

Vacuum Instability Above the LHC Scale and the Hierarchy Problem

Ervin Goldfain

Research Scholar, Ronin Institute, Montclair, New Jersey 07043

Email: ervin.goldfain@ronininstitute.org

Abstract

Built as a manifestly nonlinear quantum field theory (QFT), the Standard Model for particle physics (SM) describes the low-energy interaction of gauge and Higgs bosons with quarks and leptons. Since both Yang-Mills and Higgs bosons are self-interacting fields, their behavior is inevitably sensitive to the transition from order to chaos. Here we point out that quantum corrections to the interaction of the Higgs with gauge bosons may lower the threshold for the onset of chaos and destabilize the vacuum in the low or intermediate TeV scale. The inability of the vacuum to survive in this energy region hints to a straightforward solution for the gauge hierarchy problem. It also implies that perturbative estimates on vacuum stability well above the Large Hadron Collider (LHC) scale may need to be revisited.

Key words: Quantum Field Theory, Standard Model, vacuum stability, fine-tuning problem, gauge fields, Higgs boson, order-to-chaos transition.

1. Introduction

By construction, the SM represents a *nonlinear field theory* in which Yang-Mills (YM) and Higgs bosons are self-interacting. Nonlinear dynamics of such objects is present at

both classical and quantum levels. The chaotic attributes of YM fields have been known and studied since the beginning of the eighties [1-11]¹. Chaos was first analyzed in the classical limit of the YM theory and it was shown to exist in both the continuum and lattice formulations of the theory. Particularly, for homogeneous gauge field configurations, it was found that spontaneous symmetry breaking triggers the *transition to chaos* (TC) with the rise of the energy density, whereas the dynamics of YM fields in the absence of spontaneous symmetry breaking remains chaotic at any density of energy. The emergence of chaos in classical dynamics of the $SU(2) \times U(1)$ theory was numerically explored in [9]. Follow-up research was focused on understanding the TC in the semi-classical regime of quantum mechanics (QM) as well as in quantum field theory (QFT). The investigation of chaos in classical gauge theory has later targeted on a number of specific problems. One of them was confirming the effect induced by the Higgs on the chaotic dynamics of classical YM theory. It was shown that the Higgs scalar regularizes the dynamics of gauge fields at low energy densities [4, 7-8]. It was also discovered that quantum fluctuations leading to symmetry breaking via the Coleman–Weinberg mechanism tend to stabilize chaotic dynamics of spatially homogeneous systems of YM and Higgs fields at low energy densities [7-8, 10-11]. The connection between the chaotic dynamics of a classical field theory and the instability of the one-loop effective action of the associated QFT was analyzed in [10].

Surprisingly, aside from studies like the ones previously cited, most theoretical models in particle physics have overlooked the implications of chaos in QFT. The goal of our report is to contribute to a reversal of this trend. We emphasize here that one-loop

¹ Due to the large number of contributions on this topic, we have opted to list only a representative sample of recent publications. As a result, many relevant references are not included here.

corrections to the classical interaction of the Higgs with W, Z bosons or photons are likely to lower the threshold for the TC and destabilize the vacuum somewhere in the TeV range of scales. *By default, a rapidly decaying vacuum in this energy region explains away the long-standing problems of fine-tuning and ultraviolet stability of the SM.*

Our report is organized as follows: section two reviews the Higgs potential and the fine-tuning problem of the SM. Estimates on vacuum stability based on extrapolation of the SM near the Planck scale are briefly addressed in section three. Section four highlights details on the TC that may be relevant for the dynamics of the di-boson and di-photon decay channels. Concluding remarks are presented in the last section. This work represents a continuation of several studies initiated by the author in [12-15]. It is preliminary in nature and calls out for further clarifications and revisions. Concurrent efforts may refute, refine or consolidate our findings.

