Observing the Movement of Prime Numbers in Prime-based Number Shapes

Marc Schofield

TappTappNeuroAid Ltd. (UK)
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Abstract:

A Pascal's Triangle was mapped to prime numbers and the placement of prime numbers propagated in the triangle was observed as a fluctuating offset was applied to the top of the triangle.

Methodology:

A spreadsheet was populated following the propagation rule we are familiar with in Pascal's Triangle, namely that each cell is the sum of the cell above and the cell to its right. (the two cells diagonally above when viewed from a 45° counterclockwise angle). Consecutive prime numbers were mapped into row 1 and column A, and cell B2 has been highlighted as the offset. This offset was manually manipulated to view how this affected where prime numbers were propagated. Figure 1 shows a nominated offset of 3 and an illustration of how cell G3 is a summation of cells G2 and F3, a non-absolute rule copied across all cells except for column A, row 1 and cell B2.

SUM | × ✓ fx | =F3+G2

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1		1	1	2	3	5	7	11	13	17	19	23	29	31	37	41
2	1	3	4	6	9	14	21	32	45	62	81	104	133	164	201	24
3	1	4	8	14	23	37	=F3+G2	90	135	197	278	382	515	679	880	112
4	2	6	14	28	51	88	146	236	371	568	846	1228	1743	2422	3302	442
5	3	9	23	51	102	190	336	572	943	1511	2357	3585	5328	7750	11052	154
6	5	14	37	88	190	380	716	1288	2231	3742	6099	9684	15012	22762	33814	492

Sheet1 ⊕

Figure 1 – Pascal's Triangle mapped to successive primes and given a nominated offset.

In order to check for prime numbers, a separate table on the same spreadsheet was made referring back to the number grid, using the function shown below to populate the cells with a '+' to denote the presence of a prime number in the reference cell, or to fill the cell with a blank space (" ") to indicate a non-prime integer in the reference cell. The following function was used to check whether a number was prime:

=IF(A1=2,"+",IF(AND(MOD(A1,ROW(INDIRECT("2:"&ROUNDUP(SQRT(A1),0))))<>0),"+"," ")).

This formula was modified slightly from the formula discussed on www.excelexchange.com/prime_number_test.html.

This source explains that **"this is an array formula** (which) **relies on Excel's MOD function to calculate the remainder on every whole number divisor from 2 to the square root of the test number. If there is always a remainder then the number is prime. Excel's ability to calculate the remainder fails when the test number exceeds 268,435,455."**

The pattern seen when odd numbers are highlighted (a modulo 2 plot of a binomial triangle) is a fractal known as Sierpiński's Triangle. (Bannink & Buhrman, 2007). Additional to this second table showing the position of primes in the first table, conditional formatting using

the formula =MOD(A1,2)=1 shows how the position of primes propagated within the triangle move about these set odd number areas (the light green cells) as the offset is manipulated. Graphs were observed for all offsets between -30 and 30.
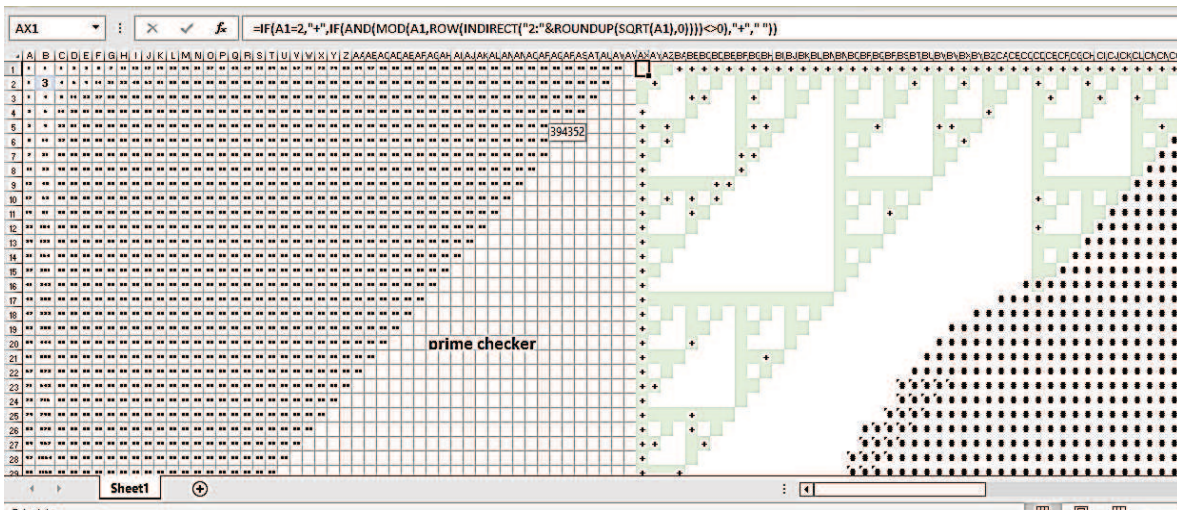

Figure 2 – showing the position of primes for offset= 3, within the confines of where odd numbers can propagate (shown as light green coloured cells in the rightmost graph, which is also reporting primes in the binomial triangle).

Findings:

It was found that mapping the triangle to primes in the way described above did not affect the odd/even distribution of the numbers propagated, since the binomial triangle does still have symmetry. It was further found that manipulating the offset (cell B2) did not change this odd/even distribution: the conditional formatting highlighting a modulo 2 plotting of the propagated numbers showed the same familiar Sierpiński fractal pattern as the offset was manually altered.

It was found that with an offset of -1 primes populated around the top of the odd/even distribution with a line of primes leading along and down from the input prime 13, as well as primes surrounding the smaller upper even areas.
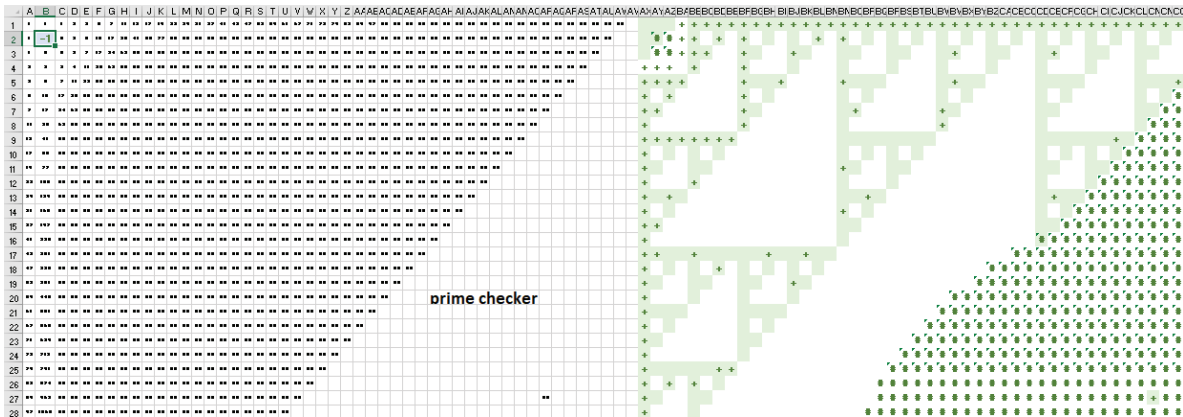
Figure 3 – showing the position of primes for offset= -1, within the confines of where odd numbers can propagate.

1	1	1	2	3	5	7	11	13					2	3	5	7	11	13
1	-1	0	2	5	10	17	28	41					2	5		17		41
1	0	0	2	7	17	34	62	103					2	7	17			103
2	2	2	4	11	28	62	124	227		2	2	2		11				227
3	5	7	11	22	50	112	236	463		3	5	7	11					463
5	10	17	28	50	100	212	448	911		5		17						911
7	17	34	62	112	212	424	872	1783		7	17							1783
11	28	62	124	236	448	872	1744	3527		11								3527
13	41	103	227	463	911	1783	3527	7054		13	41	103	227	463	911	1783	3527	

Figure 4 – showing rows/columns 1-9 of a prime mapped binomial triangle with a -1 offset, with and then without non-prime integers.

