Direction and the Relativity of Probability

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Abstract

Probabilities sometimes depend on the direction in which a situation is being observed or projected. Different directions can produce different probabilities for one and the same situation. Those probabilities have to be described differently. This is demonstrated by Bell-test experiments.

Introduction

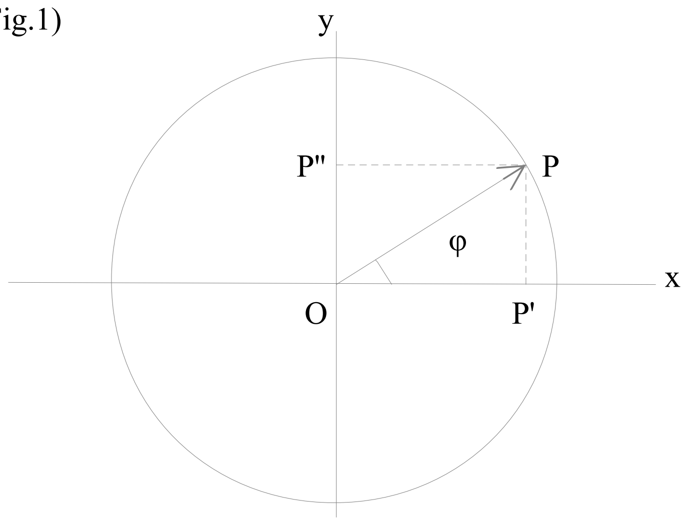
The main idea in this paper is that a probability is a ratio of numbers. In geometry a projection is a ratio of lengths or surfaces. Sometimes probabilities can be represented by projections.

In Bell-test experiments the correlations are calculated from probabilities. These probabilities can be represented by projections of vectorspaces. These vectorspaces are determined by the positions and adjustments of Stern Gerlach devices The projections of the vectorspaces depend on the direction of projection. Different directions yield different projections from one and the same vectorspace. In the same way the different directions from which the vectorspace is being observed, need different probability definitions. Vectorspaces observed by detectors who's observation direction doesn't correspond to the direction of movement of the particles (the line of motion), yield Bell probabilities. These probabilities cannot be detected. Vectorspaces observed by detectors who's observation direction does correspond to the line of motion, yield Quantum Mechanic (QM) probabilities. These probabilities can only be detected indirectly. It will become clear what detectors can and cannot detect.

Probabilities

Let us look at a reference system with origin O, a horizontal x-axis and a vertical y-axis. In it is a circle with its centre in O and radius 1. P is a point on the circumference of the circle and P moves through the points of the circle, counterclockwise and starting at the positive x-axis. OP is a vector. The angle between the vector and the positive x-axis is φ. P' is the projection of P onto the x-axis. P'' is the projection of P onto the y-axis. OP' is $\cos\varphi$. OP'' is $\sin\varphi$. According to Pythagoras $\sin^2\varphi + \cos^2\varphi$ = radius = 1. (See fig.1)).

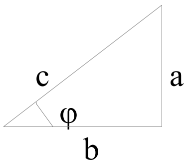
Fig.1)