2. Stability of the Higgs potential and the fine-tuning problem

Electroweak (EW) symmetry in the SM is broken by a scalar field having the following doublet structure [16]:

$$\Phi = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}[(H + v) + iG^0] \end{pmatrix} \quad (2.1)$$

Here, G^+ and G^0 represent the charged and neutral Goldstone bosons arisen from spontaneous symmetry breaking, H is the SM Higgs boson, $v \approx O(M_{EW}) = 246 \text{ GeV}$ is the Higgs vacuum expectation value (vev) and M_{EW} stands for the Fermi scale. Symmetry

breaking is caused by the Higgs potential, whose form satisfies the requirements of renormalizability and gauge-invariance:

$$V = \mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2 \quad (2.2)$$

with $\lambda \approx O(1)$ and $\mu^2 \approx O(M_{EW}^2)$. A vanishing quartic coupling ($\lambda = 0$) represents the critical value that separates the ordinary EW phase from an unphysical phase where the Higgs field assumes unbounded values. Likewise, the coefficient μ^2 plays the role of an order parameter whose sign describes the transition between a symmetric phase and a broken phase. Minimizing the Higgs potential yields a vev given by:

$$v^2 = -(\mu^2 / \lambda) \quad (2.3)$$

where the physical mass of the Higgs is:

$$M_H^2 = -2\lambda v^2 = 2\mu^2 \quad (2.4)$$

The renormalized mass squared of the Higgs scalar contains two contributions:

$$\mu^2 = \mu_0^2 + \Delta\mu^2 \quad (2.5)$$

in which μ_0^2 represents the ultraviolet (bare) value. This mass parameter picks up quantum corrections $\Delta\mu^2$ that depend quadratically on the ultraviolet cutoff Λ of the theory. Consider, for example the contribution of radiative corrections to μ^2 from top quarks. The complete one-loop calculation of this contribution reads:

$$\Delta\mu^2 = \frac{N_c \lambda_t^2}{16\pi^2} [-2\Lambda^2 + 6M_t^2 \ln(\frac{\Lambda}{M_t}) + \dots] \quad (2.5)$$

in which λ_t and M_t are the Yukawa coupling and mass of the top quark. If the bare Higgs mass is set near the cutoff $\mu_0^2 = O(\Lambda^2) = O(M_{Pl}^2)$, then $\Delta\mu^2 \approx -10^{35} \text{ GeV}$. This large correction must precisely cancel against μ_0^2 to protect the EW scale. This is the root cause of the *fine-tuning problem*, which boils down to the implausible requirement that μ_0^2 and $\Delta\mu^2$ should offset each other to about 31 decimal places.

Closely related to the fine-tuning problem is the question of whether the SM remains valid all the way up to the Planck scale (M_{Pl}). This question is non-trivial because it depends on how the Higgs quartic coupling λ behaves at high energies. Competing trends are at work here, namely [17-19]:

- 1) Radiative corrections from top quarks *drop* λ at higher scales, while those from the self-interacting Higgs *grow* λ at higher scales.
- 2) If $\lambda(M_{EW})$ is too large, the Higgs loops dominate and λ diverges at some intermediate scale called the *Landau pole*. However, if $\lambda(M_{EW})$ is too small, the top loops dominate, λ runs negative at some intermediate scale which, in turn, makes the potential unbounded from below and destabilizes the vacuum.

3. Radiative corrections and the vacuum stability

Within the SM, the value of the physical Higgs mass $M_H \approx 125 \text{ GeV}$ hinted by recent LHC data falls at the border of vacuum stability which, in turn, implies a vanishing quartic coupling near M_{Pl} . A recent study [17] has undertaken a complete perturbative analysis on the vacuum stability, including the two-loop threshold correction to λ at the

EW scale due to QCD and top quark couplings. This analysis has unveiled the following outcomes:

- 1) Vacuum instability develops around a Renormalization Group scale on the order of $\mu_{cr} = O(10^{11})$ GeV.
- 2) Both parameters of the Higgs potential (2.2) assume near-critical values about μ_{cr}

$$\mu^2 \ll M_{Pl}, \quad \lambda(\mu_{cr}) \approx 0 \quad (3.1)$$

It was concluded that (3.1) hints at the possibility that the SM behaves as a statistical system approaching criticality at μ_{cr} [17]. The next section attempts to refute this conclusion. We find that critical behavior and the approach to chaos in the SM are likely to occur at a scale *appreciably lower* than μ_{cr} .

4. Transition to chaos in Higgs channels

The goal of this section is to briefly survey several scenarios describing the TC in systems comprising YM or Maxwell fields (EM) in interaction with the Higgs scalar. These scenarios may be relevant for the dynamics of decay channels in which Higgs breaks up in pairs of YM bosons or photons.

