

# Multi-Fold Black Holes: Entropy, Evolution and Quantum Extrema

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October 31, 2020

## Abstract:

*In a multi-fold universe, gravity emerges from Entanglement through the multi-fold mechanisms. As a result, gravity-like effects appear in between entangled particles; that they be real or virtual. Long range, massless gravity results from entanglement of massless virtual particles. Entanglement of massive virtual particles leads to massive gravity contributions at very small scales. Multi-folds mechanisms also result into a spacetime that is discrete, with a random walk fractal structure, and non-commutative geometry, that is Lorentz invariant, and where spacetime locations, and particles, can be modeled with microscopic black holes. All these recover General relativity at large scales, and semi-classical models remain valid till smaller scale than usually expected. Gravity can therefore be added to the Standard Model. It can contribute to resolving several open issues with the Standard Model without new Physics other than gravity. These considerations hint at an even stronger relationship between gravity and the Standard Model.*

*In our original work on multi-fold universe, we derived area laws for black hole, and considerations on the black hole paradoxes. Our analysis of evolution of charged black holes was used to derive a new unification model based on democracy of forces and particles: the Ultimate Unification (UU), significantly different from conventional GUTs. The discrete structure of spacetime, and multi-fold mechanisms, also ensure the absence of gravitational or cosmological singularities*

*Recently, progress has been made with conventional modeling of black holes, and with the AdS/CFT conjecture, it is believed to be very close to demonstrate a resolution (and/or the absence) of the information paradox. Something we had already hinted. Doing so, they also added to the modeling of the interior of black holes based on quantum considerations.*

*Considering how we have so far been able to recover many conventional models in multi-fold universes, typically with variations, factual statements, and physical (and often microscopic) explanations of the effects, this paper is focused on deriving the equivalent model for the interior and evolution of black holes. It includes recovering the Page curve for multi-fold black holes, the quantum extremal surface inside a black hole, and resolution of the information paradox with a solution consistent with the conventional approach, but in 4D, and without dependency on the AdS/CFT correspondence, or the ER=EPR conjectures, and its wormholes. This last statement about dependencies is not totally honest, as the latter two conjectures are inherently factual in multi-fold universes, and multi-fold may be, or correspond to, wormholes.*

*Anecdotally, we also obtain, that black holes behave as if they had singularities, despite the absence of singularity in a multi-fold universe. The quantum extremal surface plays a key role in significant recovery of information, and in explaining the Page curve, although different from conventional models.*

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# 1. Introduction

The preprint [1] proposes contributions to several open problems in physics like the reconciliation of General Relativity (GR) with Quantum Physics, explaining the origin of gravity proposed as emerging from quantum (EPR - Einstein Podolsky Rosen) entanglement between particles, detailing contributions to dark matter and dark energy and explaining other Standard Model mysteries without requiring New Physics beyond the Standard Model other than the addition of gravity to the Standard Model Lagrangian. All this is achieved in a multi-fold universe, that may well model our real universe, which remains to be validated.

With the proposed model of [1], spacetime and Physics are modeled from Planck scales to quantum, and macroscopic scales, and semi classical approaches appear valid till very small scales. In [1], it is argued that spacetime is discrete, with a random walk-based fractal structure, fractional, and noncommutative at, and above, Planck scales (with a 2-D behavior and Lorentz invariance preserved by random walks till the early moments of the universe). Spacetime results from past random walks of particles. Concretized spacetime locations and particles can be modeled as microscopic blackholes (Schwarzschild for photons and spacetime coordinates, and metrics between Reiser Nordstrom [2] and Kerr Newman [3] for massive and charged particles, possibly extremal). Although surprising, [1] recovers results consistent with other like [4], while also being able to justify the initial assumptions of black holes from the gravity or entanglement model. The resulting gravity model recovers General Relativity (GR) at larger scale, as a 4-D process, with massless gravity, but also with massive gravity components, at very small scale, that make gravity significant at these scales. Semi-classical models also work well till way smaller scales than usually expected.

In this paper, we remain at a high level of discussion of the analysis and references are generic for the subjects. It makes the points accessible to a wider audience and keeps the door open to further papers or discussions devoted to details of interest. Yet, it requires the reader to review [1], as we do not revisit here all the details of the multi-fold mechanism or reconstruction of spacetime. More targeted references for all the material discussed here are compiled in [1].

## 2. Multi-fold Black Holes

### 2.1. Particles and Concretized Spacetime location as Black Holes

In its reconstruction phase, [1] models particles and spacetime locations as microscopic (and extremal) black holes. The origins of these black holes comes from the observation that multi-fold create a black hole structure surround every particle / energy location. Doing so they also create AdS(5), as dual or cotangent space to the particles. It is the origin of the AdS/CFT factual correspondence in Multi-fold universes.

### 2.2 No Gravity or Cosmology Singularity

With multi-fold mechanisms, we also determine that no singularity exist. It results from the fact that:

- Multi-fold mechanisms (can) contribute torsion within matter. It ensures the absence of singularity (because, by definition, torsion requires paths from a location ending at another location, and, therefore, it is incompatible with a point singularity). [5] derives this result as a conventional classical or semi classical version.
- Multi-fold mechanisms contribute a positive cosmological constant, that we believe can explain or at least contribute to dark energy effects [6]. These contributions, in the presence of a lot of system interacting with each other (and therefore entangled) over small distance, will result into a significant additional outbound pressure.

