THE COSMOS DYNAMIC WITH NAVIER STOKES

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Abstract

Human beings have always felt the need to know and predict events future, using Science. Many natural phenomena are explained by numerical models; but there are as many numerical models as there are phenomena; this is problem, a big problem; ideally, many events should be explained using for this, the least amount of mathematical models.

Navier Stokes equations have been used for many years to simulate fluid dynamics. There are many particular cases in which these equations describe the dynamics of various phenomena as different as economics and meteorology. Most attempts to use these equations in a variety of fields, lie in properly defining the variables involved, giving them a physical explanation. This is an exciting challenge, of course ant that, is the main goal for this article: applying these equations, for explaining the Cosmos dynamic.

If we look at many natural events, we will see that they evolve as a fluid: flocks of birds, vehicular or pedestrian traffic, are typical cases of analysis, but we can also observe this dynamic in events such as the stock market, the economy or even human relations.

1. Introduction

Cosmos (Carl Sagan): “Cosmos is all in the past, present and future.”

All the matter in the Cosmos seems to behave in the same way: its dynamics is based with the same or similar laws, whatever the scale (not atom scale).

If it want to explain how the cosmos works, it is necessary:

- To detect patterns in events.
- To detect patterns between events.

And what is more important and transcendental:

- To detect patterns between numeric models that describes the events.

It will find similarities between phenomenon’s and numeric models. This is one the main goal.

We mathematicians speak a language. In language made up of words, groups of words, sentences, paragraphs and even feelings.

Mathematics can also be applied to solve physical problems, simulating "analogies"; for example, the case of traffic jams in cars, using the analogy of a water hammer in pipes: a multi-lane road can be installed to mitigate a traffic jam, in the same way that a water tank is installed, to reduce or eliminate water hammer (water tank and multiple roads, simulating a “expansion”).

In a particle group and its movement, there is a force that push every particle; this force is the result of several forces (friction, magnetic, pressure, Coriolis, etc), which, acts on the particle. It is incredible, but this occurs in each of the particles of a group, and each of them, completely unaware of the forces and directions of other particles. This mutual ignorance perhaps is the main reason in the generation of precious figures or geometry and global structures.

The main goal for any mathematician is create numeric models about nature phenomenon. For that, is necessary discovery (or create artificially) patterns, and if it is possible lineally, but that, is not easy, and normally not real. As a writer, a mathematician thinks with languages and as all language have their rules, their pretty rules…. It is very nice is front a white paper and write ideas and translating dreams….

The human being expresses himself with numerous languages in the many places of the world where Mathematics is another language, with its rules. Any language serves to express feelings and facts; if the Mathematical language is capable of having words that express many feelings under many hypotheses or contexts, it will be able to express more faithfully or really, a natural fact. If Mathematics is not capable of expressing a phenomenon, it simply means that language must be advanced and improved until it is capable of modeling it.

A Mathematician therefore explains and describes "Reality" with a characteristic language. There are phenomena in Nature that are not yet explainable or easily explained. This is because there are not yet any mathematical words or sentences for it. The work of a good mathematician is double:

- To know how to use all the mathematical language that already exists.
- To create new words to explain phenomena.

For example, the continuity equation or divergence zero, means that the density does not vary.

We, as human beings, have had the feeling on many occasions, of wanting to express certain feelings, ideas or dreams, but we cannot: there are no words that can express them. We use then, unions of already existing words that normally express badly what we want to express. The fact of call one event as unpredictable is to assume ignorance. The goal so, is know the evolution in any coordinate of any object or event, from similarities. In the nature, there are a lot of think very weird, about patterns and data series:

- Benford law, applied for example, in distances of galaxies from earth (Timoteo Briet Blanes – 2017): (brown=Benford law, yellow=data 4.000 galaxies data founds from Internet).

There are many fields where this Benford law applies, such as fraud detection (invoices, etc...). But also in other fields such as brightness in objects captured by the Fermi telescope, or other cases.

- Generation on Lissajous curves from different cases: for example in lift against position in a vibrating wing, etc… ([30] How to design a Race Car, Step by Step; Timoteo Briet Blanes, Amazon).

- Even, Vortex Street is sometimes formed when the wind from a star flows past a neutron star companion…. And also, Vortex Karman street, in different scales as a turbulences in cylinders, atmospherics events, tail in striped galaxies, etc….

- It´s possible to see the vortices-disturbances created from the Kelvin-Helmholtz effect, also in different scales: clouds, Orion Nebula.

Many times, it is not easy to observe a well-defined or clear pattern. It´s therefore necessary to analyze data or geometries, perhaps indirect to the phenomenon, which may give rise to a possible pattern.

For example, in a Meteorites rain (Quadrantides – temporal data from J. M. Trigo - January 1992), it´s possible detect and analyze one geometry multifractal (may be because there is a random variable….) ([16] Structure multifractal in the galaxies distribution; Vicent Martínez García; University Valencia – Spain; 1987).

And more: it´s possible to see some phenomenon or properties as a fluid, in objects or particles dynamic.

For example, Bernouilli principle in accumulation-aggregation or exit of people from sport stadium, also sheep out of a stable, even it is possible apply fluids theories in vehicles traffic in cities, etc….

About the phenomenon prediction, if there are few laws which define him, it will be more complicated to know the evolution (chaos essence).

From all that is necessary to ask us, if there some think common for all these cases, some law able to means these examples.

That is the main goal for me: know how the nature think and decides, and create language or concept mathematics, pretty and simple, in order to explain any event, as a fluid or as a particle.

To know the evolution of any event means the introduction may be of a probability of to be or not to be. That is very important.

Can you fight the flow of a brave river trying to reach the edge? Surely it will be useless, but you can try....

Every person has his own will and is able to choose his destiny or displacement as a decision or choice, but the group dilutes that will; It might even alter your environment, but only the environment ....

A person solitary, is unlikely to originate or produce a "different" evolution of the whole; but it will be able to do so, only in the case of being able to generate a great impact that affects many people: the union, it makes force. When one speaks of "power," power is the ability to influence large numbers of people. The birds, don’t know what is the geometry of a flock, but hi flights and moves....

Who, when a very dear one has died, has not thought that the world is going to stop, that the sun will not come out any more, or that everything will change, or that he will telephone on your birthday to congratulate you. Really the sun does not come out the same way and with the same beauty, but the world follows, and despite what happened, everything remains the same.... and never phone.... I need understand the Carl Sagan cosmos, but I and my actions, are very and quite insignificants….

There is a special relation between sloth or minimum energy principle and fluid dynamics: If I must to go from here to there, yes; I will go: but, with the minimum energy,,,, If it ask question about universe, it would be able to understand it….

Some variables found in the Navier Stokes Equations have been adapted to analyze problems of accumulation of people in premises, as in ([8] Kazunori Shinohara and Serban Georgescu) without offering a clear and well-defined protocol for analyzing any problems related to pedestrians; in fact, in other articles on pedestrians, a protocol is drawn up but adapted to each problem, without defining a generic one. Furthermore, there is no discussion of the possibility of working with these equations, in "n" dimensions greater than 3.

In ([3] Jakimowicza and J. Juzwiszynb), the possible spirals or vortices that would be formed in problems are commented economic, if studied in 3 dimensions; but they do not offer either a clear and well-defined, for all kinds of economic problems. Same as above, Nor is there any discussion of the possibility of working with such equations, in "n" dimensions greater than 3.

In short, all the articles and research I have been able to obtain and analyze have the same problems:

- They offer a numerical model and/or protocol of action, adapted to each problem.
- They do not work with Navier Stokes Equations in dimensions greater than 3.

What is attempted in this Article is to solve both problems.

First, the variables used in the numerical models are defined; then the methods by which matter accumulates are studied, to then describe numerical models for the dynamics of a group of particles; finally, the Navier Stokes Equations are analyzed, proposing an application protocol to describe and predict events, in general.

2. Definitions

2.1. Event

It cans considerer an Event, as any concept, dependent of time: Event, Phenomenon, Particle, Success, is the same concept; as Carl

Sagan understood it: "all that has been, is and will be". Fluid is a group of particles.

There are 2 types of events:

- Continuous events: those events that at any moment have a value. For example, the price of oil; in this case, a price increase is established, so that as long as the price does not vary more than this increase, the unit of discretion does not vary either.

- Occasional events: those are not continuous. For example bomb explosion, volcano eruption, etc.

2.2. Dependence, dimension and representation

A coin is thrown: what is the probability that it comes out face? The answer seems pretty obvious. But, and if it is known that previously the same coin has been launched 50 times and has always has face? The answer is no longer so simple, besides that there are some explanations mathematically (Markov chain, etc….). Also analyze Bayes, Pascal and Anchenwall. Does it therefore influence what is known a priori of an event in order to predict it? Does knowledge influence? Yes that influences, indeed: if you ask us if it's going to rain an hour, just look at the sky and know if there are many clouds....

If an experiment is measured, it can affect the development of that experiment; let us suppose the following situation: we roll a die; what's the probability of it being a "6"? And if we know that before, it has been thrown 50 times and a "6" has always come out? We will answer the same thing? The previous knowledge, influences this test?

Be 2 events; it is assumed that one of them varies and it is observed that the other event also varies or responds to the variation of the first. Are both events therefore dependents? One could say yes, as long as these mutual variations are known over a suitably long time, since, perhaps, the second event varies "coincidentally"....

A group of events can be represented by their relations between them, in the following ways:

- Springs, dampers, shock absorbers, fix bars or nothing:

- Fixed bar (positive or negative): one event moves in the same proportion as another to the same direction.
- Spring: it is defined analogously to the bar, but with a force of repulsion or attraction, as a spring.
- Damper: it is a displacement damper, applicable to bars and springs. Is a try to enter the variable "time” and velocity.
- Inerter. Is a try also, to enter the variable "time" and acceleration.
- Nothing: if are not dependents.
- Etc….

It is possible to apply "mass" (size) to the event, as "importance" or "transcendence" or “weight”.

The options, therefore, of connections between events, are endless. All these relationships can work under linear and non-linear functions.

- An event is represented according to different "Coordinates", which are the variables on which the Event depends. The "Dimension" of the event is defined as the number of variables on which it is possible to represent it.

In the face of the evolution of an economic crisis, it always asks us: "until when?"

It do not know at all, when it will stop downloading, or when it will stop uploading in your case; but one thing is clear: at some point it will stop going down.

There is nothing that goes up or down forever; like a diver, no matter how deep the waters you dives, "always" there will be a time when you touch the bottom or reach your maximum depth.

To say that the economy rises and falls alternately, like a saw tooth, is to admit our ignorance of how it evolves; besides, if he did not do it, it would be absolutely incredible to go up or down constantly ... Sure it would be surprised.

And another question:

Is there any merit in "leaving" that some stones, thrown into the sea, reach the bottom, is there merit in saying that they will reach the bottom?

Imagine a pool like an ocean; if we open the drain, sooner or later, it will empty...

The question always arises: "What to do?".

All governments "try" to mitigate the effects of the crisis, "doing things" under the options and criteria, more or less successful, that mark or govern their ideologies.

But also, we can all verify that these actions either have no appreciable effect, or are slightly appreciable in the very long term. If indeed it can see some effect, it is simply because the previous diver was already close to the bottom....

The world economy or global dynamics is the one that always prevails; it's like wanting to empty the sea, from glass to glass.

It´s true that before a small action, as it is to cover the drain of the ocean, we make it never empty; but we will know that it is not going to be emptied, in a very long time. It´s more: there are actions that do not affect "absolutely" in anything; therefore, it has 3 possibilities:

1. Do something and see its possible consequences in many and many years.
2. Do something that does not affect anything (and people see that something is done).
3. Let the global dynamics prevail and flow...

What is the best choice? The 3; At least, let's dedicate ourselves to enjoyment and that other rights are not affected. Sup 3 events (A, B and C) (Fig. (1)); "A" fixed; then if "C" moves, "B" will move; but the greater "b" and the smaller "c", keeping "a" constant, the displacement of "B" will be less.

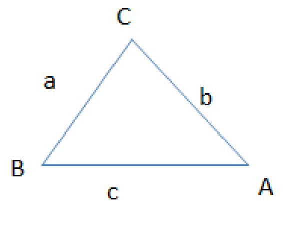


Fig. (1)

It is an example to observe that although we have 3 dependent events, certain displacements of one of them, may have very little importance on the others.

Any government that takes credit for taking a country out of a crisis lies: it simply has been lucky to be at the right time.

It can define "being alive" to that substance that is able to have notion or consciousness of the passage of time.

It is possible to perceive time in a different way; in fact, when it is sleeping or when it is older, it does so.

It´s time the necessary variable for there to be a dynamic? If everything were causal, the existence of time would not be necessary, since "everything" would already be defined and marked until eternity. It is also true that, as we have already seen, in the dynamics of a set of phenomena, only one of them lacks the power to modify fully; it is the randomness that marks this effect or influence.

Randomness is necessary in the universe, for whatever reason, but it is necessary.

