PLANCK SCALE QUANTUM GRAVITY

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Abstract. In this short paper i present a proof that Planck scale is self consistent with quantum physics and lead naturally to gravity if i abandon Lorentz transformation and need that speed of light stays constant.

1. Quantizing Gravity

1.1. **Coordinate transformation.** Planck scale is goal of quantum gravity, in this paper I will present an alternative idea to gravity model of General Relativity that allows faster than speed of light movement and acceleration. First I discard Lorentz transformation of space-time and create a new one that allow for faster than speed of light movement. Next i need to quantize them to get rid of infinity and agreement with Planck scale. Let's say i have object in frame that moves with speed $\dot{x}$ in x direction i label time as t i can transform from one frame of reference to other by taking speed of light and dividing it by speed of object and multiplying it by direction x, so i just take a ratio between speed of light and object speed, where speed has direction towards light or away from it. I can write it:

$$x' = \frac{cx}{\dot{x}} \tag{1.1}$$

It means i divide speed of light cone (movement of light) by speed of object so speed of light is no longer a constant for all observers. This idea has obvious problem when speed goes to zero coordinate blows to infinity, so zero speed is problematic, that's why there is a need for minimum speed to get rid of that infinity. I take radius of universe as maximum length and take inverse of it as minimum length. Its smallest distance i can travel, by this way it looks pure random but it leads to something if i take Planck length as smallest length it's inverse is maximum length. Now I need only to define minimum speed, if object travels by one Planck length in time universe exist i have minimum speed. And that time can be maximum speed of light times universe radius so i get:

$$t_{max} = \frac{c}{l_P} \tag{1.2}$$

$$\dot{x}_{\min} = \frac{l_P}{t_{max}} \tag{1.3}$$

Now i can calculate coordinate transformation for minimum speed that gives:

$$x' = \frac{cx}{\dot{x}_{\min}} = l_P^2 x \tag{1.4}$$

That is correct solution to that problem each meter changes into Planck length and for whole universe there is one Planck length, that means there is one Planck length for whole universe, so if i take x as universe radius i get:

$$x'_{universe} = \frac{l_P^2}{l_P} = l_P \tag{1.5}$$

So it encodes that universe natural unit is Planck length. And it means i can't take less than universe radius as coordinate x. It gives dependence on x coordinate and speed, for object to use units of distance for example one meter i need at least speed of one Planck length per one second and so on. Now i go back to time. Time is inverse of it, first i write it as:

$$t' = \frac{\dot{x}t}{c} \tag{1.6}$$

So it's distance object travels divided by speed of light, meaning of it is that those transformations are set to make speed of light movement as unit movement so its unit coordinate. Now i have minimum and maximum length and time, and coordinate transformation that makes speed of light unit speed it means Planck units are natural units where speed of light is unit speed. From it i can move to gravity.

1.2. **Gravity as change in coordinates.** Gravity can be understood as acceleration. From coordinates transformation i need to create first a transformation rule for acceleration next space-time geometry so called metric tensor. First step is straight simple and if i generalize it its easy to go to step two. If i take into account just speed in one direction i need to take how it changes in time to arrive at acceleration, so i turn normal coordinates into change of them so i get straight simple calculations:

$$\dot{x}' = x \to \dot{x} = \frac{c\dot{x}}{\ddot{x}} \tag{1.7}$$

$$\dot{t}' = t \to \dot{t} = \frac{\ddot{x}\dot{t}}{c} \tag{1.8}$$

Now i can see that both of them depend on acceleration, new term is that they depend now on change of coordinates. But same thing with space, how does it change with respect to time? So i get only time direction acceleration. This leads to clear picture that i need to take how it changes with respect to that coordinate not to time only. So i write a new coordinate transformation that can be generalized for any coordinate as change in that coordinate:

$$\dot{x}' = x \to \partial_x x = \frac{c\partial_x x}{\partial_x \dot{x}_x} \tag{1.9}$$

$$\dot{t}' = t \to \partial_t t = \frac{\ddot{x}_t \dot{t}}{c} \tag{1.10}$$

So i can generalize it for any coordinate and any change so i create four order mixed tensor. Upper indexes are components of acceleration, down ones are direction that object changes with respect to. I can write it as :

$$G^{\alpha\beta}_{\mu\nu}(x^\mu) = \partial_\mu \partial_\nu G^{\alpha\beta}(x^\mu) \tag{1.11}$$

