Lectures on Physics - Chapter V Relativity, EM waves and radiation

Jean Louis Van Belle, Drs, MAEc, BAEc, BPhil

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Abstract

The special problem we try to get at with these lectures is to maintain the interest of the very enthusiastic and rather smart people trying to understand physics. They have heard a lot about how interesting and exciting physics is—the theory of relativity, quantum mechanics, and other modern ideas—and spend many years studying textbooks or following online courses. Many are discouraged because there are really very few grand, new, modern ideas presented to them. Also, when they ask too many questions in the course, they are usually told to just shut up and calculate. Hence, we were wondering whether or not we can make a course which would save them by maintaining their enthusiasm. This paper is a draft of the fifth chapter of such course. It offers a comprehensive overview of the complementarity of wave- and particle-like perspectives on electromagnetic (EM) waves and radiation.

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Basics of relativity

A charge moves along a *geodesic*, i.e. a line (trajectory) which we describe in a three-dimensional Cartesian space (i.e. an empty *mathematical* space) by measuring its distance from a chosen point of origin at successive points in time *t*.

The point of origin of the reference frame is usually the position of the observer, and the clock time will also be measured using the clock of the observer. Hence, all measurements of distance and time intervals are relative to the observer, whose position in spacetime is, therefore, given by the four-vector (x = 0, y = 0, z = 0, t = t). This frame of reference is the *inertial* frame of reference, in which the observer moves along his own timeline only. The reference frame of the charge itself is referred to as the moving reference frame, in which position and time will be measured using primed space and time coordinates x', y', z', t'.

Because there is no preferred origin, the coordinate values (x, y, z, t) and (x', y', z', t') have no essential meaning: we are always concerned with *differences* of spatial or temporal coordinate values belonging to two events, which we will label by the subscript 1 and 2. This *difference* is referred to as the *spacetime interval* Δs , whose squared value is given by:

$$(\Delta s)^2 = (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2 - (\Delta ct)^2$$
$$= (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2 - c^2(t_2 - t_1)^2$$

In the context of this expression, *c* should be thought of as an invariable mathematical constant which allows us to express the time interval $\Delta t = (t_2 - t_1)$ in equivalent distance units (*meter*). The same spacetime interval in the moving reference frame is measured as:

$$(\Delta s')^2 = (\Delta x')^2 + (\Delta y')^2 + (\Delta z')^2 - (\Delta ct')^2$$
$$= (x'_2 - x'_1)^2 + (y'_2 - y'_1)^2 + (z'_2 - z'_1)^2 - c^2(t'_2 - t'_1)^2$$

The spacetime interval is invariant and $(\Delta s)^2$ is, therefore, equal to $(\Delta s')^2$. We can, therefore, combine both expressions above and write:

$$(\Delta x)^{2} + (\Delta y)^{2} + (\Delta z)^{2} - (\Delta ct)^{2} = (\Delta x')^{2} + (\Delta y')^{2} + (\Delta z')^{2} - (\Delta ct')^{2}$$
$$\Leftrightarrow [(\Delta x)^{2} - (\Delta x')^{2}] + [(\Delta y)^{2} - (\Delta y')^{2}] + [(\Delta z)^{2} - (\Delta z')^{2}] = c^{2}[(\Delta ct)^{2} - (\Delta ct')^{2}]$$

This equation shows two observers – in relative motion one to another – can only meaningfully talk about the spacetime interval between two events if they agree on (1) the reality of the events¹, (2) a common understanding of the measurement units for time and distance as well as a conversion factor between the two units so as to establish equivalence.

