to three different things:

1. A complex-valued wavefunction which, for all practical purposes, can always be written as the product of a spatial and time-dependent part – something like this¹¹:

$$\psi(\mathbf{x}, t) = a \cdot e^{i\theta} = a \cdot e^{i\left(\frac{E \cdot t}{\hbar} - \frac{\mathbf{p} \cdot \mathbf{x}}{\hbar}\right)}$$

2. The (real-valued) coefficient in front of the complex exponential(s) in the wavefunction, so that is the a of the $a \cdot e^{i\theta}$ function.

3. The amplitude to go from one state to another, which is given by the fraction of (1) a cycle time (or half or a quarter of it) and (2) the time unit.

These concepts are effectively very different. The complex-valued wavefunction describes a state and is used in a system of Hamiltonian equations which describes all possible states. The amplitude to go from one state to another is a *related* concept but its nature is very different: it is like a probability. Indeed, you should note that the concept of a probability (usually) also involves some fraction of a period or a time unit: if we say that the probability of being in either of two states is 1/2, then we mean to say that it is in this or that state either of the two states about half of the time. Also, if we would know that the

¹¹ Having a plus or a minus sign in the (complex-valued) exponential ($e^{-i\theta}$) is a mere matter of *convention* for most physicists. We think that is a grave mistake: the plus or minus sign is related to the two possible *directions* of spin of a particle. See our paper on [Euler's wavefunction and the double life of -1](#). We will also see the sign in front of the $\mathbf{p} \cdot \mathbf{x}/\hbar$ term represents the direction of travel, which can be positive or negative. The plus or minus sign is, therefore, not random. That is also why we prefer to write $\mathbf{p} \cdot \mathbf{x}$ as a vector (dot) product.

electron is in one of two possible states, then the probability of it going into the other state would be calculated as a function of (1) the time that has elapsed already, and (2) the (normal or average) cycle time. So, yes, it would involve that amplitude to go from one state to another!

Let us not worry about these concepts too much right now. We might come back to it. Let us continue to try to describe what might be happening in a semiconductor.

The elementary wavefunction

Planck's quantum of action (h or, in its reduced form, \hbar) models an elementary cycle in Nature. What's an elementary cycle? It can be anything: we think of an electron itself as an oscillating charge¹², a photon, a ring current in a superconductor is an oscillation too, an atomic or molecular orbital obeys the same law, an oscillation in a two-state system, etcetera. Regardless of the specifics of the situation, the frequency of these oscillations is always given by the Planck-Einstein relation: $E = h \cdot f \Leftrightarrow f = 1/T = E/h$.

The specifics of the situation here may look very different: we are, effectively, *not* looking at some stationary system here. Our electron moves, in either of two directions depending on the potential difference that is being applied. However, all motion is relative and so we might consider using relativity theory to analyze what may or may not be going on in the frame of reference of the electron itself.

Huh? Yes. Just hang in there. It will be alright. Let us consider the idea of a particle (it is our electron, of course) traveling in the positive x -direction at constant speed v . This idea implies a pointlike concept of position: we think the electron will be *somewhere* at some point in time. The *somewhere* in this expression does not mean that we think the particle itself is dimensionless or pointlike. In fact, we think it is *not*. In this particular case, we think of the electron as orbiting around a nucleus¹³, so it takes up a rather large space. To be precise, the order of magnitude of the *radius* of this oscillation will be that of the Bohr radius of an atom, so that is about 150 times the electron's Compton radius.¹⁴ Hence, the idea of position here only implies that we can think of some *center* of this oscillation, which we think of as being *physical*: a pointlike charge going round and round in a rather regular fashion (otherwise our idea of a cycle time and an orbital would not make no sense whatsoever). Hence, we have a *quantization* of space here but it is very different from, say, the quantization of space that results from applying the Planck-Einstein relation to the idea of a free pointlike charge (i.e. the *Zitterbewegung* or ring current model of an electron).

OK. We get this. Let us now go back to the *motion* of our electron. Two extreme situations may be envisaged: $v = 0$ or $v = c$. However, let us consider the more general case inbetween. In our reference

¹² We refer to [our interpretation of the Zitterbewegung theory](#) of an electron here, which we believe to be true because it explains all of the properties of an electron: its energy (and, hence, its equivalent mass), its magnetic moment (including the small anomaly which, therefore, is not an anomaly at all), and, last but not least, its angular momentum (spin).

¹³ It is an *extra* electron so we have a negative ion instead of a neutral atom. However, this does not fundamentally alter the analysis.