In fact, let's think of 2 different phenomena (water flow and galaxy formation): time scales and time are different; the time is other coordinate so.

It is as if the dynamics of the universe invite us or force us to standardize time and its scale, in order to be able to compare.

It defines dimension of event, as a coordinates number of event; that is: every coordinate is a factor which the event depend. From these coordinates, it´s possible so, representing the event.

2.3. Velocity V

Velocity of an event "E in a dimension "d" is defined as the number of events with respect to that direction; that is:

$$V = \frac{\partial E}{\partial d} \qquad \text{Eq. (1)}$$

2.4. Density ρ

Density "ρ" is defined as the number of particles per unit volume or time interval (or any group of coordinates).

In general, Density is defined as the quotient between the number of particles enclosed in a ball of determined radius "R" and center of particle, and the volume of the ball. This definition is extended to "n" dimensions, defining the volume of a ball of "n" dimensions as:

$$\frac{\pi^{n/2} R^n}{\Gamma(n/2+1)} \quad \text{Eq. (2)}$$

"z" is an integer and "Γ" being the Gamma function:

$$\Gamma(z) = \int_0^\infty t^{z-1} e^{-t} dt$$

The time will be the most used dimension, but density can be calculated in any coordinate or direction.

The definition of Density in a "$\vec{V}$" direction of dimension "n": "N_v" number of events in the "$\vec{V}$" direction and "Vol" the volume of dimension "n":

$$\rho = \frac{N}{Vol} \quad \text{Eq. (3)}$$

2.5. Pressure P

First, it can think about pressure "P" as a definition in Kinetic theory of gasses (proportional to "m" total mass particles, "1/Vol" volume ("Vol"), "u" average velocity of particles, impulse (m*u) and "N" number particles); "ρ" is a local value, that is: around a point, "K" is a constant):

$$P \propto K \frac{mNu^2}{Vol} = K * \rho * u^2 * N \propto \rho u^2$$

Eq. (4)

This concept is very important in galaxies formation and evolution or in general in cosmology. In this case, "P" is called "Ram Pressure", and very similarly, to Einstein equation (simplified) $E=mc^2 (m \leftrightarrow \rho)$ ("c" speed of light, "m" mass, "E" energy):

$$E = mc^2 = \frac{m}{Vol} c^2 u \propto \rho * Vol$$
$$E \propto P * Vol$$

Eq. (5)

2.6. Compressibility

It can have a fluid with compressibility but is necessary to know the velocity for this compression or expansion. This value is the divergence of velocity; that is: the variation of volume, and may be positive or negative. It suppose that events group, may be different pressure against the time or other variable. That is: 2 events in a fluid (as a particles set) non incompressible:

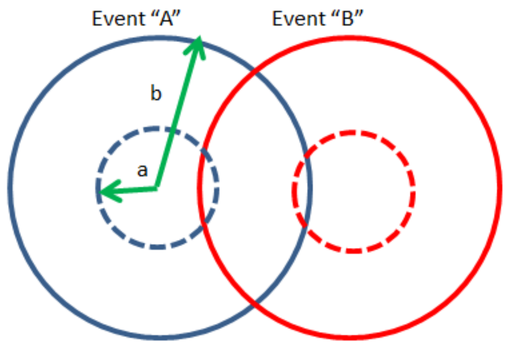


Fig. (2)

The 2 events "A" and "B" (Fig. (2)) cannot be less than a distance "a" or more than a distance "b". In case of being more than "b", the events can be considered independent, in the first phase. These distances "a" and "b" can are different depending on temperature, pressure and density, for the same fluid-group of events (depends of a fluid type). The speed of compression and dilatation ("a" and "b", is function of spring-damper system, or other combination between forces, velocity, acceleration, etc…). The density of a fluid formed by particles depends directly on the compressibility and vice versa; compressibility is defined as the force applied to 2 particles to bring them closer together. Be a closed box full of billiard balls; if it tries to move the balls, it will be absolutely impossible.

But if there is some kind of compressibility, the balls will tend to move and pass one another.... (Tennis ball, for example). From this reasoning, any people can perfectly understand Pascal's principle, or the transfer of forces between communicating vessels.

Let us suppose circular particles (2 dimensions); in an enclosure with such particles, there will be a certain pressure and a certain density; there could be another enclosure with other particles of a larger radius, with the same density, but the pressure will also be higher: the pressure depends on the size of the particles.

The compressibility can be defined also by:

- By “Z”: “Vol_m “is a molar volume, “R” is a fluid constant ”T” is the temperature:

$$Z = \frac{PVol_m}{RT}$$ Eq. (6)

“R” depend of “a” and “b” (size movement particle).

- By “1/β” (“Vol” is a volume):

$$\beta = \frac{\partial P}{\partial Vol} Vol$$ Eq. (7)

- By Pressure or Density variation (in 1 dimension or direction “x”):

$$\frac{\partial P}{\partial x} = \frac{\partial \rho}{\partial x}$$ Eq. (8)

- By diameter “a”.

2.7. Temperature

In the expression for pressure before, calculate it for a 1 mole; then (“N_A“ is Avogadro number, “M” molecular mass, “K” is a constant, “Rg” is universal constant of gasses, “u” the velocity “T” the temperature):

$$P * Vol = m u^2 N_A = Rg * T$$
$$T = K * M * u^2 \propto u^2$$ Eq. (9)

In this case, “u” is a average if velocity because: It is always said that the displacement of the particles or molecules of a fluid is something unpredictable: in the Brownian displacement, the particles vibrating (Temperature), produce a variation of position, and this position unpredictable, produce an evolution unpredictable.

Refractive index values are usually determined at standard temperature. A lower temperature means the liquid becomes denser and has a higher viscosity, causing light to travel slower in the medium. This results in a larger value for the refractive index due to a larger ratio.

2.8. Viscosity μ

The Viscosity seems a friction force in order to stop the dynamic or particle movement.

For example, whether "A" is an event (oil price), represented by its phase diagram, depends on 2 variables: productivity (number of barrels "Nb"), "Kw" (Kilowatts) per day produced by alternative energies; Time "t" is always present:

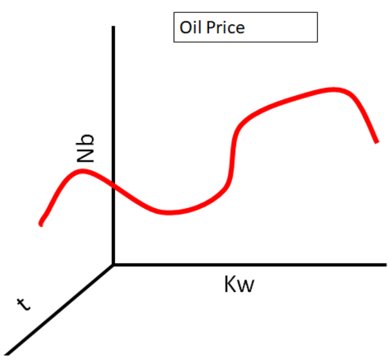


Fig. (3)

As it knows before with all definitions, depending on which coordinate or direction "V" the dynamics of the price of a barrel of oil is studied, there will be more or less resistance to its dynamics. So as before, the viscosity is calculated in a direction “V”.

When starting a car when the traffic lights turn green, it does so after some time after the car that precedes it moves (delay time or "T_d").

It also happens when the price of oil changes due to the index variation of the New York Stock Exchange-Market; it does not do it immediately. It defines "Viscosity= $\mu = 1/T_d$". In a fluid incompressible, ρ = Constant if and only if $\mu = \infty$.

It can see the same delay or gap time, in a typical prey-predator numeric model, between input and output (excitation and response - pick to pick) or in oil price against politic decision:

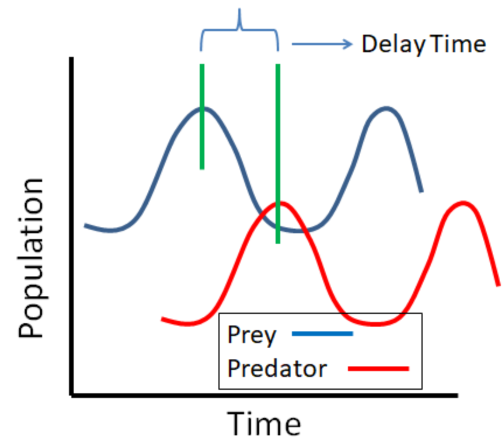


Fig. (4)

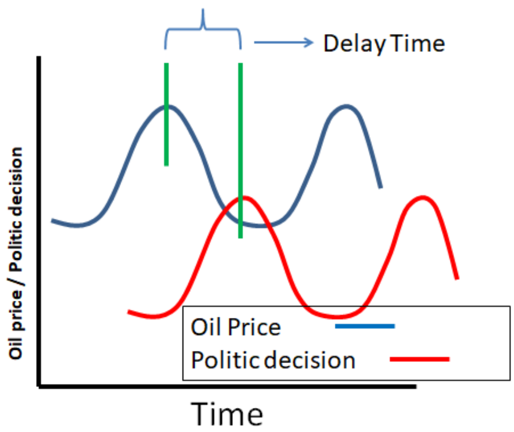


Fig. (5)

1/Viscosity, as reaction time or gap time, brings together the reaction times of all the forces involved in the displacement of a particle: the force or gravitational field, induces a reaction time, the same as the magnetic field, pressure and others; the "final" viscosity, is the reaction time of a particle, before all the force fields that work or act on the particle (adding all delay times).

It defines Sound vibration or Sound Wave: in a fluid formed by particles or mutually dependent events, a Sound Wave is defined as the evolution in a direction "V" of a variation between the particles.

Viscosity = 1 / delay time between molecules in a fluid, in order to transmit the sound in a direction "V"; it is a way to classify different fluids or events.

➔ Particular case:

Calculate now, the reaction time between 2 particles of a fluid (in this case, fluid is a fluid "traditional"), for transmit a sound wave. Coordinates of this event: "C" is the speed of sound (wave shock) in a fluid, "R" is the fluid constant, "x" is the average displacement of particles (as a Brownian movement), "t" is time and "N_m" the number particles in 1 lineal meter, "N_A" is Avogadro number:

$$T_d = \frac{1/C}{N_m} = \frac{1}{C\sqrt[3]{\frac{P}{RT}N_A}} = $$

$$= \frac{1}{C\sqrt[3]{\rho N_A}} \qquad \text{Eq. (10)}$$

Einstein viscosity value is:

$$\mu_E = \frac{RT}{N_A}\frac{1}{6\pi Dr} / x^2 \propto D*t$$

Eq. (12)

"D" is Diffusivity and "r" radio molecules or particles. So:

$$T_d = \sqrt[3]{\frac{\mu_E \, 6\pi Dr}{P\,C^3}} \qquad \text{Eq. (11)}$$

It is possible so, in this moment, to do a fluids classification against "T_d". For that, is necessary calculate all with the same pressure and temperature. The sound speed "C", for every fluid, depends of variation of pressure, against density; that is:

$$C \propto \sqrt{\frac{\partial P}{\partial \rho}}$$ Eq. (12)

This expression is equivalent to: the sound speed, depend of temperature "T". That is very important:

$$C \propto T$$ Eq. (13)

From another point of view, it has a particles group and between them, there is a spring between particles (or full fluid volume) with a constant "K"; from Hookes law, it is ("x" displacement, "u" velocity", "t" time, "m" the mass):

$$F = Kx = m\frac{u}{t}$$

$$K = \frac{mC^2}{N_A \mu_G} \propto \frac{m}{\mu_G}$$ Eq. (14)

This "delay time" or "delay phase" (between input and output signal), can produce Lissajous curves: for example: in a flapping wing case, show a position against lift generate by wing, for a one frequency; show "input" and "output" and delay time between them ([17] Timoteo Briet Blanes):

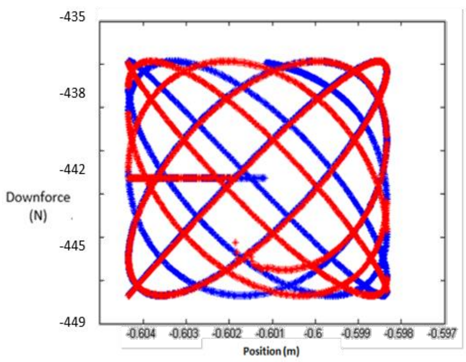


Fig. (6)

What is the Diffusivity "D" as a fluid property? It´s the tendency to fade. If it has a spherical particles group ("r" radio particle, "K" is a constant). When the viscosity and radio particle is greater, the diffusivity is less, if "T" (temperature) is greater the diffusivity also; that is:

$$D = K * \frac{T}{\mu r}$$ Eq. (15)

Comparing this, with the Einstein relation for diffusivity ("K_B" is a Boltzmann constant) (very similar):

$$D = \frac{K_B T}{6\pi\mu r}$$ Eq. (16)

The viscosity is a function of Density; that is: the Viscosity, depends of Density by a function "f":

$$\mu = f(\rho)$$ Eq. (17)

Even is possible to know the viscosity, density and viscosity/density of dark matter (plus baryonic matter) in a galaxy, in order to create a "real" velocity rotation curve ([10] Timoteo Briet Blanes).

2.9. Similarity number

In order to be able to compare phenomena with each other or simply to know limit or transition values between different dynamic states, a value is needed. This value is denoted as "S_n".