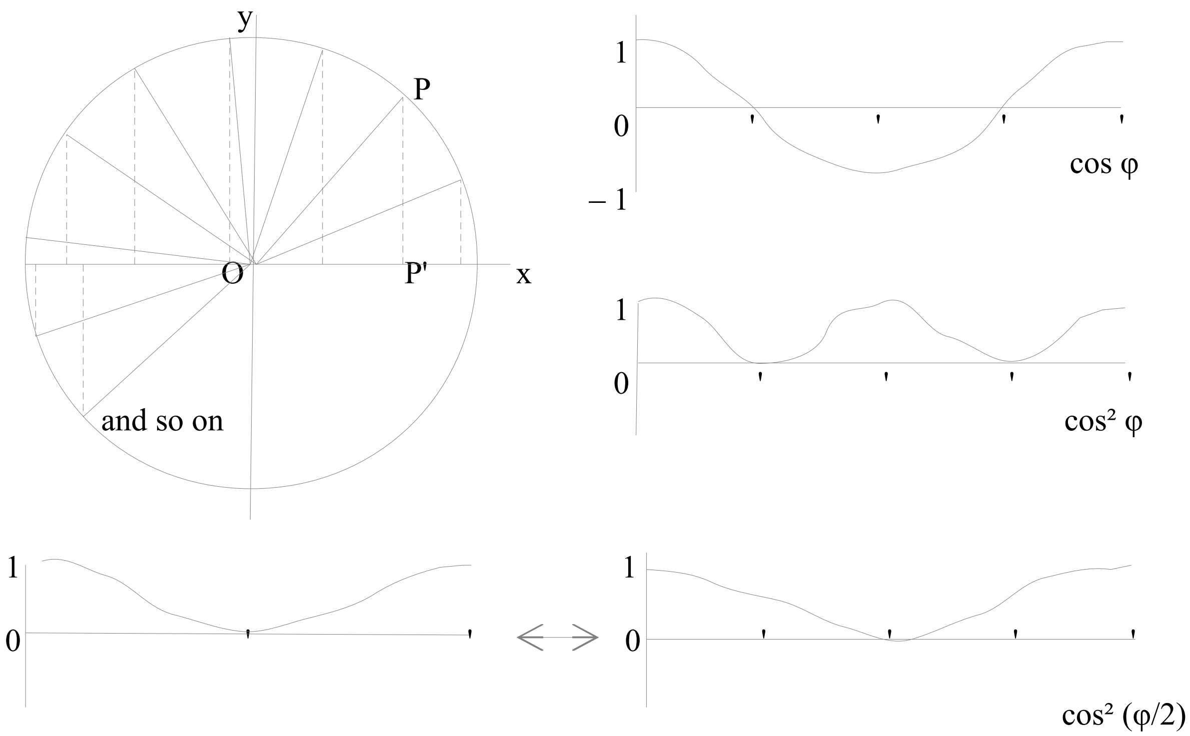
There is a correspondence between the projection density of the points P' on the x-axis and the probability for a point P of the circle to arrive, by projection, at a particular point P' on the x-axis. (This correspondence is $\cos^2(\varphi/2)$). As for $90° < \varphi < 270°$ $\cos \varphi$ is negative and probabilities cannot be negative, $\cos \varphi$ is squared. Now the period is halved and to dubble it again, the angle φ is divided by 2. (See fig.2)).

Fig.2)

Definition cosine:



$$\cos \varphi = b / c = OP' / OP = OP' / 1 = OP'$$



projection density distribution

probability

Equally distributed points P of the circle are being projected onto the x-axis and then the projection density distribution of the points P' is considered: there is a correspondence between the projection density and the probability for a point P of the circle to arrive, by projection, at a particular point P' on the x-axis. The correspondence is $\cos^2 (\varphi/2)$.
At the right hand side the probability is deduced.

So we have found that the probability of a particular point on the circumference of a circle to arrive, by projection, at a particular point on the horizontal diameter of the circle is $\cos^2 (\varphi/2)$.

We now expand the x,y-plane to the x,y,z-space, adding a z-axis to the reference system, perpendicular to the x- and y-axes. The vector OP now describes a sphere with radius 1. A point Q is chosen on the sphere. This point Q, together with the z-axis, determines one plane. This plane, together with the x,z-plane, divides the sphere in four spaces: the vectorspaces V1, V2, V3 and V4. The vectorspaces are all bounded by the two planes and the surface of the sphere.

