# Twin primes and other related prime generalization

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**Abstract**

I developed a very special summation function by using the floor function,
which provides a characterization of twin primes and other related prime generalization.

$$\pi(x)=\sum_{k=1}^{x-2}\left(\left\lfloor\frac{x+1}{k}\right\rfloor-\left\lfloor\frac{x}{k}\right\rfloor+\left\lfloor\frac{x-1}{k}\right\rfloor-\left\lfloor\frac{x-2}{k}\right\rfloor\right)\qquad \text{Where } x\geq 4$$

the initial idea was to use my function to prove the twin primes conjecture but then I started to look into other stuff

for the twin primes conjecture the idea was to check this:

when $\pi(x)=2$ then $x-1$ and $x+1$ are (twin) primes
when $\pi(x)$ is an odd value then those cases (of the function getting odd values) are rarer then the twin prime case

Let d be an odd value and $\pi(6a)=d$ , $\pi(6b)=2$, $\pi(6c)=d$ where $a<b<c$
meaning there is a twin prime case between two odd values cases of the function
so you just need to show that one of the odd value cases will continue to infinity and this automatically will prove the twin prime conjecture

http://myzeta.125mb.com/100000_first_values.html

http://myzeta.125mb.com/100000_odd_only.html
http://myzeta.125mb.com/100000_odd_twin.html

http://myzeta.125mb.com/1000000_odd_twin.html

other then that , this function has a very nice, weird and sometimes gives totally unexpected results

for instance , look what happens when $\pi(x)=7$

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If p is a prime (bigger then 3) such that $\sqrt{9p-2}$ is also a prime then $(1+\sqrt{p})/2$ is a continued fraction with a period 6

$$\frac{1+\sqrt{19}}{2}=2+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{8+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{3+\ddots}}}}}}\qquad \frac{1+\sqrt{59}}{2}=4+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{14+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{7+\ddots}}}}}}$$

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If p is a prime such that $\sqrt{49p+2}$ is also a prime then $(1+\sqrt{p})/2$ is a continued fraction with a period 8

$$\frac{1+\sqrt{71}}{2}=4+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{16+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{7+\ddots}}}}}}}}\qquad \frac{1+\sqrt{383}}{2}=10+\cfrac{1}{3+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{38+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{19+\ddots}}}}}}}}$$

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$\pi(x) = 2$

x = 4 , 6 , 12 , 18 , 30 , 42 , 60 , 72 , 102 , 108 , 138 , 150 , 180 , 192 , 198 , 228 , 240 , 270 , 282 , 312 , 348 , 420 , 432 , 462 , 522 , …

https://oeis.org/A014574 Side Note: [A014574] [A129297] [A167777]

n = 4 , 6 , 12 , 18 , 30 , 42 , 60 , 72 , 102 , 108 , 138 , 150 , 180 , 192 , 198 , 228 , 240 , 270 , 282 , 312 , 348 , 420 , 432 , 462 , 522 , …

Average of twin prime pairs. $\pi(n) = 2$

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$\pi(x) = 3$

x = 8 , 10 , 24 , 48 , 168 , 360 , 840 , 1368 , 1848 , 2208 , 3720 , 5040 , 7920 , 10608 , 11448 , 16128 , 17160 , 19320 , 29928 , 36480 , 44520 , 49728 , 54288 , 57120 , 66048 , 85848 , 97968 , …

Exceptions: $\pi(10)$

https://oeis.org/A062326 Side Note: [A062326] [A049002]

$p = \sqrt{x+1} =$ 2 , 3 , 5 , 7 , 13 , 19 , 29 , 37 , 43 , 47 , 61 , 71 , 89 , 103 , 107 , 127 , 131 , 139 , 173 , 191 , 211 , 223 , 233 , 239 , 257 , …