$$G^{\alpha\beta} = \begin{pmatrix} \frac{\dot{x}^{00}}{c^2} & \frac{\dot{x}^{01}}{c^2} & \frac{\dot{x}^{02}}{c^2} & \frac{\dot{x}^{03}}{c^2} \\ \frac{\dot{x}^{10}}{c^2} & \frac{c^2}{\dot{x}^{11}} & \frac{c^2}{\dot{x}^{12}} & \frac{c^2}{\dot{x}^{13}} \\ \frac{\dot{x}^{20}}{c^2} & \frac{c^2}{\dot{x}^{21}} & \frac{c^2}{\dot{x}^{22}} & \frac{c^2}{\dot{x}^{23}} \\ \frac{\dot{x}^{30}}{c^2} & \frac{c^2}{\dot{x}^{31}} & \frac{c^2}{\dot{x}^{32}} & \frac{c^2}{\dot{x}^{33}} \end{pmatrix} \tag{1.12}$$

Now this tensor represents each speed in each direction and how it changes in any direction. It has 256 components, so i need to change it to metric tensor that has 16 components, way to do it is very simple, i just use flat Minkowski Space-Time metric tensor:

$$g_{\mu\nu}(x^{\mu\nu}) = G^{\alpha\beta}_{\mu\nu}(x^{\mu})\eta_{\alpha\beta} = \partial_{\mu}\partial_{\nu}G^{\alpha\beta}(x^{\mu})\eta_{\alpha\beta} \tag{1.13}$$

But it discards 12 components of that G tensor so i use same tensor but i fill all zeroes with either one or minus one so that tensor in matrix form is equal to, where i use metric signature $(+---)$:

$$\eta_{\alpha\beta} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & -1 \\ 1 & -1 & -1 & -1 \\ 1 & -1 & -1 & -1 \end{pmatrix} \tag{1.14}$$

Now i have full defined metric, but i dont have energy defined so metric is undefined. And to solve that equation i need connection to some other equation that is equal to it. To do it I need to derive energy in Planck scale.

1.3. **Energy and spin.** From this logic comes naturally very simple energy tensor, first i take a look at Planck's energy its written as:

$$E_P = \frac{\hbar}{t_P} = \frac{\hbar c}{l_P} \tag{1.15}$$

That's time from transformation and length from transformation of space-time. Its maximum time and minimum length. So time of whole universe generates Planck energy and minimum length does so to. That lead to very simple principle, Planck acceleration is equal to either Planck length or maximum time. So if i get acceleration equal to Planck energy i need it to match with Planck length or maximum time, where Planck length can be thought as length of connection between two objects. Same with maximum time it can be thought as connection between two objects but in time not space. That connection is just a tensor that points from one object to another, it has sixteen components pointing in direction of that tensor, for example component x^{01} is just tensor in direction time and first coordinate of space. For each point of space-time i need to have this kind of tensor field that can't have value less than one and more than maximum time and maximum length. I only need to match it with energy that is very simple i just take that connection and if its time connection i multiply it by reduced Planck's constant squared or if its space connection i divide reduced Plank's constant squared by it and multiply by speed of light squared. I can write it formally as energy tensor:

$$T^{\alpha\beta} = \begin{pmatrix} \hbar^2 T^{00} & \hbar^2 T^{01} & \hbar^2 T^{02} & \hbar^2 T^{03} \\ \hbar^2 T^{10} & \frac{c^2\hbar^2}{T^{11}} & \frac{c^2\hbar^2}{T^{12}} & \frac{c^2\hbar^2}{T^{13}} \\ \hbar^2 T^{20} & \frac{c^2\hbar^2}{T^{21}} & \frac{c^2\hbar^2}{T^{22}} & \frac{c^2\hbar^2}{T^{23}} \\ \hbar^2 T^{30} & \frac{c^2\hbar^2}{T^{31}} & \frac{c^2\hbar^2}{T^{32}} & \frac{c^2\hbar^2}{T^{33}} \end{pmatrix} \tag{1.16}$$

Only thing left is to connect all those ideas and add spin. Its simple to see that those two tensors are equal if i add how they change in any direction:

$$\kappa T^{\alpha\beta}_{\mu\nu}(x^\mu)\eta_{\alpha\beta} = G^{\alpha\beta}_{\mu\nu}\eta_{\alpha\beta}(x^\mu) \tag{1.17}$$

Where constant is just $\kappa = \frac{1}{\hbar^2 c^2}$ to match acceleration in space units. Where connections are dimensionless. Now when connection is equal to maximum time for time components it gives Planck energy, where its equal to to minimal length it gives Planck energy for space connections. Before i go to spin last thing is to define G tensor field. I can write it as before:

$$\dot{x}^{\alpha\beta}(x^{\mu}) := \dot{x}^{\alpha\beta}(x^{\mu}) \mapsto \dot{x}^{\alpha\beta}\left(x^{\mu} + dx^{\mu}\right) \tag{1.18}$$