If the x,y-plane is considered in this space, the circle now is an intersection of the x,y-plane and the sphere and the probability description for points on this circle is of course the same as previous described. To find the probability description for other points on the surface of the sphere, projected onto the x,z-plane, we can choose a random point P at the surface and consider a plane, parallel to the x,y-plane, intersecting P. The intersection of this plane with the sphere is a circle with P on it. For each point on this circle the same probability description is valid as previous described, so also for P. So the probability for P to arrive at its corresponding point P' at the x,z-plane, by projection onto the x,z-plane, is $\cos^2(\varphi/2)$, in which φ is the angle between the x,z-plane and the plane intersecting P and the z-axis. As this goes for any P, it goes for every point of the surface of the sphere. So the probability description, $\cos^2(\varphi/2)$,is valid for the projection of all points of the sphere onto the x,z-plane.

The vectorspaces V1 and V3 and the vectorspaces V2 and V4 are opposite of each other in respect of the z-axis, like parts of an orange. We have seen that the projection of these spaces onto the x,z-plane is given by $\cos^2(\varphi/2)$. The projection of these spaces onto the x,y-plane is different of course (projection from another direction). This projection is proportional to φ, not to $\cos\varphi$. It looks like an orange cut in halve, perpendicular to its axis.

We now have a sphere, divided in four vectorspaces with a common axis: the z-axis, and a vector OP. If V1 is the vectorspace between the x,z-plane and the plane intersecting the z-axis and Q, and the angle between these planes is φ (another φ), then we can ask what the probability is for a particular vector OP to be in V1. By projecting the vectospaces onto the x,y-plane, and varying φ, we can see that the size of V1 is proportional to φ and thus the probability for the vector OP to be in V1 is also proportional to φ.

We can also project the vectorspaces onto the x,z-plane. We then find the probability $\cos^2(\varphi/2)$. This is another probability. This probability answers the question what the probability is for a particular vector to belong to one of the vectors in V1. This is a different question, answered with a different probability.

Exactly this situation occurs at the heart of a Bell-test experiment.

Bell-test experiments

In Bell-test experiments pairs of entangled particles with opposite spin are being produced. Spin can be represented by an axial vector in a random direction, independent of the direction of movement of the particles. The vector keeps its direction in space. The vectors of a pair are exactly opposite. The particles move in opposite directions (line of motion). This line of motion is chosen to be the reference direction.

Each of the particles is being detected by and subjected to the influence of a Stern Gerlach device. For our purpose we will consider the Stern Gerlach devices as consisting of two parts: the magnets and the detectorplate. The magnets are oblong and differently shaped. Because of this the magnets produce an inhomogeneous magnetic field. The particles are being sent through this field. The position of the magnets is along the line of motion. The direction of the field can be adjusted by rotating the magnets around the line of motion.

The detectorplates only record the places where the particles arrive at the plates. So one detector detects one particle of each pair. The inhomogeneous field between the magnets deflects the particles on their way to the detectorplate. The particles are being deflected in the direction of the spincomponent that corresponds to the direction of the field. Is the field direction for example vertical, then the particles can only be deflected upwards or downwards. It is as if there is a central perpendicular plane in the devices in respect of the field direction. Particles with spin 'up' in respect of the field direction arrive above the central perpendicular plane at the detectorplate and particles with spin 'down' arrive beneath it. The field direction is the only thing with which the devices can distinguish between particles with spin 'up' and particles with spin 'down' in respect of the field direction.

If we stretch out the central perpendicular planes of device A and device B to infinity, they intersect in the line of motion (see fig.3)). The planes divide the space in four subspaces, two by two opposite of each other in respect of the line of motion. The position of the source, producing the entangled pairs, is in the middle between A and B on the line of motion.