4.1) Let us begin by bringing up a numerical study of the TC for the classical and homogeneous SU(2) x U(1) theory [9]. It was found there that the critical energy density for the onset of chaos is given by:

$$\varepsilon_c = \left(\frac{4M_W^2}{\sqrt{2}G_F} \right) \varepsilon \quad (4.1)$$

where M_W is the mass of the W boson, G_F the Fermi constant and ε the energy density of the system of Yang-Mills and Higgs bosons. It follows from (4.1) that $\varepsilon_c \approx 2 \times 10^{10}$ GeV/fm³ or $\varepsilon_c \approx 1.529 \times 10^8$ GeV⁴. Multiplying this energy density by the volume corresponding to the EW interaction range $V_{EW} \approx r^3$ with $r = O(10^{-3} \text{ fm})$, results in a critical energy for the TC on the order of 100 GeV $\approx O(M_{EW})$.

4.2) We turn next to the case of massless scalar electrodynamics coupled to the Higgs scalar. One-loop corrections to the classical interaction of this system lead to the following critical energy density [10]:

$$\varepsilon_c \approx \frac{11}{108} \lambda v^4 - \frac{4}{81\pi^2} (\lambda^2 + \frac{3e^4}{10}) v^4 \quad (4.2)$$

If the quartic coupling is evaluated at the pole top mass [17] and the system is assumed to be near the EW scale, that is, if $\lambda = 0.126$ and $v \approx 246$ GeV, (4.2) leads to $\varepsilon_c = 4.658 \times 10^7$ GeV⁴ and a critical energy for the TC on the order of 10 GeV.

4.3) Finally, let us consider a system of EW bosons coupled to a massless Higgs scalar [11]. The critical energy density defined there includes a contribution from radiative corrections to the Higgs potential (ε_c^0) and a contribution from the vacuum energy density (ε_v). These are estimated to be, respectively:

$$\varepsilon_c^0 = \frac{3\eta^4}{64\pi^2} \exp[2(\alpha_w - \frac{\lambda}{g^4} \beta_w)] (1 + \frac{1}{2\cos^4\theta_w}) [1 - 7 \exp(-48\Lambda_w \beta_w)] \quad (4.3)$$

$$\varepsilon_v = O(\lambda\eta^4)$$

Here η stands for the renormalization scale, g for the weak coupling of the EW bosons, θ_w is the Weinberg angle and $\alpha_w, \beta_w, \Lambda_w$ are numerical coefficients dependent on θ_w . Taking $\lambda = 0.126$, assuming $\eta \approx O(v)$ and adding the two contributions gives $\varepsilon_c = 4.581 \times 10^8 \text{ GeV}^4$ and again a critical energy on the order of $100 \text{ GeV} \approx O(M_{EW})$.

The table below summarizes all results discussed in this section.

Relation	Context	Critical energy density (GeV ⁴)	Critical energy (GeV)
(4.1)	classical YM-Higgs	1.529×10^8	$O(10^2)$
(4.2)	quantum EM-Higgs	4.658×10^7	$O(10)$
(4.3)	quantum YM-Higgs	4.581×10^8	$O(10^2)$

4. Conclusions

Our findings suggest that the transition from order to chaos in classical and quantum systems of gauge and Higgs fields is prone to occur at a scale substantially lower than $\mu_{cr} = O(10^{11}) \text{ GeV}$. Quantum corrections from the Higgs quartic coupling and from the interaction of the Higgs with heavy particles become irrelevant as the vacuum loses stability and dies out. The inability of the vacuum to survive not too far above the LHC scale explains away the fine-tuning problem and signals the breakdown of the SM in this region.

This conclusion, albeit preliminary, sheds light on the problem of *extrapolating* field theories in the deep ultraviolet sector, specifically near the Planck scale. As it is well known, General Relativity (GR) is exclusively an “effective” low-energy framework and

efforts to develop perturbative quantization of classical gravity result in non-renormalizable theories [20]. So far, the ultraviolet completion of GR has been approached from two different directions. Whereas string theories pursues introduction of new fields and symmetries beyond the SM, some quantum gravity theories retain all fields and symmetries of GR and treat gravity as a fundamental non-perturbative interaction. Since gravity cannot be decoupled from the energy-momentum of matter at any level of description, vacuum instability in the TeV sector points out that both approaches to the ultraviolet completion of GR may be missing a critical piece of the puzzle.