Because the speed of light is an invariant constant – the *only* measured velocity which does not depend on the reference frame² – it will be convenient to measure distance in light-seconds (the distance travelled by light in one second, i.e. 299,792,458 meter *exactly*³). This, of course, assumes a common definition of the second which, since last year's revision of the international system of units (SI) only, can be defined with reference to a standard frequency only. This standard frequency was *defined* to be equal to 9,192,631,770 Hz (s⁻¹), *exactly*⁴, which is the frequency of the light emitted by a caesium-133 atom when oscillating between the two energy states that are associated with its ground state at a temperature of 0 K.⁵

The rather long introduction on relativity illustrates that two observers need to agree on (1) the use of a (physical) clock to count time and (2) the invariance of the speed of light. The reality of light effectively corresponds to a succession of events – a photon travels the distance Δx over a time interval Δt – which are separated by *invariant* spacetime intervals

$$\Delta s = \Delta s' = \sqrt{(\Delta x)^2 - c^2 (\Delta t)^2} = \sqrt{(\Delta x')^2 - c^2 (\Delta t')^2}.$$

It should be noted that the expression under the square root sign cannot be negative because the photon does not travel at superluminal velocity. In fact, for a photon traveling from point A to B the expression above can be multiplied by Δt and $\Delta t'$ respectively so as to yield the following:

$$\frac{\Delta s}{\Delta t} = \frac{\sqrt{(\Delta x)^2 - c^2 (\Delta t)^2}}{\Delta t} = \sqrt{\frac{(\Delta x)^2}{(\Delta t)^2} - \frac{c^2 (\Delta t)^2}{(\Delta t)^2}} = \sqrt{c^2 - c^2} = 0$$

¹ Both observers need to agree on measuring time along *either* the positive *or* negative direction of the time scale because the *order* of the events (in time) cannot be established in an absolute sense. Even if one reference frame assigns precisely the same time to two events that happen at different points in space, a reference frame that is moving relative to the first will generally assign different times to the two events (the only exception being when motion is exactly perpendicular to the line connecting the locations of both events).

² The velocity of light does *not* depend on the motion of the source.

³ It was only in 1983 – about 120 years after the publication of Maxwell's wave equations, which showed that the velocity of propagation of electromagnetic waves is always measured as c – that the *meter* was redefined in the International System of Units (SI) as the distance travelled by light in vacuum in 1/299,792,458 of a second.

⁴ This value was chosen because the caesium 'second' equaled the limit of human measuring ability around 1960, when the caesium atomic clock as built by Louis Essen in 1955 was adopted by various national and international agencies and bodies (e.g. USNO) for measuring time.

⁵ These two energy states are associated with the hyperfine splitting resulting from the two possible states of spin in the presence of both nuclear as well as electron spin. Spin is measured in units of *h*: the nuclear spin of the caesium-atom is 7/6 units of *h*, while the total electron spin is (because of the pairing of electrons and the presence of one unpaired electron) is equal to *h*/2. Depending on the energy, nuclear and electron spin will either have opposite or equal sign. Hence, the two energy states are associated with total spin value (nuclear and electron) F = 7/6 - 1/2 = 3 or F = 7/6 + 1/2 = 4.

$$\frac{\Delta s'}{\Delta t'} = \frac{\sqrt{(\Delta x')^2 - c^2 (\Delta t')^2}}{\Delta t'} = \sqrt{\frac{(\Delta x')^2}{(\Delta t')^2} - \frac{c^2 (\Delta t')^2}{(\Delta t')^2}} = \sqrt{c^2 - c^2} = 0$$

This establishes the light cone separating time- and spacelike intervals for both observers. We will now no longer be worried about the relativity of the reference frame and assume the reader will understand what is relative and absolute in the description.⁶

Charges, energy states, potentials, fields, and radiation

Position, time and, therefore, motion is relative. However, charge is not relative and different observers should, therefore, also agree on a measurement unit for charge, which we may equate to the elementary charge e. This is the charge of a proton or the (negative) charge of the electron. A charge fills empty spacetime (all of it) with a potential which depends on position and evolves in time. This potential is, therefore, also a function of *x*, *y*, *z*, and t.⁷ Two equivalent descriptions are possible:

- A description in terms of the electric and magnetic field vectors E(x, t) and B(x, t); and
- A description in terms of the scalar and vector potential $\phi(\mathbf{x}, t)$ and $\mathbf{A}(\mathbf{x}, t)$ respectively.

