Goldbach’s Conjecture Algorithm Addendum

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Abstract: This is an addendum to the work I previously submitted at https://vixra.org/abs/1904.0227

Do prime numbers follow any pattern? If they did, it would be easier to prove Goldbach’s Conjecture.

Other than their spacing, Prime numbers do not display any easily identifiable patterns. However, patterns do exist between the primes; across the composites. This is certain because algorithms exist which systematically eliminate the composites, leaving only the primes. My definition of “pattern” is predictable behavior that can be demonstrated with an algorithm.

How does this relate to Goldbach’s Conjecture? Seeing that there is a systematic way to eliminate composites (Sieve of Eratosyntesis), I then wondered if there was a way to eliminate all sums that are not the sum of two primes. Said differently, to eliminate all prime + composite and all composite + composite sums. I found that there is. I created an algorithm that eliminates all prime + composite sums and leaves prime + prime sums remaining. Since Goldbach’s Conjecture states that all evens are the sum of two primes, it follows that it would be easier to prove such a case if we can find patterns.

First think about even integers that are the sum of two primes. The first even we think of in general is the number 2. However, 1 is a not a prime (1+1), so 2 is not where I want to start my studies. The next even is the number 4. Yes, 4 is the sum of two primes because 2 is a prime (2+2), however, 2 + any other prime will yield an odd sum and the focus is on even sums. 6 is the even integer that is a sum of two primes (3+3). When adding 3 to all other primes, we end up with all even sums, so this paper begins with 6 as its starting point.

<u>Observations</u>

Starting with 3 as the base prime to add to all other primes, we see

3+**3**= 6, 3+**5**=8, 3+**7**=10, 3+**11**= 14, 3+**13**=16, 3+**17**=20, 3+**19**=22, 3+**23**=26, 3+**29**= 32,

We already see something interesting happening. The sums 12, 18, 24, 30 are not present. Multiples of 6 are not present. It makes sense that this will continue indefinitely because when you subtract 3 from multiples of 6, the difference will be a multiple of 3. If it is a multiple of something, it is not prime. 3 + a multiple of 3 is not a sum of two primes. Multiples of 6 from 6 (not including 6) will never be the sum of 3 + another prime. Therefore, those sums can be eliminated and will be eliminated indefinitely. Here is where I start what is called my first “strand”. A strand is a set of numbers that are eliminated. This first set, or first “strand”, will eliminate every 6 even integers from 6. When you subtract 3 from 12 (the first point that gets eliminated in this first strand, or first set of numbers to eliminate, from 6), the difference is 9, which is 3^2.[1]

[1] This will be addressed in the section about “Significance points”.

We also see that 28 is not present. To further investigate we continue adding.

3+**31**=34, 3+**37**= 40, 3+**41**=44, 3+**43**=46, 3+**47**=50 , 3+**53**=56, 3+**59**=62, 3+**61**=64, 3+**67**=70, 3+**71**=74, 3+**73**=76, 3+**79**=82, 3+**83**=86

Here again multiples of 6 continue to not exist for the reason mentioned above.

Also, numbers that end in 8 are not included in the above list of even sums. The reason for this is because a sum that ends in 8 minus 3 will end with the number 5 which is a multiple of 5. Again, if it is a multiple of something, it is not prime. 3+ an odd multiple of 5 is not a sum of two primes. This will continue indefinitely. Multiples of 10 from 8 (not including 8) will never be the sum of 3+ another prime. Ironically, when you subtract 3 from 28 (the first point that gets eliminated in this second strand, or second set of numbers to eliminate), the difference is 25, which is the same as 5^2.

So let's take a look at what types of sums are being eliminated so far (remember, eliminated means not the sum of 3 + another prime). We have started from 6, eliminating every 6th number. This is what I refer to as a "strand" in my algorithm. This is the first set of numbers to eliminate. Next, we have starting from 8, eliminate every 10th number. This is the second "strand", or second set of numbers to eliminate.

