Toroidal model of leptons

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Abstract

Toroidal models for the electron are given by several authors. Essential ingredients in these models are: a radius r_1 of the torus, a radius r_2 of the tube of the torus and a toroidal factor N, defined as the angular frequency of the charge rotating around the centre of the tube divided by the angular frequency of the charge rotating around the centre of the torus. The proposed model is extended to the muon and the tau lepton, as well as to the three observed neutrinos. Furthermore, the total energy of all leptons is split into two parts: the first part depends on radius r_1 and the second part on radius r_2.

Agreement between predicted and observed magnetic dipole moments, first order anomalous correction included, is obtained for all charged leptons. In addition, for all these leptons the same ratio between the radii r_1 and r_2, depending on the fine-structure constant, is found. Moreover, the same value $N = 1$ is compatible with the observed magnetic dipole moment of all charged leptons.

Using recently proposed theoretical neutrino masses m_i ($i = 1, 2, 3$) and magnetic dipole moments $\mu(i)$, the toroidal model can also be applied to neutrinos. For neutrino 1 a value of $N = 1$ is obtained, whereas values of $N = 4.9$ and $N = 32$ are found for neutrinos 2 and 3, respectively. Finally, the toroidal moments of all leptons are calculated. The magnitude of the toroidal moment of the neutrinos increases with increasing value of N.

1. Introduction and general formalism

Toroidal models for the electron have been investigated by Hu [1], Marinov *et al.* [2], Sbitnev [3], Hestenes [4] and Consa [5–7]. Different assumptions were applied by these authors and agreement was reached between the observed and predicted value of the magnetic dipole moment, first order anomalous contribution included. Up to now, these semiclassical models have almost exclusively been applied to the electron. In these models the electric charge is assumed to be concentrated in a single point. The topology of this charge point is described by different sets of Cartesian coordinates, depending on two angular frequencies, i.e., ω and $N\omega$ and two geometric parameters r_1 and r_2.

In this work we partly follow the toroidal solenoid model of Consa [5, 6] and postulate the following basic equations for all leptons

$$\begin{aligned} x(t) &= (r_1 + r_2 \cos N\omega t)\cos\omega t, \\ y(t) &= (r_1 + r_2 \cos N\omega t)\sin\omega t, \\ z(t) &= -r_2 \sin N\omega t, \end{aligned} \tag{1.1}$$

where r_1 is the radius of the torus and r_2 is the radius of the tube (see figure 1). The factor N is a measure for the strength of the toroidal moment and may be denoted as the toroidal factor. It is noticed that the equations (1.1) contain four unknown quantities: r_1, r_2, ω and N for every lepton. Compared to the set of equations given by Marinov *et al.* [2], their radius $-\,r_2$ has been replaced by $+\,r_2$ and vice versa in our eq. (1.1) (see for motivation of this choice below eq. (4.6)). Hu [1] choose a related set of equations with $N = ½$, whereas Sbitnev [3] also considered values of $r_2 > r_1$. Hestenes [4] discussed the angular frequency of the Zitterbewegung, but he did not give an explicit set of basic equations. By postulating a general expression for radius r_1 the number of unknown parameters is reduced by one. Further analysis then leads to explicit values for the radius r_2 and the toroidal factors N for every lepton.

In this work the toroidal model is not only applied to the electron, but also to the muon and tau lepton. In addition, the model is extended to the three observed neutrinos with mass m_1, m_2 and m_3, respectively. Using the corresponding magnetic dipole moments $\mu(1)$, $\mu(2)$ and $\mu(3)$, previously deduced by Lee and Shrock [8] and Fujikawa and Shrock [9], values for all masses m_1, m_2 and m_3 have recently been calculated by Biemond [10, 11].

Analogously to the charged leptons, where the charge is thought to be concentrated in a single point, it is assumed that the mass of the neutrinos is also concentrated in a single point. For clarity reasons it is further assumed that all these leptons as a whole display no translational motion.

The following speed squared can be calculated from (1.1)

$$\dot{r}(t)^2 = \omega^2 r_1^2 \left\{ 1 + 2\frac{r_2}{r_1}\cos N\omega t + \frac{r_2^2}{r_1^2}(\cos N\omega t)^2 + N^2\frac{r_2^2}{r_1^2} \right\}. \tag{1.2}$$

It is postulated that the integrated value of $\dot{r}(t)^2$ of (1.2) over a period $T = 2\pi/\omega$ equals the speed of light squared. This choice implies that the averaged value of $\dot{r}(t)^2$ cannot be superluminal. The integrated value of $\dot{r}(t)^2$ is then given by

$$\frac{1}{T}\int_0^T \dot{r}(t)^2 dt = \omega^2 r_1^2 \left(1 + N^2\frac{r_2^2}{r_1^2} + \tfrac{1}{2}\frac{r_2^2}{r_1^2} \right) = c^2. \tag{1.3}$$

This result applies to the following values of N: $N = ½, 1, \frac{3}{2}, 2, \ldots$. The factor between parentheses in (1.3) will be defined here as

$$g \equiv \sqrt{1 + N^2\frac{r_2^2}{r_1^2} + \tfrac{1}{2}\frac{r_2^2}{r_1^2}}. \tag{1.4}$$

Consa [5, 6] assumed that $r_1 >> Nr_2$ and neglected the term $½\, r_2^2/r_1^2$ in the helical g-factor (1.4). Since the value $N = 1$ plays an important role in the sequel of this work, we use the full expression for g.