For example, in evacuation systems pedestrians, is possible to define other phenomenon number; this is a case particular: pedestrian group in a room with exit of "A" dimensions; red arrow is the people direction evacuation:

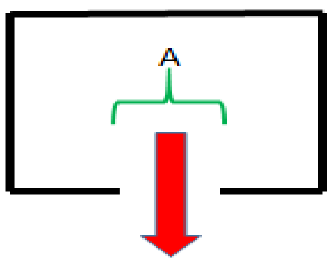


Fig. (7)

"V" velocity of pedestrian, "A" length of door, "ρ" density of pedestrian group, "T_d" delay time or reaction time between pedestrians; this value, it can apply to fluid in a duct:

$$S_n = \frac{\rho V A}{\mu}$$ Eq. (18)

This value, not have dimensions; must to be as that: density → number people per square meter, Velocity → meter per second, "A" → meters, "μ" → 1/time.

Is necessary now, calculate the value for the Similarity number, from which there is a accumulation of peoples dangerous in exit (may be change from laminar to turbulent).

For calculating the gap time: given a group of people, we push or move one of them; the time it takes (on average) for the people around it, will be the gap time "T_d"; this test, in the same density conditions, as the problem to solve. In short, it is a problem of calculating the speed of transmission of a pressure wave.

Another case where the use of this dimensionless value can be observed: it is assumed that the appearance of an event in a given time window is analyzed; the resulting value is dimensionless ("A" in time).

2.10. Tension and Expansion

The tension "TE", is the force that exerts a fluid to not expand; can be understood as the force that the molecules perform, not to expand. As always, the tension and the expansion are calculated in a direction "V".

In the case of matter (fluid) in space, the same concept can be understood as gravity, since the greater the gravity, the greater the force to be expanded.

There is an expression very important (general expression), in order to have a relation between surface tension, compressibility and density; that is ("TE" is a tension, "Z" compressibility, "ρ" density, "K" a constant):

$$TE\left(\frac{Z}{\rho}\right)^{1/2} = K$$ Eq. (19)

In fact, the tension is a drag force, as a viscosity, to movement.

Can be defining the tension, by ("T" temperature, "P" pressure, "Vol" volume):

$$\frac{\partial P}{\partial T}\ or\ \frac{\partial Vol}{\partial P}$$ Eq. (20)

There is other parameter very important, with very relation with the tension; it´s the Expansion force "α" of fluid ("Z" compressibility, "Vol" volume, "T" temperature, "ρ" density,"$(M)_n$" variable "M" with "n" constant:

$$\left(\frac{\partial P}{\partial T}\right)_{Vol} = \frac{\alpha}{Z}$$ Eq. (21)

$$\alpha = \frac{1}{Vol}\left(\frac{\partial Vol}{\partial T}\right)_P =$$

$$= -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_P$$ Eq. (22)

$$Z = \frac{1}{Vol}\left(\frac{\partial Vol}{\partial P}\right)_T$$

3. Matter aggregation

3.1. Viscosity and Tension / Expansion

The viscosity with the tension and also with the density, are a seed in lot phenomenons of matter aggregation:

Dust and lint at home, dispersion of tree leaf by the wind, clouds or plastics in sea:

Fig. (8)

A parabolic velocity profile, in a people group walking in street (there are boundary layer in lateral walls):

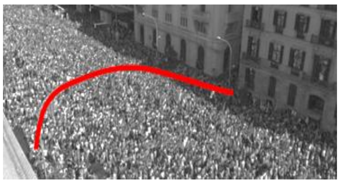


Fig. (9)

Fingers geometries, in lava, honey, glass, beach, etc:

Fig. (10)

In evolution of supernovas nebulas: ring or bubble uniform and pearls (expansion with limit):

Fig. (11)

The environmental, can be also the raison for matter accumulation; for example in street (seeds) or in a beach (people searching sun); even the feelings of people, help to join each other: chess clubs, etc); these feelings, can be viscosity but working with environmental:

Fig. (12)

Even, the feelings (friction and others factors) can be creating a group of be alive; for example bird flocks or sheep's (it can see in sheep image, the Bernoulli effect; amazing):

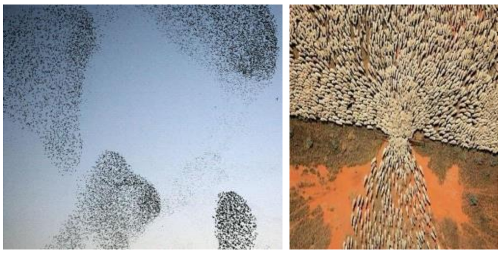


Fig. (13)

The aggregation can create folders, as Jupiter atmosphere or even in mountains:

Fig. (14)

Filaments in explosion of lava, oil in sea or supernovas:

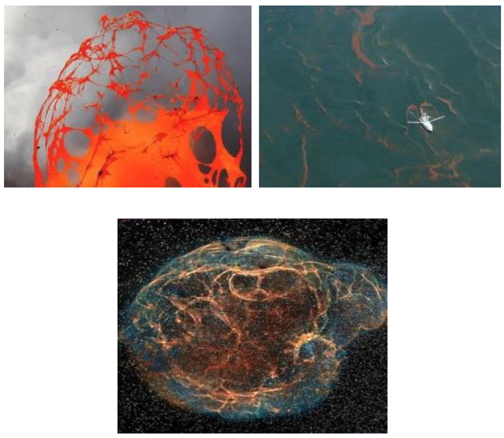


Fig. (15)

This “dispersion” of matter produce in rotation, the galaxy arms.

The concept of instability is very important in order to understand the matter aggregation process; the concept of instability is analogous to the concept of non-homogeneity: from instability, is created differences in geometry.

There are a lot types: Helmholtz, Rayleigh-Taylor, etc:

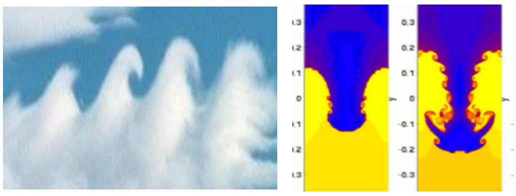


Fig. (16)

All these processes are essentials in order to explain the formation, rotation and interaction between galaxies:

Arms formation: the rotation galaxy, drag the matter:

Fig. (17)

It´s possible see that, also by density profile in rotation in coffee or chocolate:

Fig. (18)

But also is possible the arms creation, in picks of waves (high pressure):

Fig. (19)

Finally an explanation very good about the galaxies arms generation.

Galaxy tails: low pressure tube and/or matter stripped:

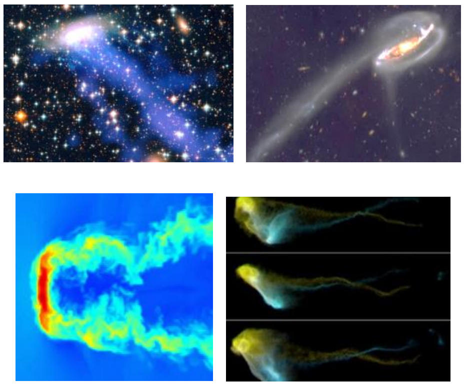


Fig. (20)

About the interaction between galaxies, the galaxy tails (high or low density), also evolve, move and change. The reason is that the environment affects them: other tails, zones of low and high density, etc.... Also affect to galaxy geometries.

Fig. (21)

The rotation sense of a galaxy is very important in order to know the evolution and interaction with other’s (paths and geometries) ([13] J. H. Lee et al) ([14] Francesca Fragkoudi); these interactions between galaxies are like two pendulums swinging close together: they affect each other; let’s think of a submerged pendulum. It makes it swing.

Fig. (22)

It will be able to see that the pendulum will stop oscillating almost immediately. This is due to the opposition of the water molecules which act on it. In fact, the more density/viscosity the fluid has (less compressibility), the less time the initial oscillation will take to stop.

Now, let's think of two identical pendulums immersed in a fluid and with opposed oscillations.

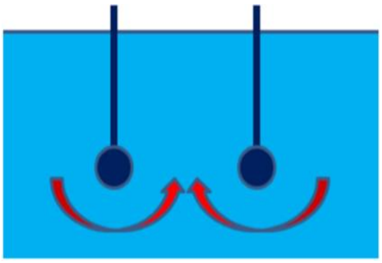


Fig. (23)

After a short time, both pendulums will oscillate in the same direction and with the same frequency!!!

Why does this fact happen?

Because the density/viscosity of the fluid, because its variations and the forces transmission trough the particles. On the moon, this wouldn't happen, due to the air absence; that is Magnus effect.

For example, opposite senses between 2 galaxies and main direction paths against rotation sense:

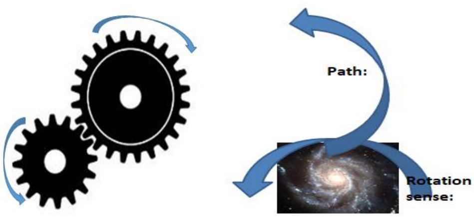


Fig. (24)

So, if it knows the velocity as a vector, of group galaxies, it´s possible to know the path in the past and the future of every galaxy.

There are lot relations between rotation velocity, luminosity, quantity matter, distance, age, arms number:

Luminosity against rotation velocity:

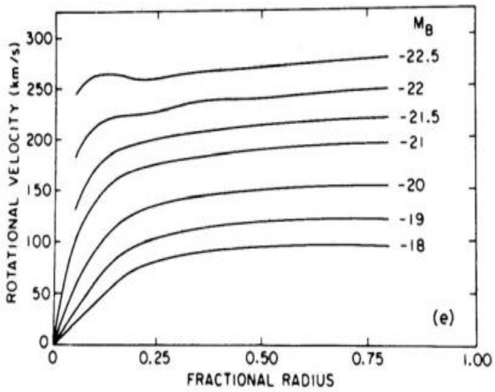


Fig. (25)

Luminosity against type:

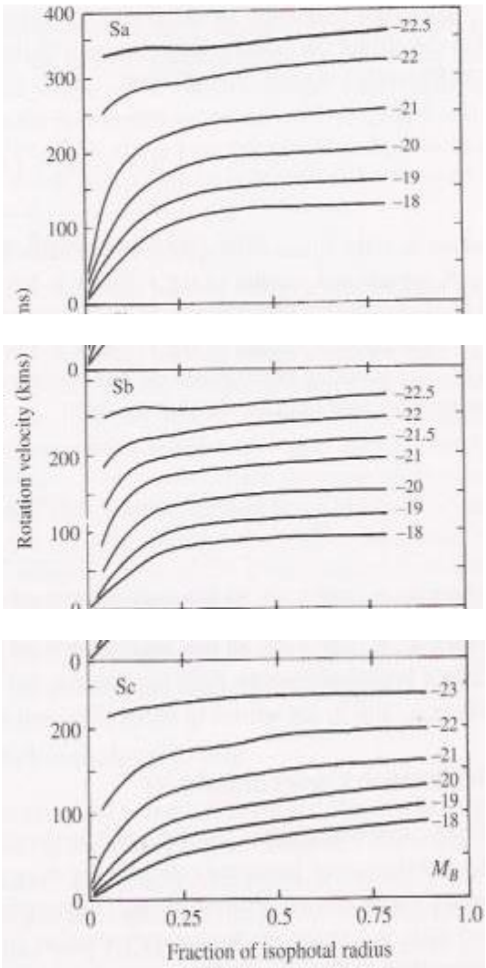


Fig. (26)

Velocity rotation against distance:

Galaxy	RA hh mm ss	Dec dd mm ss	PA d	D Mpc	Type	m_{abs}	$\log D_{25}$	$\log v_m$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
NGC 522	01 24 45.91	09 59 40.5	33.3	39.0	Sbc	−20.53	1.44	2.252
NGC 684	01 50 14.03	27 38 44.2	88.8	50.4	Sb	−21.53	1.53	2.368
MCG-01-05-047	01 52 49.01	−03 26 51.2	161.4	71.5	Sc	−21.77	1.47	2.410
NGC 781	02 00 09.02	12 39 21.5	13.0	49.8	Sab	−20.87	1.18	
NGC 2654	08 49 11.91	60 13 13.9	65.0	19.2	SBab	−20.09	1.62	2.295
UGC 4906	09 17 39.94	52 59 34.3	49.0	32.6	Sa	−20.26	1.30	2.228
NGC 2862	09 24 55.10	26 46 29.0	114.0	58.5	SBbc	−21.44	1.41	2.464
NGC 3279	10 34 42.61	11 11 50.7	152.0	19.9	Scd	−19.27	1.44	2.208
NGC 3501	11 02 47.35	17 59 22.6	28.0	16.2	Sc	−19.05	1.54	2.147
NGC 5981	15 37 53.55	59 23 30.9	139.5	36.1	Sc	−20.61	1.43	2.424
NGC 6835	19 54 33.09	−12 34 02.5	72.0	23.0	SBa	−19.55	1.38	1.803

D Mpc	Type	m_{abs}	$\log D_{25}$	$\log v_m$
(5)	(6)	(7)	(8)	(9)
39.0	Sbc	−20.53	1.44	2.252
50.4	Sb	−21.53	1.53	2.368
71.5	Sc	−21.77	1.47	2.410
49.8	Sab	−20.87	1.18	
19.2	SBab	−20.09	1.62	2.295
32.6	Sa	−20.26	1.30	2.228
58.5	SBbc	−21.44	1.41	2.464
19.9	Scd	−19.27	1.44	2.208
16.2	Sc	−19.05	1.54	2.147
36.1	Sc	−20.61	1.43	2.424
23.0	SBa	−19.55	1.38	1.803

Fig. (27)

Analyze now, "D" and "V" relation. Relation between Rotation velocity against distance:

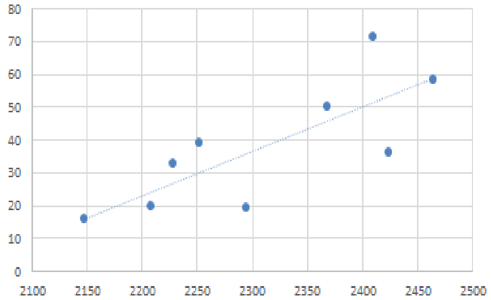


Fig. (28)

There seems a relation lineal.