Now last part is to add spin, it just rotation around connection with frequency equal to spin number times connection length. So i can write it as when i use G field notation this time:

$$\hat{R}_{pq} G^{\alpha\beta}_{\mu\nu}\left(x^{\mu\prime}\right) \hat{R}^{T}_{pq} = G^{\alpha\beta}_{\mu'\nu'}\left(\hat{R}_{pq}\left(\theta(x^{\mu})\right) x^{\mu}\right)_{p,q\neq\alpha,\beta} \tag{1.19}$$

Where σ_{μ} is spin number vector, and θ is angle that is equal to:

$$\theta(x^{\mu}) = \frac{4\pi^2}{\hbar^2 c^2}\sigma_{\mu} x^{\mu} \tag{1.20}$$

I can write full field equation again now using full version of Gravity tensor:

$$\hat{R}_{pq}\partial_{\mu}\partial_{\nu}\dot{x}^{\alpha\beta}\left(x^{\mu\prime}\right)\hat{R}^{T}_{pq} = \kappa\hat{R}_{pq}\partial_{\mu}\partial_{\nu}T^{\alpha\beta}\left(x^{\mu\prime}\right)\hat{R}^{T}_{pq} \tag{1.21}$$

$$g_{\mu'\nu'}\left(x^{\mu\prime}\right) = \hat{R}_{pq}\partial_{\mu}\partial_{\nu}\dot{x}^{\alpha\beta}\left(x^{\mu\prime}\right)\eta_{\alpha\beta}\hat{R}^{T}_{pq} = \kappa\hat{R}_{pq}\partial_{\mu}\partial_{\nu}T^{\alpha\beta}\left(x^{\mu\prime}\right)\eta_{\alpha\beta}\hat{R}^{T}_{pq} \tag{1.22}$$

Last part is to define spin number tensor components, it says what elementary particle each point has. There can be allowed in this model only four values of spin : $\pm 2, \pm 1, \pm 1/2, 0$ each represents elementary particle. Spin two is graviton, spin one is photon , spin minus one is anti-photon, spin plus one half can be proton or electron , spin minus one half can be either anti-proton or anti-electron and spin zero is Higgs field. Idea is that for each point of space-time i assign a spin number for any point in space-time, spin tensor is just a number for that point that is equal to spin when summed. So particle is sum of all spins in all directions for each point of space-time. I can write it formally as:

$$\sum_{\mu}\sigma_{\mu} = 0 \vee \frac{1}{2} \vee -\frac{1}{2} \vee 1 \vee -1 \tag{1.23}$$

$$\tag{1.24}$$

If object moves in positive time direction its matter if in negative its anti-matter. I can write it in table as:

± 2	1	-1	$\frac{1}{2}$	$-\frac{1}{2}$	0
G	γ	$\overline{\gamma}$	$p^{+} \vee e^{-}$	$\overline{p}^{+} \vee \overline{e}^{-}$	H^{0}

So interaction of any point of field creates one of those particles. Interaction between one point and another is taking spin number of that point in direction of that field and moving it to next point. Graviton is only particle that has no anti-particle like Higgs field, and its comes from from interaction with two photons or anti-photons.

1.4. **Probability.** Only thing left is to find formula for calculating probability, that is still very simple if i understand simple fact- each state of speed can move in any direction in space and still maintain same energy, so if i take area of distance in space-time and divide it by area of one path that is part of that area i get probability that is normalized - it comes from fact that area is always finite and metric is always finite. I can write it formally as:

$$P_L^2(ds_A^2) = \frac{\int_{L\in A} ds^2}{\int_A ds^2} \tag{1.25}$$

Where $P_L^2(ds_A^2)$ is probability of path L for given area that that path is part of A. I can rewrite this equation using formulas from before for metric tensor as:

$$P_L^2(ds_A^2) = \frac{\int_{L\in A} \kappa \hat{R}_{pq}\partial_\mu\partial_\nu T^{\alpha\beta}\left(x^{\mu\prime}\right)\hat{R}^T_{pq}\eta_{\alpha\beta}dx^{\mu'}dx^{\nu'}}{\int_A \kappa \hat{R}_{pq}\partial_\mu\partial_\nu T^{\alpha\beta}\left(x^{\mu\prime}\right)\eta_{\alpha\beta}\hat{R}^T_{pq}dx^{\mu'}dx^{\nu'}} \tag{1.26}$$