Primes p such that $p^2 - 2$ is also prime. if $p > 2$ then $\pi(p^2 - 1) = 3$

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$\pi(x) = 4$

x = 14 , 16 , 20 , 22 , 28 , 32 , 36 , 38 , 40 , 52 , 54 , 58 , 66 , 68 , 70 , 78 , 84 , 88 , 90 , 96 , 110 , 112 , 114 , 126 , 128 , 130 , 132 , 140 , 156 , 158 , 162 , 178 , 182 , 200 , 210 , 212 , 222 , 234 , 238 , 250 , 252 , 258 , 264 , 268 , 292 , 294 , 306 , 308 , 310 , 318 , 330 , 336 , 338 , 354 , 366 , 372 , 378 , 380 , 382 , 390 , 396 , 402 , 408 , 410 , 418 , 438 , 444 , 448 , 450 , 468 , …

Exceptions: $\pi(28)$ , $\pi(126)$ , … , This is for later: $\pi(6858) = \pi(19^3 - 1)$

https://oeis.org/A176686

n = 14 , 16 , 20 , 22 , 32 , 36 , 38 , 40 , 52 , 54 , 58 , 66 , 68 , 70 , 78 , 84 , 88 , 90 , 96 , 110 , 112 , 114 , 128 , 130 , 132 , 140 , 156 , 158 , 162 , 178 , 182 , 200 , 210 , 212 , 222 , 234 , 238 , 250 , 252 , 258 , 264 , 268 , 292 , 294 , 306 , 308 , 310 , 318 , 330 , 336 , 338 , 354 , 366 , 372 , 378 , 380 , 382 , 390 , 396 , 402 , 408 , 410 , 418 , 438 , 444 , 448 , 450 , 468 , …

Numbers n such that $n^2 - 1$ are products of 3 distinct primes. $\pi(n) = 4$

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$\pi(x) = 5$

x = 5 , 26 , 50 , 80 , 82 , 120 , 122 , 288 , 290 , 528 , 842 , 960 , 1370 , 1680 , 1850 , 2400 , 2808 , 2810 , 4488 , 5328 , 5330 , 6240 , 6242 , 6888 , 6890 , 9408 , 9410 , … , 14640 , … , 28560 , … , 707280 , …

Exceptions: $\pi(26)$ , $\pi(82)$

https://oeis.org/A109953 $p_1 = \sqrt{x-1} =$ 2 , 7 , 11 , 17 , 29 , 37 , 43 , 53 , 73 , 79 , 83 , 97 , …

https://oeis.org/A242260 $p_2 = \sqrt{x+1} =$ 11 , 17 , 23 , 31 , 41 , 53 , 67 , 73 , 79 , 83 , 97 , …

https://oeis.org/A154832 $p_3 = \sqrt[4]{x+1} =$ 3 , 7 , 11 , 13 , 29 , …

Primes $p_1$ such that $p_1^2 + 2$ is a semiprime. $\pi(p_1^2 + 1) = 5$

Primes $p_2$ such that $p_2^2 - 2$ is a semiprime. $\pi(p_2^2 - 1) = 5$

Primes $p_3$ such that $p_3^4 - 2$ is also prime. $\pi(p_3^4 - 1) = 5$

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$\pi(x) = 6$

x = 7 , 9 , 34 , 44 , 46 , 56 , 62 , 74 , 86 , 92 , 94 , 98 , 100 , 124 , 142 , 144 , 148 , 152 , 160 , 172 , 174 , 184 , 186 , 202 , 204 , 214 , 216 , 218 , 220 , 236 , 242 , 248 , 262 , 266 , 276 , 278 , 280 , 300 , 302 , 304 , 320 , 322 , 328 , 332 , 340 , 342 , 368 , 388 , 392 , 394 , 412 , 414 , 416 , 422 , 446 , 452 , 470 , 472 , 478 , 508 , 516 , 518 , 534 , 536 , 540 , 544 , 548 , 552 , 576 , 580 , 582 , 590 , 602 , 606 , 634 , 640 , 658 , 668 , 670 , 680 , 686

Exceptions: $\pi(44)$ , $\pi(46)$ , … alot more! (i did not pursue this matter because i am more interested when the function is getting an odd value)

https://oeis.org/A189974

n = 7 , 9 , 34 , 56 , 86 , 92 , 94 , 124 , 142 , 144 , 160 , 184 , 186 , 202 , 204 , 214 , 216 , 218 , 220 , 236 , 248 , 266 , 300 , 302 , 304 , 320 , 322 , 328 , 340 , 342 , 392 , 394 , 412 , 414 , 416 , 446 , 452 , 470 , 472 , 516 , 518 , 534 , 536 , 544 , 552 , 580 , 582 , 590 , 634 , 668 , 670 , 680 , 686