- Discreteness of spacetime, and non-commutativity, while allowing for black holes, also ensure minimum distances and hence no singularities [1,7,8].

Only at the first moment of the big bang is it possible that a singularity existed. This will be the object of future paper (track them at [52,53]).

The absence of singularity is maintained with black holes in a multi-fold universe. Note however that we are obtaining and modeling differently the incompleteness of geodesics, as will be seen in later sections: no infinite density, or point-like infinite concentration (conventional space like singularity), nor time like singularity (due to this different way to look at geodesics incompleteness). It leads to different ways to understand the singularity theorems) [44,48].

## 2.3 Area laws and Entropy

[1] derives an area dependency for the black hole entropy (due to gravity or spacetime), as well as for spacetime causal horizons. The derivation is not based on GR, but rather entirely based on the multi-fold mechanisms and quantum uncertainty. As the model focuses so far with qualitative models, quantitative proportionality is rather obtained by recovering the conventional results as we know that multi-fold mechanisms recover GR. Therefore, all its corresponding results at large enough scales must also be recovered. Note also that this reasoning is a bit simplistic because it does not model the history of the black hole (it just derive the result for a given black hole) and oversimplifies the multi-fold mechanisms inside the black hole. We will revisit a slightly more rigorous model in a later section.

The derivation, in [1], is based on the holographic principle associated to spin-2 symmetry of the multi-folds. As a result, all the effects of the mass or energy content of the black hole amount to the flow of multi-fold effects across any surface containing that mass; not really how it is distributed. So outside the black hole all the effects are due to the boundary of the black hole horizon. But the horizon blocks propagation of the entangled virtual particles responsible for the multi-folds. It implies that, from the inside, all virtual particles accumulate on the inside of the horizon, unable to get out. It is the quantum uncertainties that move particles inside the horizon occasionally outside, where they can then contribute to the potential energy of the layer outside (via exposure of the external layer to  $V_{\text{eff}}$ ). Particles outside generate multi-folds, reflecting that potential energy level and gravity expands outside the horizon. It is a similar phenomenon to the one that is discussed in [9] to ensure that gravity shields do not exist. (*Note on 5/16/21: See also [51]*).

From the outside, the multi-folds (and virtual particles) also accumulate on the horizon, for an external observer, reflecting the gravity effects from the outside. These external multi-folds have a particle falling in (and so eternally blocked on the external side of the horizon). Thus, any external gravity effect is similarly passed to the black hole when the fluctuation brings the external virtual particles inside.

As a result, the microscopic degrees of freedom are proportional to the area of the horizon [1].

Therefore, the black hole horizon is populated with internal and external particles, entangled through this kind of interaction: the fluctuations of the horizon. Note that while analogous to Hawking radiations, it is a different phenomenon. The amount of degree of freedoms are proportional to  $A \Delta_h$ , where  $\Delta_h$  denotes the thickness of the quantum fluctuation region. The entropy is therefore proportional to  $A$ , the area of the black hole. This is the well-known Hawking Bekenstein black entropy and area law of black holes.

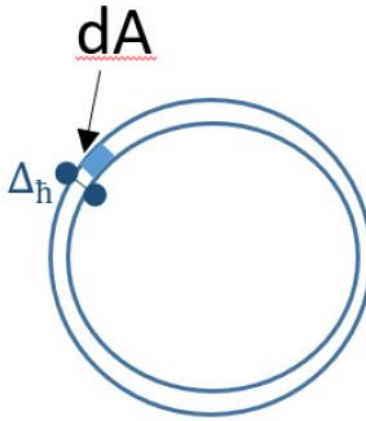


Figure 1: It shows the shell around the black hole horizon contributing to the multi-folds generated by the black hole and perceived outside. The number of involved microstates contributing to the entropy are therefore proportional to the area of the shell.

These results are for a (stationary) non-charged and not rotating black hole. Charged and rotating black holes have more complex Thermodynamics (e.g. [45]) that involves its charge(s) and angular momentum and characterized with metrics like the Kerr (no total charge) [46], Kerr-Newman [3] and Reissner-Nordstrom (no rotation/no total angular momentum) [2] metrics and a ring of singularity. The same reasoning applies, although mathematically more complicated, and with more corner cases (e.g. extremal and beyond extremal use cases) [47,48].

The area law also evolves into including a  $\ln(A)$  contribution encountered at small scales through different reasonings in [1] and [10] due to other fields (hairs and micro-hairs) from the black hole and the 2D dominant process at small scales [11]. This dependency is also recovered in conventional models [16].

## 2.4 Multi-fold Black Hole Lifecycle

Multi-fold considerations on the evaporation of charged black holes invalidate the weak gravity conjecture (WGC) [1,12] at very small scales, and shows how (charged total charge) black holes will evolve into extremal black holes, that can then ultimately break apart into smaller black holes, and eventually elementary particles. This is how all entropy (and information) is returned and the information paradox is resolved. The reasoning in [1] is mostly thermodynamic. In the upcoming sections, we will detail further what takes place.