Substitution of the factor g and a speed v_1, defined by $v_1 \equiv \omega r_1$, into (1.3) results in

$$v_1 \equiv \omega r_1 = \frac{c}{g}. \tag{1.5}$$

Introduction of v_1 and an additional speed v_2, defined by $v_2 \equiv \omega r_2$, into (1.3) yields

$$v_1^2 + \left(N^2 + \tfrac{1}{2}\right)v_2^2 = c^2. \tag{1.6}$$

Multiplication of both sides of this equation with mass m of the considered lepton shows which part of the total Einstein energy $E = mc^2$ is connected to speed v_1 and speed v_2, respectively. In addition, from definitions $v_1 \equiv \omega r_1$ and $v_2 \equiv \omega r_2$ follows $v_2 = (r_2/r_1)\,v_1$. Substitution of (1.5) into $v_2 = (r_2/r_1)\,v_1$ leads to

$$v_2 = \frac{c}{g}\frac{r_2}{r_1}. \tag{1.7}$$

Furthermore, the covered distance l of the charge along the surface of the torus during a time $T = 2\pi/\omega$ can be calculated from the expression of $\dot{r}(t)^2$ in (1.2)

$$l = \int_0^T \sqrt{\dot{r}(t)^2}\, dt = \omega r_1 \int_0^T \sqrt{\left\{1 + 2\frac{r_2}{r_1}\cos N\omega t + \frac{r_2^2}{r_1^2}(\cos N\omega t)^2 + N^2\frac{r_2^2}{r_1^2}\right\}}\, dt. \tag{1.8}$$

In the sequel of this work three limiting cases will be distinguished in the evaluation of the right-hand side in (1.8). For $r_1 >> Nr_2$ the unity term in the integrand dominates, whereas for $Nr_2 >> r_1$ the term in $N^2 r_2^2/r_1^2$ is greater. In addition, some attention will be paid to the special case $r_1 = r_2$ and $N = 1$. Since the limiting case $r_1 >> Nr_2$ may be applied to all charged leptons, and also to the neutrino of mass m_1, that case will be treated first. Series expansion of the integrand in eq. (1.8) then leads to

$$l = \omega T r_1 \left(1 + \tfrac{1}{2}N^2\frac{r_2^2}{r_1^2} + \tfrac{1}{4}N^2\frac{r_2^4}{r_1^4} - \tfrac{1}{8}N^4\frac{r_2^4}{r_1^4} + \ldots\right). \tag{1.9}$$

Subsequently, the dimensionless factor between parentheses in (1.9) will be defined as

$$g' \equiv \left(1 + \tfrac{1}{2}N^2\frac{r_2^2}{r_1^2} + \tfrac{1}{4}N^2\frac{r_2^4}{r_1^4} - \tfrac{1}{8}N^4\frac{r_2^4}{r_1^4} + \ldots\right). \tag{1.10}$$

Combination of (1.9) and (1.10), followed by evaluation, results in

$$l = 2\pi r_1 g' = 2\pi R_1, \tag{1.11}$$

where R_1 is defined by $R_1 \equiv g' r_1$. When $r_1 >> Nr_2$ the factor g' is slightly greater than unity value, so that the value of R_1 will be slightly greater than r_1. When the terms in r_2 in (1.1) are omitted, the factor g' reduces to unity value as is to be expected.

According to quantum mechanics there must exist a relation between the angular momentum L and the reduced Planck constant $\hbar$. Since it is assumed that the electric charge of the charged leptons and the mass of the neutrinos cover the circular orbit of radius r_1 with speed v_1 in time T, it will be postulated in this work that the z-component of L for all leptons, *charged* and *uncharged*, can be written as

$$L_z = m v_1 r_1 = m\frac{c}{g} r_1 = \hbar, \quad \text{or} \quad r_1 = g\frac{\hbar}{mc}, \tag{1.12}$$

where (1.5) has been substituted. For $r_2 = 0$ eq. (1.1) reduces the "ring electron model", as described by, e.g., Consa [5, 6]). Radius r_1 of (1.12) then becomes equal to the reduced Compton wavelength $\hbar/(mc)$, whereas g of (1.4) and g' of (1.10) both reduce to unity value. Combination of (1.5) and (1.12) yields the following relation for the angular frequency ω

$$\omega = \frac{c}{g r_1} = \frac{mc^2}{g^2\hbar}. \tag{1.13}$$

In this context it appears useful to introduce an energy E_1, defined by $E_1 \equiv \hbar\omega$. Combination of (1.5) and (1.13) gives for E_1

$$E_1 = \frac{mc^2}{g^2} = mv_1^2. \tag{1.14}$$

If the speed v_1 is relativistic, the energy E_1 only slightly differs from the total Einstein energy $E = mc^2$. For the energy difference, defined by $E_2 \equiv E - E_1$, follows from (1.14)

$$E_2 = mc^2\left(1 - \frac{1}{g^2}\right). \tag{1.15}$$

Combination of (1.4), (1.7) and (1.15) leads to

$$E_2 = \left(N^2 + \tfrac{1}{2}\right)mv_2^2. \tag{1.16}$$

So, for an increasing value of N the energy E_2 increases and thus energy E_1 decreases. The total energy E is thus split in a part E_1 depending on speed v_1 or radius r_1 ($v_1 \equiv \omega r_1$) and a part E_2 depending on speed v_2 or radius r_2 ($v_2 \equiv \omega r_2$).

In section 2 the formalism of this section is applied to the charged leptons, the electron, the muon and the tau lepton. Subsequently, a related procedure is applied to the three neutrinos with mass m_1, m_2 and m_3 in section 3. In section 4 the toroidal moments of all leptons are calculated. Final remarks and conclusions are given in section 5.