Numbers n such that d(n -1) = d(n +1) = 4, where d(k) is the number of divisors of k (A000005). $\pi(n) = 6$

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$\pi(x) = 7$

x = 15 , 170 , 362 , 530 , 624 , 728 , 962 , 3480 , 4490 , 5042 , 17162 , 18768 , 22202 , 24648 , 28562 , 37250 , 52442 , 83520 , 85850 , 113570 , 124608 , 130320 , 130322 , 139128 , 167282 , 175562 , 214370 , 279842 , 368450 , 380690 , 418608 , …

$p_1 = \sqrt{x-1} =$ 13 , 19 , 23 , 31 , 67 , 71 , 131 , 149 , 169 , 193 , 229 , 293 , 337 , 361 , 409 , 419 , 463 , 529 , 607 , 617 , …

p1 is not from [A109953] but what about the number 5 ?

( **Note: there is no $p_1 = 5$ because $x = 26$ was one of the exceptions at $\pi(x) = 5$ ) ( maybe also because of the 3 x 3 x 3 )

~~5^2+2 = 27 = 3 x 3 x 3~~ **
13^2+2 = 171 = 3 x 3 x 19
19^2+2 = 363 = 3 x 11 x 11
23^2+2 = 531 = 3 x 3 x 59
31^2+2 = 963 = 3 x 3 x 107
67^2+2 = 4491 = 3 x 3 x 499
71^2+2 = 5043 = 3 x 41 x 41
131^2+2 = 17163 = 3 x 3 x 1907
149^2+2 = 22203 = 3 x 3 x 2467
169^2+2 = 28563 = 3 x 9521
193^2+2 = 37251 = 3 x 3 x 4139
229^2+2 = 52443 = 3 x 3 x 5827
293^2+2 = 85851 = 3 x 3 x 9539
337^2+2 = 113571 = 3 x 3 x 12619
361^2+2 = 130323 = 3 x 43441
409^2+2 = 167283 = 3 x 3 x 18587
419^2+2 = 175563 = 3 x 3 x 19507
463^2+2 = 214371 = 3 x 3 x 23819
529^2+2 = 279843 = 3 x 93281
607^2+2 = 368451 = 3 x 3 x 40939
617^2+2 = 380691 = 3 x 3 x 42299

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.
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3691^2+2 = 13623483 = 3 x 2131 x 2131

there is no oeis for: Primes p3 such that (p3^2+2)/9 is a prime. But it seems to be a subset of [A146351]
there is no oeis for: Primes p4 such that (p4^2+2)/3 is a prime. But it seems to be a subset of [A122430]
there is no oeis for: Primes p5 such that (p5^2+2)/3 is a square of a prime. But it seems to be a subset of [A046184]

Primes $p_3$ such that $(p_3^2 + 2)/9$ is a prime. $\pi(p_3^2 + 1) = 7$

Primes $p_4$ such that $(p_4^2 + 2)/3$ is a prime. $\pi(p_4^2 + 1) = 7$

Primes $p_5$ such that $\sqrt{(p_5^2 + 2)/3}$ is a prime. $\pi(p_5^2 + 1) = 7$

https://oeis.org/A146351

p = 19 , 59 , 107 , 131 , 499 , 659 , 1627 , 1907 , 2251 , 2467 , 3803 , 4139 , 4283 , 5827 , 6779 , 9539 , 10067 , 11491 , 12619 , 13763 , 16987 , 18587 , 18803 , 19507 , 22003 , 23003 , 23819 , 24859 , 28643 , 30859 , 37507 , 40939 , 42083 , 42299 , 43403 , 43867 , 44563 , 52747 , 53507 , 55339

Primes p such that continued fraction of $(1+\sqrt{p})/2$ has period 6: primes in A146331.