As a consequence of the reasoning we introduced a new possible grand unification model: the Ultimate Unification (UU), which is quite different from conventional GUTs with uber symmetries [1,13].

## 3. Status of Conventional Resolution of the Black Hole Information Paradox

By conventional, we mean mainstream Physics, possibly including, for the purpose of this paper, and, at the difference of most of our other papers, string theory.

To our knowledge, [14] is currently the best overall review of the current status and the latest evolution, even if a popular science article. It clearly explains how multiple new results combine into the new approach. We will re-explain its overview, re-phrased our way, in order to prepare for our multi-fold analysis.

The black hole information paradox [15] results from Hawking's observations, that, if a black hole evaporates<sup>2</sup>, it will end up disappearing, and all the information in the black hole, due to all the matter previously trapped in the black hole, and modeled by the black hole entropy, will disappear and be lost. Such a conclusion violates unitarity of Quantum Physics (and its associated principle of conservation of information). It is therefore expected by many to be incorrect; yet no firm conclusion or proof has been provided so far. In [1], we argued also for indications of a resolution of this paradox based on slightly different considerations.

### 3.1 The Black Hole Entropy Page Curve

The generalized second law of Black holes [17] expresses the total entropy of the universe, with a black hole, as the external entropy plus the black hole entropy. Per thermodynamics, its variation must always be positive. With a radiating black hole model, one needs to add the entropy changes due to radiations to the black hole entropy changes (due to energy, or mass, reduction, and therefore area reductions as radiation takes place). While the universe expansion may lead to additional considerations, it does not modify the law for the universe, contrary to what has been sometimes pretended in recent publications (e.g. [55]).

Page modeled black holes with a scattering matrix ( $S$  matrix), and derived a curve that expresses the entanglement entropy of the system (as entangled particles are emitted by Hawking's radiation), that complements the black hole entropy in the generalized second law[18,19]. It resulted into the Page black hole entropy curve (see Figure 4, where we recover the same curve in multi-fold universes), showing that a decrease in the black hole entropy is matched by an increase of its entanglement entropy till they are equal. After, the entanglement entropy decreases as the black hole entropy mass continues to decrease due to the radiations, because the possible amount of black hole entropy microscopic states now limit what can be entangled.

With such a model and results, the information paradox is resolved: all entropy originally in the black hole is at the end back in the surroundings, but it is replaced by the Page paradox [14]: how can quantum effects appear at larger scale (the page time occurs when the black hole is still large / macroscopic) than intuitively expected, i.e. at quantum scales. The Page curve effects occur while the black holes are still large. Even if the reasoning that we just described seems convincing, what is really physically happening? That answer does not really exist in conventional Physics, except maybe for the next sections, which is still more a path integral mathematical formalism with some challenges (e.g. would the path integral really apply to macroscopic objects across all possible topologies). However, in upcoming sections, dedicated to multi-fold universes, we provide a multi-fold microscopic explanation.

### 3.2. Physical Explanation with Path Integrals and the AdS/CFT Correspondence Conjecture

The progresses reported in [14] are supposed to address these questions, in conventional universes:

- Using the AdS/CFT Correspondence Conjecture to match computations in AdS to computations in a flat spacetime with CFTs (Conformal Field Theory) (the dual spacetime) [20]. As CFTs are modeled with quantum physics with well understood behaviors, the theory is unitary and preserves information. Therefore, black holes in AdS must preserve information; that is:

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<sup>2</sup> In [38,39], among other things about particle in multi-fold universes, we look at Hawking's radiation from a different conventional angle: outside the horizon. The paper focuses on the horizon and inside. Original references to Hawking's work can be found in [38]. It also positions our reasoning of section 2.3, versus the additional effects of particle creation beyond the horizon; something that affects the whole system (generalized entropy) but not the black hole entropy estimates. Such extra radiation component can be considered either as associated to the black hole, as an extra radiation term in the generalized second law (section 3.1), if the external system is considered as being only the asymptotic region of the universe, or as variations of  $S_{\text{out}}$ .