Velocity rotation against mass:

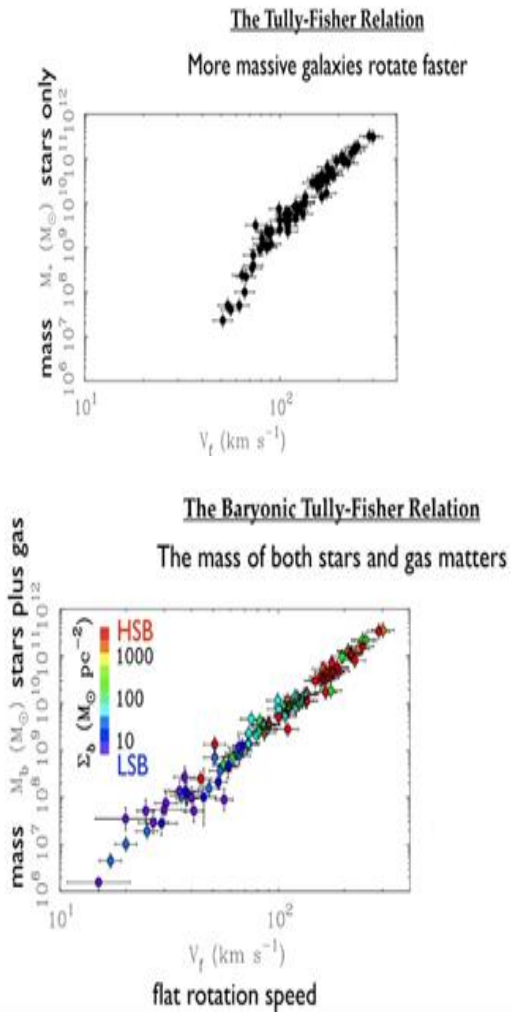


Fig. (29)

There are other´s relations, which are very interesting to analyze for me (I want to study these possible relations):

- Age against arms number.
- Old galaxies against velocity rotation and luminosity.
- Dark matter against arms number.

All these processes and properties are essentials in order to explain the formation of geometry universe in high scale.

The Background Microwave Cosmic (BMC), is a map of temperature variation in a distribution of mass in early universe (colors scale blue-red: low-high density or temperature):

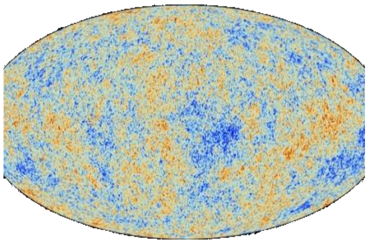


Fig. (30)

Why this special final distribution or dynamic? This distribution of zones with more and less density is normal in the nature. All explosions for example, not have an equal matter in any direction or point, fragments distribution, density or temperature (sun surface, supernovas, atomic bomb, nebulae, etc….): that, are the instabilities that it see before:

Fig. (31)

It supposes that the Universe work as a fluid with viscosity (full viscosity):

These densities-viscosities variations (CMB), origin in the future, the different galaxies cluster and matter distribution in large scale. In fact, from this BMC as a boundary condition, is possible simulate the evolution of universe: the result is very similar to universe observable today:

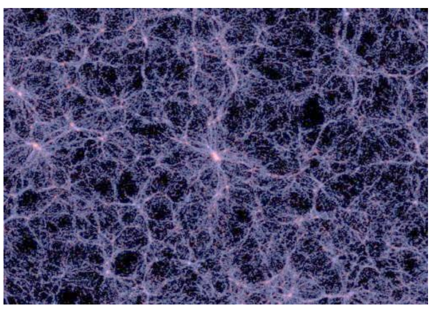


Fig. (32)

These filaments or groups, are analyzed in ([15] Y. Brenier, U. Frisch, M. H´enon, G. Loeper, S. Matarrese, R. Mohayaee, A. Sobolevski), as reverse engineering; that is: from actual geometry, create the CMB.

The formation of filaments, it´s similar to creation of filaments of lava, see before:

Fig. (33)

The formation and evolution of a galaxy and galaxies can be simulated, assuming that we are working with a fluid with certain density and viscosity conditions.

In ([16] Vicent Martínez), it analyzed the multifractal structure in geometry high scale of universe; the same, in ([9] Timoteo Briet Blanes) it analyzed the multifractal structure in meteorits rain.

3.2. Attraction

There are 2 types of attraction (may be, the viscosity may be a type also of attraction, but it´s possible isolated it):

Gravitation and Magnetism.

Obviously, the attraction, working with Viscosity, Tension and Expansion, they are the seed of universe geometry in high scale.

3.3. Low pressure

Low pressure it´s a type of matter aggregation.

4. Particles Dynamic

4.1. Introduction

When a group of particles moves, each particle does not know the movement of the rest; they are movements hidden that together, originate beautiful figures, turbulences, flows in several directions, etc...

The dynamics of a group of particles depends on the movement of each particle, but not the other way around.

To know the global or joint dynamics, it is enough to know the individual movement. This movement individual is done through a joint or resultant force, which pushes the particle towards a direction with a given acceleration.

A particle does not move randomly as would be the case with a Brownian motion; it moves by means of the so-called DLA motion or conditioned motion; this conditioning is precisely the one we want to know, which, as a result, gives the component and magnitude of the force applied to the particle; (Figures: Brownian and DLA movement by Timoteo Briet Blanes); initial particle point = (0,0) (Mathcad software):

$$x_1 := 0 \quad y_1 := 0 \qquad i := 1..100000$$

$$alea1_i := if(rnd(1) < 0.5, -0.1, 0.1)$$

$$alea2_i := if(rnd(1) < 0.5, -0.1, 0.1)$$

$$x_{i+1} := x_i + alea1_i$$

$$y_{i+1} := y_i + alea2_i$$

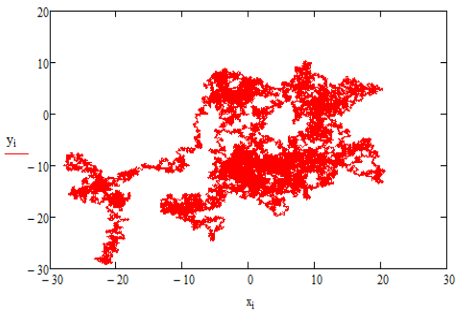


Fig. (33)

$$x_1 := 0 \quad y_1 := 0 \qquad i := 1..100000$$

$$alea1_i := if(rnd(1) < 0.505, -0.1, 0.1)$$

$$alea2_i := if(rnd(1) < 0.505, -0.1, 0.1)$$

$$x_{i+1} := x_i + alea1_i$$

$$y_{i+1} := y_i + alea2_i$$

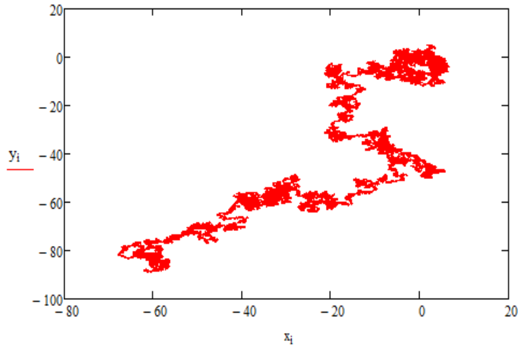


Fig. (34)

Any particle is subjected to a group of forces (pressure, magnetism, coriolis, etc.), the sum of which is the force with which it pushes the particle.

It is all a question of knowing where and with what acceleration each particle moves in its environment; that is: around the particle, depending on the size of discretization, there is the surface of a sphere; it is necessary to know a mathematical model that tells towards which point of that sphere the particle:

Fig. (35)

It can define some numerical models to solve this problem:

4.2. Paths as a Geodesies

The path always follows a minimum distance path (geodesy) between 2 points, in a given metric, in a given space or surface.

A particle follows a path towards a state of minimum energy; of all possible paths, choose the one with a minimum Action ("S"); the difference between kinetic energy "T" and potential "U" is called Lagrangian "L"; this is a basic principle ("t1" the initial time of the beginning of the movement and "t2" the final instant); "T" and "U" will change, depending on which metric or space the particle is in. If it discrete the time and space, it can tell where a particle will go at any instant; this is exactly what is needed:

$$S = \int_{t1}^{t2} (T - U) dt$$

Eq. (23)

4.3. Navier Stokes "traditionals" equations for fluids: some particulars cases

4.3.1. Pedestrians

In Research Article ([8] Kazunori Shinohara and Serban Georgescu) simulate the paths of a crown people in an aquarium.

As a viscosity, work, in this Article, as an attractive force (not separation) between people:

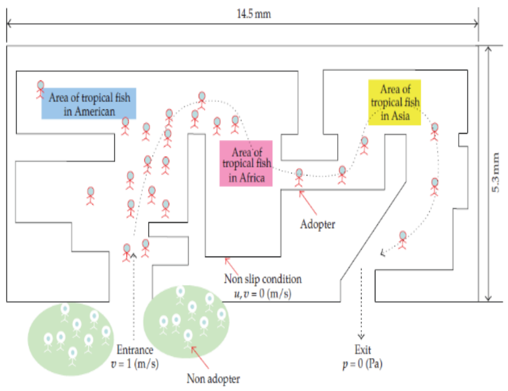


Fig. (35)

Paths people in aquarium, with velocity as a colors:

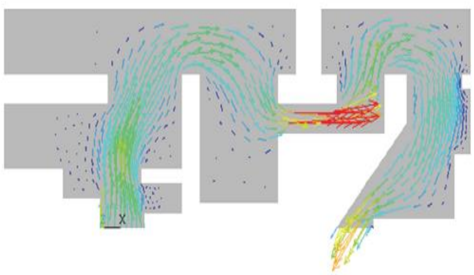
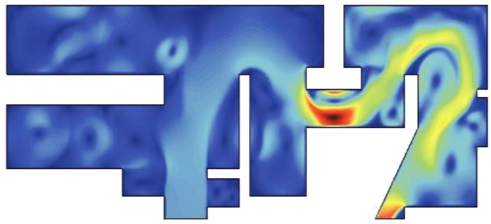


Fig. (36)

Is possible to create a simulation CFD with full Navier Stokes equations (without variations, as before) ([11] Timoteo Briet Blanes), in order to generate the solution of the same problem (Tokio Tower Aquarium) (image about velocity, streamlines and turbulences zones):



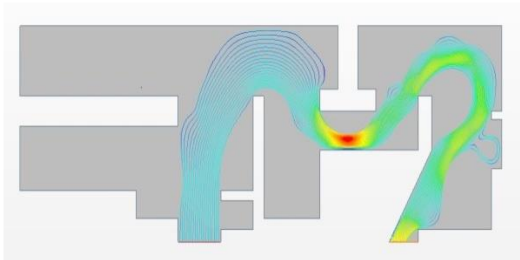


Fig. (37)

In this type or way of study, is possible calculate the pressure in any point or zone, velocities, turbulences, jams (very important), etc….