As mentioned in the first section, our conclusions need to be further scrutinized and, most importantly, confronted with the experiment. For instance, it is necessary to clarify the relationship between the standard Higgs mechanism of EW symmetry breaking and the vacuum stability problem. We anticipate that this effort may help answer the following open questions:

- a) Can one envision that the Higgs scalar is a *short-lived cluster of gauge bosons* (EW or gluons) rather than a fundamental field postulated by the theory, whose function is to preserve the consistency of the SM?
- b) Would this interpretation bring us closer to understanding the physical origin of the Higgs mass?

The outcome of this investigation will be reported elsewhere [21].

References

- [1] Biro T. S, Matinyan, S. G. and Muller B., Chaos and Gauge Field Theory, World Scientific, 1994.
- [2] Biro T. S., Muller B. and Matinyan S. G., Chaotic Quantization of Classical Gauge Fields, hep-th/0010134.
- [3] Matinyan S. G., Chaos in Non-Abelian Gauge Fields, Gravity and Cosmology, arXiv:gr-qc/0010054v1.
- [4] Matinyan S. G. and Muller B., Quantum Fluctuations and Dynamical Chaos, *Phys. Rev. Lett.* **78** (1997), 2515–2518.
- [5] Salasnich L., Quantum Signature of the Chaos-Order Transition in a Homogeneous SU(2) Yang–Mills–Higgs System, nucl-th/9707035.
- [6] Gutzwiller M. C., Chaos in Classical and Quantum Mechanics, Springer, New York, 1990;
Chaos and Quantum Physics, Editors: Giannoni M-J, Voros A and Zinn-Justin J, North-Holland, Amsterdam, 1991.
- [7] Kuvshinov V. I. and Kuzmin A. V., Order-to-Chaos Transition in SU(2) Spatially Homogeneous Model Field System, *Nonl. Phenom. in Comp. Sys.* **4** (2001), 64–66.
- [8] Kuvshinov V. I. and Kuzmin A. V., Towards Chaos Criterion in Quantum Field Theory, *Phys.Lett.* **A296** (2002), 82–86.
- [9] Berman G. *et al.*, Dynamical chaos in the SU(2) x U(1) theory, *Phys. Rev. Lett.* **A 194** (1994), 251-264.

- [10] Matynian S. G. and Muller B., Quantum Fluctuations and Dynamical Chaos: An Effective Potential Approach, Foundations of Physics, September 1997, Volume 27, Issue 9, 1237-1255.
- [11] Kuvshinov V. I. and Kuzmin A. V., The Influence of Quantum Field Fluctuations on Chaotic Dynamics of Yang–Mills System, Journal of Nonlinear Mathematical Physics Volume 9, Number 4, 2002, 382–388.
- [12] Goldfain E., On the asymptotic transition to complexity in quantum chromodynamics, Communications in Nonlinear Science and Numerical Simulation, 14, 2009, 1431-1438.
- [13] Goldfain E., Bifurcations and pattern formation in particle physics: An introductory study, EuroPhysics Lett. **82** 11001, 2008.
- [14] Goldfain E., Feigenbaum Attractor and the Generation Structure of Particle Physics, *Int. J. Bifurcation Chaos*, **18**, 891, 2008.
- [15] Goldfain E., Chaos in Quantum Chromodynamics and the Hadron Spectrum, Electronic Journal of Theoretical Physics, Vol. 7, Issue 23, 2010, 75-84.
- [16] Available at: <http://arxiv.org/abs/1205.6497>
- [18] Available at: <http://arxiv.org/pdf/1209.0393v1.pdf>
- [19] Available at: <http://www.phys.washington.edu/users/sharpe/into7/Kuti3.pdf>
- [20] Available at: <http://arxiv.org/pdf/1202.2274.pdf>
- [21] Available at:
https://www.researchgate.net/publication/346688804_Higgs_Mass_from_Topological_Condensation_of_Vector_Bosons