¹⁴ We say 150 because that is close enough to the $1/\alpha \approx 137$ factor that relates the Bohr radius to the Compton radius of an electron. The reader may not be familiar with the idea of a Compton radius (as opposed to the Compton *wavelength*) but we refer him or her to the above-mentioned *Zitterbewegung* (ring current) model of an electron.

frame¹⁵, we will have a position – a mathematical *point* in space, that is¹⁶ – which is a function of time: $x(t) = v \cdot t$. Let us now denote the position and time in the reference frame of the particle itself by x' and t' . Of course, the position of the particle in its own reference frame will be equal to $x'(t') = 0$ for all t' , and the position and time in the two reference frames will be related by Lorentz's equations¹⁷:

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{vt - vt}{\sqrt{1 - \frac{v^2}{c^2}}} = 0$$

$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Hence, if we denote the energy and the momentum of the electron in our reference frame as E_v and $p = \gamma m_0 v$, then the argument of the (elementary) wavefunction $a \cdot e^{i\theta}$ can be re-written as follows¹⁸:

$$\theta = \frac{1}{\hbar} (E_v t - px) = \frac{1}{\hbar} \left(\frac{E_0}{\sqrt{1 - \frac{v^2}{c^2}}} t - \frac{E_0 v}{c^2 \sqrt{1 - \frac{v^2}{c^2}}} x \right) = \frac{1}{\hbar} E_0 \left(\frac{t}{\sqrt{1 - \frac{v^2}{c^2}}} - \frac{\frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} \right) = \frac{E_0}{\hbar} t'$$

This is rather nice. We have just shown that the argument of the wavefunction is relativistically invariant: E_0 is, obviously, the rest energy and, because $p' = 0$ in the reference frame of the electron, the argument of the wavefunction effectively reduces to $E_0 t' / \hbar$ in the reference frame of the electron itself. However, you will say: this has nothing to do with the wavefunction of our electron in the lattice, right? Wrong.

Feynman's wavefunction for the electron in the lattice

We will not repeat Feynman's analysis here¹⁹ but so he develops a system of Hamiltonian equations, and the challenge is to find the amplitude (or wavefunction) for the electron to be at some atom or position n . He writes these amplitudes as $C_1, C_2, \dots, C_{n-1}, C_n, C_{n+1}, \dots$ (you get the idea, right?) but we might as well write them as $\psi_1, \psi_2, \dots, \psi_{n-1}, \psi_n, \psi_{n+1}, \dots$. Fancier, no? And guess what? Feynman shows these C_n or ψ_n functions must look like this:

$$e^{i[kx_n - (\frac{E}{\hbar})t]} = e^{-i(\frac{E}{\hbar}t - \frac{p}{\hbar}x_n)}$$

¹⁵ We conveniently choose our x -axis so it coincides with the direction of travel. This does not have any impact on the generality of the argument.

¹⁶ We may, of course, also think of it as a position *vector* by relating this point to the chosen origin of the reference frame: a point can, effectively, only be defined in terms of other points.

¹⁷ These are the Lorentz equations in their simplest form. We may refer the reader to any textbook here but, as usual, we like Feynman's lecture on it (chapters 15, 16 and 17 of the first volume of Feynman's *Lectures on Physics*).

¹⁸ One can use either the general $E = mc^2$ or – if we would want to make it look somewhat fancier – the $pc = Ev/c$ relation. The reader can verify they amount to the same.

¹⁹ In case you would be doubting, we are still referring to [Chapter 13 in Volume III](#).

It is the same as the elementary wavefunction above. Not approximately, but *exactly*. The minus sign has got to do with the direction of travel, as Feynman readily acknowledges: “The wave can travel toward positive or negative x depending on the sign we have picked for k .” We fully agree, but so that wave *is* the electron.

So far, so good, but we still do not have that formula for $k = p/\hbar = m_{\text{eff}}v/\hbar = \hbar v/2Ab^2$, right? Right. But we have a much better view on what is what now, and so we can see that there is no need to explain k , really: k is just the regular wave number, which we get from the second *de Broglie* relation.²⁰ What we need to explain is that formula for the effective mass:

$$m_{\text{eff}} = \frac{p}{v} = \frac{\hbar k}{v} = \frac{\hbar^2}{2Ab^2}$$

Let us first try to *read* it. The dimensional analysis comes out alright: $\left[\frac{\hbar^2}{2Ab^2} \right] = \frac{\text{N}^2 \cdot \text{m}^2 \cdot \text{s}^2}{\text{N} \cdot \text{m} \cdot \text{m}^2} = \text{N} \frac{\text{s}^2}{\text{m}} = \text{kg}$.

It also makes sense that m_{eff} is *inversely* proportional to A : if the amplitude for an electron to go from one atom to another is *large* – because the electron is only very loosely bound to the ionic atom so the potential barrier is very *low* – then the effective mass of the electron will appear to be *small*. And vice versa, of course.

But what about that b^2 factor? It is there because $x_n = n \cdot b$ ²¹ and, much more importantly, because solving that set of Hamiltonian equations gives us a formula for E . To be precise, it gives us the formula which is shown in Figure 1:

$$E = E_0 - 2A \cdot \cos(kb)$$

What’s E_0 ? It is the average of the maximum and minimum value of E . Are we joking? No. The exact *values* of E and E_0 will depend on some reference point for the energy (the zero of energy, really), but E_0 itself is just the middle value between $E_0 + 2A$ and $E_0 - 2A$.

