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Beal Conjecture Proved Very Simply

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Abstract

The author proves directly the original Beal conjecture (and not the equivalent conjecture) that if $A^x + B^y = C^z$, where A, B, C, x, y, z are positive integers and $x, y, z > 2$, then A, B and C have a common prime factor. One will let r, s and t be prime factors of A, B and C, respectively,such that $A = Dr$, $B = Es$, and $C = Ft$, where D, E and F are positive integers,. Then, the equation $A^x + B^y = C^z$ becomes $D^x r^x + E^y s^y = F^z t^z$. The proof would be complete after proving that $r^x = t^x$ and $s^y = t^y$, which would imply that $r = s = t$. The proofs of the above equalities would also be complete after showing that the ratio, $\frac{r^x}{t^x} = 1$ and the ratio, $\frac{s^y}{t^y} = 1$. The main principle for obtaining relationships between the prime factors on the left side of the equation and the prime factor on the right side of the equation is that the power of each prime factor on the left side of the equation equals the same power of the prime factor on the right side of the equation. High school students can learn and prove this conjecture as a bonus question on a final class exam.

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Option 1

Introduction

One will let r, s and t be prime factors of A, B and C, respectively, such that $A = Dr$, $B = Es$, and $C = Ft$, where D, E and F are positive integers, Then, the equation, $A^x + B^y = C^z$ becomes $D^x r^x + E^y s^y = F^z t^z$. The proof would be complete after showing that $r = s = t$. Since one would like to prove equalities from the equation, $D^x r^x + E^y s^y = F^z t^z$, one will need equalities between the powers of the prime factors on the left side of the equation and the power of the prime factor on the right side of the equation. Two approaches will be covered in finding these equalities.

Approach 1: Common Sense Approach

At a glance, and from the experience gained in solving exponential and logarithmic equations, one can identify the powers involved with respect to the prime factors, r, s, t, as r^x, s^y, and t^z. Thinking like a tenth grader, one would like to have equalities involving r^x, t^x, s^y, t^y, t^z. One will therefore, let $t^z = t^x t^{z-x}$ to introduce t^x, and $t^z = t^y t^{z-y}$ to introduce t^y. The possible equalities between the powers of the prime factors on the left side and the power of the prime factor on the right side of the equation, $D^x r^x + E^y s^y = F^z t^z$ are $r^x = t^x$, $r^x = t^z$, $s^y = t^y$ and $s^y = t^z$. Of these possibilities, only $r^x = t^x$ and $s^y = t^y$, on inspection, would lead to the conclusion, $r = t$ $s = t$, and $r = s = t$. Therefore, one conjectures the equalities, $r^x = t^x$ and $s^y = t^y$. These conjectures will be proved in the Beal conjecture proof. To prove these two equalities, one will show that the ratio, $(r^x/t^x) = 1$ and the ratio, $(s^y/t^y) = 1$. Two main steps are involved in the proof. In the first step, one will determine how r and t are related, and in the second step, one will determine how s and t are related.

Approach 2: Factorization Approach

In approach 2, one will be guided by the properties of factored numerical Beal equations.

Illustration of the equalities $r^x = t^x$ and $s^y = t^y$ of factored Beal equations

For the factorization with respect to r^x:

$$\boxed{r^x = t^x} \qquad D^x r^x + E^y s^y = F^z t^z$$

$$\underbrace{r^x}_{K}[\underbrace{D^x + E^y s^y \bullet r^{-x}}_{L}] = \underbrace{t^x}_{M}\underbrace{t^{z-x}F^z}_{P} \qquad (K = M)$$

Example 1:

$$33^5 + 66^5 = 33^6$$

$$11^5 \bullet 3^5 + 11^5 \bullet 2^5 \bullet 3^5 = 11^6 \bullet 3^6$$

$$11^5(3^5 + 2^5 \bullet 3^5) = 11^5 \bullet 11 \bullet 3^6$$

$$\underbrace{11^5}_{k}(\underbrace{3^5 + 2^5 \bullet 3^5}_{L}) = \underbrace{11^5}_{M} \bullet \underbrace{11 \bullet 3^6}_{P}$$

For the factorization with respect to s^y:

$$\boxed{s^y = t^y} \qquad D^x r^x + E^y s^y = F^z t^z$$

$$\underbrace{s^y}_{K}[\underbrace{E^y + D^x r^x \bullet s^{-y}}_{L}] = \underbrace{t^y}_{M}\underbrace{t^{z-y}F^z}_{P} \qquad (K = M)$$

Example 2:

$$34^5 + 51^4 = 85^4$$

$$17^5 \bullet 2^5 + 17^4 \bullet 3^4 = 17^4 \bullet 5^4$$

$$17^4(17 \bullet 2^5 + 3^4) = 17^4 \bullet 5^4$$

$$\underbrace{17^4}_{k}(\underbrace{17 \bullet 2^5 + 3^4}_{L}) = \underbrace{17^4}_{M} \bullet \underbrace{5^4}_{P}$$

From either Approach 1 or Approach 2, one will next prove the equalities

$\boxed{r^x = t^x}$ and $\boxed{s^y = t^y}$, and deduce $\boxed{r = s = t}$.