Fig.3)

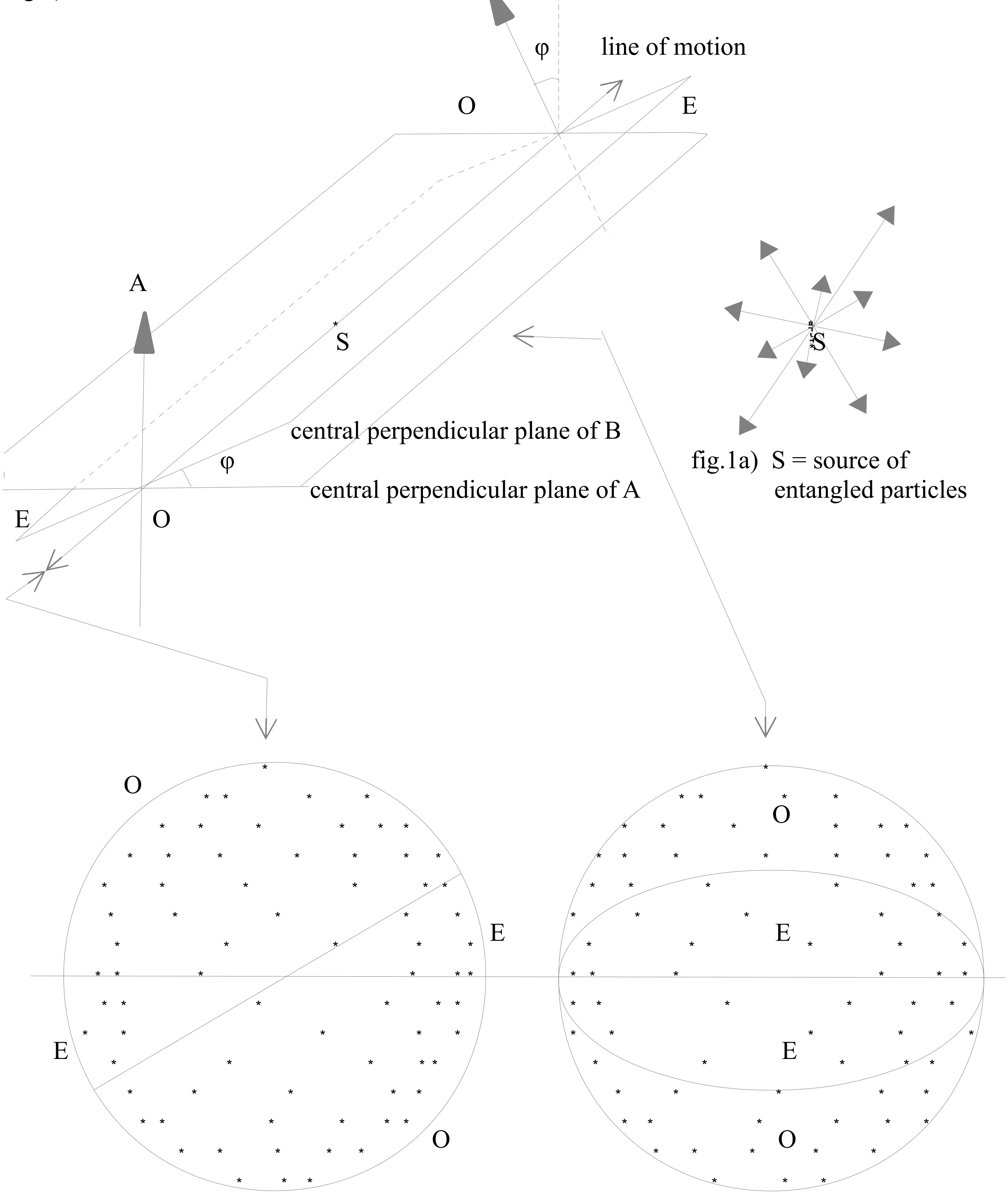


fig.1a) S = source of entangled particles

fig.1b) Bell's probabilities
Projection of the vectorspaces E and O in the directions of the line of motion of the particles onto the detectorplates.

fig.1c) QM's probabilities
The same vectorspaces E and O perceived from a direction perpendicular to the line of motion.