$$\frac{1+\sqrt{19}}{2} = 2+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{8+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{3+\ddots}}}}}}$$

$$\frac{1+\sqrt{59}}{2} = 4+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{14+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{7+\ddots}}}}}}$$

https://oeis.org/A046184

1 , 9 , 121 , 1681 , 23409 , 326041 , 4541161 , 63250209 , 880961761 , 12270214441 , 170902040409 , 2380358351281 , 33154114877521 , 461777249934009 , 6431727384198601 , 89582406128846401 , 1247721958419651009 , 17378525011746267721 , 242051628206028097081

Indices of octagonal numbers which are also square.

https://oeis.org/A122430

n = 17 , 457 , 617 , 1009 , 1777 , 2081 , 3137 , 4409 , 5897 , 9521 , 11657 , 14009 , 24481 , 25577 , 29009 , 39217 , 43441 , 47881 , 49409 , 62497 , 67801 , 75209 , 81017 , 85009 , 87041 , 93281 , 97561 , 104161 , 110977 , 120401 , 132721 , 135257 , 140401 , 159161 , 182041

Primes of the form $1+2n+3n^2$ where n is https://oeis.org/A086285

x = 15 , 170 , 362 , 530 , 624 , 728 , 962 , 3480 , 4490 , 5042 , 17162 , 18768 , 22202 , 24648 , 28562 , 37250 , 52442 , 83520 , 85850 , 113570 , 124608 , 130320 , 130322 , 139128 , 167282 , 175562 , 214370 , 279842 , 368450 , 380690 , 418608 , …

$p_2 = \sqrt{x+1} =$ 4 , 25 , 27 , 59 , 137 , 157 , 289 , 353 , 361 , 373 , 647 , …

4^2-2 = 14 = 2^4-2
25^2-2 = 623 = 5^4-2
27^2-2 = 727 = 3^6-2
59^2-2 = 3479 = 7 x 7 x 71
137^2-2 = 18767 = 7 x 7 x 383
157^2-2 = 24647 = 7 x 7 x 503
289^2-2 = 83519 = 17^4-2
353^2-2 = 124607 = 7 x 7 x 2543
361^2-2 = 130319 = 19^4-2
373^2-2 = 139127 = 23 x 23 x 263
647^2-2 = 418607 = 7 x 7 x 8543
…
961^2-2 = 923519 = 31^4-2
…
1331^2-2 = 1771559 = 11^6-2
…
1369^2-2 = 1874159 = 37^4-2
…
2801^2-2 = 7845599 = 23 x 23 x 14831
…
4489^2-2 = 20151119 = 67^4-2
…
4913^2-2 = 24137567 = 17^6-2
…
15497^2-2 = 240157007 = 23 x 23 x 453983
…
50653^2-2 = 2565726407 = 37^6-2
…
103823^2-2 = 10779215329 = 47^6-2

2,5,17,19,31,37,67,71,79,89,97,103,109,113,137,163,167,

there is no oeis for: Primes p6 such that (p6^2-2)/49 is a prime. But it seems to be a subset of [A146353]

there is no oeis for: Primes p7 such that (p7^2-2)/529 is a prime. But it seems to be a subset of [A146357]

there is no oeis for: Primes p8 such that (p8^2-2) is a prime. But it seems to be a subset of [A038600]

https://oeis.org/A146353 31 , 71 , 383 , 503 , 743 , 983 , 1327 , 2543 , 4271 , 5711 , 6151 , 8543,

Primes p such that continued fraction of $(1+\sqrt{p})/2$ has period 8; primes in A146333.

$$\frac{1+\sqrt{71}}{2}=4+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{16+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{7+\ddots}}}}}}}}$$

$$\frac{1+\sqrt{383}}{2}=10+\cfrac{1}{3+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{38+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{19+\ddots}}}}}}}}$$

https://oeis.org/A146357

103 , 127 , 239 , 263 , 479 , 887 , 1567 , 2711 , 5743 , 5903 , 8311 , 8447 , 10567 , 10847 , 12391 , 14783 , 14831 , 15887 , 18191 , 22343 , 23447 , 28151 , 31391 , 32359 , 40087 , 40343 , 42703 , 53407 , 60103 , 60623 , 64231 , 75431 , 79943 , 81559 , 83663 , 93503 , 114167 , 130199 , 135119 , 141863 , 149111 , 157823 , 158231 , 168631 , 184879 , 196519 , 197711 , 202127 , 204679 , 226783 , 233743 , 248063 , 248071 , 262807 , 273727 , 277663 , 285599 , 297503 , 298031 , 304663 , 314983 , 322519 , 322871 , 346751 , 351343 , 368327 , 377263 , 386927 , 395719 , 419711 , 420743 , 442151 , 453983 , 477991 , …