- If you believe in the AdS/CFT Correspondence Conjecture. It a priori implies an underlying superstring and supersymmetric model, which is contrary to what [14] states when suggesting that the progress would have no direct dependency on strings<sup>3</sup>. But it is granted that many have moved beyond superstrings when using the AdS/CFT Correspondence Conjecture that they rather consider as a global approach to M-theory [49].
- If you believe that black holes in AdS have any relevance to the real universe; which we know is not AdS. Attempts to shakily extrapolate to non-anti de sitter's spaces, i.e., to de Sitter universes, have usually not been that rigorous (See, for example, [27,28]), and often with controversies and mistakes.
- However, in AdS, evaporation can never complete [29], because what is emitted ends up being reflected, and, therefore, re-absorbed in the future. The resolution of this aspect requires a trick, like the introduction of the evaporon, to allow escape in an additional dimension. With the duality, it would amount to cooling down CFTs with an extra dimensional escape path.
  - It is an ad hoc, and not that well physically justified, proposal, to say the least. But then again, the whole business of dualities are like that [25].
- With the evaporon, in AdS, [30,31] recover the Page curve due to a phase transition, when a quantum extremal surface forms inside the black hole in AdS.
  - Consider the generalized entropy (coming from the surface and the external region) from the second generalized law of black Holes. For AdS black holes, the area entropy plus the entanglement entropy is the generalized entropy (to be suitably renormalized [34] in a non-discrete spacetime, which is the conventional case). The formula can be extended to other suitable surfaces, splitting a Cauchy surface in two (e.g. a causal horizon) [32]. A quantum extremal surface extremizes the generalized entropy. It is the generalization of the Ryu-Takayanagi area law for the generalized entropy [32,33].
  - If an AdS black hole is modeled with CFTs as its boundaries/horizon, when it evaporates, after the phase transition is reached, a quantum extremal surface appears within the black hole. As a causal horizon, it splits the black hole interior and we recover the information escape figure of [14]: no entanglement exists across that surface (the black hole horizon also varies a bit). The extremal surface decreases as the black hole area decreases. So after the phase transition, less of the black hole is available for entangled Hawking radiation. It explains the decreasing entanglement entropy mentioned in section 3.1, matching the decrease the black hole area entropy and the Page curve.
  - Computations estimates were done in reduced dimensions AdS (e.g. AdS(2)). Unfortunately, we do not live in a 2D spacetime, at least not at our scale, or in our current epoch. [37] can be seen as extending the ideas to higher dimensional AdS black holes.
- More rigorous computations are reported for AdS(2), in [35,36]. They rely on path integrals over all possible spacetime topologies. Interestingly the significant contributions to the integral include topologies of multiple entangled blackholes linked by wormholes, something reminiscent of the ER=EPR conjecture, that also reminds of the multi-fold mechanisms [1,25]. To some extent, one can see these results as a different derivation of the result of the previous main bullet just as the ER=EPR conjecture actually derives from the AdS/CFT correspondence conjecture.

### 3.3 Conventional End to End Black Hole Evaporation Scenario

So, the new end-to-end explanation of the information paradox, à la [14], goes as follows, in a conventional universe:

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<sup>3</sup> Our work on the subject has led us to question the adequacy of superstrings and supersymmetry to describe our real universe [1, 10,13,21-25]. Yet, [25,26] provide arguments that such superstring dualities may provide mathematically correct approximations of multi-fold universe results, and therefore, hopefully, also of the real universe.

- As Hawking's radiation takes place, entanglement appears between the black hole interior and the outside.
- After a while, a phase transition occurs where, by tension occurs between the entangled interior and exterior, and an island appears towards the center, separated by a quantum extremal surface. What is beyond, i.e. within, that surface is no more available for entanglement with the outside of the black hole.
- As radiation continues, the entangled radiation, and the black hole entropy, decrease.
- At the end, all content of the black hole has been radiated and information has been transferred, via radiation, to the outside, through the entanglement. No information has been lost.

Of course, the model is fully developed, beyond the Page reasoning, only for an AdS spacetime, and for no more than 1 or 2 spatial dimensions. Neither of the conditions matches our spacetime. We know that gravity is fundamentally different in 1D, and 2D, spatial dimensions so the arguments for this being a good indication of 4D AdS, or above, does not necessarily hold. We also know that AdS is fundamentally unstable for GR (in the presence of matter) [50], so, again, extrapolations to a flat, or positively curved spacetime are not guaranteed.

Therefore, it is fair to say that, while very impressive as a development work, we do not know how physical the theory actually is, i.e., if and how it applies to our real universe. For sure, ideas like evaporation and the replica trick with wormhole replica models in the path integrals, as used in the computations of [35-37], are speculations and relying heavily on conjectural interpretations of the AdS/CFT correspondence, itself a conjecture... Also computations, required with the approach they follow, are quite approximative. Additional criticisms, and concerns, with this story so far are discussed in [14].

An outcome of the upcoming sessions is that the results hold for multi-fold universe, through different reasonings and more detailed microscopic interpretations. It implies also support of positively curved 4D spacetimes, à la de Sitter, that maybe, who knows, more relevant to our real universe.

## 4. Multi-fold Version of Evaporation

In order to understand any impact, on black holes, from the multi-fold theory, as well as to provide a different perspective on the effects in our real universe, let us try to analyze what the scenario of section 3.3. becomes in a multi-fold universe.

### 4.1 Radiation from Changes in Curvature

As a black hole radiates, the curvature of spacetime outside its horizon also changes.

As discussed in [38,39] (see <sup>2</sup>), changes of the spacetime curvature also generate new particles. A well-known result of QFTs in curved spacetime. The effects can be seen as a significant additional non-entangled radiation (because an in-falling particle never reaches the black hole horizon). That is how we will model this contribution in the present paper, and it affects  $S_{\text{out}}$ , in the generalized second law of black holes.

As we do not care, for this paper, about the exact quantitative proportionality constant, we consider anything outside the horizon to contribute to  $S_{\text{out}}$ . It simplifies the analysis without changing the outcome.