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Option 2

Beal Conjecture Proved Very Simply

Given: $A^x + B^y = C^z$, A, B, C, x, y, z are positive integers and $x, y, z > 2$.

Required: To prove that A, B and C have a common prime factor.

Plan: Let r, s and t be prime factors of A, B and C, respectively, such that $A = Dr$, $B = Es$, and $C = Ft$, where D, E and F are positive integers, Then, the equation $A^x + B^y = C^z$ becomes $D^x r^x + E^y s^y = F^z t^z$. The proof would be complete after showing that $r = s = t$. The conjectured equalities, $r^x = t^x$ and $s^y = t^y$, which would imply that $r = s = t$, will be proved by showing that $(r^x/t^x) = 1$ and .$(s^y/t^y) = 1$. In the first step of the proof, one will determine how r and t are related, and in the second step, one will determine how s and t are related.

Proof

Step 1: The conjectured equality, $r^x = t^x$ would be true if and only if $(r^x/t^x) = 1$.

$D^x r^x + E^y s^y = F^z t^z$ (1)

$\dfrac{D^x r^x + E^y s^y}{F^z t^z} = 1$ (Dividing both sides by $F^z t^z$) (2)

Because of the equality , $r^x = t^x$, a t^x factor is needed on the right side of equation (1)

$D^x r^x + E^y s^y = t^x t^{z-x} F^z$ (In equation 1, let $t^z = t^x t^{z-x}$)

$D^x r^x + E^y s^y = r^x t^{z-x} F^z$ (Replace t^x by r^x) <-----see biconditional proofs on page 5

$D^x r^x + E^y s^y = r^x t^{-x} F^z t^z$ (Splitting t^{z-x})

$D^x r^x + E^y s^y = \dfrac{r^x}{t^x} F^z t^z$ (Positive exponents only)

$\dfrac{D^x r^x + E^y s^y}{F^z t^z} = \dfrac{r^x}{t^x}$ (solving for $\dfrac{r^x}{t^x}$)

$1 = \dfrac{r^x}{t^x}$ (From (2), $\dfrac{D^x r^x + E^y s^y}{F^z t^z} = 1$)

If $\dfrac{r^x}{t^x} = 1$, $\dfrac{r^x}{t^x} = \dfrac{D^x r^x + E^y s^y}{F^z t^z}$ $\left(\dfrac{D^x r^x + E^y s^y}{F^z t^z} = 1\right)$

$r^x F^z t^z = t^x (D^x r^x + E^y s^y)$ (cross-multiplying)

$r^x = t^x$ (Divide left side by $F^z t^z$ and right side by $D^x r^x + E^y s^y$, since $D^x r^x + E^y s^y = F^z t^z$)

If $r^x = t^x$, $r = t$. ($\log r^x = \log t^x; x \log r = x \log t; \log r = \log t;$ $r = t$)

Step 2: The conjectured equality, $s^y = t^y$ would be true if and only if $(s^y/t^y) = 1$.

$D^x r^x + E^y s^y = F^z t^z$ (1)

Because of the equality, $s^y = t^y$, a t^y factor is needed on the right side of equation (1)

$D^x r^x + E^y s^y = t^y t^{z-y} F^z$ (Let $t^z = t^y t^{z-y}$ in equation 1)

$D^x r^x + E^y s^y = s^y t^{z-y} F^z$ (Replace t^y by s^y)

$D^x r^x + E^y s^y = s^y t^{-y} F^z t^z$ (Splitting t^{z-y})

$D^x r^x + E^y s^y = \dfrac{s^y}{t^y} F^z t^z$ (Positive exponents) only)

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$$\frac{D^x r^x + E^y s^y}{F^z t^z} = \frac{s^y}{t^y} \qquad \left(\text{Solving for } \frac{s^y}{t^y}\right)$$

$$1 = \frac{s^y}{t^y} \qquad \left(\text{From (2) in Step 1, } \frac{D^x r^x + E^y s^y}{F^z t^z} = 1\right)$$