2. Toroidal model of charged leptons

In order to test the toroidal model, the observed magnetic dipole moment $\boldsymbol{\mu}(l)$ of the electron, muon or tau lepton ($l = e, \mu, \tau$) can be used. It appears that all these leptons can be described by the same limiting case $r_1 >> Nr_2$. Starting from the standard definition of $\boldsymbol{\mu}(l)$ and using Cartesian coordinates, related to that of (1.1), Marinov *et al.* [2] calculated for $\boldsymbol{\mu}(l)$

$$\boldsymbol{\mu}(l) = \frac{1}{2c}\int \mathbf{r} \times \mathbf{j}\, dV = \frac{i}{2c}\int_0^T \mathbf{r} \times \frac{d\mathbf{r}}{dt}\, dt, \tag{2.1}$$

where $\mathbf{r} = \mathbf{r}(t)$. In their deduction they assumed that a uniform current i is generated by the moving charge e. Moreover, they replaced the current element $\mathbf{j}dV$ by $i d\mathbf{r}$ in their derivation. Compared to their calculation, the variable $\varphi = \omega t$ has been replaced by t in the right-hand side of (2.1). Note that the slightly different Cartesian coordinates used in ref. [2] and in (1.1) do not change the result of (2.1). Furthermore, it is noticed that Gaussian units are used throughout this paper.

Substitution of the Cartesian components of $\mathbf{r} = \mathbf{r}(t)$ and the time derivatives $d\mathbf{r}/dt$ from (1.1) into (2.1), then yields the following result for the z-component of $\boldsymbol{\mu}(l)$

$$\mu_z(l) = \frac{i\pi r_1^2}{c}\left(1 + \tfrac{1}{2}\frac{r_2^2}{r_1^2}\right). \tag{2.2}$$

Equation (2.2) does not depend on the parameter N, but the other components of $\boldsymbol{\mu}(l)$, $\mu_x(l)$ and $\mu_y(l)$, in general do. Values of N varying from $N = ½$ up to $N = 32$ will now be considered more in detail.

Some examples of values of $\mu_x(l)$ and $\mu_y(l)$ will be given. For the lowest value $N = ½$ a contribution $\mu_x(l) = -(4i/3c)r_1r_2$ is obtained for the time interval $t = 0$ up to T, whereas a contribution $\mu_x(l) = +(4i/3c)r_1r_2$ is found for the next time interval from $t = T$ to $t = 2T$. So, the net contribution to $\mu_x(l)$ for the time interval [0, $2T$], [$2T$, $4T$], and so on, reduces to zero value. For the y-component of $\boldsymbol{\mu}(l)$, $\mu_y(l)$, follows in case of $N = ½$

$$\mu_y(l) = \frac{i\pi r_2^2}{4c}. \tag{2.3}$$

The total scalar magnetic dipole moment of $\mu(l)$ for $N = ½$ can be found by combination of (2.2) and (2.3). One obtains

$$\mu(l) = \sqrt{\mu_y(l)^2 + \mu_z(l)^2} = \frac{i\pi r_1^2}{c}\sqrt{\tfrac{1}{16}\frac{r_2^4}{r_1^4} + \left(1+\tfrac{1}{2}\frac{r_2^2}{r_1^2}\right)^2} \approx \frac{i\pi r_1^2}{c}\left(1+\tfrac{1}{2}\frac{r_2^2}{r_1^2}\right) = \mu_z(l). \tag{2.4}$$

It appears that the term $\mu_y(l)$ is small compared to $\mu_z(l)$, so that in first order the total magnetic dipole moment $\mu(l)$ is equal to $\mu_z(l)$ of (2.2).

For the next value of N, $N = 1$, calculation of the y-component of $\boldsymbol{\mu}(l)$ gives

$$\mu_y(l) = \frac{i\pi r_1 r_2}{c}, \tag{2.5}$$

whereas the x-component, $\mu_x(l)$, is zero. Combination of (2.2) and (2.5) then yields for the total magnetic dipole moment $\mu(l)$

$$\mu(l) = \sqrt{\mu_y(l)^2 + \mu_z(l)^2} = \frac{i\pi r_1^2}{c}\sqrt{\frac{r_2^2}{r_1^2} + \left(1+\tfrac{1}{2}\frac{r_2^2}{r_1^2}\right)^2}. \tag{2.6}$$

An alternative coordinate system x, y', z' can be chosen instead of the x, y, z coordinates with an angle δ between the positive z- and z'-axis. In that case tg δ is equal to $\mu_y(l)/\mu_z(l) = (r_2/r_1)/(1 + ½\, r_2^2/r_1^2) \approx r_2/r_1$ and the only surviving component $\mu_{z'}(l)$ is given by

$$\mu_{z'}(l) = \mu(l) = \frac{i\pi r_1^2}{c}\sqrt{\frac{r_2^2}{r_1^2} + \left(1+\tfrac{1}{2}\frac{r_2^2}{r_1^2}\right)^2} \approx \frac{i\pi r_1^2}{c}\left(1+\frac{r_2^2}{r_1^2}\right), \tag{2.7}$$

where the approximation $r_1 >> Nr_2$, or $r_1 >> 1 \times r_2$ in this case, has been used.

Calculation shows that for $N = 2$ and $N = 5$ the z-component of $\boldsymbol{\mu}(l)$, $\mu_z(l)$ of (2.2), is the only surviving component. Symmetry suggests that for $N = 32$ the component $\mu_z(l)$ of (2.2) is also the only non-zero component. These results for $N = 5$ and $N = 32$ are relevant because they will be used in sections 3 and 4.