### 4.2 Hawking's radiation for a Multi-fold Black Hole

The reasoning presented in [1], and in section 2.3, is, intentionally, a bit too simplistic: from the inside horizon to further inside the black hole horizon, particles plunge towards the center, accelerating until reaching again  $c$ . From that point, virtual particles that they emit towards the horizon are essentially freezing in place, while the ones towards the center contribute to more attraction towards the center: no virtual particle can really propagate with momentum components away from the black hole center: only those with momentum towards the center propagate. Conversely, if energy is accumulating at the horizon, and matter is still in move, one can see the quantum extremal surface as a black hole horizon within the black hole horizon, due now solely to the matter that is closer to the center. As illustrated in figure 2, it creates the multi-fold phenomena equivalent to the classical trapped surfaces introduced by Penrose [40] and the quantum extremal surface discussed in section 3.2. More details on the trapped surface and gravitational collapse are also provided in Appendix A.

After crossing the horizon of the black hole, something that appears to take forever to an external observer, the notion of time loses sense for an external observer (in fact time becomes imaginary [56,57]). Inside the black hole, one see that conventionally the particle falls rapidly towards the center in its proper time [43,52]. However, as in general (e.g. collapse), there is matter at the horizon, when/if crossing it due to quantum fluctuation and Hawking's radiation, then they are initially within a symmetric sphere with mass on the external shell. So the mass seen is smaller than the externally estimated black hole mass. As a result, particles accelerate and encounter a new horizon that results from a black hole effect. Think of a black hole with lesser mass. We will call this horizon the quantum extremal surface, by analogy to the concept introduced in the previous sections for conventional models. In that region, the particles will appear to take forever to cross the surface from the point of view of an observer on the inside of the horizon of the black hole.

It matters, and it is a difference from typical black hole models: when the black hole evaporates and its horizon shrinks, it will catch with these frozen particles, while the quantum extremal surface remain essentially the same (as no particle enters it until the end of time). These effects are also different from the scenarios described above:

- particles do not disappear within the quantum extremal surface
- particles take forever to reach the quantum extremal surface. This effect compensate for the difference of the previous bullet. Frankly makes much more sense as disappearing within the quantum extremal surface, does not exactly explain where/how it disappears and it seems that some information would not be exactly recovered this way.
- Even more interesting, and probably against all odds, even within the black hole horizon, the center remains hidden to the horizon (inner). More considerations on the center region are discussed in Appendix A.



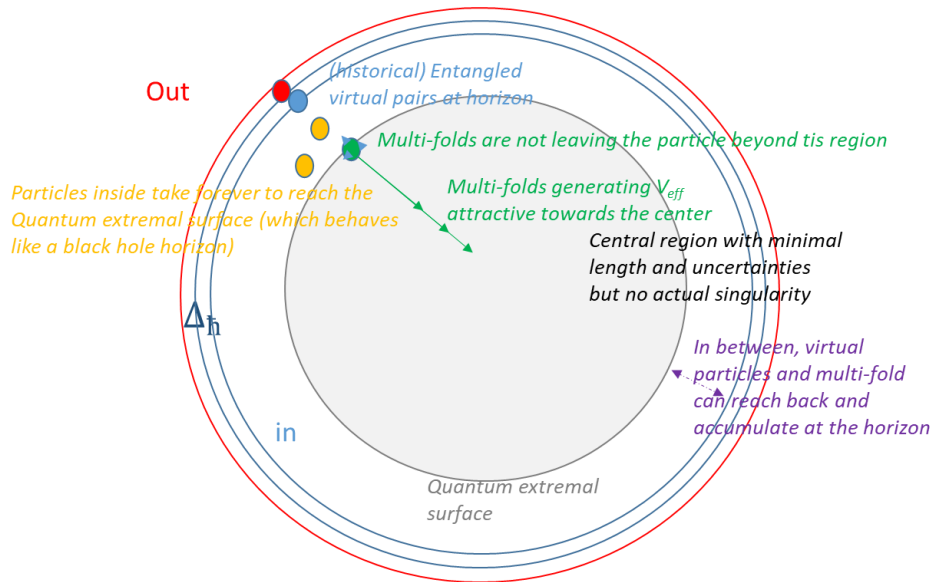


Figure 2: It illustrates the in and out virtual particles filling the black hole horizon. The ones on the inside can't escape. Near the horizon, the ones propagating externally, towards the horizon, take forever to cross it, for an external observer. Their historical impact (before a gravitational collapse or when absorbed) is reflected by the virtual particles and multi-folds on the inside of the horizon. Quantum fluctuations entangle them (the ones inside with the ones outside). This effect also increases the potential energy on the outside, resulting into virtual particles and external multi-folds responsible for the black hole gravitational effects beyond the horizon (and conversely for external gravity effects from external masses on the black hole and its inside). Particles reaching the quantum extremal surface of the black hole can only generate effective potential attractive towards the black hole center and take forever to cross that surface, resulting into the apparition of the equivalent to a quantum extremal surface. It is like the horizon of a black hole with the matter that is not outside of it and consists of anything initially within that quantum extremal surface.

Virtual particles, associated to multi-folds from inside the black hole, essentially result from when the collapse (or merger) into a black hole took place (See appendix A), and / or when later particle got absorbed, are frozen at the horizon, or at the internal quantum extremal surface. It explains why, and how, saturation of the horizon in the shell, around the horizon, of Figure 1 takes place, without over-saturation: no new contribution (as virtual particles attached to multi-folds) reaches it, from the particles trapped at the quantum extremal surface, and when a new particle is absorbed, the black hole expands, as in [42], which also explains, in our model, why some may see stringy effects on a blackhole horizon, despite no strings being involved [22].