From an image of center town of Castellón (Spain), it analyzed the dynamics of pedestrians with a series of closed streets and others open; the possibilities are immense: images of contour of speed, pressure and turbulence ([12] Timoteo Briet Blanes):

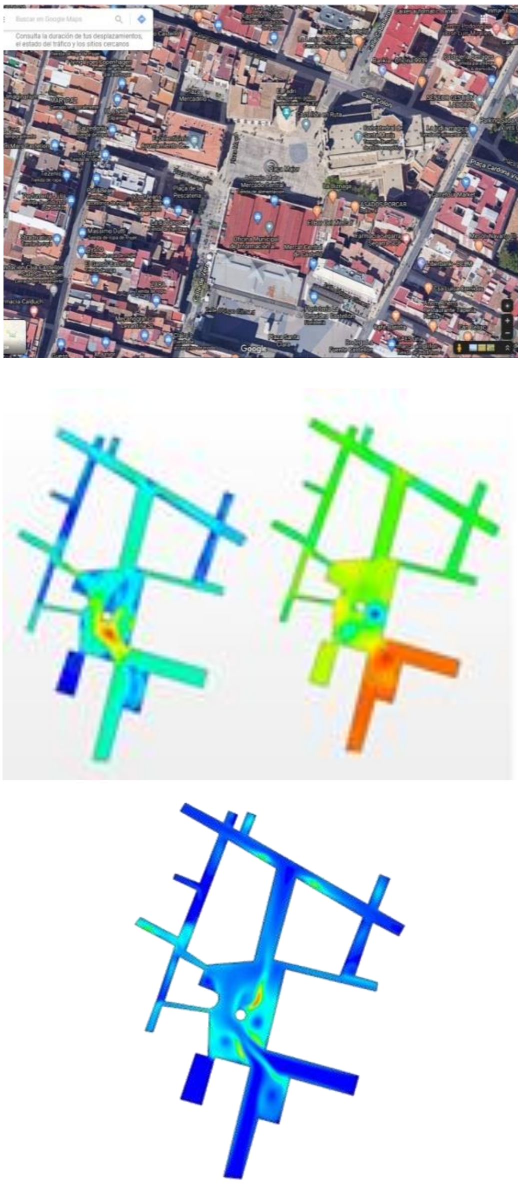


Fig. (38)

4.3.2. Expansion of Universe

Between 2 zones with different pressure, there is pressure difference which produces acceleration “a” (high to low pressure): pulling with acceleration “a” (From Navier Stokes equations), proportional to variation pressure and inverse density.

What is the origin of this Acceleration-Suction, as a Dark Energy action? It know perfectly, that the Bing Bang, is not an explosion or blast . But, is perfectly possible to assign it an analogy with a wave-shock. In any wave shock produce by a blast (big bang for example), there are a zone of high pressure (wave front) and after, other wave or zone of low pressure. This zone, produce (without any drag) one acceleration: next images about wave explosion propagation with simulation CFD techniques (test made with Star CCM+ as a CFD code: simulation in 3D (cut plane view), 10 km diameter sphere, 14.5 million mesh cells, explosion of dynamite into air dry, K-epsilon turbulence model; Timoteo Briet Blanes):

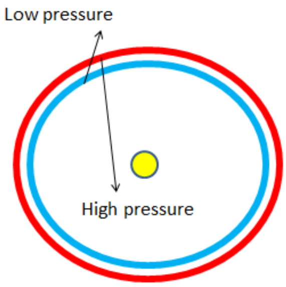


Fig. (39)

The front-shock wave, have only 2 brakes (drag): viscosity total and mass (gravity).

The pressure profile in any blast wave is (the wave may to be oscillations (positives and negatives) in time). Friedlander waveform sample for any explosion:

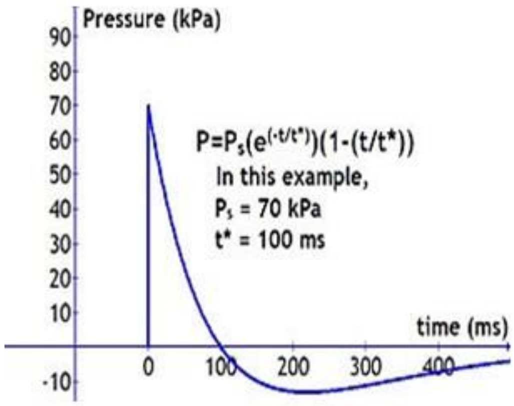


Fig. (40)

The zone or zones, with negative pressure, the “dark energy”, work (the dark energy change in time), producing acceleration-suction (with positive pressure, work but pushing).

Even, combining some hypothesis is possible that in the future, there are variations positives and negatives in dark energy:

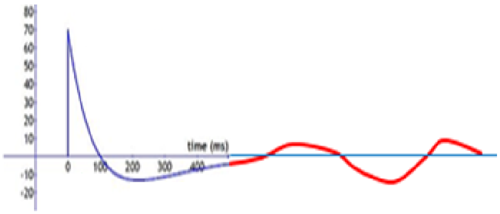


Fig. (41)

This evolution in waves or not, depend of densities in the universe (baryonic matter, radiation, dark energy, dark matter, electromagnetism).

Applying the Euler Lagrange equations ("KE" and "PE" are kinetic and potential energies, "m" mass, "P_x" is the variation pressure against "x", "x" is an direction, "ρ" density, the super index "*" is a variation against time):

$$P_x = \frac{\partial P}{\partial x} \qquad KE = \frac{1}{2} m \overset{*}{x}^{2} \qquad PE = m \frac{P_x}{\rho} x$$

So: $$\frac{\partial P}{\partial x} = \overset{**}{x} \rho \qquad \text{Eq. (24)}$$

This model of expansion of the Universe has been realized in 1 dimension, assuming that the whole Universe expands equally in any direction. But I don't think this is the case; there have been observations in various directions in the Universe, which show that it is not uniform in all directions (not equal).

For this reason, the expansion of the Universe will depend on pressure and density, so in each direction, the expansion will be different. It works with Navier Stokes equations traditionals.

Analyzing the expressions for every part or term in Navier Stokes equations:

Sup: $V=Hx^{1+\epsilon}$ → Variation of Hubble equation ("ϵ" is a value >0, "H" Hubble constant, "V" velocity, "G" gravitational constant, "g" acceleration gravity).

Analyzing the possible error with "ϵ": if ϵ=0.01, then (10^{26} meters = radio universe):

$$if \rightarrow x = 10^{26} meters$$
$$x^{2\varepsilon} \approx 1.8 (error)$$

But, the "Expansion Velocity: "V_E"" may be, depend of Pressure and Density also ("x" space length), ("a" and "b" > 0):

$$V_E = function(x, P, \rho)$$

$$V_E \propto \frac{1}{\rho^a P^b}$$

$$V_E = H(t) \frac{1}{\rho(x)^a P(x)^b} x \qquad \text{Eq. (25)}$$

Working in 1 dimension "x"; term by term, in Navier Stokes equations with V=Hx:

$$\rightarrow \frac{\partial V}{\partial t} = \overset{*}{H} x + H^2 x$$

Eq. (26)

$$\rightarrow V \frac{\partial V}{\partial x} = H^2 x \qquad \text{Eq. (27)}$$

$$g = \frac{Gm}{x^2} = \frac{4\pi G \rho x}{3}$$

→ If $V=Hx^{1+\epsilon}$

$$\mu \nabla^2 V = \mu H \varepsilon (1+\varepsilon) x^{\varepsilon - 1}$$

Eq. (27)

The next term in Navier Stokes, is the most important:

$$\rightarrow \frac{P_x}{\rho} \quad \text{value ????}$$

"Λ" cosmologic constant, "ρ_{vac}" density of vacuum, "c" speed of light:

Method 1:

$$\rho_{vac} = \frac{\Lambda c^2}{8\pi G} \qquad \Lambda = \frac{3H^2}{c^2}\Omega_\Lambda$$

$$P_{vac} = -\rho_{vac} c^2$$

$$P = -\frac{c^4}{8\pi G}\frac{3H^2}{c^2}\Omega_\Lambda = -\frac{3H^2c^2}{8\pi G}\Omega_\Lambda$$

Eq. (28)

Method 2:

It knows also, that:

$$P = \left(\frac{-\overset{*}{\rho}}{3H} - \rho\right)c^2$$

Eq. (29)

3 results for "P":

➔

Derivating the expressions for "P" (in 2 methods).

➔ Also: H=H(x), from:

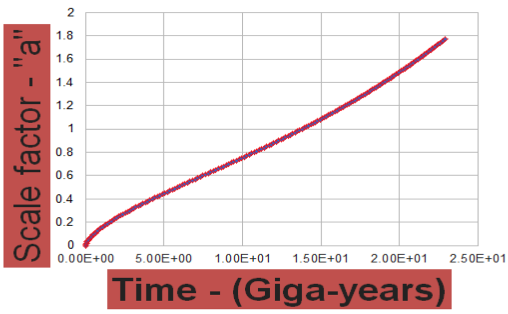


Fig. (42)

➔ From Navier Stokes, directly:

Sup: V=Hx:

$$\frac{P_x}{\rho} + g = \frac{P_x}{\rho} + \frac{4\pi G\rho x}{3} = H^2 x$$

$$P_x = \left(H^2 - \frac{4\pi G\rho}{3}\right)\rho x$$

Eq. (30)

It´s easy calculating that, with other 2 models for "H".

There is another problem, very important:

If the density or pressure is not the same in any direction or there are different densities in some places, if it calculates (from supernovas, or galaxies cluster or individual galaxies) the "H_0" Hubble constant, the results can will be different.

That is very important, in the famous "Hubble Tension" problem: in the main Friedman equation, the "H" depend of density….

From other point of view: from Euler-Lagrange equations: Sup: V=Hx ("L" is the Lagrangian):

$$L = \frac{1}{2}mV^2 + \frac{GmM}{x}$$

$$\frac{\partial L}{\partial x} = \frac{8\pi G\rho}{3}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \overset{*}{x}}\right) = \overset{**}{x}$$

$$H^2 x + \overset{*}{H} x = \frac{8\pi G\rho}{3}$$

Eq. (31)

The viscosity, it knows, it´s a friction force; so, it´s possible to simulate it by a damper: Spring-Damper model expansion universe:

Considering that the viscosity of universe works as a damper; also, the mass (gravity) and density so, work as a spring (in this special and particular case, not considering other´s forces).

So ("K_S" is a value of spring constant, and "K_D" diffusivity of Damper) (is possible that "K_S" and "K_D" non-constants) (Vacuum(Force)=Fv):

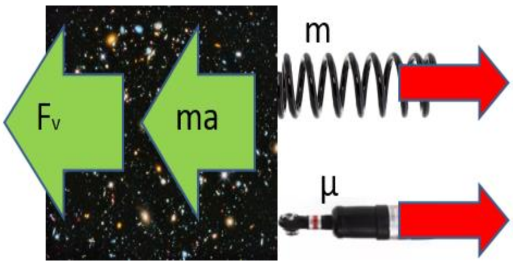


Fig. (43)

$$F_v - K_S x - K_D \frac{\partial x}{\partial t} = ma$$ Eq. (32)

About the constants (in general form) (“f” and “g” functions):

$$K_S = f(mass, x) = f(m, x)$$
$$K_D = g(viscosity, x) = g(\mu, x)$$

Eq. (33)

Is possible suppose that (reasonable option), the next values:

$$K_S = x$$
$$K_D = Velocity = V$$ Eq. (34)

Also:

$$K_D \frac{\partial x}{\partial t} = K_D V = F_{viscous}$$
$$K_S x = Force(Gravitational)$$
Eq. (35)

Also, is possible substituting the acceleration “a” in the “Force” (generate by the “full” acceleration) general expression:

$$F = ma = m\frac{\partial V}{\partial x} = m\left(\frac{P_x}{\rho} - g\right) =$$
$$= \overset{*}{H} x + H^2 x$$
$$/ V = H x$$

Eq. (36)

Very similar to Navier Stokes equations….

Applying the Euler-Lagrange equations:

$$L = \frac{1}{2} m \overset{*}{x}^2 - \frac{1}{2} K_S x^2 + K_D \overset{*}{x}$$

$$\frac{\partial L}{\partial x} = K_S x$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \overset{*}{x}}\right) = m \overset{**}{x} + K_S x$$

$$m \overset{**}{x} + K_S x + K_D \overset{*}{x} = 0$$

Eq. (37)

5. Navier Stokes equations; applications and numerical models

The main goal, in order to later advance, is to assign to an event a series of coordinates or factors dependent on the event, in order to apply Navier Stokes equations ([24] H. Lamb.) (simplifications, parts, combinations, etc), and analyze its evolution.

5.1. Introduction

The Navier Stokes equations have the next expression (without externals forces (gravity and viscosity basically) in right term):

$$\frac{\partial V}{\partial t} + (V\nabla)V = -\frac{1}{\vec{\rho}} \nabla P$$ Eq. (38)

It knows already, that:

$$P \propto \rho V^2 \rightarrow P = \frac{1}{2}\rho V^2$$ Eq. (39)

Supposing Density constant (in 1 Dimension):

$$\frac{\partial P}{\partial x} = \rho u \frac{\partial u}{\partial x} \quad \text{Eq. (40)}$$

The acceleration "a" is ("t" time):

$$a = \frac{\partial u}{\partial t}$$

Also, the units of next expression are acceleration:

$$\frac{\frac{\partial P}{\partial x}}{\rho} = \frac{P_x}{\rho} \quad \text{Eq. (41)}$$

So finally (for simplifying: "g" as gravity acceleration, is null now (Navier Stokes equations):

$$\frac{\partial u}{\partial t} = -\frac{\frac{\partial P}{\partial x}}{\rho} - u\frac{\partial u}{\partial x} + F(viscous)$$

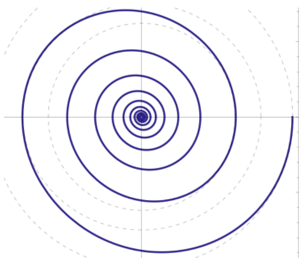
Eq. (42)

If the density is not constant:

$$\frac{\partial P}{\partial x} = \frac{1}{2}\frac{\partial \rho}{\partial x}V^2 + \rho V\frac{\partial V}{\partial x} \quad \text{Eq. (43)}$$

Is possible here, substituting the Hubble universe expansion equation (V=Hx).