The magnetic dipole moment $\mu_z(l)$ of (2.2) for $N = 1$ will be evaluated first. Insertion of (1.5) yields

$$\mu_z(l) = \frac{i\pi r_1^2}{c}\left(1+\tfrac{1}{2}\frac{r_2^2}{r_1^2}\right) = \frac{e}{2c}\frac{2\pi r_1}{T} r_1\left(1+\tfrac{1}{2}\frac{r_2^2}{r_1^2}\right) = \frac{e}{2c} v_1 r_1\left(1+\tfrac{1}{2}\frac{r_2^2}{r_1^2}\right) = \frac{e}{2}\frac{r_1}{g}\left(1+\tfrac{1}{2}\frac{r_2^2}{r_1^2}\right). \tag{2.8}$$

Substitution of radius r_1 from (1.12) further transforms (2.8) into

$$\mu_z(l) = \frac{e\hbar}{2m_l c}\left(1+\tfrac{1}{2}\frac{r_2^2}{r_1^2}\right). \tag{2.9}$$

Note that both the components $\mu_z(l)$ and L_z are defined in the coordinate system x, y, z of (1.1). From combination of (2.9) and (1.12) then follows for the gyromagnetic ratio $\mu_z(l)/L_z$

$$\frac{\mu_z(l)}{L_z} = \frac{e}{2m_l c}\left(1+\tfrac{1}{2}\frac{r_2^2}{r_1^2}\right). \tag{2.10}$$

Going from the coordinate system x, y, z to the system x, y', z', the angular momentum L_z transforms in a similar way as $\mu_z(l)$. As a result, it can be shown that the gyromagnetic ratio $\mu_{z'}(l)/L_{z'}$ is equal to $\mu_z(l)/L_z$ of (2.10). Therefore, we continue our treatment with the expression for $\mu_z(l)$ from (2.9) for $N = 1$. Since the limiting case $r_1 >> r_2$ is adopted, the ratio r_2^2/r_1^2 is much smaller than unity value. Then, the factor $(1 + \frac{1}{2} r_2^2/r_1^2)$ in (2.9) approximately equals the factor g' of (1.10) for $N = 1$. So, $\mu_z(l)$ can then be written as

$$\mu_z(l) = \frac{i\pi r_1^2}{c}\left(1+\tfrac{1}{2}\frac{r_2^2}{r_1^2}\right) = \frac{e\hbar}{2\,m_l\,c}\left(1+\tfrac{1}{2}\frac{r_2^2}{r_1^2}\right) = \frac{e\hbar}{2\,m_l\,c}\,g_l{}'. \tag{2.11}$$

In the treatment of neutrinos below, the factor g' of (1.10) will be generalized to $g_i{}'$ in order to include neutrinos.

A comparison with the analysis of Consa [5, 6] may be useful here. Following the method of Marinov *et al.* [2], he obtains the same z-component of magnetic dipole moment $\mu_z(e)$ of the electron for all values of N

$$\mu_z(e) = \frac{i\pi r_1^2}{c}\left(1+\tfrac{1}{2}\frac{r_2^2}{r_1^2}\right) = \frac{e\hbar}{2\,m_e c}\,\frac{1}{g}\left(1+\tfrac{1}{2}\frac{r_2^2}{r_1^2}\right), \tag{2.12}$$

See for the calculation of the right-hand side of (2.12) eq. (50) of ref. [5] and section 6 in ref. [6]. Since Consa uses another postulate for radius r_1, i.e., relation $r_1 = \hbar/(m_e c)$, different from our radius r_1 of (1.12), he consequently finds a result different from our expression (2.9). Insertion of factor g from (1.4) into (2.12) yields

$$\mu_z(e) = \frac{e\hbar}{2\,m_e c}\left(1+\tfrac{1}{2}\frac{r_2^2}{r_1^2}\right)\Bigg/\sqrt{1+\frac{N^2 r_2^2}{r_1^2}+\tfrac{1}{2}\frac{r_2^2}{r_1^2}} \approx \frac{e\hbar}{2\,m_e c}\left(1+\tfrac{1}{2}\frac{r_2^2}{r_1^2}\right)\Bigg/\sqrt{1+\frac{N^2 r_2^2}{r_1^2}}\,, \tag{2.13}$$

where the term $\frac{1}{2}\, r_2^2/r_1^2$ has been neglected compared to the term $N^2 r_2^2/r_1^2$. Subsequently, Consa introduces the approximation (compare with section 6 of ref. [6])

$$\left(1+\tfrac{1}{2}\frac{r_2^2}{r_1^2}\right)\Bigg/\sqrt{1+\tfrac{1}{2}\frac{N^2 r_2^2}{r_1^2}} \approx \sqrt{1+\tfrac{1}{2}\frac{N^2 r_2^2}{r_1^2}}. \tag{2.14}$$

This relation, however, only applies for $N = 1/\sqrt{2}$, not for high values of N.

In the standard theory, the gyromagnetic ratio of a charged lepton $\mu_z(l)/S_z$ is usually written as

$$\frac{\mu_z(l)}{S_z} = \frac{\mu_z(l)}{\frac{1}{2}\hbar} = \frac{g_l\, e}{2\,m_l\, c}, \tag{2.15}$$

where $\mu_z(l)$ is the z-component of the magnetic dipole moment of $\boldsymbol{\mu}(l)$, g_l an empirical factor and $S_z = \frac{1}{2}\hbar$ is the z-component of the spin angular momentum of charged lepton l, as has been discussed by Pauli [12]. Furthermore, combination of (2.11) and (2.15) shows that $g_l = 2\, g_l{}'$. Usually, the factor g_l is written as a series expansion like

$$g_l \equiv 2\left(1+\frac{\alpha}{2\pi}+...\right)=2\left(1+0.001\ 161\ 4+...\right), \tag{2.16}$$

where $\alpha = e^2/\hbar c = 1/137.0359991$ is the fine-structure constant at low energy. The leading term in the series expansion of g_l, $g_l = +2$, has been deduced by Dirac [13]. Later on, Schwinger [14] gave the first and largest one-loop correction α/π to g_l, deduced from quantum electrodynamics (QED). A discussion of higher order corrections to g_l for the electron has recently been given by Consa [7]. In this work they will not be considered.