These following graphics show a Sierpiński's Triangle overlaid onto the number grid (first row/column through to row/column starting 29) resulting from a -1 offset in cell B2. The colour of the areas containing primes have been inverted to show their positioning along the edges of the areas populated by even numbers. For clarity, 2's have not been highlighted as prime in these graphics, and propagated primes have been shown in green.
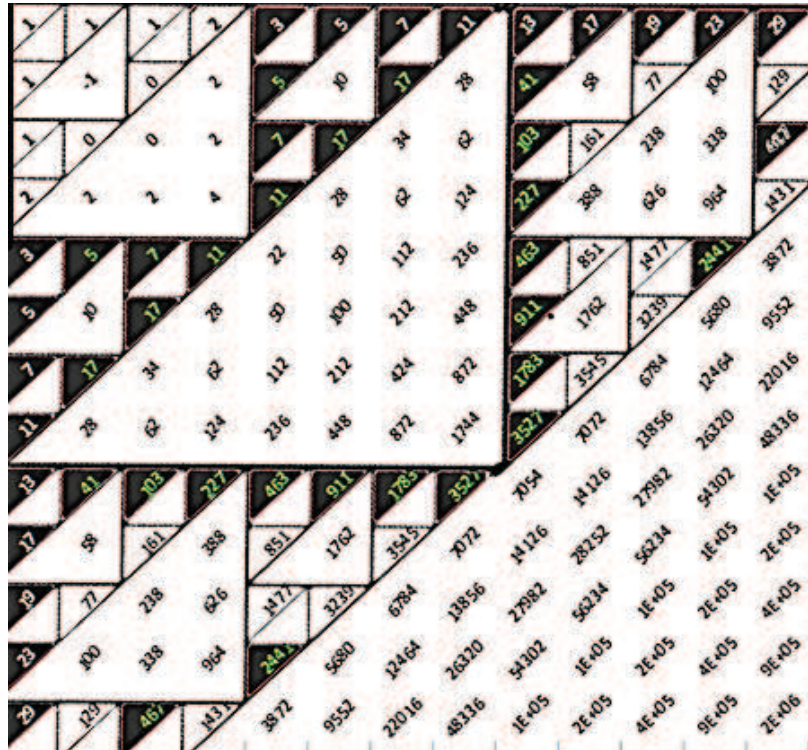
Figure 5 – Sierpiński's Triangle overlaid upon the number grid shown in fig. 3
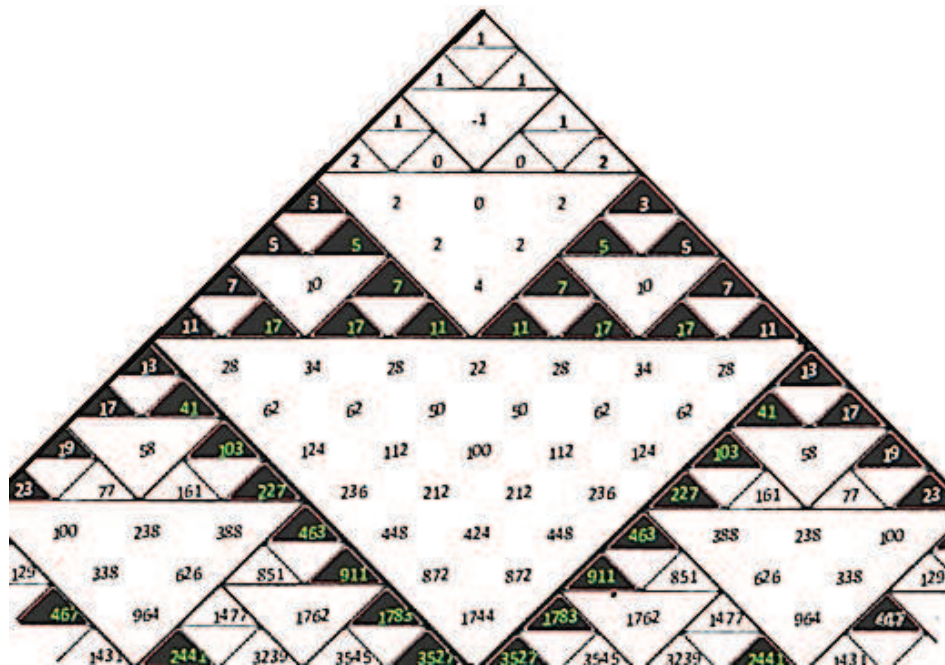

Figure 6 -Sierpiński's Triangle overlaid upon the number grid shown in fig. 3. and given a 45° clockwise rotation.

Further investigations:

Next a spreadsheet was made as shown below. Again the grid follows Pascal's progression and the light green conditional format shows the odd/even distribution of the numbers propagated. For the purpose of this work, we will look at each 1 at the side of a Pascal's triangle as an input.

- Row 1 and column A contained the sequence 1,2,3,4,5,6,7… to help view each 1 as an input, and to allow for each input to be 'turned on' by the contents of a separate cell, A1.
- Cells C2 – AN2 had the formula =IF(A1=C1,1,0) copied across it.
- Similarly, cells B2 – B40 had copied down them the formula =IF(A1=A2,1,0).
- Cell A1 was used as a selector and allowed the viewing of the progression added by each input 1 in a Pascal's triangle.

Figure 7 – illustrating how the Sierpiński pattern grows in diagonally as the 'inputs' are turned on sequentially by manually increasing cell A1..

An experiment not pictured here for brevity's sake explained why the 'offset' used in the first investigations revealed lines of primes. This was achieved by setting A1 to 0 and putting 1 in cell C3. The resulting pattern was what was being taken away by that -1 'offset'. Investigations into manually changing numbers within polynomial progressions were abandoned (no more 'offsets' from here on in) and mapping triangles to primes was shelved for a while. We will get back to doing that again later.

As before, using the formula

=IF(C3=2,C3,IF(AND(MOD(C3,ROW(INDIRECT("2:"&ROUNDUP(SQRT(C3),0))))

<>0),C3,"")) copied down through a second grid, a map of primes was made as the inputs were

sequentially turned on. A mod2 conditional formatting showed a fractal pattern for each input in

grid 1. Manual formatting was used in the second and third grid to track and show the movement

of prime numbers, parallel to the input 1s selected by cell A1. Cell A1 was changed from 1

through to 38 and formatting of the second and third grids were updated with the placement of

propagated primes for each input 1. The diagonal black lined cells in the second grid show where

the 2s propagate for each input 1, these 2s being the corner of the constituent Pascal's triangle

for each input 1. After the pattern blue/white pattern was traced, the process of cycling A1

through 1 to 38 was repeated and the primes for each value of A1 was added into the third grid.

The second grid is a tracking of propagated primes in grid 1 as the input moves along and down.
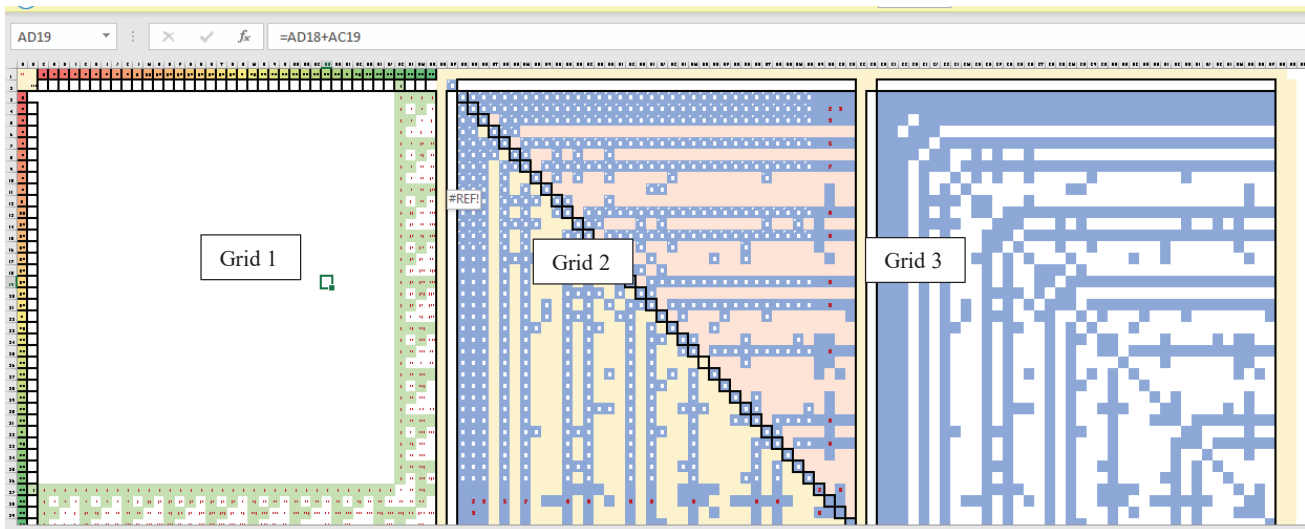