The field vectors **E** and **B** have three components⁸ and we, therefore, have six dependent variables $E_x(\mathbf{x}, t)$, $E_y(\mathbf{x}, t)$, $E_z(\mathbf{x}, t)$, $B_x(\mathbf{x}, t)$, $B_y(\mathbf{x}, t)$, and $B_z(\mathbf{x}, t)$. In contrast, the combined scalar and vector potential give us four dependent variables $\phi(\mathbf{x}, t)$, $A_x(\mathbf{x}, t)$, $A_y(\mathbf{x}, t)$, and $A_z(\mathbf{x}, t)$ only. For the time being, however, we will stick to a description of the fields in terms of the *E* and **B** fields.

We refer to Feynman's Lectures⁹ for a clear and complete derivation of these functions out from Maxwell's equations for a *single* charge q moving along any arbitrary trajectory, as illustrated below (Figure 1). The point to note is that the electric and magnetic field at point (1) *now* will be written as a function of the position and motion of the charge at the *retarded* time t - r'/c. The relevant distance is, therefore, also the *retarded* distance r', which is the distance between (1) and (2') – which is *not* the charge's position at time. The latter position is point (2): it is separated from position (2') by a time interval equal to t - (t - t'/c) and a distance interval which depends on the velocity \mathbf{v} of the charge q which will be generally much less than c.

⁶ For a short (15 minutes) brief, we refer the reader to our <u>YouTube video on reality</u>, philosophy, and physics.

⁷ We will no longer be worried about the relativity of the reference frame and assume the reader will understand what is relative and absolute in our description.

⁸ **Boldface** symbols denote vector quantities, which have both a magnitude and a direction. Scalar quantities only have a magnitude. However, depending on the reference point for zero potential energy, the *potential* energy of a charge in a potential field may be negative. Potential *energy* is – just like a distance – measured as a *difference*. The plus or minus sign of the potential *energy*, therefore, depends on the *direction* in which we would be moving the charge.

⁹ Richard Feynman, II-26, *Solutions of Maxwell's equations with currents and charges*.



Figure 1: The concepts of retarded time, position an distance (Feynman, II-21, Fig. 21-1)

To be precise, Feynman writes the *E* and *B* field vectors as:

$$\boldsymbol{E}(1,t) = \frac{q}{4\pi\epsilon_0} \left[\frac{\boldsymbol{e}_{r\prime}}{r'^2} + \frac{r'}{c} \frac{d}{dt} \left(\frac{\boldsymbol{e}_{r\prime}}{r'^2} \right) + \frac{1}{c^2} \frac{d^2}{dt^2} \boldsymbol{e}_{r\prime} \right]$$
$$c\boldsymbol{B}(1,t) = \boldsymbol{e}_{r\prime} \times \boldsymbol{E}(1,t)$$

The second and third term are, obviously, equal to zero if the charge is *not* moving, in which case the charge comes with a static (i.e. non-varying in time) Coulomb field only: in this case, the *retarded* field is just the Coulomb field *tout court*. The scalar product which defines the magnetic field is equal to the product:

- the magnitude of the unit vector e_r, whose origin is (2') and which points to (1) and whose magnitude is equal to 1;
- (2) the *magnitude* of the electric field vector at point (1) at time *t*;
- (3) the cosine of the angle between $e_{r'}$ and E(1, t), which is zero *if the second and third term are zero*: in other words, a static electric field does *not* come with a magnetic field.

However, when the charge is moving, the first- and second-order derivative of the $e_{r'}$ will not be equal to zero:

(i) The second term then corresponds to what Feynman refers to as a *compensation* for the retardation delay, as it is the product of (*a*) the *rate of change* of the retarded Coulomb field multiplied by (*b*) the retardation delay (the time needed to travel the distance r' at the speed of light *c*). In other words, the first two terms correspond to computing the retarded Coulomb field and then extrapolating it (linearly) toward the future by the amount r'/c – which is right up to time t.¹⁰ It should be noted that both terms are inversely proportional to the squared distance r'^2 .