Continuing our studies of which sums do not exist when adding 3 to another prime: 52 also does not exist and it is neither a multiple of 6 nor does it end in 8. Following the pattern from the other two strands, I deduced that this would fall under 3+ multiples of 7, which means it would not be the sum of two primes and will continue indefinitely. It also followed that starting with +2 from the last start point, eliminate multiples of (the next odd *2). So: starting from 10, we can eliminate every 14th number. 24 and 38 are eliminated from the first two strands. The third strand does follow what's happening so far: when you subtract 3 from 52, the difference is 49, which is the same as 7^2.

84 is missing, starting from +2 from the last start point, eliminate multiples of (the next odd *2): start with 12 and eliminate every 18th number. 30,48, 66, are all eliminated by the first three strands. Following the same pattern above, when you subtract 3 from 84, the difference is 81, which = 9^2, which is where the fourth strand starts and will continue indefinitely because this is 3 + multiples of 9, multiples = not prime.

This brings me to "significance points". Essentially, I realized that every so often, new strands, or sets of numbers to eliminate, become necessary for eliminating all evens that are not the sum of 3 + another prime. I looked to see if there was a pattern, and indeed found that the difference between the last significance point and the next was equal to the last significance point + multiples of 8, and that it corresponded to which strand I was on.

So, going from 12 to 28. 12 is the starting point for the first set of numbers to be eliminated (or first strand). 28 is the starting point for the second set of numbers to be eliminated. In the algorithm, it says "second strand starts with 8, skips by 10". This is still accurate, but it is also extra work since 18 is eliminated in the first strand, or first set of numbers to be eliminated. So, the first number that is not eliminated by strand one, is 28. 28 is the "significance point" for where to start eliminating the second set of numbers if you want to save yourself time.

Back to the point: I want to know where the second strand starts. So, the difference of 28-12= 16. What about the third strand? It starts at 52. 52 is where the third set of numbers to be eliminated becomes necessary (because the numbers before it that are crossed off from performing the "starts with 10, skips by 14" were eliminated by strands 1 and 2). What is the difference of 52-28? 52-28 is 24. There I saw that the difference between the significance points were multiples of 8.

To recap: to find the *second* significance point, I needed to add 8***2** to the last point. 12 + (8*2).

To find the *third* significance point, I needed to add 8***3** to the last point. 28 + (8*3) and so on.

This continues indefinitely, because I later found out that the significance points are actually just (odds^2) +3. Which .. goes on forever. 3^2+3= **12**, 5^2+3= **28**, 7^2+3=**52,** 9^2+3=**84**… and so on.

I also found out that some of the strands are redundant. For example: the fourth strand "starts with 12, skips by 18" will cross off only multiples of 6, which the first strand already eliminated. Essentially, the strands where the significance points are a (composite ^2) + 3 are not necessary. This is because composites are multiples of the primes, and the multiple of primes already eliminate the evens that are multiples of composites. So again, to save yourself time, if you know the starting point is (composite^2)+3, then you can skip it and go on to the next strand (Like 9^2 + 3; 84 which is a significance point but also redundant).

From our observations the pattern, or algorithm is now:

First strand: Start with 6, eliminate every 6^{th}

Second strand: Start with 8, eliminate every 10^{th}

Third strand: Start with 10, eliminate every 14^{th}

Fourth strand: Start with 12, eliminate every 18^{th}

So the overall pattern, or algorithm is start with +2 from the last starting point, eliminate multiples of the (next odd *2).

Aka

Next strand: Start with +2 more than last, eliminate multiples of +4 more than last.