Figure 8- Grids 2 and 3 were manually formatted based on the position of primes in each iteration of the first grids manipulation via cell A1.
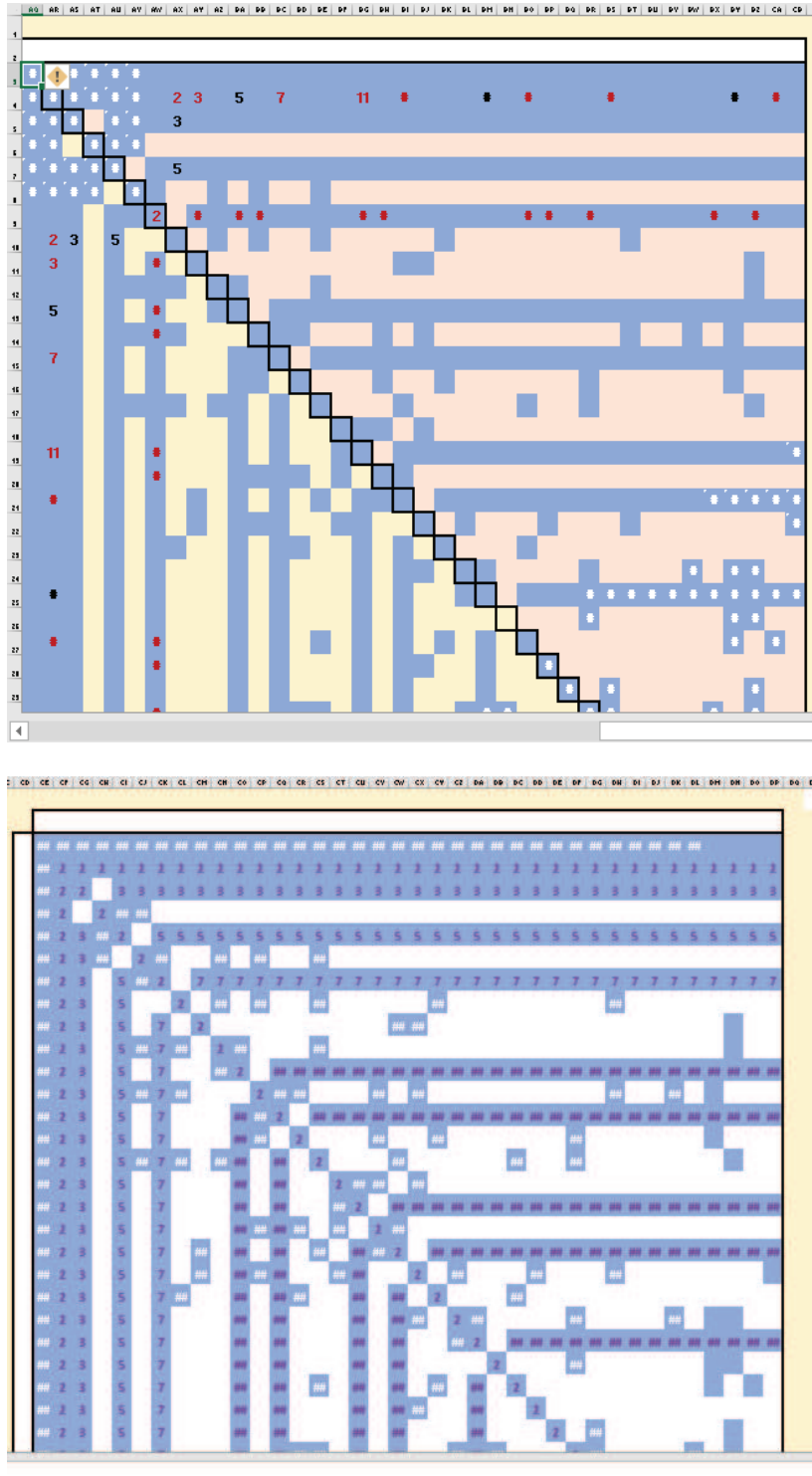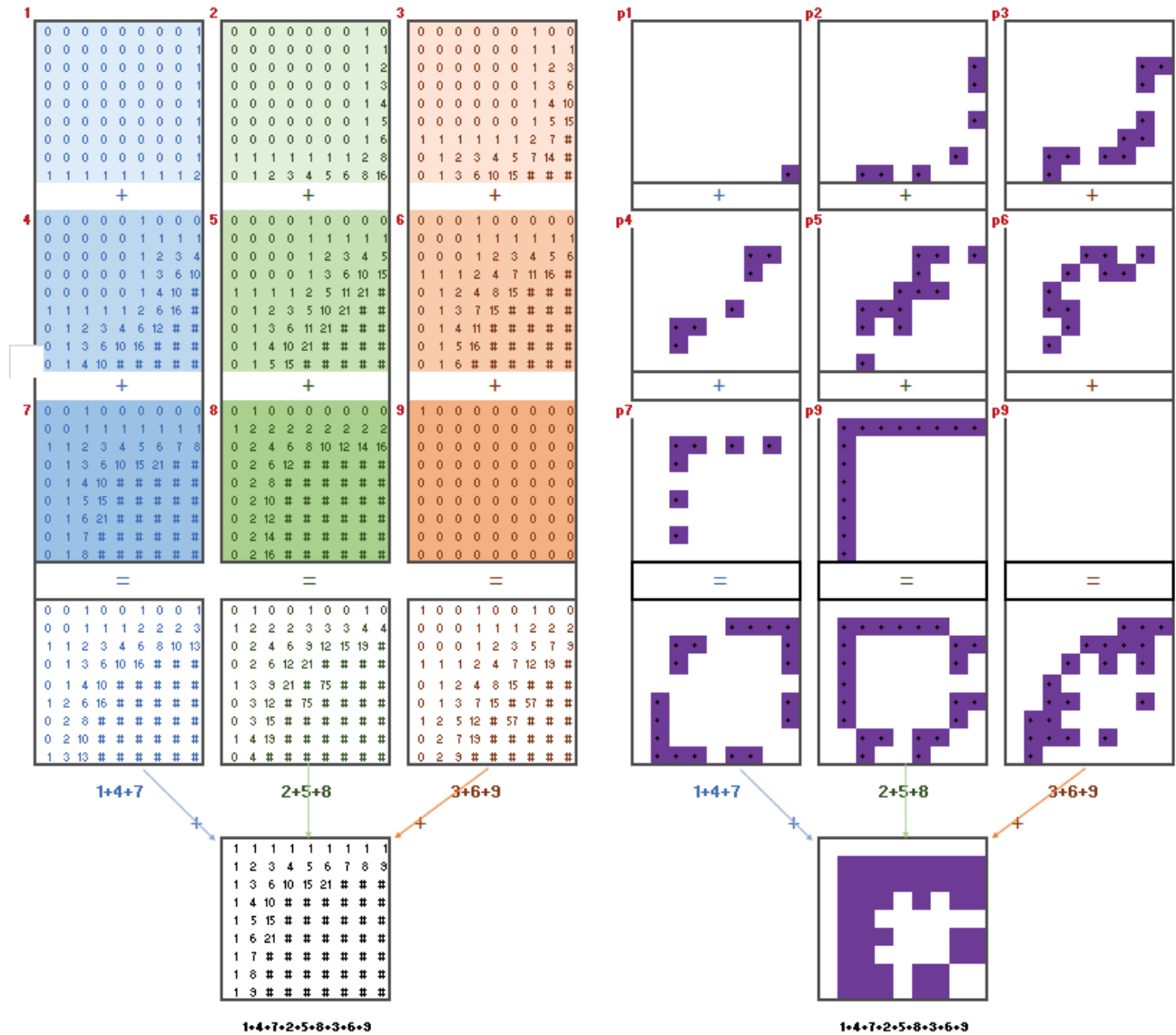
Figure 9- a closer look at the second two grids in fig. 8 and transferring the primes propagated for all input 1s into Grid 3.