¹⁰ We apologize for quoting quite literally from Feynman's *exposé* here, but we could not find better language.

(ii) The third term – the second-order derivative $d^2(\mathbf{e}_r)/dt^2$ – is an acceleration vector which – because of the origin of the unit vector $\mathbf{e}_{r'}$ is fixed at point (2') – can and should be analyzed as the sum of a transverse component and a radial component.¹¹

In the chapters where Feynman first introduces and uses these equations (Vol. I, Chapters 28 and 29 as well as Vol. II, Chapter 21), the assumption is that the transverse piece is far more important than the radial piece, but such statement crucially depends on the assumption that the charge is moving at a more or less right angle to the line of sight, which is not necessarily the case. Feynman corrects for this assumption in Chapter 34 of Vol. I, in which he gives the reader a full treatment of all 'relativistic effects' of the motion of a charge.

Feynman also associates the third term with *radiation* which, as we now know, consists of a stream of photons carrying energy. We must, effectively, assume the charge does not only generate a potential but moves in a potential field itself. Its energy, therefore, must also continually change. To be specific, *in free space*, we must assume the charge will *lower* its total energy (kinetic *and* potential) by moving along a path which minimizes the (physical) action $S = \int_{t_1}^{t_2} (\text{KE} - \text{PE}) dt$. This is just an application of the (classical) *least action principle*.¹²

Of course, in classical physics, potential energy must be converted to kinetic energy and vice versa: the total energy of the charge (the *sum* of KE and PE) does, therefore, not change: only its *components* KE and PE, which depend on its velocity and its position respectively, but they add up to a constant. In other words, the situation which we have been considering up to this point, is that of *a charge whose energy state does not change*.

Such energy state may be the energy state of a free electron or of an electron in a bound state, i.e. an electron in an atomic or molecular orbital. *If and when an electron moves from one energy state to another*, as it does when hopping from one atomic or molecular orbital to another. Indeed, as the electron moves as a proper *current* in a conductor¹³ – whose direction is from high to low potential – it should emit *photons* which will be packing a *discrete* amount of energy which is given by the Planck-Einstein relation:

$\Delta E = h \cdot f = h/T$

The frequency f of the photon is, obviously, the inverse of the *cycle time* T, and the Planck-Einstein relation may, therefore, also be written as $h = \Delta E \cdot T$. Because the drop in potential from one atomic or molecular orbital in a crystal structure – i.e. along the conductor – is extremely small, power lines – whether they be high-voltage DC or low-voltage AC lines – emit only extremely low frequency (ELF) *radiation*. Such low-frequency radiation is associated with heat radiation at very low temperature: a

¹¹ In the chapters where Feynman uses these equations (Vol. I, Chapters 28 and 29 as well as Vol. II, Chapter 21), he assumes the transverse piece is far more important than the radial piece, but such statement crucially depends on the assumption that the charge is moving at a more or less right angle to the line of sight, which is not necessarily the case. Feynman corrects for this assumption in Chapter 34 of Vol. I, in which he gives the reader a fuller treatment of the 'relativistic effects in radiation'.

¹² See: Feynman's *Lectures, The Least Action Principle* (Vol. II, Chapter 19).

¹³ For a distinction between the concepts of current, electrical *signal*, and (probability) amplitudes, see our paper on <u>electron propagation in a (crystal) lattice</u> (November 2020).

photon frequency of 300 Hz, for example, is associated with a wavelength that is equal to $\lambda = c/f \approx (3 \times 10^8 \text{ m/s})/(300 \text{ s}^{-1}) = 1 \times 10^6 \text{ m} = 1000 \text{ km}.^{14}$

Hence, yes, we finally dropped the word: *radiation*. Electrons who stay in the same energy state – in a bound atomic or molecular state, for example – do not emit radiation and, hence, do not lose energy. Likewise, the orbital motion (*spin*) of the charge inside a stationary charge does *not* cause any radiation and, therefore, the energy does not leak out.