Strand 1				
Skip By 6 starting with 6 not including 6				
2	22		62	82
4		44	64	
6	26	46		86
8	28		68	88
10		50	70	
	32	52		92
14	34		74	94
16		56	76	
	38	58		98
20	40		80	100

Strand 2				
Skip By 10 starting with 8 not including 8				
2	22	42	62	82
4	24	44	64	84
6	26	46	66	86
8				
10	30	50	70	90
12	32	52	72	92
14	34	54	74	94
16	36	56	76	96
20	40	60	80	100

Strand 3				
Skip By 14 starting with 10 not including 10				
2	22	42	62	82
4		44	64	84
6	26	46		86
8	28	48	68	88
10	30	50	70	90
12	32		72	92
14	34	54	74	
16	36	56	76	96
18		58	78	98
20	40	60		100

Strand 4				
Skip By 18 starting with 12 not including 12				
2	22	42	62	82
4	24	44	64	
6	26	46		86
8	28		68	88
10		50	70	90
12	32	52	72	92
14	34	54	74	94
16	36	56	76	96
18	38	58	78	98
20	40	60	80	100

Strands 1-4 (Overlay)				
Overlay of Above 4 Strands				
2	22		62	82
4		44	64	
6	26	46		86
8				
10		50	70	
	32			92
14	34		74	
16		56	76	
20	40			100

The beautiful thing is when you have a list of evens and you cross off all evens that are not a sum of 3 + other primes, what's left is all evens that are the sum of 3+ other primes. If you subtract 3 from these evens, the sums are the consecutive list of primes.

Answering Questions

Is this algorithm only applicable to 3+ other primes or just primes or all odd numbers?

It is applicable to all odd numbers.

Say I want to know what 11 + all other primes will be. The only thing that changes is the first starting point. Since 3 is the first prime that when added to 11 will equal an even sum, the first starting point for eliminating evens that are not the sum of two primes is 11+3. From there we still eliminate multiples of 6, 10, 14, 18 … and so on (+4 more than the last) because they are still even multiples of the odds, and multiples are not prime.

This brings me to my next point:

This algorithm shows that sums of prime + prime and prime + composite follow the same pattern. No matter what odd we apply the algorithm to, when we subtract the algorithm number (the number we chose to see all the sums of) from the evens that are left over, the difference will always be a consecutive list of primes.

Example:

On the 3's:

6-3= **3**, 8-3=**5**, 10-3= **7**, 14-3= **11** and so on

On a composite (9 for this example):

12-9= **3**, 14-9= **5**, 16-9=**7**, 20-9=**11** and so on

Applying the algorithm to a prime or composite will both have a difference of consecutive primes when subtracting the chosen prime or composite from the evens that have not been eliminated. Having the same set of differences no matter which type of odd the algorithm is applied to, seemed like a pattern.

This led me to believe that since prime + prime and prime + composite follow the same pattern, proving that all evens are the sum of prime + composite would also mean that all evens are the sum of two primes.

I came up with this argument:

All evens ≥ 12 are the sum of prime + composite. The largest number of consecutive primes is 3: 3,5,7. The largest gap in composites is 2 (where the twin primes occur). Using 3,5,7 alone, each composite can make three even sums per 1 prime + composite combination. When primes are added over the gap, there will be no gap in consecutive even sums. There is one composite before the gap and one composite after. Over the course of 4 odd numbers with the gap on numbers 2 and 3, there are 6 consecutive sums. Example: list of odd numbers with a twin prime gap: 15,17,19,21. 3+15=18, 5+15= 20, 7+15= 22, 3+21= 24, 5+21= 26, 7+21= 28. There is no break in the even sum between 7+15 and 3+21 because the number of consecutive primes is larger than the number in gap. This will always be true because every third odd number greater than 3 is divisible by 3. The gap will never exceed 2 and every composite starts with being added to 3,5, and 7 to account for all possible prime+ composite combinations. With no gap in even sums of prime + composite addends ≥ 12, all evens ≥ 12 are the sum of prime + composite.

I said ≥ 12 because the evens below 12 are not prime + composite since 1 does not count as prime or composite. The rest of the odds that can be added together to equal an even sum below 12 are prime.

While I very much liked this argument and wanted it to work, I realized composites are far more dense than primes. The redundancies and overlaps are the reason it's easy to show that all evens are the sum of prime + composite. Even though both primes and composites go on infinitely, it seems that it would be harder to prove that the redundancy in sums of prime + composites also necessarily means redundancies in sums of two primes infinitely.

So, I came up with a different argument that I think works much better logically.