A spreadsheet was then used to split a 9x9 binomial triangle into 9 component matrices, with only one input 1 on each upper and leftmost axes. These are accompanied by prime reporting squares, to view the logical addition of prime patterns.

1
```
0 0 0 0 0 0 0 0 1
0 0 0 0 0 0 0 0 1
0 0 0 0 0 0 0 0 1
0 0 0 0 0 0 0 0 1
0 0 0 0 0 0 0 0 1
0 0 0 0 0 0 0 0 1
0 0 0 0 0 0 0 0 1
0 0 0 0 0 0 0 0 1
1 1 1 1 1 1 1 1 2
        +
```

2
```
0 0 0 0 0 0 0 1 0
0 0 0 0 0 0 0 1 1
0 0 0 0 0 0 0 1 2
0 0 0 0 0 0 0 1 3
0 0 0 0 0 0 0 1 4
0 0 0 0 0 0 0 1 5
0 0 0 0 0 0 0 1 6
1 1 1 1 1 1 1 2 8
0 1 2 3 4 5 6 8 16
        +
```

3
```
0 0 0 0 0 0 1 0 0
0 0 0 0 0 0 1 1 1
0 0 0 0 0 0 1 2 3
0 0 0 0 0 0 1 3 6
0 0 0 0 0 0 1 4 10
0 0 0 0 0 0 1 5 15
1 1 1 1 1 1 2 7 #
0 1 2 3 4 5 7 14 #
0 1 3 6 10 15 # # #
        +
```

4
```
0 0 0 0 0 1 0 0 0
0 0 0 0 0 1 1 1 1
0 0 0 0 0 1 2 3 4
0 0 0 0 0 1 3 6 10
0 0 0 0 1 4 10 #
1 1 1 1 2 6 16 #
0 1 2 3 4 6 12 #
0 1 3 6 10 16 # #
0 1 4 10 # # # #
        +
```

5
```
0 0 0 0 1 0 0 0 0
0 0 0 0 1 1 1 1 1
0 0 0 0 1 2 3 4 5
0 0 0 1 3 6 10 15
1 1 1 1 2 5 11 21 #
0 1 2 3 5 10 21 # #
0 1 3 6 11 21 # # #
0 1 4 10 21 # # # #
0 1 5 15 # # # # #
        +
```

6
```
0 0 0 1 0 0 0 0 0
0 0 0 1 1 1 1 1 1
0 0 0 1 2 3 4 5 6
1 1 1 2 4 7 11 16 #
0 1 2 4 8 15 # # #
0 1 3 7 15 # # # #
0 1 4 11 # # # # #
0 1 5 16 # # # # #
0 1 6 # # # # # #
        +
```

7
```
0 0 1 0 0 0 0 0 0
0 0 1 1 1 1 1 1 1
1 1 2 3 4 5 6 7 8
0 1 3 6 10 15 21 # #
0 1 4 10 # # # # #
0 1 5 15 # # # # #
0 1 6 21 # # # # #
0 1 7 # # # # # #
0 1 8 # # # # # #
        =
```

8
```
0 1 0 0 0 0 0 0 0
1 2 2 2 2 2 2 2 2
0 2 4 6 8 10 12 14 16
0 2 6 12 # # # # #
0 2 8 # # # # # #
0 2 10 # # # # # #
0 2 12 # # # # # #
0 2 14 # # # # # #
0 2 16 # # # # # #
        =
```

9
```
1 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0
        =
```

```
0 0 1 0 0 1 0 0 1
0 0 1 1 1 2 2 2 3
1 1 2 3 4 6 8 10 13
0 1 3 6 10 16 # # #
0 1 4 10 # # # # #
1 2 6 16 # # # # #
0 2 8 # # # # # #
0 2 10 # # # # # #
1 3 13 # # # # # #
```
1+4+7

```
0 1 0 0 1 0 0 1 0
1 2 2 2 3 3 3 4 4
0 2 4 6 9 12 15 19 #
0 2 6 12 21 # # # #
1 3 9 21 # 75 # # #
0 3 12 # 75 # # # #
0 3 15 # # # # # #
1 4 19 # # # # # #
0 4 # # # # # # #
```
2+5+8

```
1 0 0 1 0 0 1 0 0
0 0 0 1 1 1 2 2 2
0 0 0 1 2 3 5 7 3
1 1 1 2 4 7 12 19 #
0 1 2 4 8 15 # # #
0 1 3 7 15 # 57 # #
1 2 5 12 # 57 # # #
0 2 7 19 # # # # #
0 2 3 # # # # # #
```
3+6+9

```
1 1 1 1 1 1 1 1 1
1 2 3 4 5 6 7 8 3
1 3 6 10 15 21 # # #
1 4 10 # # # # # #
1 5 15 # # # # # #
1 6 21 # # # # # #
1 7 # # # # # # #
1 8 # # # # # # #
1 3 # # # # # # #
```
1+4+7+2+5+8+3+6+9

p1 +   p2 +   p3 +
p4 +   p5 +   p6 +
p7 =   p9 =   p9 =

1+4+7   **2+5+8**   **3+6+9**

1+4+7+2+5+8+3+6+9

Next two binomial progressions were based on inputs '101010101…' (graph 1) and '010101010…' (graph 3). Graphs 1 and 3 were then recombined to form a Pascal's triangle (graph 5). Graphs 2, 4 & 6 report the primes in graphs 1, 3 & 5. Graph 6 can be explained by each coloured cell being the result of a logical OR decision being applied based on the corresponding cells in graphs 2 & 4.
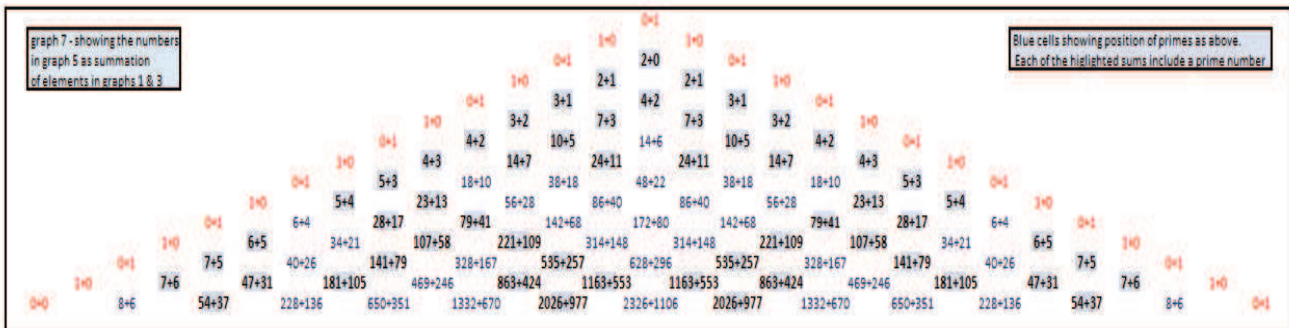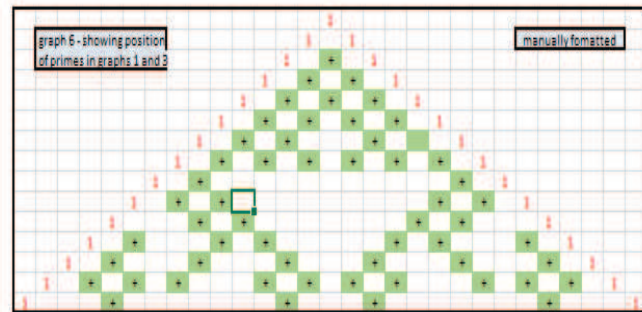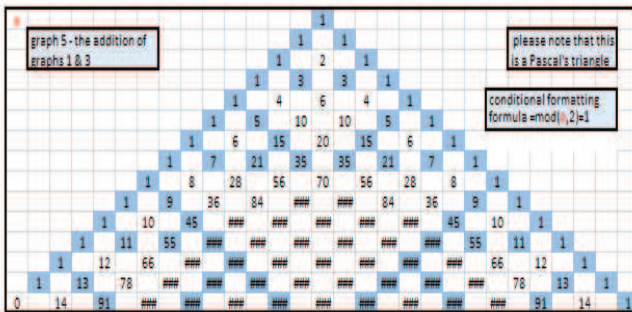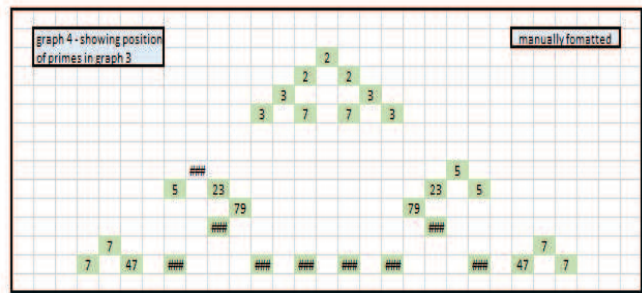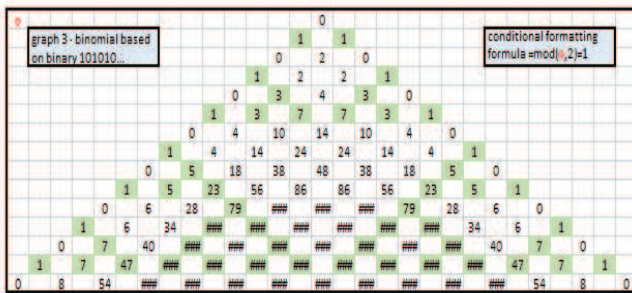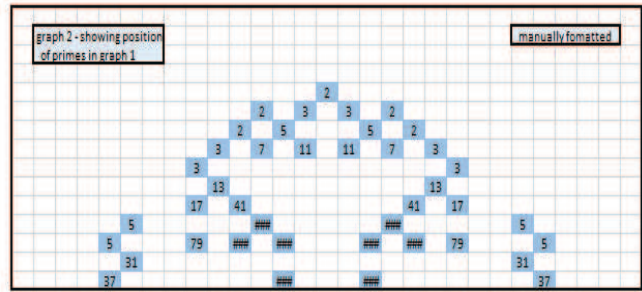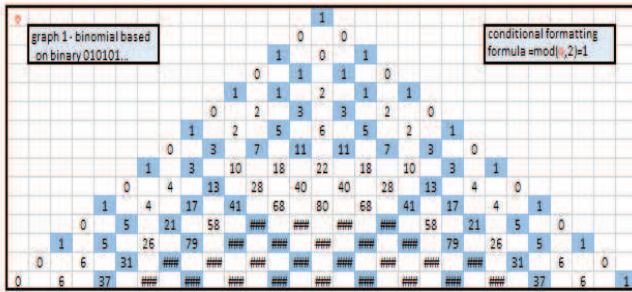
Next, a large trinomial grid was explored. Each cell other than the input 1s are a summation of the three above it, as is usual with trinomial triangles.