The deviation of the Dirac value $g_l = +2$ is usually expressed in terms of the so-called magnetic moment anomaly a_l ($l = e, \mu, \tau$) defined by

$$a_l \equiv \frac{g_l-2}{2}. \tag{2.17}$$

Current experimental values and uncertainties have recently been summarized by Zyla *et al.* [15]

$$\begin{aligned} &a_e = 0.001\ 159\ 652\ 180\ 91\ (26),\\ &a_\mu = 0.001\ 165\ 920\ 9\ (6),\\ &-0.052 \le a_\tau \le +0.013. \end{aligned} \tag{2.18}$$

In this work the value of a_l for electron, muon and tau lepton will be approximated by the same value, i.e., $a_e = a_\mu = a_\tau = \alpha/2\pi$. According to (2.17), the g_l values for all charged leptons g_e, g_μ and g_τ are then equal, although higher order terms are neglected. In that case combination of (2.9), (2.15) and (2.16) leads to the same ratio r_2/r_1 for all charged leptons

$$\frac{r_2}{r_1} \approx \sqrt{\frac{\alpha}{\pi}} = 0.04820 = \frac{1}{20.75}. \tag{2.19}$$

As has been assumed, this result meets the limiting case $r_1 >> r_2$.

Applying the same method as in the deduction of (2.9), the following result can be calculated from (2.4) for the value $N = ½$

$$\mu_z(l) = \frac{i\pi r_1^2}{c}\left(1+\tfrac{1}{2}\frac{r_2^2}{r_1^2}\right) = \frac{e\hbar}{2m_l c}\left(1+\tfrac{1}{2}\frac{r_2^2}{r_1^2}\right). \tag{2.20}$$

In this case a value $g' \approx (1 + \tfrac{1}{8} r_2^2/r_1^2)$ is obtained from (1.10), different from the factor $(1 + \tfrac{1}{2} r_2^2/r_1^2)$ in (2.20). So, the value $N = ½$ proposed by Hu [1] is not possible, when our relation for r_1 of (1.12) is valid.

Utilizing (1.7) and (2.19), calculation of the energy E_2 from (1.16) gives for $N = 1$

$$E_2 = \tfrac{3}{2} m_l v_2^2 = \tfrac{3}{2}\frac{r_2^2}{r_1^2}\frac{m_l c^2}{g^2} \approx \tfrac{3}{2}\frac{\alpha}{\pi} m_l c^2. \tag{2.21}$$

This relation predicts that for charged leptons the energy E_2 is about two orders of magnitude smaller than energy $E_1 \approx m_l c^2$ from (1.14).

For the value $N = 1$ and the limiting case $r_1 >> r_2$, figure 1 is given as an illustration for charged leptons. A positive radius r_1 has been chosen in the set of basic equations (1.1), so that a positive sign is obtained for the z-component of the magnetic dipole moment $\boldsymbol{\mu}_z(l)$ for a positive charge e (see also comment below eq. (4.6)). Furthermore, the y-component of the magnetic dipole moment $\boldsymbol{\mu}_y(l)$ and the total dipole moment $\boldsymbol{\mu}(l)$ are shown. The numbers 1, 2, 3 and 4 denote the location of charge e at time $t = 0$, $t = ¼\, T$, $t = ½\, T$ and $t = ¾\, T$, respectively. Note that the speeds vary at these different times, e.g., at position 1: $\dot{\mathbf{x}}(t) = \dot{\mathbf{x}} = 0$, $\dot{\mathbf{y}}(t) = \dot{\mathbf{y}} = \mathbf{v}_1 + \mathbf{v}_2$ and $\dot{\mathbf{z}}(\mathrm{t}) = \dot{\mathbf{z}} = -\mathbf{v}_2$. Although the positions 1, 2, 3 and 4 are lying in the same plane, the orbit of a positive charge e (drawn in red) is not completely flat.

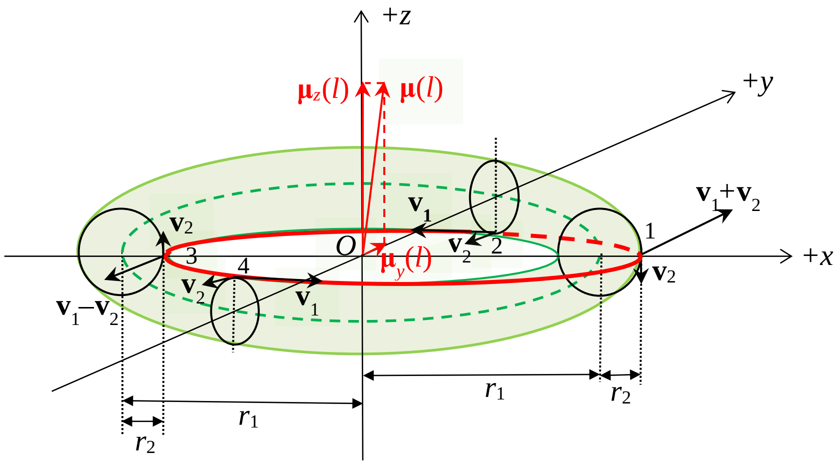

Figure 1. Toroidal model of charged leptons, according to eq. (1.1) for $N = 1$ and $r_1 >> r_2$. When O is the origin of the coordinate system, the location of a positive charge e is fixed by the Cartesian coordinates $x(t) = x$, $y(t) = y$ and $z(t) = z$. The positive charge e moves with an average speed v_1 in a ring of radius r_1 and a speed v_2 ($v_1 >> v_2$) in a circle of radius r_2. The green blocked line is a circle with radius r_1 in the x-y plane and the orbit of e is drawn in red. For clarity reasons the values of r_1, r_2, v_1 and v_2 are not drawn to scale. The vectors of the y- and z-component of the magnetic dipole moment $\boldsymbol{\mu}(l)$ of charged lepton l are also shown. See text for further comment.