If $\frac{s^y}{t^y} = 1$, $\frac{s^y}{t^y} = \frac{D^x r^x + E^y s^y}{F^z t^z}$ $\left(\frac{D^x r^x + E^y s^y}{F^z t^z} = 1\right)$

$s^y F^z t^z = t^y (D^x r^x + E^y s^y)$ (cross-multiplying)

$s^y = t^y$ (Divide left side by $F^z t^z$ and right side by $D^x r^x + E^y s^y$, since $D^x r^x + E^y s^y = F^z t^z$)

If $s^y = t^y$, $s = t$. ($\log s^y = \log t^y$; $y \log s = y \log t$;= $\log s = \log t$; $s = t$)

Step 3: It has been shown in Step 1 that $r = t$, and in Step 2 that $s = t$; therefore, $r = s = t$.

Since $A = Dr$, $B = Es$, $C = Ft$ and $r = s = t$, A, B and C have a common prime factor, and the proof is complete.

Discussion

The above proof is beautiful mathematics because of the symmetric structure of the proof, One can observe that Step 2 could be viewed as a duplication of Step 1 with r^x replaced by s^y, and t^x replaced by t^y. The beauty continues when $r^x = t^x$ and $s^y = t^y$ imply that $r = t$ and $s = t$, respectively, resulting in the conclusion, $r = s = t$, In the previous papers, viXra:2001.0694, viXra:2012.0041), the conjecture of these equalities was based on only the properties of the factored numerical Beal equations. In the present paper, a common sense approach as well as a factoring approach was the basis.

About biconditional proofs

The conjectured equality, $r^x = t^x$ would be true if and only if $(r^x/t^x) = 1$. could be written **biconditionally** as $r^x = t^x$ if and only if $(r^x/t^x) = 1$. .One will prove the following two **conditional** statements::.

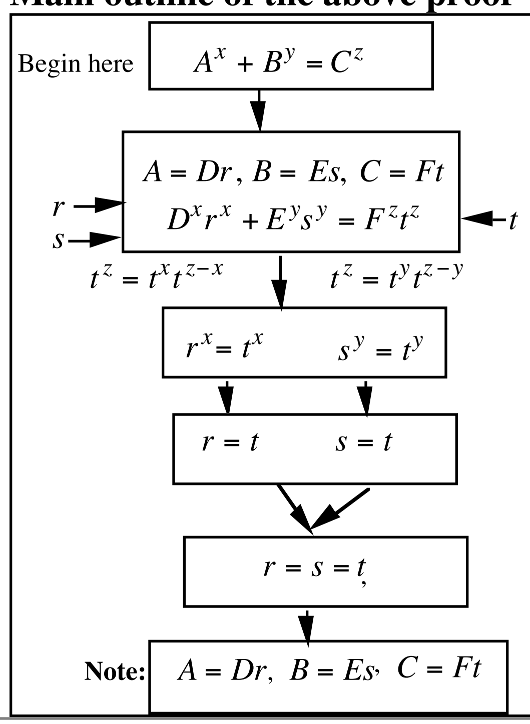
1. If $r^x = t^x$, then $(r^x/t^x) = 1$
2. If $(r^x/t^x) = 1$, then $r^x = t^x$

In the first statement,.assume $r^x = t^x$, and show that $(r^x/t^x) = 1$.

In the second statement,, assume $(r^x/t^x) = 1$, and show that $r^x = t^x$

After proving both statements, one would have proved that $r^x = t^x$ if and only if $(r^x/t^x) = 1$.

Main outline of the above proof



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Conclusion

In this paper, the author is happier to have proved the original Beal conjecture and **not** the equivalent conjecture. The proof was based on the two equalities, $r^x = t^x$ and $s^y = t^y$. which were conjectured and proved. These equalities were conjectured using common sense as well as the factorization properties of the factored numerical Beal equations. From these. equalities, it was concluded that $r = s = t$, (where r, s and t are prime factors of A, B and C, respectively), establishing the truthfulness of the Beal conjecture. High school students can learn and prove this conjecture as a bonus question on a final class exam.

Extra: Fermat's Last Theorem can be proved by modifying the above proof as follows: For the hypothesis, let $x, y, z = n > 2$, $r \neq s \neq t$ and prove by contradiction (see viXra:2003.0303).

PS: Other proofs of Beal Conjecture by the author are at viXra:2001.0694, viXra:1702.0331; viXra:1609.0383; viXra:1609.0157; viXra:2012.0041

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