Figure 2 also illustrates the multi-fold behavior of the quantum extremal surfaces. introduced by the papers discussed in section 3.2. Within the horizon, some of the energy is concentrated near the horizon. So crossing it imply first the possibility to emit, towards the horizon, virtual, or real, particles that are able to reach it. After a while, a region with a shorter radius behaves like a new horizon. Indeed, as particles cross it, they can no longer emit real or virtual particles with momentum components away from the black hole center: they effectively stop contributing new entanglement (or other new causal effects) with the horizon (and of course beyond). This effect occurs as soon as the black hole is formed. It is not due to a later phase transition, or occurring just at or after Page time, which is due to other effects. But, at a later stage, this effect starts dominating.

With quantum uncertainties, particles, real or virtual, inside or outside the black hole horizon, can temporarily be inside or outside the horizon. As such, virtual particles are entangled with each other, on either side of the horizon. When Hawking's radiation takes place on the horizon, one of the particle inside on the horizon is entangled with it, and receives an opposite momentum kick, to maintain a null total momentum. As the internal particle moves in, it is captured by the black hole until it reaches the quantum extremal surface, where it can no

longer generate new virtual particles reaching the horizon, or new entanglement (beyond the existing one with the evaporated particle). So the process and budget are:

- 2 virtual particles on the horizon (entangled) disappear
- One is evaporated, it is entangled with the other one: Entanglement receives one more particle. Entanglement entropy increases by one particle effect.
- That second one disappears within the inside of the blackhole and once it reaches the quantum extremal surface, it can't contribute new entanglement: the black hole mass is decreased by one the effect of one particle. Black hole entropy is reduced by one particle effect (The quantum extremal surface grows towards the horizon. As it is crossed, entangled particles with the outside no disappear in the inside, therefore reducing the entangled radiation / entropy).
- The horizon area decreases, reducing available virtual particles on the inside and outside of the horizon. It can catch up with virtual or real particles absorbed earlier due to Hawking's radiation and frozen due to the quantum extremal surface effect.
- The process repeats.
- When entanglement entropy matches the black hole (area) entropy (i.e.  $\sim$  at Page time), pairs on the horizon start to also reduce existing entanglement: one lost entangled pair (with the outside) per radiated particle with the process above: entanglement entropy and black hole entropy drop at the same rate or radiation no destroy entanglement by expelling from the black hole real particles not entangled that haven't entangled with outside of horizon (they already entangled with the previously evaporated particle) till there is nothing any more (and not lost information). While it occurs already before Page time, it is at Page time that the process starts to dominate as there are not really not enough option any more for entanglement building radiations.
- Alternatively, and/or, at some point, black holes splits could occur for (charged) black hole (as in [1,13]).
- The latter steps progressively become more dominant: it is not a strict transition as in the model reviewed by [14].

In terms of the quantum extremal surface and its content, particles that were within the surface, when it formed, may include a few particles entangled with the outside of the black hole. These are not to be considered as information that entered the black hole later. When the evaporating black hole has its horizon shrinking to the quantum extremal surface, only this entanglement remains (any entanglement with particles in the gap between the horizon of the black hole and the quantum extremal surface is now gone, or, probably in rarer cases, in entanglement with the outside of the black hole). It then continues, mostly through a gap then catching up towards the central region. There may be cases where an interest step includes more quantum extremal surfaces with a same process; but it shouldn't be in general. Meanwhile, black hole splits as described in [1,13] can also finalize faster the total release of information. It is also the basis for hints of the Ultimate Unification (UU) [1,13].

With the above, we recover the Black hole entropy (or information) Page curve. The process and curve are illustrated in Figures 3 and 4.