=BP5+BQ5+BO5

BO	BP	BQ
	1	
1	1	1
2	3	2
6	7	6
16	19	16
45	51	45
126	141	126
357	393	357

Figure 8 – the start of our trinomial triangle
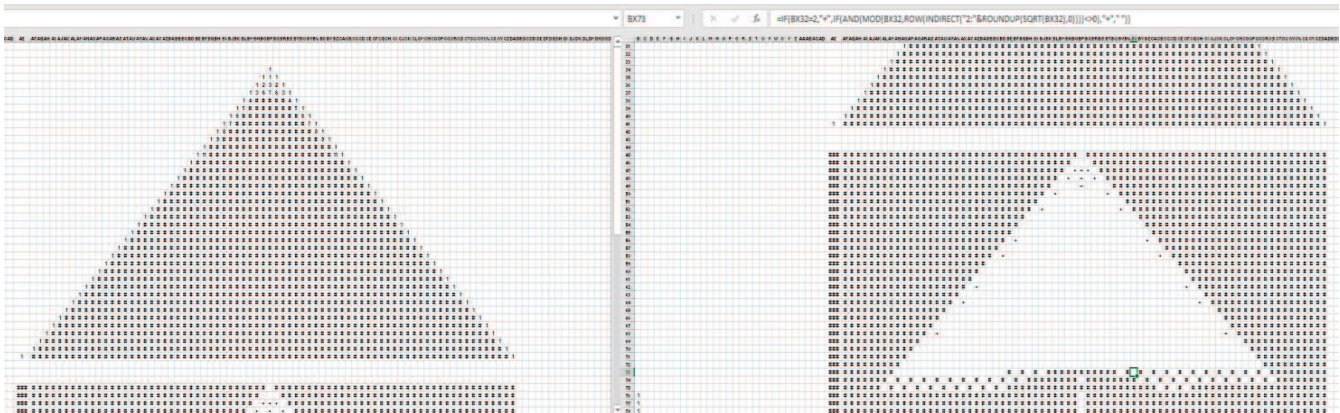
Figure 9 – rightmost grid reporting primes in the left.

It was found that the only primes propagated were in place amongst all the other integers along the first row beyond the inputs of '1,2,3,4,5,6,7,8,…'.
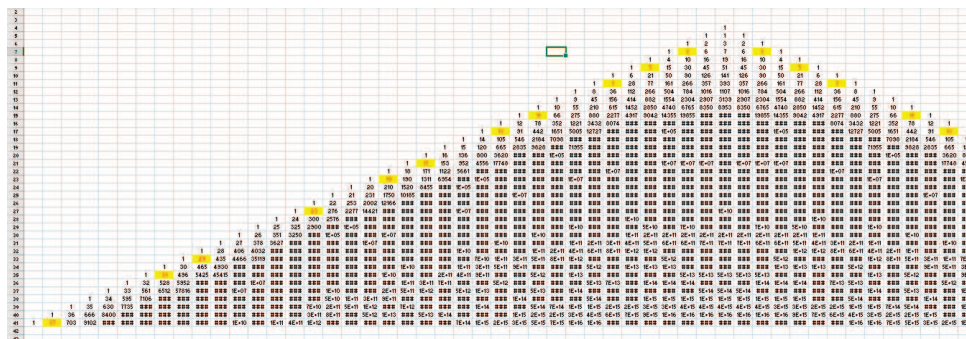
Figure 10 – the leftmost triangle in fig.9 with all propagated primes highlighted in yellow

Splitting this trinomial triangle in the same way the as binomial triangle was split into constituent progressions using a pattern of 101010101…, it was observed that the numbers, including primes, seemed to be propagating perpendicular to the line of inputs on each side, illustrated by blue lines here:
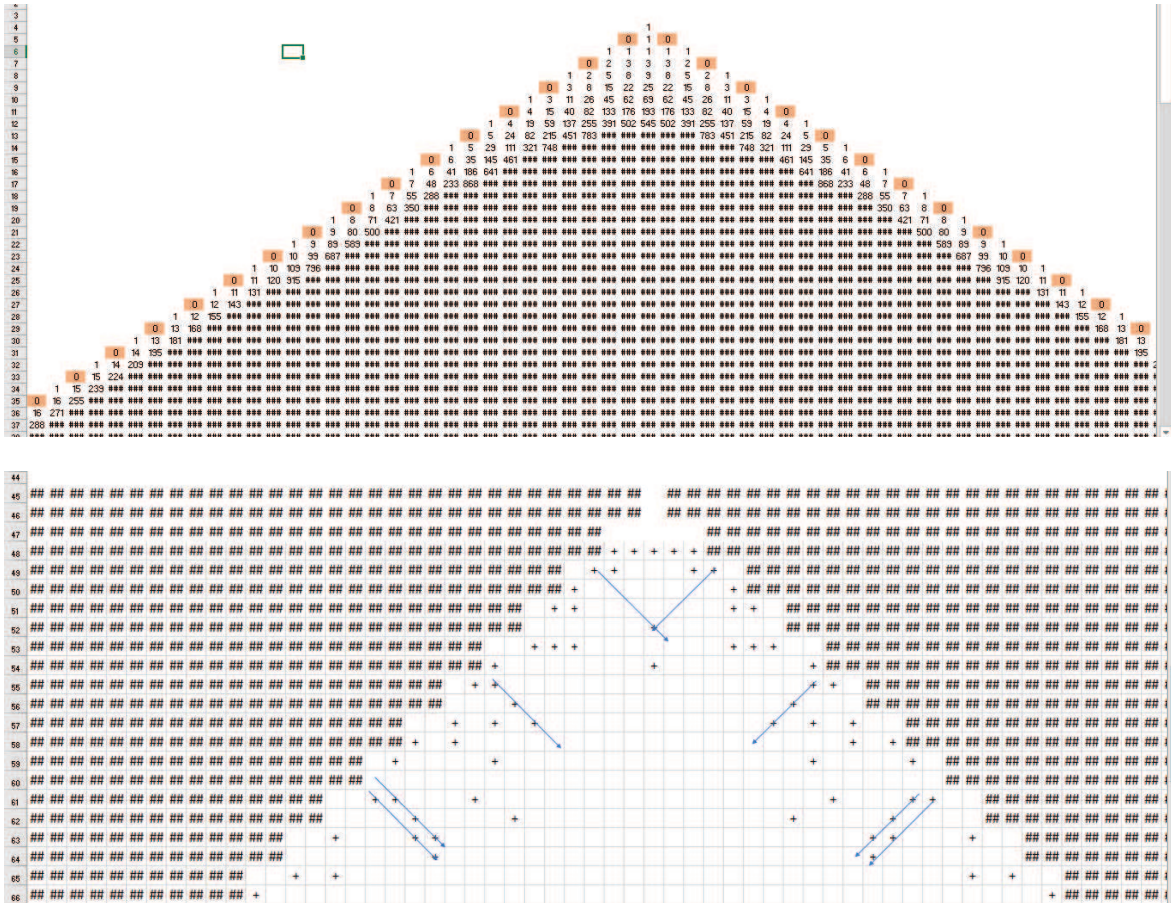
Figure 11 – showing lines of primes perpendicular to the line of inputs 10101010… on each side