This, then, combines Maxwell's equations with the Planck-Einstein relation which tells us *energy* comes in quantized packets whose integrity is given by Planck's quantum of action (h). We now have the trio of physical constants in electromagnetic theory (classical as well as quantum physics): c, e, and h.

Photons and fields

1. In 1995, W.E. Lamb Jr. wrote the following on the nature of the photon:

"There is no such thing as a photon. Only a comedy of errors and historical accidents led to its popularity among physicists and optical scientists. I admit that the word is short and convenient. Its use is also habit forming. Similarly, one might find it convenient to speak of the "aether" or "vacuum" to stand for empty space, even if no such thing existed. There are very good substitute words for "photon", (e.g., "radiation" or "light"), and for "photonics" (e.g., "optics" or "quantum optics"). Similar objections are possible to use of the word "phonon", which dates from 1932. Objects like electrons, neutrinos of finite rest mass, or helium atoms can, under suitable conditions, be considered to be particles, since their theories then have viable non-relativistic and non-quantum limits."¹⁵

The opinion of a Nobel Prize laureate carries some weight, of course, but we think the concept of a photon makes sense. As the electron moves from one (potential) energy state to another – from one atomic or molecular orbital to another – it builds an oscillating electromagnetic field which has an integrity of its own and, therefore, is not only wave-like but also particle-like.

The photon carries no charge but carries energy. We should probably assume its kinetic energy is the same at start and stop of the transition. In other words, at point t_1 and t_2 , (KE)₁ and (KE)₂ are assumed to identical in the (physical) action equation which we introduced above:

$$S = h = \int_{t_1}^{t_2} (\text{KE} - \text{PE}) dt = \int_{t_1}^{t_2} (\text{KE}) dt - \int_{t_1}^{t_2} (\text{PE}) dt$$

This, of course, does not mean that the $\int_{t_1}^{t_2} (\text{KE}) dt$ integral vanishes: it only does so when assuming the velocity in the KE = $m_e v^2/2$ formula¹⁶ is zero everywhere, which cannot be the case because – when

¹⁴ ELF radiation is usually defined as radiation with a (photon) frequency below 300 Hz. Typical field strength near a high-voltage power is typically 2-5 kV/m (1 V/m = 1 J/C·m = 1 N/C) for the electric field strength and up to 40 μ T (1 T = 1 (N/C)·(s/m), with the latter factor reflecting the 1/c scaling factor and the orthogonality of the *E* and *B* vectors) for the magnetic field but – as the equations show – diminish rapidly with distance. The typical range for low-voltage lines is 100-400 V/m and 0.5-3 μ T, respectively. See, for example: https://ec.europa.eu/health/scientific_committees/opinions_layman/en/electromagnetic-fields07/l-2/7-power-lines-

elf.htm

¹⁵ W.E. Lamb Jr., <u>Anti-photon</u>, in: Applied Physics B volume 60, pages 77–84 (1995).

¹⁶ We use the non-relativistic kinetic energy formula here because the *drift velocity* of the electron is very low. Also, the rather low energy levels involves ensure a particle with rest mass of about 0.51 MeV/ c^2 should not reach

everything is said and done – the electron does move from one cell in the crystal lattice to another. However, we will leave it to the reader to draw possible KE, PE and total energy graphs over the electron transition from one crystal cell to another. Such graphs should probably be informed by a profound analysis of the nature of the photon.

We mentioned a photon carries energy, but no charge. While carrying electromagnetic energy, a photon will only exert a force when it meets a charge, in which case its energy will be absorbed as kinetic energy by the charge. In-between the emission and absorption of the photon, we should effectively think of the photon as an oscillating electromagnetic field and, hence, such field can usefully be represented by the electric and a magnetic field vectors **E** and **B**. The magnitudes should not confuse us: field vectors do not take up any space, although we may want to think of them as a force without a charge to act on. Indeed, a non-zero field at some point in space and time – which we describe using the (x, y, x, t) coordinates – tell us what the force *would* be *if* we would happen to have a unit charge at the same point in space and in time. This is reflected in the electromagnetic force formula: the Lorentz force equals $\mathbf{F} = \mathbf{q} \cdot (\mathbf{E} + \mathbf{v} \times \mathbf{B})$. Hence, the electromagnetic force is the sum of two (orthogonal) *component* vectors: $\mathbf{q} \cdot \mathbf{E}$ and $\mathbf{q} \cdot \mathbf{v} \times \mathbf{B}$.