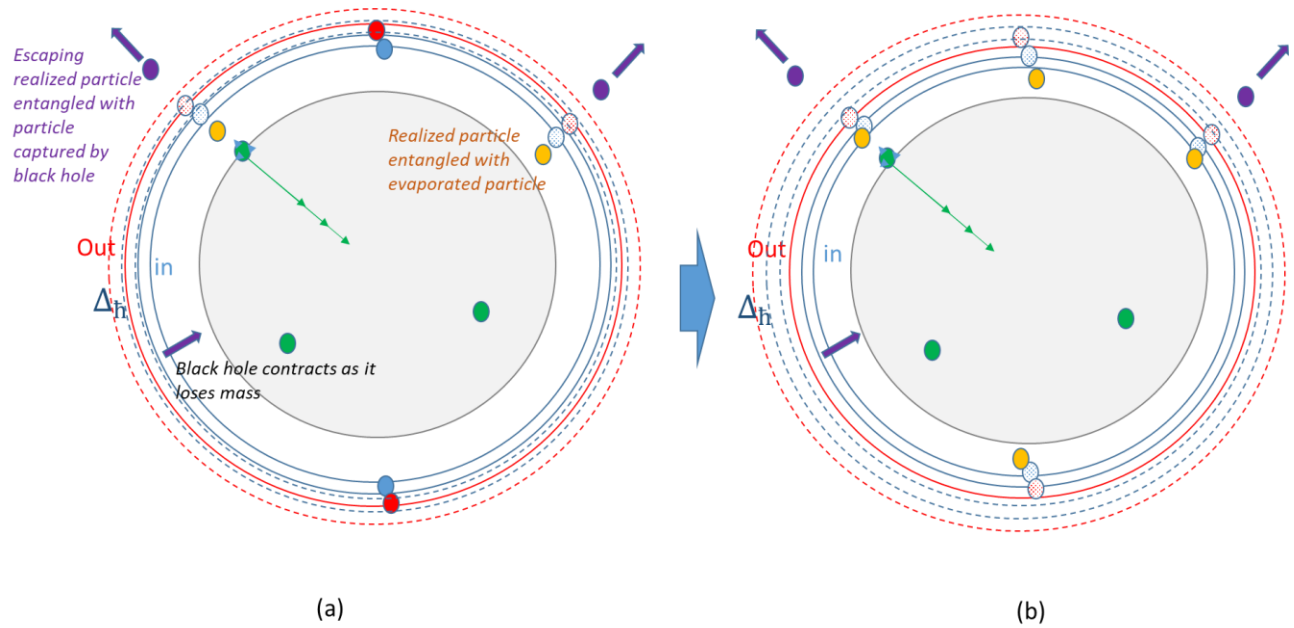


Figure 3: (a) illustrates the Hawking evaporation before reaching Page time: the majority of the radiation contributions come from entangled virtual particles in and out of the horizon. Evaporation increases the entanglement entropy. (b) illustrates what happens, all the time, but especially when most entangled pairs are already radiated. Now real particle entangled with the evaporated particles are radiated. That reduces the entanglement entropy instead of increasing it. It resulted from the contraction of the black hole horizon possibly catching up with these particles or rather them disappearing within the quantum extremal surface. Of course, in between, these two phases, a mix occur and empty microstates may be repopulated with new multi-folds (caught up by the contracting horizon, from history, or from accompanying a caught up real particles) and entangled in and out. However, as the area reduced, less microstates are available leaving the door to radiating caught up real particles frozen in place by the quantum extremal surface. Also, particles reaching within the horizon will rapidly fall to the center [43,52]. If reached by the horizon contraction before, they can evaporate as soon that a quantum fluctuation lets them outside the horizon. It is a faster process than also having to be “realized” for virtual particles, which usually rather first get entangled rather than evaporated. So, in general, these particles are entangled with previously evaporated entangled particles and take that contribution out of the entanglement entropy of the black hole, or, through the fluctuations and evaporation they lose their entanglement and do not re-entangle with a particle on the outside of the horizon or by disappearing behind the quantum extremal surface. At the horizon, they evaporate immediately when given the opportunity to escape. When the horizon reaches the quantum extremal surface, the process continues till all particles are radiated or the black hole splits.

Within the quantum extrema surface, the matter/particles/energy, that made it as the black hole and the surface formed, are not encountering a singularity (space like) because of the arguments of section 2.2, and nothing can have a momentum component towards the surface. Therefore, matter accumulates (finite amount) in the center region, frozen in place: the geodesics terminate (but do not converge to a singular point – i.e. no time-like or space-like singularity)).

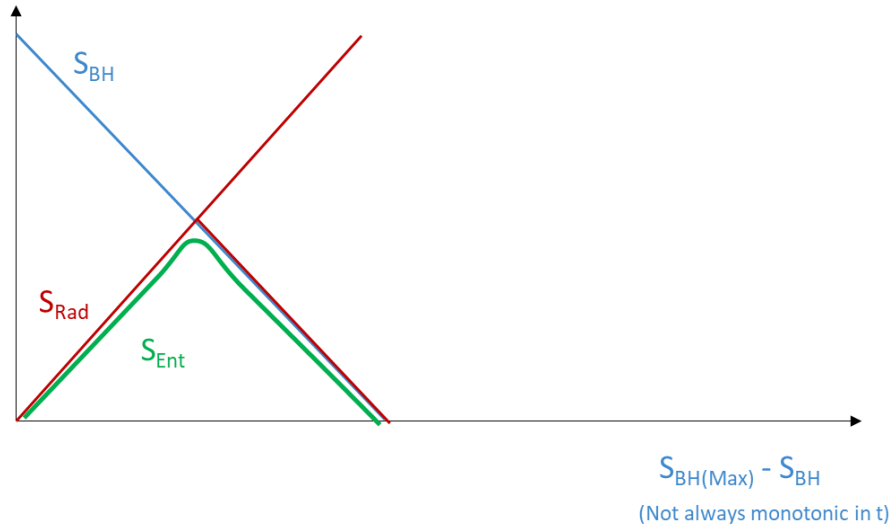


Figure 4: Page curve recovered by the process described for multi-fold black holes. Note the time scale can vary beyond the Page time (intersection of  $S_{Rad}$  and  $S_{BH}$ ) [19].  $S_{BH(max)}$  is the initial entropy of the black hole (initial condition), which here means at the moment when we start modeling radiation. This value and its evolution in time depends on the black hole history (e.g. primordial or formed by mergers, splits or star collapses).