It was decided that now we are looking at trinomials, a change in the shape of cells deciding the progression would move these lines of primes. We could not do this with a binomial triangle.
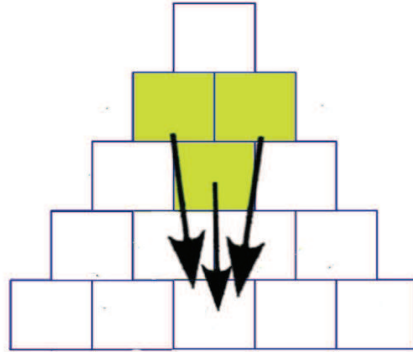
Figure 12 – showing the revised sum progression used in all grids to follow.

Moving back to and updating the spreadsheet used earlier (shown in fig.7 with accompanying prime tracker, Grid 2 on fig.8). Grid 3 on this spreadsheet was deleted and Grid 2 (tracking primes in Grid 1) was cleared of all formatting. Please note the revised shape of the three cells that make each calculation up, as shown in fig.12. This decision was made after viewing the positions of primes in the 10101010… trinomial triangle (fig.11) and wishing to bend them into something with straight lines of primes. This starts with cell D4 being a summation of C3, C2 & B3, and this formula copied to every second cell.

Figure 13 – progression based on D4=C3+C2+B3, coloured with a gradient conditional on cell value.

We only get 3s propagated here. Changing the inputs from 1s to a progression of integers allows us to look at how those 3s numbers were propagated down through the previous grid. It was noted that for this progression of integers, no prime numbers were propagated within the grid except for the diagonal line of 5s and on the first row and column, in place among a rational line of odd numbers. This is further shown in fig. 14.
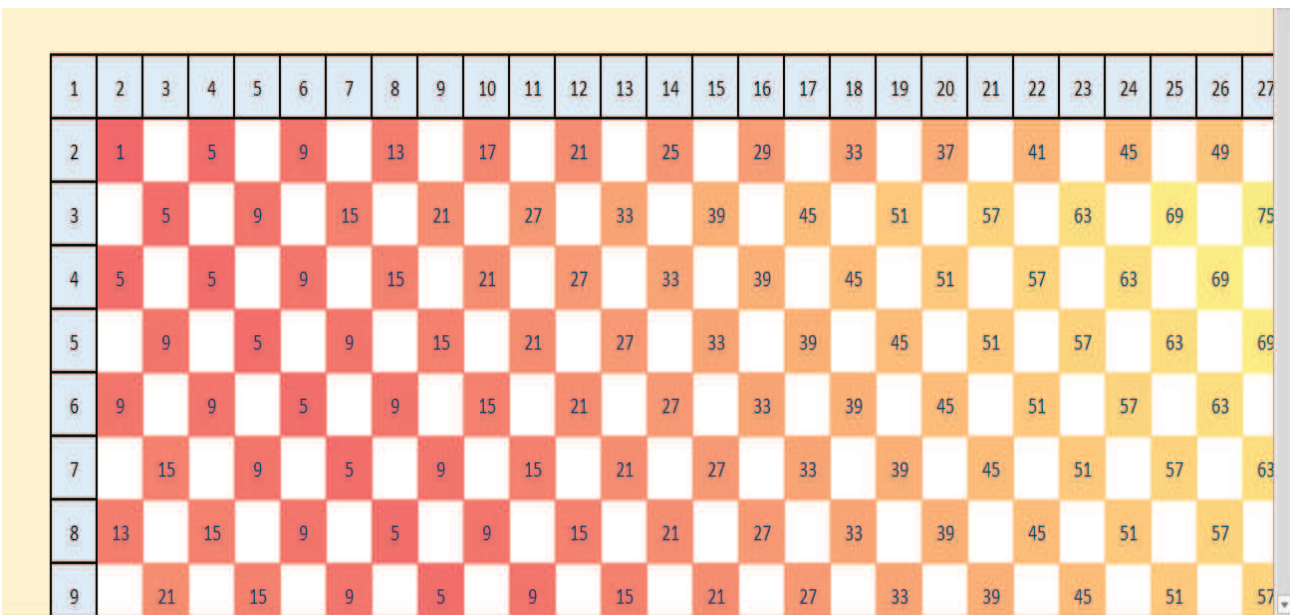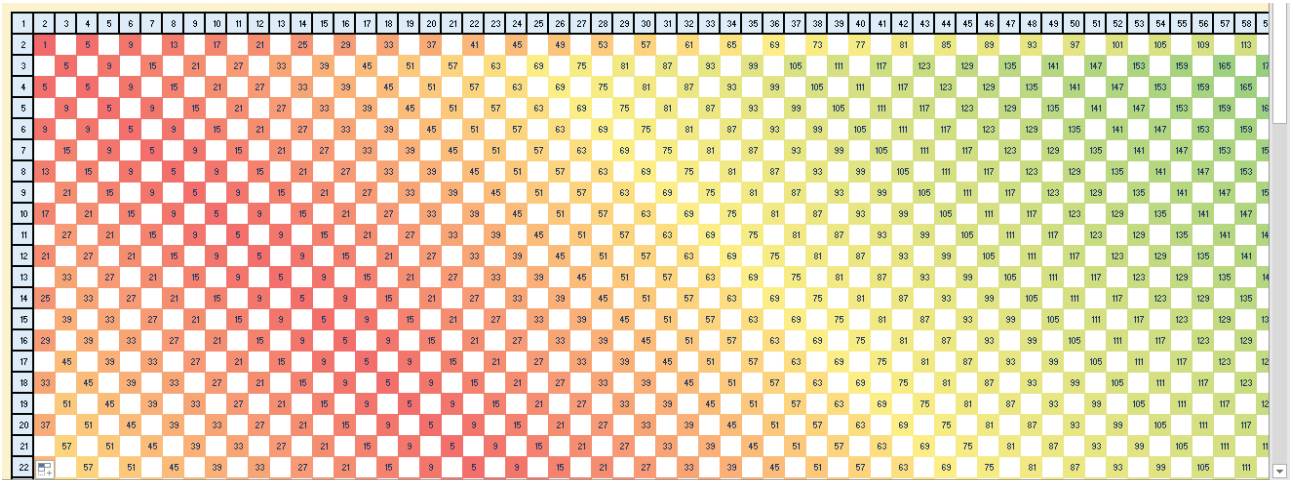

Figure 14 - a progression based on all integers.

Basing the grid on prime numbers allowed rules to be written for the primes propagated.

Figure 15 – mapping our grid to prime numbers

Figure 16 – looking at the primes propagated in the grid shown in fig.18, all 'inputs' on and prime.