Summing up, the toroidal model for electron, muon and tau lepton may be compatible with measured magnetic dipole moment values in case of $N = 1$. In that case the same value for the ratio r_2/r_1 of (2.19), depending on the fine-structure constant α, is obtained. It is noticed that in the present treatment the magnitudes of the masses of the charged leptons itself are not predicted. In section 3 the toroidal model will now be applied to the neutrinos of mass m_1, m_2 and m_3.

3. Toroidal model of neutrinos

An expression for the magnetic dipole moment of massive Dirac neutrinos has previously been deduced by Lee and Shrock [8] and Fujikawa and Shrock [9], in the context of electroweak interactions at the one-loop level. The predicted magnetic dipole moment $\mu(i)$ ($i = 1, 2, 3$) of the neutrino was found to be proportional to its mass m_i. In addition, another magnetic dipole moment $\mu(i)$ arising from gravitational origin is predicted by the so-called Wilson-Blackett formula [10, 11]. The latter formula may also be deduced from a gravitomagnetic interpretation of the Einstein equations [16–18]. By combination of equivalent magnetic dipole moments for $\mu(1)$ a value of 1.530 meV/c^2 is obtained for mass m_1 of neutrino 1. In addition, from recent observed values of $\Delta m_{21}^2 \equiv m_2^2 - m_1^2$ and $\Delta m_{32}^2 \equiv m_3^2 - m_2^2$ the values of the other two masses m_2 and m_3 can also be calculated [11]

$$m_1 = 1.530 \text{ meV}/c^2, \quad m_2 = 8.79 \text{ meV}/c^2 \quad \text{and} \quad m_3 = 50.5 \text{ meV}/c^2. \tag{3.1}$$

These results have been deduced for normal ordering.

Assuming analogous relations for the gyromagnetic ratio (2.15) of charged leptons and the ratio $\mu_z(i)/S_z$ for neutrinos, one obtains for the latter ratio [10, 11]

$$\frac{\mu_z(i)}{S_z} = \frac{\mu_z(i)}{½\hbar} = g_i \frac{G^{½}}{2c}. \tag{3.2}$$

Here $\mu_z(i)$ is the z-component of the magnetic dipole moment $\boldsymbol{\mu}(i)$, g_i an empirical factor and $S_z = ½\,\hbar$ is the z-component of the spin angular momentum of neutrino i, as has been discussed by Pauli [12].

Neutrino 1 will now be treated first. Using a gravitomagnetic approach [10], a factor $g_1 = 2$ has been deduced for neutrino 1 from the Dirac equation. Analogous to the electromagnetic case of (2.16), a first order correction term might be adopted for neutrino 1 too

$$g_1 \equiv 2\left(1 + \frac{\alpha_W}{2\pi}\right), \tag{3.3}$$

where $\alpha_W = g^2/\hbar c$ is the electroweak coupling constant at low energy. It is noticed that a possible relation between neutrino mass and α_W has recently been discussed in refs. [10, 11]. Taking an illustrative value of $\alpha_W = 1/32.0$, one obtains a value $g_1 = 2.010$ from (3.3). It is noticed that in the calculation of mass m_1 of (3.1) a value of $g_1 = 2$ has been assumed.

Furthermore, according to the gravitomagnetic approach [10, 11, 16–18], a moving electrically neutral mass m_i may be considered as a mass current i_m that generates a gravitomagnetic moment $\boldsymbol{\mu}(i)$. Analogously to the derivation of the z-component of the electromagnetic dipole moment $\boldsymbol{\mu}(l)$ for $N = 1$ in (2.11), the z-component of the gravitomagnetic dipole moment, $\boldsymbol{\mu}$(l) of neutrino 1 can be calculated to be

$$\mu_z(1) = \frac{i_m \pi r_1^2}{c}\left(1 + \tfrac{1}{2}\frac{r_2^2}{r_1^2}\right) = \frac{G^{½} m_i}{2c}\frac{2\pi r_1}{T} r_1 \left(1 + \tfrac{1}{2}\frac{r_2^2}{r_1^2}\right) = \frac{G^{½}\hbar}{2c}\left(1 + \tfrac{1}{2}\frac{r_2^2}{r_1^2}\right) = \frac{G^{½}\hbar}{2c} g_1'. \tag{3.4}$$

Note that (3.4) can be found from the analogous relations (2.8) and (2.9) by replacing the charge e by the quantity $G^{½}m_i$, where the quantities e and $G^{½}m_i$ possess the same dimension. In addition, combination of (3.2) and (3.4) shows that $g_i = 2g_i'$. Furthermore, combination of (3.2), (3.3) and (3.4) leads to the following ratio r_2/r_1 for neutrino 1

$$\frac{r_2}{r_1} \approx \sqrt{\frac{\alpha_W}{\pi}} = 0.10 = \frac{1}{10}, \tag{3.5}$$

whereas a value $g_1' = 1.005$ follows for g_1'. Since $N = 1$ and the limiting case $r_1 >> r_2$ may be applicable to neutrino 1, figure 1 may also serve as an illustration for neutrino 1. In that case the magnetic dipole moment $\boldsymbol{\mu}(l)$ of lepton l in figure 1 has to be replaced by the magnetic dipole moment $\boldsymbol{\mu}(1)$ of neutrino 1.

According to the theoretical prediction from refs. [8, 9], the magnetic dipole moments $\mu(i)$ are proportional to m_i. Assuming a value $g_1' = 1$ for neutrino 1, the magnetic dipole moments $\mu_z(i)$ (i = 2, 3) then can be written as

$$\mu_z(2) = \frac{G^{½}\hbar}{2c}\frac{m_2}{m_1} = \frac{G^{½}\hbar}{2c}5.75, \quad \mu_z(3) = \frac{G^{½}\hbar}{2c}\frac{m_3}{m_1} = \frac{G^{½}\hbar}{2c}33.0, \tag{3.6}$$

where the masses from (3.1) have been substituted.