$$P_L^2(ds_A^2) = \frac{\int_{L\in A} \hat{R}_{pq}\partial_\mu\partial_\nu \dot{x}^{\alpha\beta}\left(x^{\mu\prime}\right)\eta_{\alpha\beta}\hat{R}^T_{pq}dx^{\mu'}dx^{\nu'}}{\int_A \hat{R}_{pq}\partial_\mu\partial_\nu \dot{x}^{\alpha\beta}\left(x^{\mu\prime}\right)\eta_{\alpha\beta}\hat{R}^T_{pq}dx^{\mu'}dx^{\nu'}} \tag{1.27}$$

$$P_L^2(ds_A^2) = \frac{\int_{L\in A} g_{\mu'\nu'}\left(x^{\mu\prime}\right)dx^{\mu'}dx^{\nu'}}{\int_A g_{\mu'\nu'}\left(x^{\mu\prime}\right)dx^{\mu'}dx^{\nu'}} \tag{1.28}$$

That makes this model complete. Before measurement state of particle is given by all possible states so its equal to ds_A^2 afterwards it changes to ds_L. There is need that both probabilities have same sign and first one can't be greater than first one. I can write it formally as:

$$0 \leq P_L^2(ds_A^2) \leq 1 \tag{1.29}$$

$$0 \leq \frac{\int_{L\in A} g_{\mu'\nu'}\left(x^{\mu\prime}\right)dx^{\mu'}dx^{\nu'}}{\int_A g_{\mu'\nu'}\left(x^{\mu\prime}\right)dx^{\mu'}dx^{\nu'}} \leq 1 \tag{1.30}$$

That's final need in theory to work. It does account spin states- spin changes with direction positive direction give positive spin, negative direction negative spin. So finally i can write that state of system changes from A state to L state after measurement with probability P:

$$M_A : P_A^2(ds_A^2)g_{\mu'\nu'}\left(x^{\mu\prime}\right)\Big|_{x^\mu\in A} \rightarrow P_L^2(ds_A^2)g_{\mu'\nu'}\left(x^{\mu\prime}\right)\Big|_{(x^\mu)\in L} \tag{1.31}$$

So metric wave function is just equal to sum of all of those states, that means integral:

$$\Psi(x^{\mu'}) = P_L^2(ds_A^2)\int_{L\in A} g_{\mu'\nu'}\left(x^{\mu\prime}\right)dx^{\mu'}dx^{\nu'} \tag{1.32}$$

And that is wave function that has gravity build in it. In simple terms if Planck scale is fundamental that is only theory of gravity that is self consistent that can be created out it.

1.5. **Many universes Planck scale.** In first subsection I said that I take minimum length as inverse of universe maximum radius. This principle can be generalized without breaking rules of Planck scale for any number of universes, it means that minimum length no longer is Planck length but it to power of number of universes. What defines objects gravity in this way of thinking is its energy. Laws for any scale are same but energy limit increases it means there has to be more dimensions than four each representing one universe. So N universes means $4N$ dimensions of space-time and N times repeated Planck Scale. There is no reason to think that is number does not goes to infinity, and if it does there has to be infinite number of space-time dimensions. But still to reach infinity you need all energy of all universes combined so its not possible to get out of all set of universes that makes this idea self consistent as expected. Math is like before very simple i just increase number of dimensions:

$$\kappa^n T^{\alpha_1\beta_1\ldots\alpha_n\beta_n}_{\mu'_1\nu'_1\ldots\mu'_n\nu'_n}\left(x^{\mu'_1\ldots\mu'_n}\right)\eta_{\alpha_1\beta_1\ldots\alpha_n\beta_n} = G^{\alpha_1\beta_1\ldots\alpha_n\beta_n}_{\mu'_1\nu'_1\ldots\mu'_n\nu'_n}\left(x^{\mu'_1\ldots\mu'_n}\right)\eta_{\alpha_1\beta_1\ldots\alpha_n\beta_n} \tag{1.33}$$

Out of it i can create wave function of all possible universes:

$$\Psi^n\left(x^{\mu'_1\ldots\mu'_n}\right) = P^{2n}_{L_n}(ds^{2n}_{A_n})\int_{L_n\in A_n} g_{\mu'_1\nu'_1\ldots\mu'_n\nu'_n}\left(x^{\mu_1\ldots\mu_n\prime}\right)dx^{\mu'_1\mu'_2\ldots\mu'_n}dx^{\mu'_1\nu'_1\ldots\nu'_n} \tag{1.34}$$