Primes p such that continued fraction of $(1+\sqrt{p})/2$ has period 12 : primes in A146336.

$$(1+\sqrt{263})/2=[8;1,1,1,1,4,32,4,1,1,1,1,15]$$
$$(1+\sqrt{14831})/2=[61;2,1,1,3,1,242,1,3,1,1,2,121]$$
$$(1+\sqrt{453983})/2=[337;2,1,1,3,1,1346,1,3,1,1,2,673]$$

https://oeis.org/A154934

3 , 11 , 17 , 37 , 47 , 59 , 67 , 127 , 139 , 173 , 241 , 367 , 373 , 383 , 431 , 523 , 541 , 569 , 613 , 631 , 673 , 683 , 691 , 829 , 967 , 977 , 1019 , 1063 , 1163 , 1213 , 1249 , 1291 , 1301 , 1303 , 1327 , 1367 , 1439 , 1483 , 1487 , 1601 , 1607 , 1609 , 1733 , 1747 , 1789 , 1801 , 1823 , 1907

Primes p such that $p^6-2$ is also prime. $\pi({p_x}^6-1)=7$

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Primes $p$ such that $p^1-2$ is also prime. $\pi(p^1-1)=2$ first at: $\pi(5^1-1)=\pi(5)=2$ [Twin Prime Case!]

Primes $p$ such that $p^2-2$ is also prime. $\pi(p^2-1)=3$ first at: $\pi(3^2-1)=\pi(8)=3$ (because $p^n-1\geq 4$)

Primes $p$ such that $p^3-2$ is also prime. $\pi(p^3-1)=4$ first at: $\pi(19^3-1)=\pi(6858)=4$

Primes $p$ such that $p^4-2$ is also prime. $\pi(p^4-1)=5$ first at: $\pi(3^4-1)=\pi(80)=5$

Primes $p$ such that $p^5-2$ is also prime. $\pi(p^5-1)=6$ first at: $\pi(3^5-1)=\pi(242)=6$

Primes $p$ such that $p^6-2$ is also prime. $\pi(p^6-1)=7$ first at: $\pi(3^6-1)=\pi(728)=7$

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Generalization for $p^n-2$: Primes $p$ such that $p^n-2$ is also prime. $\pi(p^n-1)=n+1$

$$\pi(p^n-1)=\sum_{k=1}^{p^n-1-2}\left(\left\lfloor\frac{p^n-1+1}{k}\right\rfloor-\left\lfloor\frac{p^n-1}{k}\right\rfloor+\left\lfloor\frac{p^n-1-1}{k}\right\rfloor-\left\lfloor\frac{p^n-1-2}{k}\right\rfloor\right)$$ Where $p^n-1\geq 4$

Primes $p$ such that $p^n - 2$ is also prime.

$$\pi(p^n - 1) = \sum_{k=1}^{p^n - 3} \left( \left\lfloor \frac{p^n}{k} \right\rfloor - \left\lfloor \frac{p^n - 1}{k} \right\rfloor + \left\lfloor \frac{p^n - 2}{k} \right\rfloor - \left\lfloor \frac{p^n - 3}{k} \right\rfloor \right) = n + 1 \qquad \text{Where } p^n \geq 5$$

now that is interesting …

I am stopping here (for now anyway) because I don’t have a lot of free time on my hands and I am working all alone with no help …

I started 2 months ago … wanted to try and check the twin prime conjecture … I wasn't expecting it to be such a time consuming lol

I have other materials on the twin primes conjecture that maybe I will upload soon as well

(I hope someone will read this and will continue from were I stopped)

yours truly,
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