## 5. Conclusions

In the present paper, we showed that we can recover much of the results of [14] in a multi-fold universe with some variations:

- We recover the Page curve of entropy/information evolution (with some twist on what the Page time means)
- Black holes have properties that remind of the quantum extremal surface, throughout their cycle, except in cases where all matter is already at the center (e.g. if horizon reached the quantum extremal surface).
- At some point, the radiated particles are no more entangled with the black hole, and rather destroys existing entanglement by taking away from the black hole entangled particles. These effects indeed contribute growing non entangled radiations (or non-entangled contributions to  $S_{out}$ )
- The black hole entropy will go to zero, with all information in radiated (entangled and non-entangled) recovered outside the black hole ( $S_{out}$ ) over the course of lifetime of the radiation. It resolves and explains the information paradox.
- At small sizes, and beyond Page time, one would expect that the phenomena of black hole break down, discussed in [1.13] could also occur. If, and when they happen, these splits produce new black holes and starts anew the process with each black hole, with a new corresponding  $S_{BH}$  for each of them.
- The entanglement between the black hole and external particles, or regions, is associated to multi-folds. It hints at the ER=EPR conjecture [42] and its wormholes [1,21,23,24,25].

The above ensures no information paradox in a multi-fold universe, and consistency with UU.

We believe that our derivation is very general, explains the physical phenomena that take place in multi-fold universe, and may enlighten what happens behind the models reviewed in [14], especially as it works for a 4D spacetime with positive curvature. It may well describe the real universe processes, if the real universe is a multi-fold universe, or with the point of view on dualities that we reached in [25,26].

In a multi-fold universe, no singularity (zero size concentration or time-like) exist; yet the black holes behaves as if they existed as in a continuous universe.

## Appendix A: Star Collapse and Trapped Surface in Multi-fold Universes

The collapse of a star into a black hold is sketched in Figure 5.

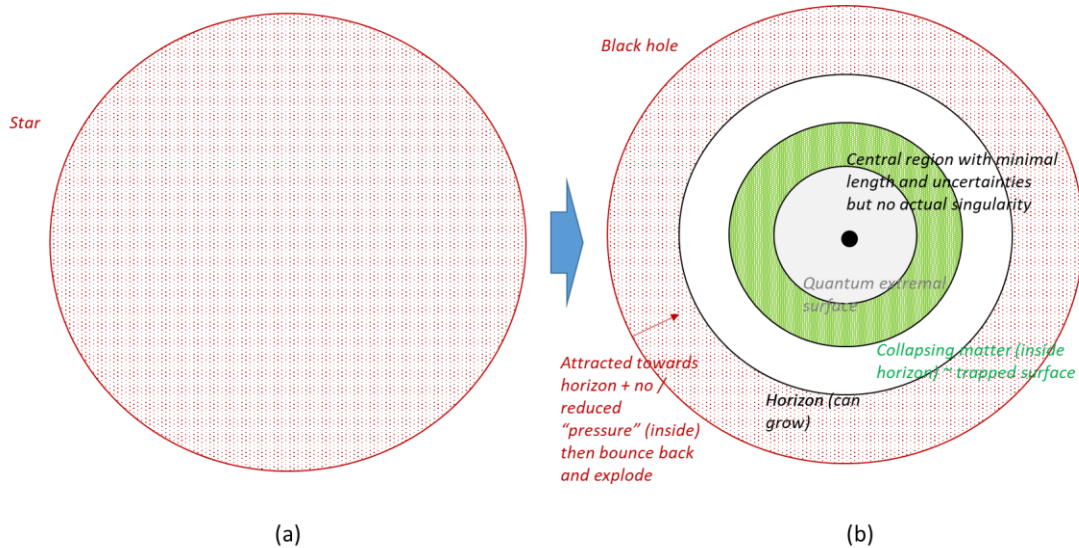


Figure 5: It sketched how a star (a) can collapse (b), and an horizon and trapped surface (matter within the horizon) appear as well as the quantum extremal surface. The greyed-out region is the quantum extremal surface.

Figure 5 is consistent with the classical derivation of gravitational collapse and trapped surface introduced by Penrose [40]. Any curve below the resulting black hole horizon is a trapped surface. It captures the multi-fold version of Penrose collapse and singularities theorems.

Within the quantum extremal surface, multi-folds and associated massless virtual particles towards the horizon are frozen in place. Even if other photons / massless particles are directed towards the center, they can't move past and outside the uncertainty / minimum length region, again freezing in place when trying to do so. However per [1], no infinite curvature occurs. All particles, other than the ones frozen in place, converge towards a minimal region from which they can't escape. They can't move out anymore, which means that after moving towards the center (or central region), they freeze in place, if they have momentum components towards the horizon, something that always happens in that region. It is the end of their story, space-like (massive) and time-like (massless). In conventional GR, one would say that the geodesics become incomplete. Here, we have also a space-like and time-like misbehaving region, even without a zero-length region: within a multi-fold universe, the absence of physical singularity does not prevent the semblance of singularity, time-like and space-like [44,48].

The same properties are true for non-stationary black holes.

Beyond gravitational collapse of stars, [1,13] described black hole splits and UU. Mergers and collisions of black holes are more complicated and today mostly modeled with numerical GR [58,60] (*Note added on 5/23/21: and interesting point like approximations as in [61,62]*), but one would expect that it would also result into apparition of quantum extremal surfaces. More on this in future work.

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