The gravitomagnetic approach leading to (3.4) may be extended to neutrinos 2 and 3 for $N \geq 2$. Using (1.5) and (1.12), one then obtains

$$\mu_z(i) = \frac{i_m \pi r_1^2}{c}\left(1+\tfrac{1}{2}\frac{r_2^2}{r_1^2}\right) = \frac{G^{1/2} m_i}{2c}\frac{2\pi r_1}{T} r_1\left(1+\tfrac{1}{2}\frac{r_2^2}{r_1^2}\right) = \frac{G^{1/2}\hbar}{2c}\left(1+\tfrac{1}{2}\frac{r_2^2}{r_1^2}\right). \tag{3.7}$$

Combination of (3.6) and (3.7) leads to the following results for the ratios r_2/r_1 for neutrino 2 and 3, respectively

$$\left(1+\tfrac{1}{2}\frac{r_2^2}{r_1^2}\right) = 5.75, \quad \text{so that} \quad \frac{r_2^2}{r_1^2} = 9.5, \quad \frac{r_2}{r_1} = 3.1, \tag{3.8}$$

$$\left(1+\tfrac{1}{2}\frac{r_2^2}{r_1^2}\right) = 33.0, \quad \text{so that} \quad \frac{r_2^2}{r_1^2} = 64, \quad \frac{r_2}{r_1} = 8.0. \tag{3.9}$$

Note that no explicit values of N occur in eqs. (3.8) and (3.9).

Analogously to (3.4) for $N = 1$, eq. (3.7) for neutrinos 2 and 3 may also be written as

$$\mu_z(i) = \frac{G^{1/2}\hbar}{2c}\left(1+\tfrac{1}{2}\frac{r_2^2}{r_1^2}\right) = \frac{G^{1/2}\hbar}{2c} g_i'. \quad (i = 2,3) \tag{3.10}$$

Since $r_2 >> r_1$ for neutrino 2 and 3, the limiting case $Nr_2 >> r_1$ must be used. From series expansion of the integrand in (1.8) then follows for the distance l [1]

$$l = \omega T r_2 g_i' = 2\pi R_2, \tag{3.11}$$

where $R_2 \equiv g_i' r_2$ (i = 2, 3). Calculation of g_i' gives

$$g_i' = N\left(1+\tfrac{1}{2}\frac{1}{N^2}\frac{r_1^2}{r_2^2}+\tfrac{1}{4}\frac{1}{N^2}-\tfrac{3}{8}\frac{1}{N^4}\frac{r_1^2}{r_2^2}-\tfrac{1}{8}\frac{1}{N^4}\frac{r_1^4}{r_2^4}-\tfrac{3}{64}\frac{1}{N^4}+\ldots\right). \tag{3.12}$$

Since (3.10) implies that $g_i' = (1 + ½ r_2^2/r_1^2)$, an additional condition is imposed to g_i' by (3.12). From this extra condition the value of N can be calculated. Combination of (3.8) and (3.12) gives a value $N = 4.9$ for neutrino 2, whereas combination of (3.9) and (3.12) yields a value $N = 32$ for neutrino 3. It is noticed that Sbitnev [3] also discussed examples with $N > 1$ and $r_2 > r_1$.

From the obtained values of N all radii $r_1 = r_1(i)$ (i = 1, 2, 3) can now be calculated. For neutrino 1 combination of (1.4), (1.12) and (3.5) gives for $N = 1$

$$r_1(1) = g(1)\frac{\hbar}{m_1 c} \approx 1.007\frac{\hbar}{m_1 c}. \tag{3.13}$$

For neutrino 2 combination of (1.4), (1.12), (3.1) and (3.8) yields for $N = 4.9$

$$r_1(2) = g(2)\frac{\hbar}{m_2 c} = g(2)\frac{m_1}{m_2}\frac{\hbar}{m_1 c} = 2.7\frac{\hbar}{m_1 c}. \tag{3.14}$$

Likewise, combination of (1.4), (1.12), (3.1) and (3.9) gives for $N = 32$

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[1] For the special case $N = 1$ and $r_1 = r_2$, series expansion of the integral of (1.8) up to sixth order terms in $\cos\omega t$ leads to $l = 2\pi\, r_1 \times 1.514$. So, in this case g_i' is equal to 1.514.

$$r_1(3) = g(3)\frac{\hbar}{m_3 c} = g(3)\frac{m_1}{m_3}\frac{\hbar}{m_1 c} = 7.8\frac{\hbar}{m_1 c}. \tag{3.15}$$

These high values of the radii $r_1(i)$ are remarkable and require further investigation.

Analogously to the electromagnetic case, combination of (1.7), (1.16) and (3.5) leads to the following expression for the energy E_2 of neutrino 1 for $N = 1$

$$E_2 = \tfrac{3}{2} m_1 v_2^2 = \tfrac{3}{2}\frac{r_2^2}{r_1^2}\frac{m_1 c^2}{g^2} \approx \tfrac{3}{2}\frac{\alpha_W}{\pi} m_1 c^2. \tag{3.16}$$

This relation predicts that the energy E_2 for neutrino 1 is about two orders of magnitude smaller than the corresponding energy $E_1 \approx m_1 c^2$ from (1.14).