The velocity vector **v** in the equation shows *both* of these two component force vectors depend on our frame of reference. Hence, we should think of the separation of the electromagnetic force into an 'electric' (or electrostatic) and a 'magnetic' force *component* as being somewhat artificial: the *electromagnetic* force is (very) *real* – because it determines the *motion* of the charge – but our cutting-up of it in two separate components depends on our frame of reference and is, therefore, (very) *relative*. We should refer to our remarks on the relative *strength* of the electric and magnetic field, however: the reader should not think in terms of the electric or magnetic force being more or less important in the analysis and always analyze *both* as aspects of one and the same *reality*.

Let us get back to our photon: we think the photon is pointlike because the **E** and **B** vectors that describe it will be zero at each and every point in time *and* in space *except if our photon happens to be at the* (x, y, z) *location at time t*.

[...] Please read the above again: our photon is pointlike because the electric and magnetic field vectors that describe it are zero everywhere except where our photon happens to be.

2. At the same time, we know a photon is defined by its *wavelength*. So how does that work? What is the *physical* meaning of the wavelength? It is, quite simply, the distance over which the electric and magnetic field vectors will go through a full *cycle* of their oscillation. That is all there is to it: nothing more, nothing less.

That distance is, of course, a *linear* distance: to be precise, it is the distance Δs between two points (x_1 , y_1 , z_1) and (x_2 , y_2 , z_2) where the **E** and **B** vectors have the same value. The photon will need some time Δt to travel between these two points, and these intervals in time and space are related through the (constant) velocity of the wave, which is also the velocity of the pointlike photon. That velocity is, of course, the speed of light, and the time interval is the cycle time T = 1/f. We, therefore, get the equation that will be familiar to you:

relativistic velocity levels. The non-relativistic formula simply defines the kinetic energy as the difference between the total energy and the potential energy.

$$c = \frac{\Delta s}{\Delta t} = \frac{\lambda}{T}$$

We can now relate this to the Planck-Einstein relation. Any (regular) oscillation has a frequency and a cycle time T = $1/f = 2\pi/\omega$. The Planck-Einstein relation relates *f* and T to the energy (E) through Planck's constant (*h*):

$$\mathbf{E} = h \cdot f = \hbar \cdot \omega \Leftrightarrow \mathbf{E} \cdot \mathbf{T} = h$$

The Planck-Einstein relation does not only apply to matter-particles but also to a photon. In fact, it was *first* applied to a photon.¹⁷ Think of the photon as *packing* not only the energy E but also an amount of *physical action* that is equal to h.

3. We have not talked much about the meaning of *h* so far, so let us do that now. *Physical action* is a concept that is not used all that often in physics: physicists will talk about energy or momentum rather than about physical action.¹⁸ However, we find the concept as least as useful. Physical action can *express* itself in two ways: as some energy over some time (E·T) or – alternatively – as some momentum over some distance (p· λ). For example, we know the (pushing) momentum of a photon¹⁹ will be equal to p = E/c. We can, therefore, write the Planck-Einstein relation for the photon in two equivalent ways:

$$\mathbf{E} \cdot \mathbf{T} = \frac{E}{c} \cdot c\mathbf{T} = h \iff \mathbf{p} \cdot \lambda = h$$

We could jot down many more relations, but we should not be too long here.²⁰

The near- and far-fields

The picture above is quite clear and consistent: a conductor – or a crystal lattice – emits electromagnetic waves as photons, who should be thought of as self-perpetuating through the interplay of the electric and magnetic field vector.²¹ The *direction* of propagation equals the line of sight (more or less²²) and a

¹⁷ The application of the Planck-Einstein relation to matter-particles is *implicit* in the *de Broglie* relation. Unfortunately, Louis de Broglie imagined the matter-wave as a linear instead of a circular or orbital oscillation. He also made the mistake of thinking of a particle as a wave *packet*, rather than as a single wave! The latter mistake then led Bohr and Heisenberg to promote uncertainty to a metaphysical principle. See our paper on the meaning of <u>the *de Broglie* wavelength</u> and/or <u>the interpretation of the Uncertainty Principle</u>.