Suppose only one pair of particles is being produced. Suppose spin of each particle is a vector with length 1. The vectors of both particles point in opposite directions. And suppose the vectors are in the vectorspaces E. Now the situation is a one to one match with the previous described probability situations. The line of motion corresponds to the z-axis. The vectorspaces E correspond to V1 and V3. The vectorspaces O correspond to V2 and V4. The only difference is the direction in which the vectors are being projected. In the probability description they were being projected from a vertical direction onto the x,z-plane. In Bell-test experiments they are being projected in the direction of the line of motion onto the detectorplates (horizontal direction). This means that cos must be replaced by sin and that sin must be replaced by cos in the expressions for the probabilities.

If one particle of an entangled pair has its spin direction in space E, the other particle of the same pair has its spin direction in the other space E because the spin directions are opposite and the vectorspaces are opposite. This means that both spin directions are at the same side of the central perpendicular planes for both detectors, yielding combinations of equal spin results. If the spin directions of the particles of one pair are in the spaces O, then they are at the opposite side of the central perpendicular planes for both detectors, yielding combinations of opposite spin results.

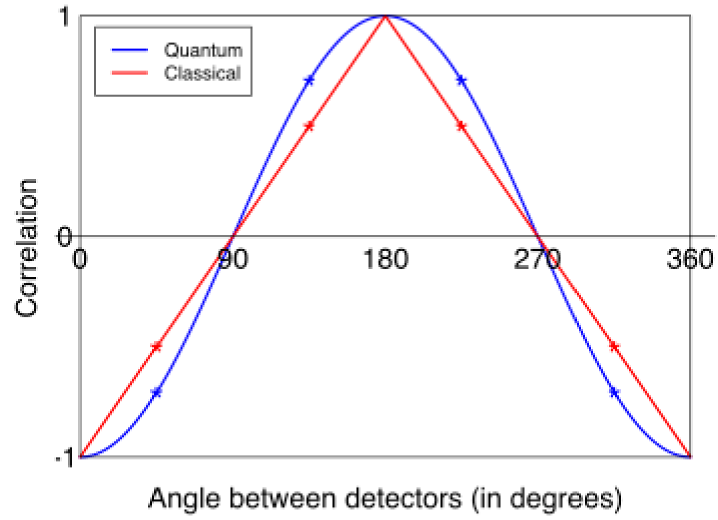
Because the line of motion corresponds to the z-axis, the detectorplates correspond to planes parallel to the x,y-plane in the probability description. From the observation direction of the detectorplates, the probability of a pair to have its spin direction in E is proportional to φ. This indeed is the probability Bell calculated (see diagram). This probability is not found in experiments and it is also not predicted by Quantum Mechanics. There are two reasons for this.

The first is that the Stern Gerlach devices cannot detect pairs of particles and they cannot detect vectorspaces. They only can distinguish an upper and a lower hemisphere in respect of their field direction by deflecting particles up- or downwards. As a result of this one detectorplate can only show the place of arrival on the detector for only one particle of a pair in a 50 % probability to be 'up' or 'down'. This is all one device can do. Two devices together cannot do much better. They still cannot detect pairs of particles and vectorspaces. They both produce lists of results and these lists are being compared afterwards. It is only then that a number of combinations of equal spin results is found. With this number the probability is calculated and one still would expect to find Bell's probability. This is not found because of the second reason.

The second reason, at least as important, is that the detectorplates have been placed perpendicularly on the line of motion, the reference direction. Theoretically this cannot be done at random. One has to start placing the detectorplates along the reference direction and then rotate them 90° to get them perpendicular on the line of motion. This rotation must be taken into account. This means that the observation directions of the detectors don't correspond any longer to the reference direction (the line of motion). They are the same directions but they don't correspond. This is very confusing. Taking into account the rotation, the observation directions of the detectorplates correspond to a direction perpendicular to the line of motion of the particles. Looked at the vectorspaces E from this direction the probability $\sin^2(\varphi/2)$ emerges (φ being the angle between the adjustments of A and B and so also between the central perpendicular planes of A and B). This indeed is the QM probability for combinations of equal spin results.