So far, so good. Now how do we get that m_{eff} formula out of that energy formula? Feynman gets it through (1) the use of the small angle approximation for the cosine factor ($\cos\theta \approx 1 - \theta^2/2$) and (2) choosing $E_0 - 2A$ as the zero of energy (or, what amounts to the same, *defining* E_0 as $E_0 = 2A$). The formula above then effectively reduces to:

$$E = E_0 - 2A \cdot (1 - k^2b^2/2) = E_0 - 2A + Ak^2b^2 = Ak^2b^2$$

Is that legitimate? Let us first think about the second assumption: what *is* that energy E anyway? It is the *extra* energy which an electron gets *as it moves through the lattice*. The energy must, therefore, be *kinetic* (as opposed to potential), and we may effectively use the non-relativistic formula as an excellent approximation because we know those drift velocities are non-relativistic: we are talking like a millimeter per second, or even less. This may be surprising but it is what it is: the electric *signal* is lightning fast, but the electrons in the lattice themselves are covering very small distances only.²² Hence,

²⁰ In case you wonder what the first *de Broglie* relation might be: that is just the Planck-Einstein relation. See the *de Broglie* relations in [the Wikipedia article on the matter wave](#).

²¹ The b is the (linear) distance between two successive atoms: $x_{n+1} = x_n + b$ and x_0 is *defined* as $x_0 = 0$.

²² You can *google* this, or check [the Wikipedia article on drift velocities](#). If you want an analogy, think of the musical chairs game but with the chairs on a line and all players agreeing to kindly move to the next chair for the new

$E = mv^2/2$ and we can now also understand Feynman's choice of the zero of energy: when $v = 0$, E will be zero. The *minimum* value for E must, therefore, be zero too! Hence, this is not just some random assumption: it is part and parcel of the *reality* of what we are modeling here.

This is interesting. We should, of course, note that we are still not sure whether or not we can apply that small angle approximation, but let us revisit that later. Let us also quickly check the logic of the $E = m_{\text{eff}}v^2/2 = Ak^2b^2$ and $p = \hbar k = m_{\text{eff}}v$ equations:

$$m_{\text{eff}}v = \hbar k \Leftrightarrow \frac{2Ak^2b^2}{v} = m_{\text{eff}}v \Leftrightarrow Ak^2b^2 = E = \frac{m_{\text{eff}}v^2}{2}$$

We still do not have that formula for m_{eff} , or for v . At this point, our *realist* interpretation of quantum mechanics differs again from Feynman's. Feynman calculates the velocity v by applying the formula for the *group* velocity of a wave packet:

$$v = \frac{d\omega}{dk} = \frac{d\left(\frac{E}{\hbar}\right)}{dk} = \frac{d(Ak^2b^2)}{\hbar dk} = \frac{2Ab^2}{\hbar} k$$

We do not see a wave packet. We see one wave, representing one particle. The only velocity that matters is the *phase* velocity, which is given by the simple ratio of ω and k :

$$v = \frac{\omega}{k} = \frac{Ak^2b^2}{\hbar k} = \frac{Ab^2}{\hbar} k$$

The difference is a factor 2. It gives a difference in the formula for the effective mass, of course. *My* formula does *not* have that factor 2:

$$m_{\text{eff}} = \frac{p}{v} = \frac{\hbar k}{v} = \frac{\hbar^2 k}{Ab^2 k} = \frac{\hbar^2}{Ab^2}$$

Hence, *my* calculation of the effective mass gives a value which is *twice* that of Feynman:

$$m_{\text{eff}} = \frac{p}{v} = \frac{\hbar k}{v} = \frac{\hbar^2 k}{2Ab^2 k} = \frac{\hbar^2}{2Ab^2}$$

Who is right? We think we are, of course! Why? Because the original 1963 print edition of Feynman's Lectures says this:

“Don't forget that m_{eff} has nothing to do with the real mass of an electron. It may be quite different – although in real crystals it often happens to turn out to be the same general order of magnitude, about 2 to 20 times the free-space mass of the electron.”²³

In our *realist* interpretation of quantum physics, m_{eff} *must* be related to the free-space mass of the electron, and it must be 1 to 10 times (or more) the free-space mass of the electron. 😊

arrival and – importantly – the last person on the last chair agreeing to leave the game to get a beer. The *signal* moves fast, from start to end, but the persons themselves move only from one chair to the next.

²³ See footnote 4. We realize the reader may find this difficult to check, but we trust he or she will trust what we write here.

Conclusions

This paper applied our realist interpretation of quantum mechanics to an analysis of the mechanics of electron propagation through a crystal. We derived a formula for the *effective* mass of an electron which differs by a factor 2 from Feynman's. We think this solves his rather weird remark on the relation between the effective and free-space mass of an electron, which says the effective mass turns out to be 2 to 20 times the free-space mass of the electron. Our calculations imply the effective mass equals the free-space mass in the absence of a potential barrier between successive atoms in an lattice, which is what is to be expected.

In the next version of this paper, we will comment on Feynman's development of the Schrödinger equation in the context of simple linear motion.

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