In the Article ([3] A. Jakimowicza and J. Juzwiszynb), the formation of vortices in the evolution of economic parameters is analyzed; it is appreciated in the image depending on the section (coordinates) that is observed, the helix of the vortex appears; in these type or vortex, there is an attractor:



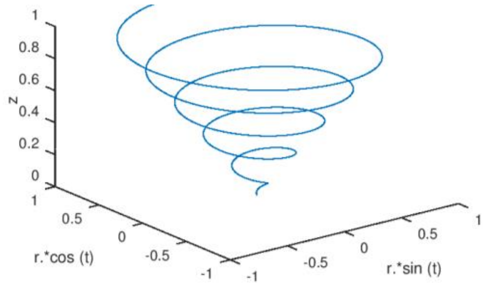


Fig. (44)

Also is necessary transforming the coordinates in order to show attractors:

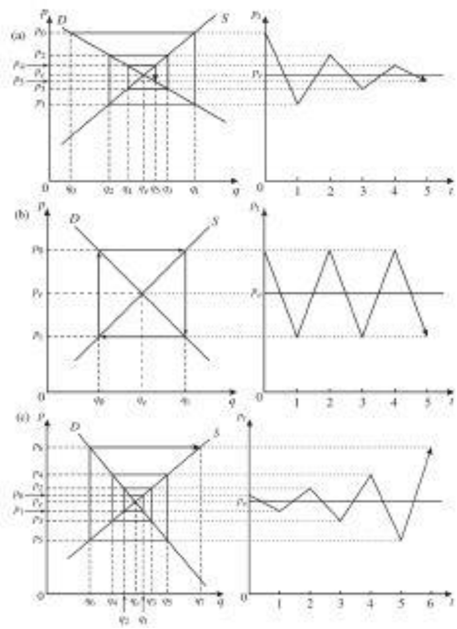


Fig. (45)

5.2. Numerical Analogies in Nature

There are a lot phenomena's in Nature that can be explained by numerical equations, very similar to Navier Stokes equations or simplifications:

- Prey a predator model:

Is a model very simple with "x" and "y" (prey and predator) initials and point fix (a/b, c/d):

$$\frac{dx}{dt} = ax - bxy$$
$$\frac{dy}{dt} = -cy + dxy \qquad \text{Eq. (44)}$$

In this model, there is an attractor:

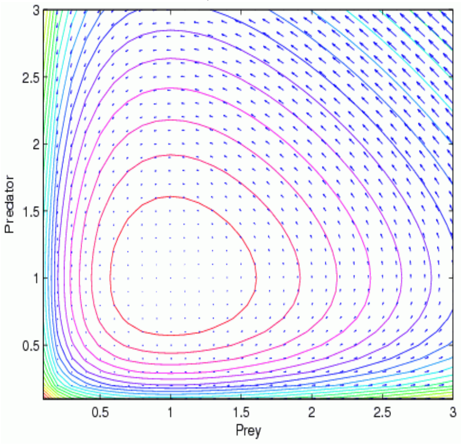


Fig. (46)

- Romeo and Juliet model:

Love equations between two peoples (Romeo "R" and Juliet "J" model):

$$\frac{dR(t)}{dt} = aR(t) + bJ(t)$$
$$\frac{dJ(t)}{dt} = cR(t) + dJ(t) \qquad \text{Eq. (45)}$$

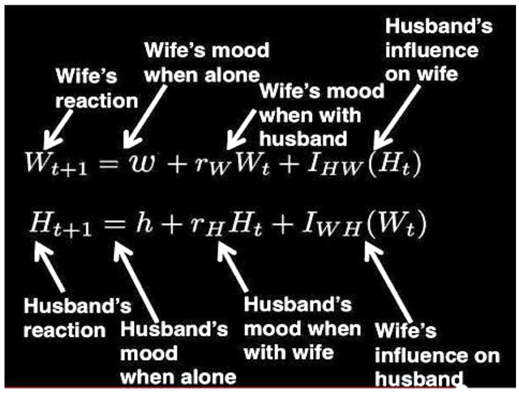
Second order derivatives can be added that specify functions that act as catalysts by accelerating or decelerating sentiment, such as economic stability, gender and family opposition, and include partial derivatives so that "R" and "J" do not depend only on "t".

I love more a girl (H) if the girl (M) loves me:

$$\frac{dH(t)}{dt} = aM(t)$$
$$\frac{dM(t)}{dt} = -bH(t) \qquad \text{Eq. (46)}$$

That is: the variation of my love to you, depend of your love to me.

There are other´s equations of love, one bit more complicate (Hannah Fry), but basically, are the same:



Eq. (47)

- Lanchester model:

In the Second World War, the Lanchester equations, for predicting an air combat ("A" and "B", number aircraft of two parts):

$$\frac{dA(t)}{dt} = -bB(t)$$
$$\frac{dB(t)}{dt} = -aA(t) \qquad \text{Eq. (48)}$$

So, are the Prey and Lanchester equations, some similarities as a phenomenon? Are the Prey, Lanchester and Love, events similar? There are also, equations for war "guerrillas":

$$\frac{dA(t)}{dt} = -bA(t)B(t)$$
$$\frac{dB(t)}{dt} = -aA(t)B(t) \qquad \text{Eq. (49)}$$

If the phenomenon is the "same", the numeric model also, but vice versa, is not necessary….

In the Lanchester case eat aircrafts, and in the Prey case, eat animals, and if one go up, the other go down, with a gap or delay time.

Basically, prey model and Lanchester model, are the same. It can transform:

$$ax - bxy \rightarrow x(a - by) \qquad \text{Eq. (50)}$$

If it put the external forces (only viscosity) in Navier Stokes equations, the expression is:

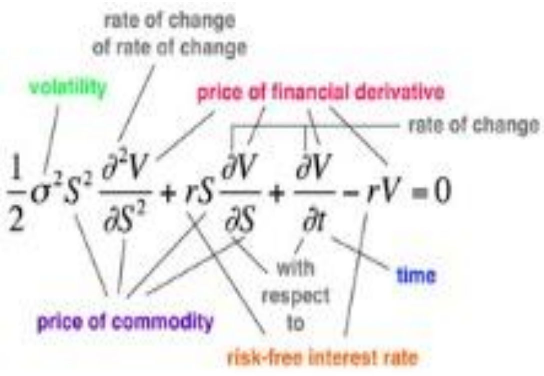
$$\frac{\partial V}{\partial t} + (V\nabla)V = -\frac{1}{\vec{\rho}}\nabla P + \mu \nabla^2 V$$

Eq. (51)

- Black-Scholes model:

It´s a model for analyze the behavior of Stock Market (sell and call), and predict some prices in the future.

The expression is very similar to Navier Stokes equations:



Eq. (52)

Other expression for Navier Stokes equations:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} - fv = -\frac{1}{\rho}\frac{\partial p}{\partial x} + K\frac{\partial^2 u}{\partial x^2}$$

Eq. (53)

And its similarity with Black-Scholes:

1. $\frac{\partial V}{\partial t} \sim \frac{\partial u}{\partial t}$
2. $\frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} \sim K\frac{\partial^2 u}{\partial x^2}$
3. $rS\frac{\partial V}{\partial S} \sim u\frac{\partial u}{\partial x}$
4. $-rV \sim -fv + \frac{1}{\rho}\frac{\partial p}{\partial x}$

Eq. (5)

- Schroeringer model:

$$i\hbar\frac{\partial}{\partial t}\psi(\mathbf{r},t) = -\frac{\hbar^2}{2m}\nabla^2\psi(\mathbf{r},t) + V(\mathbf{r},t)\psi(\mathbf{r},t)$$

Eq. (55)

It can work so or considerer as a wave (there is a wave expression in Schroeringer equation), all event in the universe.

$$\Psi(x,t) = Ae^{i(kx-\omega t)} = A[\cos(kx-\omega t) + i\sin(kx-\omega t)]$$

Eq. (56)

The similarities between the Schrodinger and Navier Stokes equations are evident:

$$\underbrace{\frac{-\hbar^2}{2m}\nabla^2\Psi(\mathrm{r})}_{\text{Kinetic Energy}} + \underbrace{V(r)\Psi(\mathrm{r})}_{\text{Potential Energy}} = \underbrace{E\Psi(\mathrm{r})}_{\text{Total Energy}}$$

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u} = -\frac{\nabla P}{\rho} + \nu\nabla^2\mathbf{u},$$

Eq. (57)

It is even possible to identify parameters and variables between both equations.

Therefore, it is possible that there are phenomena that can be explained by means of the 2 equations or numerical models; and vice versa: if 2 phenomena can be explained by both equations, both phenomena must be "similar".

- Alan Turing biology evolution model:

Is a numeric model in order to predict the formation of patterns in the Nature as a spot in tigers for example (there is a wave expression in equations model):

$$\frac{\partial u}{\partial t} = D_u \frac{\partial^2 u}{\partial x^2} + f(u,v)$$
$$\frac{\partial v}{\partial t} = D_v \frac{\partial^2 v}{\partial x^2} + g(u,v)$$

Eq. (58)

"u" and "v" are the concentrations of 2 axis. "D_u" and "D_V" are the coefficients of diffusion of "u" and "v", and "f" and "g" the reaction between.

It can see perfectly, the heat equation (diffusion), into Alan Turing equations.

This evolution model, is similar to Voronoi scheme evolution

- Heat convection model:

"T" is the temperature, "V" the velocity vector, "a" the acceleration vector and "x" and "y", the coordinates in 2D:

$$\vec{V} = (u, v)$$

$$\vec{a} = \frac{D\vec{V}}{Dt} = \frac{\partial \vec{V}}{\partial t} + \frac{\partial x}{\partial t}\frac{\partial \vec{V}}{\partial x} + \frac{\partial y}{\partial t}\frac{\partial \vec{V}}{\partial y}$$

$$\frac{\partial T}{\partial t} + U\frac{\partial T}{\partial x} + V\frac{\partial T}{\partial y} = a\frac{\partial^2 T}{\partial y^2}$$

Eq. (59)

- Conclusions about these last models:

In these models before, the numerical models are very similar, so the phenomenon must to be also (may be….).

Schröeringer, Black-Scholes, Alan Turing: in these 3 equations, we can see the diffusion equation (heat equation). This diffusion part also is in Navier Stokes equations. If in Navier Stokes equations, the extern forces are zero, is possible create and apply the Alan Turing model.

¿¿ It´s possible so, apply Schröeringer equation, to Stock Market evolution calculation….

5.3. Parts, terms, models, combinations and cases

5.3.1. Low pressure model

It´s the most simple and easy model:

The direction of one particle is the direction with the minimum pressure. Also, the pressure work as a density; one particle will be where there is less density. Give a particle and give "sectors" in a sphere with center the particle (discretizing the time: "u" is a velocity, "u^n" is a velocity in instant "n", "P" pressure and "ρ" density):

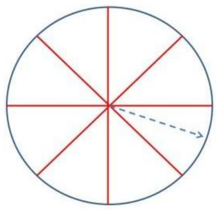


Fig. (47)

$$u^{n+1} = u^n - \frac{1}{\rho}\nabla P$$ Eq. (60)

The particle will move toward the half angle line of sector, with the least density (pressure); the direction of movement is "x"; this displacement, with a delay time (viscosity) (step by step).

The particle acceleration will be "a" (in direction "x"); this expression is a term in Navier Stokes equations: simplification so:

$$a = \frac{1}{\rho}\frac{\partial P}{\partial x}$$ Eq. (61)

If "$P=\rho u^2$"; in 1 dimension "x"; density not constant:

$$\frac{1}{\rho}\frac{\partial P}{\partial x} = \frac{\partial \rho}{\partial x}\frac{u^2}{\rho} + 2u\frac{\partial u}{\partial x}$$ Eq. (62)

u_i^t is the velocity in position “i” and time “t”. So:

$$u_i^{t+1} - u_i^t = \frac{(\rho_{i+1}^t - \rho_i^t)\left(u_i^t\right)^2}{\rho_i^t} + 2u_i^t\left(u_{i+1}^t - u_i^t\right)$$

Eq. (63)

When a particle or group of particles (as a galaxy for example) moves, his path is a depression zone; this zone is an attractor for any particle around; this depression tubes, create vortices around:

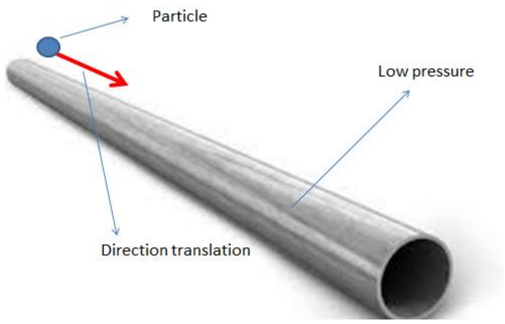


Fig. (48)

Fig. (49)

This tube does not rotate. The only think that rotates is the matter around it, sucked in by the tube.