The probabilities can be defined. Bell's probability is the probability for a particular particle to have its spin direction in vectorspace E. This probability cannot be detected as we have seen. The QM probability is the probability for a particular particle that its spin direction belongs to one of the vectors in E, meaning that the particle belongs to one of the particles that have their spin direction in E. As devices can only detect one particle of a pair, they, together, can produce these numbers indirectly, by comparing the lists afterwards.

So Bell's probabilities cannot be detected because pairs of particles and vectorspaces cannot be detected by one detector (and also not by two detectors) and because the observation directions of the detectors don't correspond to the line of motion. If the observation directions of the detectors would correspond to the line of motion, they certainly would yield Bell's probabilities. The QM probabilities also cannot be detected directly by the detectors because their observation directions don't correspond to the line of motion of the particles. To find the QM probabilities one can put the detectorplates in their starting positions, along the line of motion, and from that position let them observe the vectorspaces. One would object that the detectors cannot detect particles in this position and this is exactly the reason why the QM probabilities can only be obtained indirectly by comparing the lists of results afterwards. The defined QM probabilities correspond perfectly to this situation.



The best possible local realist imitation (red) for the quantum correlation of two spins in the singlet state (blue), insisting on perfect anti-correlation at 0°, perfect correlation at 180°. Many other possibilities exist for the classical correlation subject to these side conditions, but all are characterized by sharp peaks (and valleys) at 0°, 180°, and 360°, and none has more extreme values (±0.5) at 45°, 135°, 225°, and 315°. These values are marked by stars in the graph, and are the values measured in a standard Bell-CHSH type experiment: QM allows $\pm 1/\sqrt{2} = \pm 0.7071\ldots$, local realism predicts ±0.5 or less.

Source: Wikipedia

Summary and conclusion

Correlation in Bell-test experiments is calculated from probabilities. The probabilities are number ratio found by comparing the lists of results of the detectors afterwards. The results represent particle strikes at the detectors and the places of strikes on the detectorplates are related to spin of the particles. There appear to be combinations of equal spin results although spin of each pair is opposite. This is due to different angles of detection by the devices.

The compared results of the detections represent the probability for a particular particle to belong to the particles that have their spin direction in vectorspaces determined by the position and adjustment of the Stern Gerlach devices. This is the QM probability.

The compared results of the detections do not represent the probability for a particular particle to have its spin direction in those vectorspaces. This would be Bell's probability. These are different probabilities. Bell,s probabilities do exist but cannot be detected.

The QM probabilities can be found by taking into account the rotations needed to get the detectors perpendicular on the line of motion. These probabilities can be perceived by looking at the vectorspaces from a direction perpendicular to the line of motion, the reference direction, the direction in which the detectors detect the particles (perpendicular because of the rotation). This is why the probabilities cannot be measured directly.

Correlation is defined as the probability of a combination of equal spin results subtracted by the probability of a combination of opposite spin results. The probability for a particular particle to belong to a pair, having their spin directions in vectorspaces E, is $\sin^2(\varphi/2)$. So the probability for a particular particle to belong to a pair, having their spin directions in vectorspaces O, is $1 - \sin^2(\varphi/2) = \cos^2(\varphi/2)$. Correlation then is $\sin^2(\varphi/2) - \cos^2(\varphi/2) = -\cos\varphi$.

Bell-test experiments are all about direction. Time does'nt play a role. There is no information exchange between A and B, so the search for loopholes is useless. The experiments are completely according to local-realism and thus the Bohr – Einstein quantum debate has finally ended in the advantage of Einstein.

Reference:

https://www.youtube.com/watch?v=g1quDMTEIFE (video)

Addendum

Time and direction play a similar role in the universe, both being equivalent elements of spacetime coming together in movement. Movement is change of position. Movement can change in time or in direction or both. It is impossible to be at rest.
(It takes the universe to know one's place).