Looking at the first three rows as far along as those first 7 propagated primes on row 2. For the purposes of calculations row 2 = Output 2. Since the 3 in the first cell on the row 2 was only a copy of the input 3, it was not used in the following calculation relating to the numbers on row 2.

	i/p	o/p1	o/p2	o/p3	o/p4	o/p5	o/p6	o/p7	o/p8	o/p9	o/p10	o/p11	o/p12	o/p13	o/p14	o/p15	
i/p	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	
o/p1	5	3		12		24		36		52		68		84		100	
o/p2	7		13		23		41		59		83		109		131		

Input: primes

Output 1: 12,24,36,52,68,84,100

$$a_n = \frac{n^5 - 25\,n^4 + 245\,n^3 - 215\,n^2 - 366\,n - 720}{30\,(2\,n - 5)} \quad \text{(for all terms given)}$$

Output 2: 13,23,41,59,83,109,131,159

Changing the inputs in the graph shown in figure 19 to text 'n1, n2, n3 ...' and changing the grids addition formula from *'D4=C2+C3+B3'* to

'D4 = C2&"+"&C3&"+"&B3' allowed for the following table to be made. This table has been manually tidied from the results of using this formula.

n1	n2	n3	n4	n5	n6	n7	n8	n9	n10	n11	n12	n13	n14	n15	n16	n17	n18	n19
n2	n1		n3++n2		n4+n5		n6+n7		n8+n9		n10+n11		n12+n13		n14+n15		n16+n17	
n3		n1+n2+n2		n2+n3+n4		n4+n5+n6		n7++n6+n8+		n8+n9+n10		n10+n11+n12		n13+n12+n14		n14+n15+n16		n16+n17+n18

3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	0	0	0
5	3		5+7		11+13		17+19		23+29		31+37		41+43		47+53		0 59	
7		3+5+5		5+7+11		11+13+17		17+19+23		23+29+31		31+37+41		41+43+47		47+53+59		0 59 0
		13		23		41		59		83		109		131		159		

It was noted that except for the initial 13 which had two n2 terms, the primes being added to give the propagated primes were in a very pleasantly consecutive order: - 5,7,11   11,13,17

17,19,23   23,29,31   31,37,41   41,43,47   47,53,59

Figure 17 – looking at those sums, and the flow of primes through them.

Visualising these summations with the constituent primes moving begged the question how many there would be of each, for instance how many 5's will there be? Maybe 5?

Figure 18 – colours added to help see how lines of primes might be extrapolated.

There are too many 5s.

It was decided to make a 'triangle' with each prime shown as many times as its value, to block in the

prime trios from above and to compare lines drawn between the trios and their summations on a linear line above the prime shape. This shape, plenty explored before I am sure, will not be a triangle, but the lines we are going to draw coming off of the prime 'triangle' will also not be linear as a result of the shapes irregularity. It can be expected that if there is a congruence that this will be evident in the angles of the lines drawn to connect the trios and their sums on a linear line of numbers, in accordance with the work of Thales of Miletus.
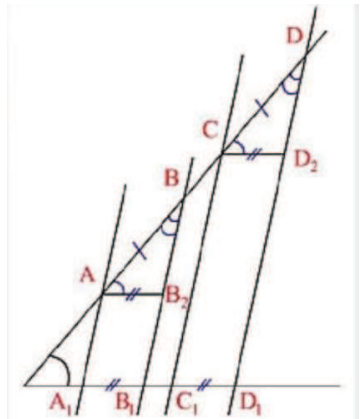
Figure 19 – an inspiration
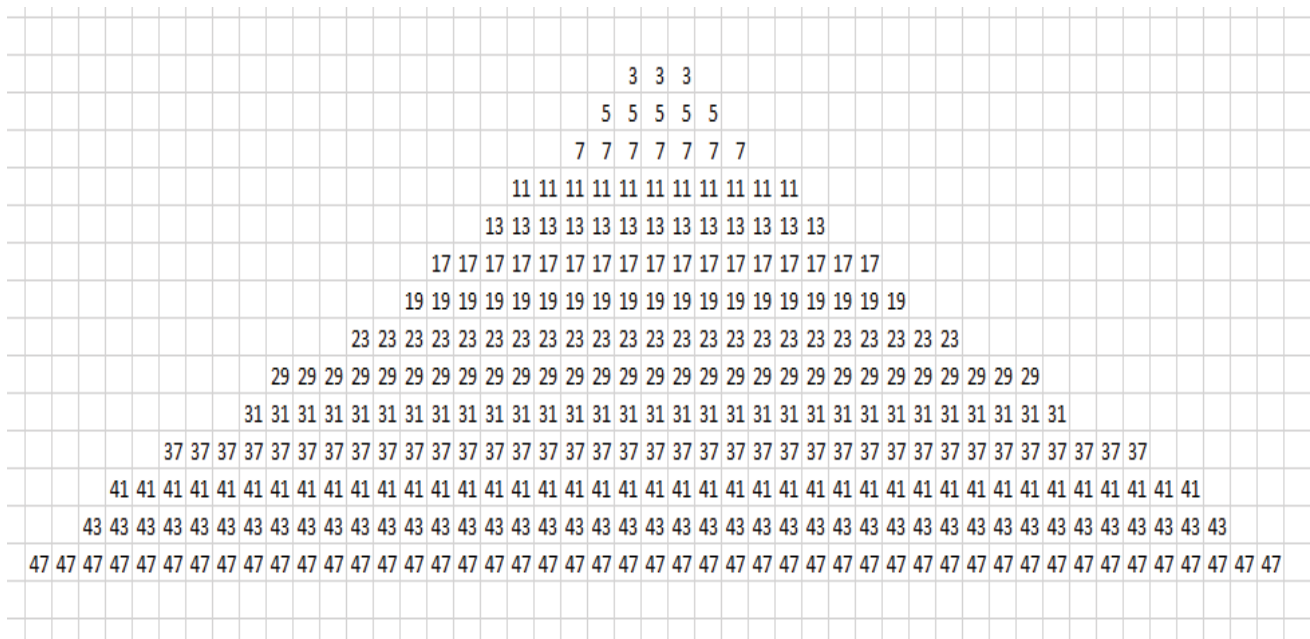
Figure 20 – the prime shape mentioned above; our 'triangle'

Not all of the summations add to prime numbers, for instance 3+5+7 = 15 and 13+17+19=49. The

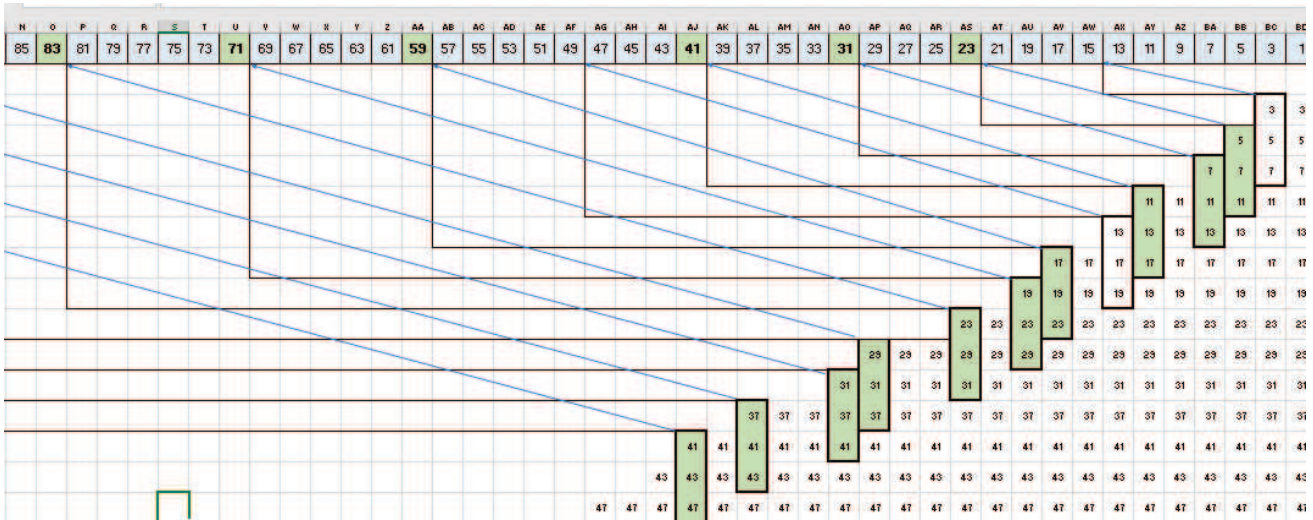cases where the summations are prime have been highlighted in green:

Figure 21 - consecutive prime trios showing an angular correspondence to the value of their

summations on a linear line of odd numbers.