This low pressure, is present also around each particle in displacement, so, others particles and also others paths (galaxies for example), are attracted ([17] Roberto Camassa, Daniel M. Harris, Robert Hunt, Zeliha Kilic & Richard M. McLaughlin).

Fig. (50)

Even, between galaxies, can exist star bridges, ancients low pressure paths or stripped matter paths ([18] Ekta Patel and other´s).

Fig. (51)

5.3.2. Laplacian term

Laplacian measures the "curvature"; it measures how much the difference between the value of the field, with its average value measured over the surrounding points.

Basically it is said to measure the minimums of a point or the concavity of that point. The laplacian operator is the divergence of the gradient. I understand the intuitive meanings of both. The gradient when dotted against a unit vector gives the rate of change in that direction. The divergence is the flow in or out of an infinitesimal sphere surrounding a point:

1 dimension “x”:

$$\frac{\partial u}{\partial t} = \nu \Delta u = \nu \frac{\partial^2 u}{\partial x^2} \qquad \text{Eq. (64)}$$

- If the laplacian is positive at one point, the mean value of the function on a very small sphere with a center at that point will be greater than the value of the function at the same point.

- If it is negative, it will contract, that is: the average will be lower.

- If it is zero, the mean will be the same; the function is harmonic.

If it works with temperature, then there is more heat exchange in regions where the temperature is very variable, and vice versa:

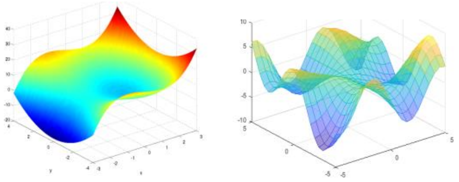


Fig. (52)

Discretizing the Laplacian expression:

$$u_i^{n+1} = u_i^n + \nu \frac{\Delta t}{\Delta x^2}\left(u_{i+1}^n - 2u_i^n + u_{i-1}^n\right)$$

Eq. (65)

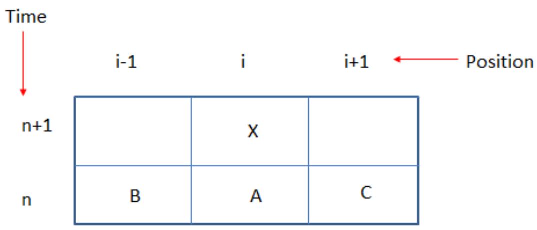


Fig. (53)

$$x = A + \nu \frac{\Delta t}{\Delta x^2}(C - 2A + B) \qquad \text{Eq. (66)}$$

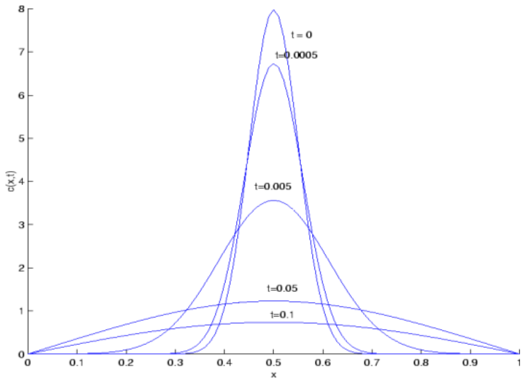


Fig. (54)

C-2A+B:

- C-2A+B = 0, if the A,B,C in progression lineal (Arithmetic progression).
- >0 if is crescent and <0 in other case.
- Will be a magnitude bigger, when the variation is bigger.
- The lower the "nu", the less heat transfer.
- C-2A+B=(B-A)-(A-C) : that is: variation average between distances in A, B and C.

Sample:

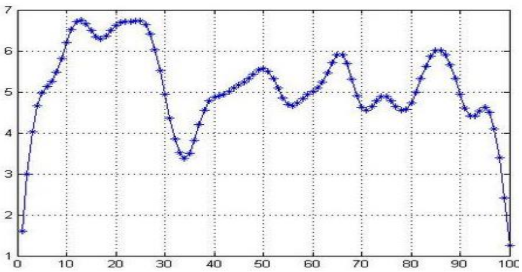
(C,A,B)=(2,8,48)
2 by 4 = 8
8 by 6 = 48
A-C=6 / B-A=40
(B-A)-(A-C)= 34

Matlab code and sample, Flow Diffusion using Crank Nicholson:

```
clc
clear
M=100;
N=10;
LX=1;
LY=1;
TIME0=0;
TIME=1;
tt=1000;
Dt=(TIME-TIME0)/tt;
D=12e-4;
DX=LX/M;
DY=LY/N;
mu=D*Dt/(DX)^2;
%Initilization Matrix
for t=1:1;
   for i=2:M-1;
         U(i,t)=10*rand(1,1);
   end
end
%Boundary Conditions
for t=1:1;
```

```
U(1,t)=0;
U(M,t)=0;
end
for t=1:1;
   for i=1:1;
         d(i,t)=mu*U(i+1,t)+(1-2*mu)*U(i,t);
   end
end
for t=1:1;
   for i=2:M-1;
         d(i,t)=mu*U(i+1,t)+(1-
2*mu)*U(i,t)+mu*U(i-1,t);
   end
end
for t=1:1;
   for i=M:M;
         d(i,t)=(1-2*mu)*U(i,t)+mu*U(i-1,t);
   end
end
%Constructing the Diagonal Matrix
a=ones(M-1,1)
b=ones(M,1)
g=(1+2*mu)*diag(b)-mu*diag(a,-1)-
mu*diag(a,1)
gg=g^-1
for t=1:1;
U(:,t)=gg*d(:,t)
end
for t=1:tt;
   for i=1:1;
         d(i,t)=mu*U(i+1,t)+(1-2*mu)*U(i,t);
   end
   for i=2:M-1;
         d(i,t)=mu*U(i+1,t)+(1-
2*mu)*U(i,t)+mu*U(i-1,t);
   end
   for i=M:M;
         d(i,t)=(1-2*mu)*U(i,t)+mu*U(i-1,t);
   end
U(:,t+1)=gg*d(:,t)
end
for t=1:tt;
plot(U(:,t),'-*')
grid on
pause(0.4)
close
end
```

Give a function, to apply the Diffusion equations:



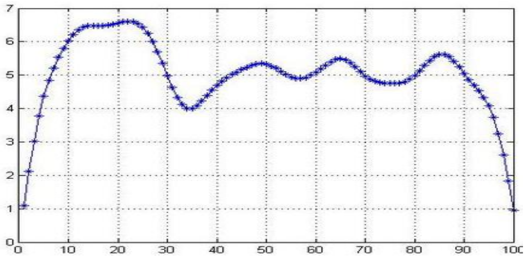


Fig. (55)

5.3.3. Advection lineal equation 1-D; Transport with velocity "c"

$$\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = 0$$

$$u_i^{n+1} = u_i^n - c\frac{\Delta t}{\Delta x}\left(u_i^n - u_{i-1}^n\right)$$

Eq. (67)

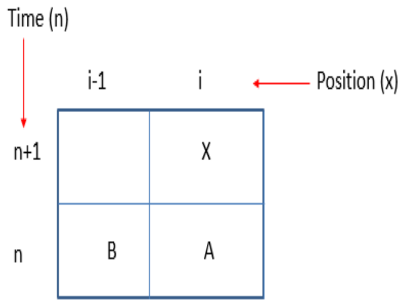


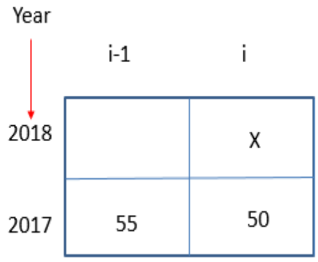
Fig. (56)

$$X = A - c\frac{\Delta t}{\Delta x}(A - B)$$

Eq. (68)

- If A>B, then x<A, independently of what scheme work for finites differences (forward, backward, central, etc….).
- X is "A" plus a value, function of a variation (plus or minus).
- If Δt is bigger, the variation is more important (more incorrect) (bigger). That is the basic concept for a inter and extrapolation.
- If Δx is bigger, the variation is smaller.
- "c" is the "risk factor"; if "c" is smaller, the variation is smaller.

Sample (money invests for a “i” and “i-1” people):



$$X = 50 - \frac{1}{10}\frac{10}{1}(50-55) = 55$$

$$X = 50 - \frac{1}{10}\frac{10}{5}(50-55) = 50$$

Fig. (57)

c=0.1

In the first case, “x-1” is a good friend of “x”(Δx=1). In the second case, is a friend not close (Δx=5).

5.3.4. Advection lineal in 2-D

$$\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} + c\frac{\partial u}{\partial y} = 0$$

$$u_{i,j}^{n+1} = u_{i,j}^{n} - c\frac{\Delta t}{\Delta x}\left(u_{i,j}^{n} - u_{i-1,j}^{n}\right) - c\frac{\Delta t}{\Delta y}\left(u_{i,j}^{n} - u_{i,j-1}^{n}\right)$$

Eq. (69)

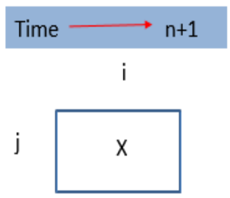


Fig. (58)

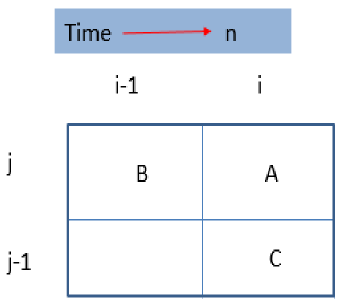


Fig. (59)

$$X = A - c\frac{\Delta t}{\Delta x}(A-B) - \frac{\Delta t}{\Delta x}(A-C)$$

Eq. (70)

5.3.5. Advection non lineal in 1-D; transport with velocity “u”; turbulence formation

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = 0$$

$$u_x^{n+1} = u_x^{n} - u_i^{n}\frac{\Delta t}{\Delta x}\left(u_x^{n} - u_{x-1}^{n}\right)$$

Eq. (71)

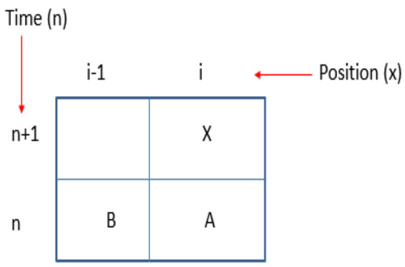


Fig. (60)

$$X = A - A\frac{\Delta t}{\Delta x}(A-B)$$

Eq. (72)

This model, allow the turbulence or non-linearity:

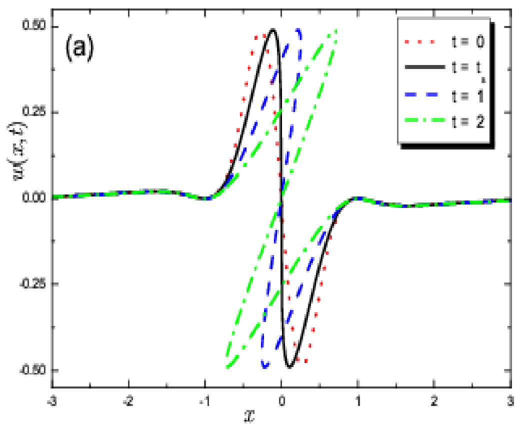


Fig. (61)

5.3.6. Burgers equation

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \nu \frac{\partial^2 u}{\partial x^2}$$

$$u_i^{n+1} = u_i^n - u_i^n \frac{\Delta t}{\Delta x}\left(u_i^n - u_{i-1}^n\right) + \nu \frac{\Delta t}{\Delta x^2}\left(u_{i+1}^n - 2u_i^n + u_{i-1}^n\right)$$

Eq. (73)

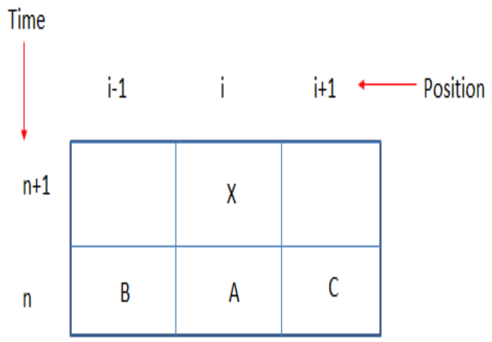
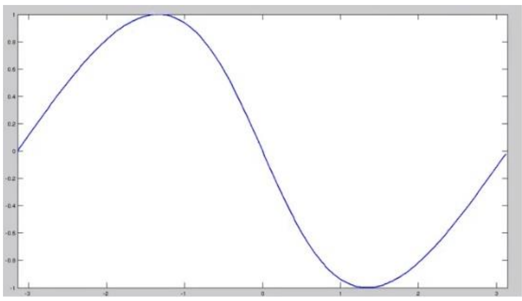