 ¹⁸ We think the German term for physical action – *Wirkung* – describes the concept much better than English.
¹⁹ For an easily accessible treatment and calculation of the formula, see: <u>Feynman's Lectures</u>, Vol. I, Chapter 34, section 9.

²⁰ We may refer the reader to <u>our manuscript</u>, our paper on <u>the meaning of the fine-structure constant</u>, or various others papers in which we explore <u>the nature of light</u>. We just like to point out one thing that is quite particular for the photon: the reader should note that the $E = mc^2$ mass-energy equivalence relation and the p = mc = E/c can be very easily related when discussing photons. There is an easy *mathematical* equivalence here. That is not the case for matter-particles: the *de Broglie* wavelength can be interpreted geometrically but the analysis is somewhat more complicated—not impossible (not at all, actually) but just a bit more convoluted because of its circular (as opposed to linear) nature.

²¹ We skipped a discussion on photon spin: we think of photon spin as angular momentum, and it is always plus or minus one unit of *h*. Photons do not have a zero-spin state.

²² Because the lattice consists of several layers, one may think an electron may not always move to the crystal cell right next to it. This is true, it may deviate to left, right, up, or down while moving through the lattice. On the other

crystal lattice (conductor) acts as a series of point sources or oscillators. By modulating the voltage (AC or DC), frequency and – and taking into account the spacing and properties of the crystal lattice – one gets photon beams in all directions, whose intensity and energy depends on the above-mentioned factors and – important – can carry a signal through frequency or amplitude modulation (AM or FM). We take, once again, an illustration from Feynman to show how this works (**Figure 2**). It should be noted that the *interference* pattern that emerges does not result from random indeterminism but from an interplay of regular and statistically determined photon emissions from each of the crystals in the conducting lattice. As such, the addition or superposition of photons, electromagnetic waves and probabilities amounts to the same – with the usual *caveat* for the photon picture, of course, which – as particles – do not engage in constructive or destructive interference. The *complementarity* of the different viewpoints, perspectives or *representations* of the same reality is, therefore, clear.



Figure 2: The intensity pattern of a continuous line of oscillators (Feynman, I-30, Fig. 30-5)

However, by way of conclusion, we must probably say something about the oft-used distinction between near- and far-fields. In order to do so, we ask the reader to, once again, carefully look at the relevant equation(s) for the *E* and *B* field vectors:

$$\boldsymbol{E}(1,t) = \frac{q}{4\pi\epsilon_0} \left[\frac{\boldsymbol{e}_{r'}}{r'^2} + \frac{r'}{c} \frac{d}{dt} \left(\frac{\boldsymbol{e}_{r'}}{r'^2} \right) + \frac{1}{c^2} \frac{d^2}{dt^2} \boldsymbol{e}_{r'} \right]$$
$$c\boldsymbol{B}(1,t) = \boldsymbol{e}_{r'} \times \boldsymbol{E}(1,t)$$

The reader will note the magnitude of the (retarded) Coulomb effect (the first term) diminishes with distance following the inverse square law (~ $1/r'^2$) while the second term involves only inverse proportionality (~ $1/r^2$).²³ Finally, the third term does not fall off with distance at all! It is this what gives rise to the very different *shape* of the near-field versus the far-field waves, with a transition zone inbetween.

hand, the conducting electrons will repel each other and will, therefore, tend to travel on the surface of the crystal, which is in agreement with standard theory.

²³ The coefficient r'/c and the $1/r'^2$ in the argument of the first-order derivative combine to give us a rather straightforward 1/r' factor.