Drawing the blue lines as shown above, and the black lines showing the blue lines as vectors of

height and length, it was found that that the lines drawn between groups of consecutive prime

trios towards the value of their summations on a linear line of odd numbers showed an angular

congruence.

It was further found (and illustrated on the other side of the same prime 'triangle', highlighted in

yellow in the following graphic) that lines drawn between groups of four consecutive primes

to the value of their summations on a linear line of even numbers also showed an angular
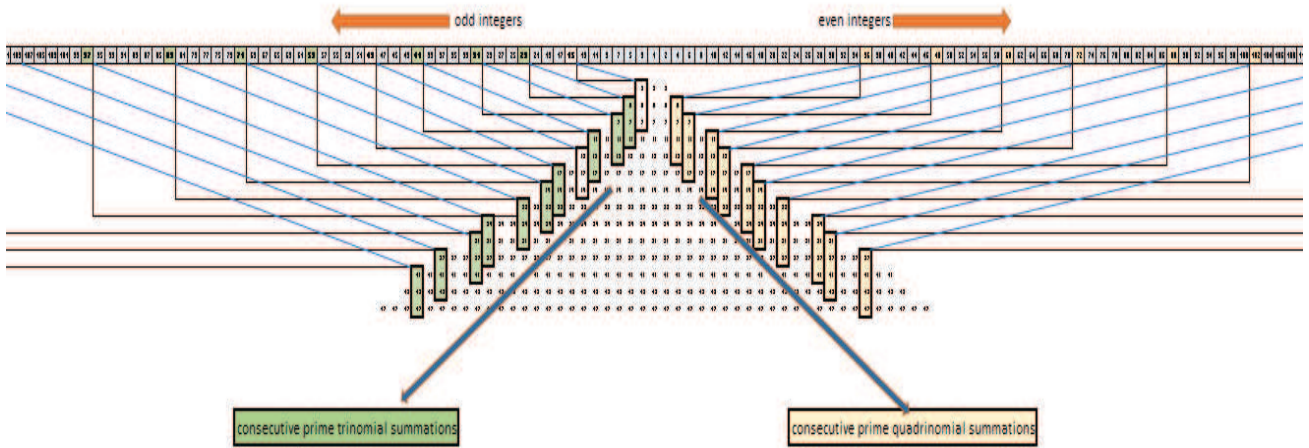
similarity.

Figure 22 – showing angular similarity in trinomial and quadrinomial summations
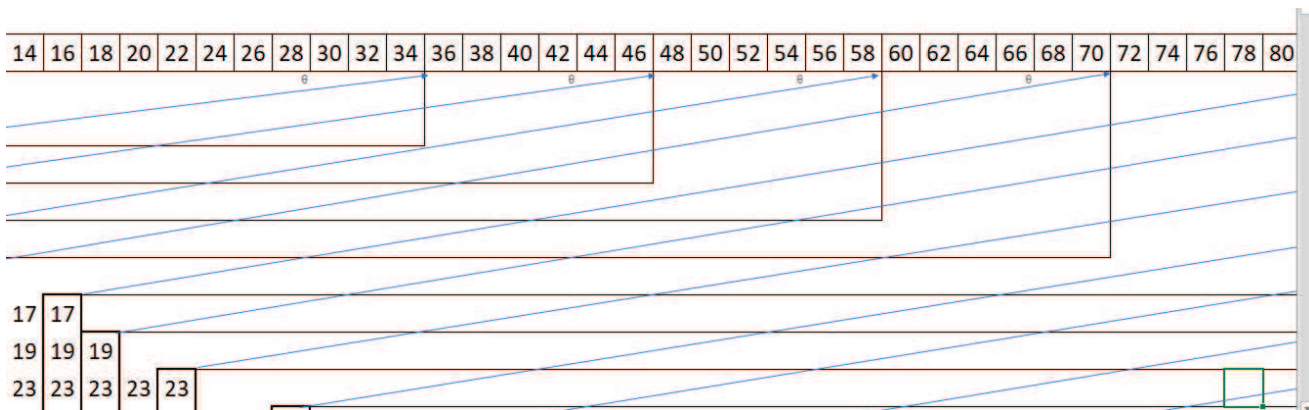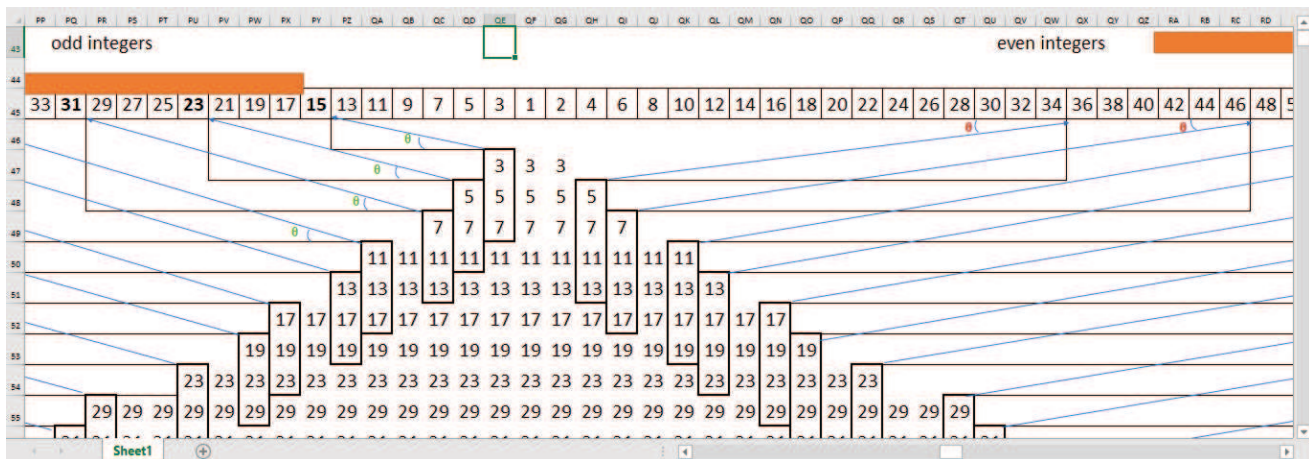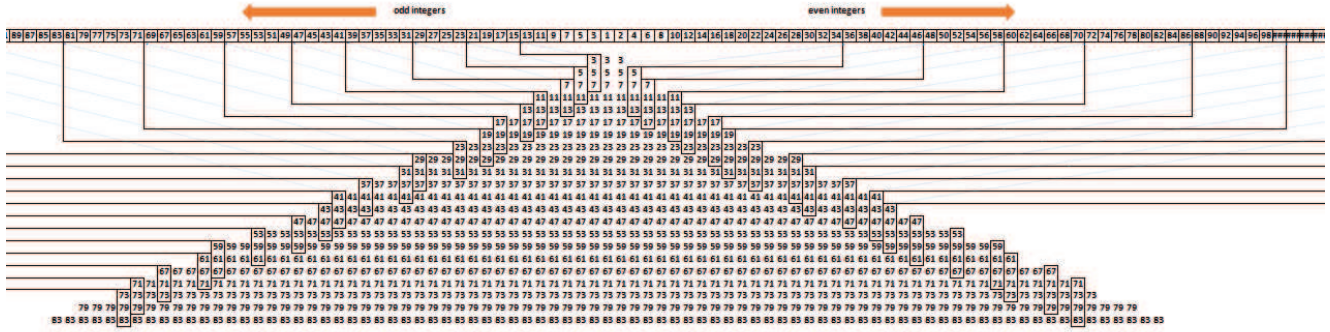

Figure 26 – zooming in

References:

Bannink, T and Buhrman, H (2017) *Quantum Pascal's Triangle and Sierpinski's carpet.*

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Marc Schofield   20/12/2020