Fig. (62)

$$X = A - A\frac{\Delta t}{\Delta x}\left(A - B\right) + \nu \frac{\Delta t}{\Delta \chi^2}\left(C - 2A + B\right)$$

Eq. (74)

In the case of Burgers equation without viscosity, the instability increases (tents to instability or not continuous):



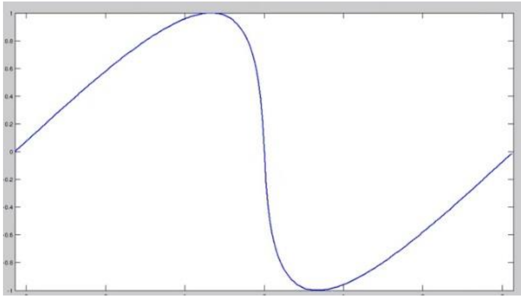


Fig. (63)

On the other hand, if the term diffusive is incorporated, the curve becomes more stable and smooth:

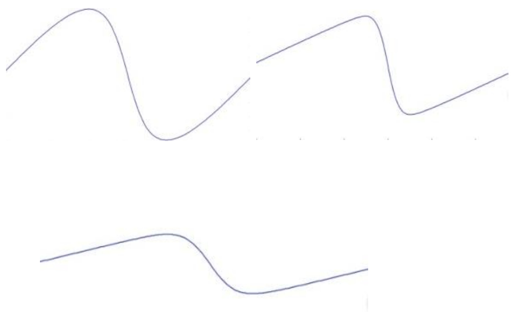


Fig. (64)

A stochastic term (η) can also be added to the Burgers equation, to add a term called "noise"; it is a kind of random Brownian signal:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \frac{\mu}{\rho}\frac{\partial^2 u}{\partial \chi^2} - \lambda \frac{\partial \eta}{\partial x}$$ Eq. (75)

5.3.7. Euler equation

It´s a particular case, against, for the Navier Stokes equations: without externals forces and viscosity ([23]): in 1 dimension “x”:

$$\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x} = \frac{1}{\rho}\frac{\partial P}{\partial x}$$ Eq. (76)

It´s one the most simply models to simulate a gas dynamic for example.

In 1 dimension “x”; (“u” is the velocity):

$$\left(u_i^{t+1}-u_i^{t}\right)+\left(u_{i+1}^{t}-u_i^{t}\right)u_i^{t}=-\frac{(P_{i+1}^{t}-P_i^{t})}{\rho_i^{t}}$$

Eq. (77)

5.3.8. Navier Stokes equations

Combinations between models analyzed before (“V” velocity, “P” pressure, “ρ” density, “t” time, “υ” is viscosity/density):

$$\frac{\partial \vec{v}}{\partial t}+(\vec{v}\cdot\nabla)\vec{v}=-\frac{1}{\rho}\nabla p+\nu\nabla^2\vec{v}$$

Eq. (78)

With a study of accelerations, in Navier Stokes equations, it has 4 very different parts (accelerations):

A. Total acceleration of the particle, as a sum of 2 accelerations.

B. Acceleration of the particle, produced by a low pressure; this low pressure sucks in the particle, adding an acceleration to it. This is the most important term in the Navier Stokes equations, as this is the maximum acceleration that will occur on the particle, in the absence of any forces.

C. Forces that oppose suction by depression, such as friction or Viscosity, etc.... These forces produce a reduction or subtraction of acceleration.

D. Forces that help suction by depression, such as gravity depending on the direction of its vector. These forces produce an increase or summation of acceleration.

With the variation of speed with respect to space, multiplied by the speed, a convective acceleration is obtained. The rest of the terms are accelerations.

Navier Stokes Equations, incorporate vectors of "n" dimensions; the most typical and "real" case is to work with the 3 dimensional space, with "t" the fourth dimension; but it is possible to work with more dimensions; the representation of this space of phases, is logically complicated and it is necessary to have ad hoc tools.

5.3.9. Pedestrians

In fact, these models can be applied in other´s samples or fields as a fluids, traffic, birds, etc….

5.3.9.1. Case 1

The simplest case of Navier Stokes equations is applied to the movement of pedestrians; it is a matter of applying the movement towards the minimum pressure.

The potential “U” of this dynamic is defined as the pressure variation, divided by density; this value is a potential, i.e. the maximum energy value that can potentially reach a particle (1 dimension or direction “x”):

$$U=\frac{\partial P/\partial x}{\rho}$$ Eq. (79)

Therefore, the action "S", defined as (“T” kinetic energy):

$$S=\int_{t1}^{t2}\left((T-U)dt\right)$$ Eq. (80)

In this case particular (“V” velocity):

$$T=V^2$$ Eq. (81)

For the action to be minimal, the potential has to be maximum; this coincides with previously explained that the particle will go towards the maximum pressure variation (divided by density) possible.

Note: remember that “V”, “ρ”, “P” are vectors, so “T” and “U” also.

It´s possible in this simple and first case, apply the Navier Stokes equations, adding more terms as externals forces.

5.3.9.2. Case 2

This numerical model can be improved by adding to the potential, the friction energy that opposes the movement of the particle; that is: the Viscosity, that depend of the density:

$$U = \frac{\partial P / \partial x}{\rho} + ...$$ Eq. (82)

5.4. Govern measures

Front bulb in ship:


Fig. (65)

Nowadays, big ships and also small ones, have the lower part that is submerged, a bulb in the front. The function of this bulb is to create a series of waves or turbulence, which when joined with the waves generated by the boat itself, are annulled or at least almost eliminated.

In this way, they greatly reduce the drag of the boat. This front bulb, is placed in front of the boat, as a kind of advance, as opening the way, as smoothing the way of the boat that comes behind. In real life, as for example in the implementation of an economic or political measure in any country, it is necessary to carry out a series of smaller measures before the main measure. In this way, the harmful effects are softened or mitigated.

5.5. Dynamic sloth

The universe cools; less energy and more laziness; despite this principle the galaxies are moving away from each other, and increasingly faster .... Suppose a spiral pipe; at the extreme, the fluid will leave with a tendency to follow a spiral path; but, the fluid, "hardly" will take anything to follow a straight path. To the dynamics of the fluid, it does not cost him anything to become dissatisfied with a certain dynamic that "forces" him to "something". An economic measure will remain in time (its effects), if the means are put periodically, so that it lasts or remains. If it wants to divert a flow of fluid to a very "far" point, we have to place several "corrective" devices or adapters "along the trajectory, to reach our final objective, not just a device (or corrector) initially.

5.6. Measures from country govern – aggregation, people groups

The government of a country can to make political, economic or social measures, which allows the non-creation or creation of groups of people who share the same hobby, or who belong to the same religion, or who share ideals of many kinds. This can be applied to combat terrorism or to provide measures that help the group. That is: the conditions in grass:

Fig. (66)

If the goal is the aggregation, a solution may be, increase the viscosity.

5.7. Bernoulli effect

Exit of sheep’s:

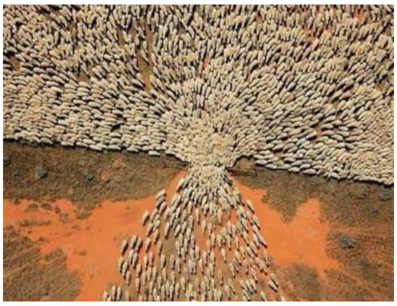


Fig. (67)

This geometry is very similar to nozzle exhaust; and not only the geometry, also the velocity field:

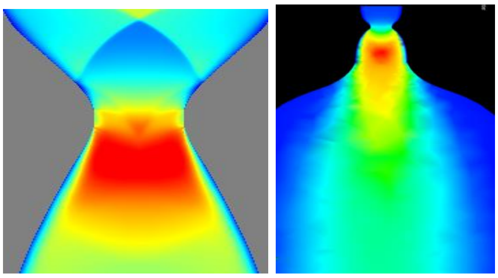


Fig. (68)

5.8. Main goal: model - 1

As an example, an event, y 2 coordinates "x" and "y" on which the event depends.

A Potential is defined which will be incorporated into the term Pressure in the equations of Navier Stokes; this Potential, is the expression (combination of parameters or values) by which the event evolves over time (suction).

The Navier Stokes equations are solved (with specials initials and boundary conditions), ignoring time; you will have a map of pressures (e.g.) in 2 dimensions in which by choosing a point, it will obtain a streamline. This streamline or path, will be the evolution of the event with respect to time; the potential will be varied to adapt the calculated evolution to the real evolution.

If suddenly, there is a factor impossible to determine or know that affects the evolution of the event, the path is recalculated, introducing a new real seed point, from which, another path will be obtained. It is also possible to change the potential, to make it more suitable.

In both cases, the aim is to improve the model and/or the path.

The streamlines, may form spirals or vortices or deviate from a high or low pressure zone/point for example; but the Time, is the third coordinate….:

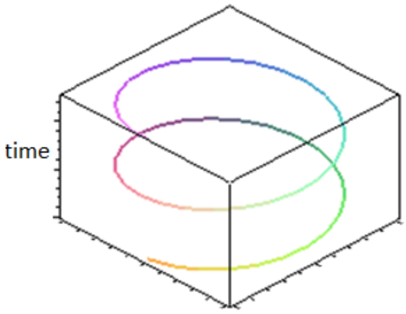


Fig. (69)

Depending the seed point, it creates different paths or streamlines (blue: low pressure):

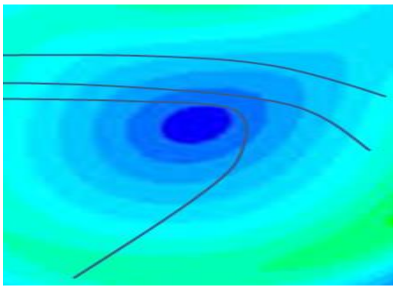


Fig. (70)

5.9. Particular case: Economy

If the objective is to analyze an economic event, it must be defined:

- The coordinates: for example supply, demand, time, price, etc.

- A potential or an expression that quantifies the acceleration or speed of the event.

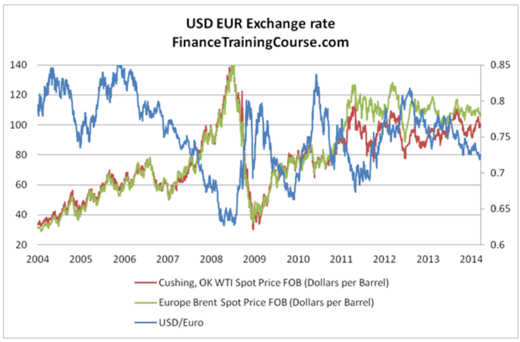
Regardless of the event to be analyzed, a potential could be:

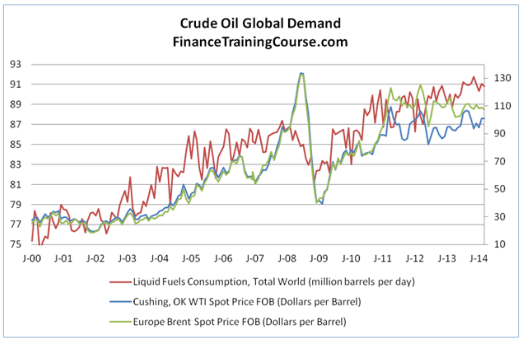
➔ The greater the difference between supply and demand, the greater the speed.

With this, the Pressure and Pressure variation has already been defined. The other parameters needed to define, were already defined at the beginning of this Article, minus the Viscosity. This slows down the speed of the event.

- Viscosity depends on which plane the dynamics of the event is analyzed; that is, on which coordinates evolution is analyzed. The viscosity is proportional to the gap-time (if a study coordinate is time) between variables. Viscosity can be a function of several variables, in order to respond well to sudden or unexpected changes.

For example, you can see graphs in which it is easy and difficult to calculate this gap-time:





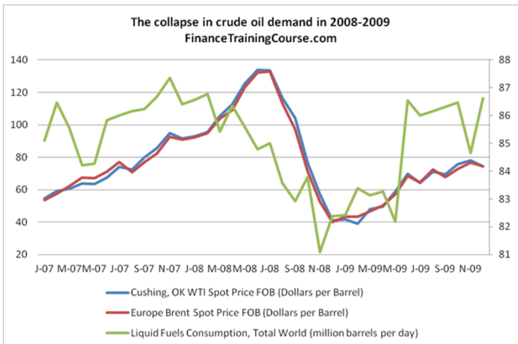


Fig. (71)

- Finally, also it´s necessary to define ideal initial and boundary conditions (known data) to fit the generated model.

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7. Conclusions

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