Since the algorithm works for all primes, each prime's starting point for applying the algorithm is that prime +3, and the algorithm always eliminates the same group of multiples: each prime starts the pattern over. So every even sum that is the sum of 3 + another prime is a starting point for the pattern. The same pattern starts over and overlaps itself at every point where 3 is added to another prime. Knowing this, we can then pay attention to the spacing of the primes. Every odd is a certain number of spaces from 3 (keep in mind when I say "spaces from 3", I'm only including odd numbers since we're looking for even sums and 3 + an even integer is odd).

For instance: 5 is one space from 3. 7 is two spaces from 3. 9 is 3 spaces from 3. 11 is four spaces from 3. 13 is five spaces from 3. 15 is six spaces from 3. Here we see the pattern we saw earlier but in a different way: Numbers that are a multiple of 3 from 3 are not prime. This is true and will always be true because multiples are not prime. As we continue, it will always be the case that if the distance from 3 is a multiple of 3, it will not be prime.

There are certainly other odds that are not prime, like multiples of 5 from 5, multiples of 7 from 7 and so on. These are displayed by the Sieve of Eratosynthesis. When counting backwards using only the knowledge of distances from 3 is sufficient, since any number we're going to land on is a sum of 3 + another prime and it will either be a multiple of 3 spaces back from the random even, or it will not. If not, then it is the even sum/ 3+ prime we're looking for. This can be used as shorthand since 3 is the smallest prime multiple, the frequency will occur most often.

The algorithm is groundwork for showing that the same pattern is repeated on top of itself at every point of 3 + another prime, and the pattern continues infinitely. Because the algorithm is applied in least-to-greatest fashion going forward,and each prime +3 is the starting point for the algorithm, we can now say that if we can select an even number at random and count backwards, we will land on an even that is the sum of 3 + another prime. Because primes are not evenly spaced, we will eventually land on a 3 + prime that is not the distance of a multiple of 3 backwards. When we land on that point, the corresponding even - 3 will be one of the primes in the addend. We can subtract that prime from our random even to find the other prime. We can also remember that every odd is a certain number of spaces from 3, and the number of spaces we counted backwards to land on the 3+ prime that is not the distance of a multiple of 3 backwards, corresponds to the number of spaces from 3 (the distance counted backwards = the distance from 3); the corresponding prime in that distance will also equal the same difference as subtracting the the prime in that spot from our random even.

Example:

My random even: 98

Counting backwards, there is a 3 + prime three spaces back (3 + 89). We cannot use this because it is 3 spaces back from the even, which means 3 spaces forward from 3, which = 9 and is not prime.

There is another 3 + prime six spaces back (3+83). We cannot use this because it is a multiple of 3 spaces back from the even; 6 spaces back is the same as 6 spaces from 3, which = 15 and is not prime.

Finally, we land on a 3+ prime that is 8 spaces back (not a *multiple of 3* spaces), therefore we can use this even. It happens to be 3+ 79, and so 79 is one of the primes in our addend to obtain the sum of 98. We can subtract 79 from 98 and find that the sum is 19. 19 also equals 8 spaces back from 98, which corresponds to 8 spaces forward from 3, which = 19.

Example #2:

My random even: 544

The first 3+ prime back is 9 spaces back. 9 spaces is a multiple of 3, so we can't use this. However, the next 3+ prime back is 10 spaces back from 544, which isn't a multiple of 3 and happens to be 3+ 521. 10 spaces forward from 3 is 23. Again: we can either subtract the prime 3 was added to, to get 23, or we can subtract 23 from 544 to get 521. It checks out both ways.

Conclusion:

Because the algorithm continues indefinitely forward, overlapping itself, each new occurrence of the pattern starting at every 3+ prime: applying to the algorithm on 3's, one will always be able to count backwards and land on a point in the algorithm. Either it will be a multiple of 3 spaces back or it will not. Multiples of 3 spaces back are common occurrences and primes are not orderly, so it is easy to find a match. This algorithm shows that prime numbers are predictable both forward and backward, and can therefore be used to support Goldbach's Conjecture.