On a differential equation of Lienard type with strong nonlinearities

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Abstract

A differential equation of Lienard type with strong nonlinearities is proposed in this work. The equation admits exact and general periodic solution expressible in terms of trigonometric functions.

Keywords: Lienard equations, strong nonlinearity, general periodic solutions, trigonometric function.

Theory

Let us consider the differential equation stated in [1] as

$$\ddot{x}+\frac{q}{\ell x}\left(b x^{-q}-a x^{\alpha-q}\right)^{\frac{2}{\ell}}+\frac{a\alpha}{\ell}x^{\alpha-q-1}\left(b x^{-q}-a x^{\alpha-q}\right)^{\frac{2-\ell}{\ell}}=0 \quad (1)$$

where a, b, ℓ, q and α, are arbitrary parameters, and overdot denotes differentiation with respect to time. The application of $\ell=1$, leads to

$$\ddot{x}+(q-\alpha)a^2 x^{2(\alpha-q)-1}-(2q-\alpha)ab\,x^{\alpha-2q-1}+b^2 q\,x^{-2q-1}=0 \quad (2)$$

Substituting $b=-a(q+1)^2$, and $\alpha=2(q+1)$, into (2) yields

$$\ddot{x}-2a^2(q+1)^2 x+a^2 q(q+1)^4 x^{-2q-1}-a^2(q+2)x^{2q+3}=0 \quad (3)$$

The equation (3) is the proposed nonlinear differential equation of Lienard type with strong nonlinearities. Using the corresponding first order differential equation [1]

$$\dot{x}x^q+a\,x^{2q+2}=-a(q+1)^2 \quad (4)$$

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one may obtain

$$\int \frac{x^q}{1+\left(\frac{x^{q+1}}{q+1}\right)^2}dx = -a\left(q+1\right)^2\left(t+K\right) \tag{5}$$

where K is an arbitrary constant. From (5) one may secure the exact and general periodic solution of (3) in the form

$$x(t) = \left\{\left(q+1\right)\tan\left[-a\left(q+1\right)^2\left(t+K\right)\right]\right\}^{\frac{1}{q+1}} \tag{6}$$

where $q \neq -1$.

Reference

[1] M. D. Monsia, Analysis of a purely nonlinear generalized isotonic oscillator equation, Math.Phys.,viXra.org/2010.0195v1.pdf (2020).