Furthermore, combination of (1.4) and (1.14) with values $r_2^2/r_1^2 = 9.5$ from (3.8) and $N = 4.9$ for neutrino 2 leads to the following energy E_1

$$E_1 = \frac{m_2 c^2}{g^2} = \frac{m_2 c^2}{234}. \tag{3.17}$$

Likewise, from the values $r_2^2/r_1^2 = 64$ from (3.9) and $N = 32$ for neutrino 3 one finds for energy E_1

$$E_1 = \frac{m_3 c^2}{g^2} = \frac{m_3 c^2}{65600}. \tag{3.18}$$

Contrary to neutrino 1, the energy E_1 is now much smaller than E_2 for both neutrinos 2 and 3. Thus, for these two neutrinos E_2 is the dominant energy.

4. Toroidal moment of charged leptons and neutrinos

Apart from a magnetic dipole moment, the coordinates of $\mathbf{r} = \mathbf{r}(t)$ of (1.1) imply that charged leptons also possess a toroidal moment $\mathbf{T}(l)$ ($l = e, \mu$, or τ). The latter quantity can be calculated from the standard definition of $\mathbf{T}(l)$ [2, 19, 20]

$$\mathbf{T}(l) = \frac{1}{10c}\int\left\{\mathbf{r}(\mathbf{j}\cdot\mathbf{r}) - 2r^2\mathbf{j}\right\} dV, \tag{4.1}$$

where $r(t)^2 = r^2$ is given by $r^2 = x^2 + y^2 + z^2$. Marinov *et al.* [2] assumed that a uniform current i is generated by the moving charge e and that the current element $\mathbf{j}dV$ may be replaced by $i d\mathbf{r}$. In that case (4.1) can be rewritten as

$$\mathbf{T}(l) = \frac{i}{10c}\int\left\{\mathbf{r}\left(\frac{d\mathbf{r}}{dt}\cdot\mathbf{r}\right) - 2r^2\left(\frac{d\mathbf{r}}{dt}\right)\right\} dt. \tag{4.2}$$

In the right-hand side of (4.2) the variable $\varphi = \omega t$ of ref. [2] has again been replaced by t in our representation. Using the relation $\mathbf{r}(\mathbf{r}\cdot d\mathbf{r}/dt) = ½\,\mathbf{r}(dr^2/dt)$, eq. (4.2) transforms into

$$\mathbf{T}(l) = \frac{i}{10c}\int\left\{\frac{1}{2}\mathbf{r}\left(\frac{dr^2}{dt}\right) - 2r^2\left(\frac{d\mathbf{r}}{dt}\right)\right\} dt. \tag{4.3}$$

The components of $d\mathbf{r}/dt$ and the quantity $r^2 = r_1^2 + 2\, r_1 r_2 \cos N\omega t + r_1^2$ can also be calculated from (1.1).

For $N = 1$ the following components of $\mathbf{T}(l)$ can be calculated from (4.3)

$$T_x = 0, \quad T_y = -\frac{i\pi r_1^2 r_2}{2c}, \quad T_z = +\frac{i\pi r_1 r_2^2}{2c}. \tag{4.4}$$

Transformation of these vector components T_x, T_y and T_z from the coordinate system x, y, z to the system x, y', z' gives

$$T_x = 0, \quad T_{y'} = -\frac{i\pi r_1^2 r_2}{2c}\sqrt{1+\frac{r_2^2}{r_1^2}}, \quad T_{z'} = 0. \tag{4.5}$$

It appears that for all values of N the following expression for component T_z is obtained

$$T_z = +N\frac{i\pi r_1 r_2^2}{2c}, \quad N = \tfrac{1}{2}, 1, \tfrac{3}{2}, 2, \ldots. \tag{4.6}$$

This formula was earlier deduced by Marinov *et al.* [2]. When a positive charge e and the set of coordinates of (1.1) are chosen, a positive sign for T_z is obtained. Note that T_z only differs from zero value, when both r_1 and r_2 differ from zero value. Further evaluation of T_z from (4.4), followed by insertion of (1.5), (1.12) and (2.19) yields

$$T_z = \frac{i\pi r_1 r_2^2}{2c} = \frac{e}{4c}\frac{2\pi r_1}{T}r_2^2 = \frac{e}{4c}\frac{c}{g}\frac{r_2^2}{r_1^2}r_1^2 \approx \frac{e}{4}\frac{\alpha}{\pi}\frac{\hbar^2}{m_l^2c^2} \approx \frac{\alpha}{2\pi}\mu_z(l)\frac{\hbar}{m_l c}. \tag{4.7}$$

Likewise, evaluation of T_y from (4.4) yields

$$T_y = -\frac{i\pi r_1^2 r_2}{2c} = -\frac{e}{4c}\frac{2\pi r_1}{T}\frac{r_2}{r_1}r_1^2 \approx -\frac{e}{4}\sqrt{\frac{\alpha}{\pi}}\frac{\hbar^2}{m_l^2c^2} \approx -\tfrac{1}{2}\sqrt{\frac{\alpha}{\pi}}\mu_z(l)\frac{\hbar}{m_l c}. \tag{4.8}$$

Attempts to connect the toroidal moment $\mathbf{T}(l)$ to the magnetic field B_{tor} of a toroidal solenoid have been made by several authors, e.g., [5, 19]. This field can be calculated from the formula

$$B_{tor} = \frac{2Ni}{c\, r_1}, \tag{4.9}$$

where N is the number of windings. Although the field B_{tor} only gives an approximate description of the magnetic field in case of $N = 1$, B_{tor} may be illustrative for the strength of this field. Using (1.5) and (1.12), calculation of the absolute value of B_{tor} for an electron from (4.9) yields

$$B_{tor} = \frac{2i}{c\, r_1} = \frac{|e|}{\pi c}\frac{2\pi r_1}{T}\frac{1}{r_1^2} = \frac{|e|}{\pi c}\frac{c}{g}\frac{1}{r_1^2} \approx \frac{|e|}{\pi}\frac{m_e^2c^2}{\hbar^2} = 1.03\times 10^{11}\,\text{G}. \tag{4.10}$$

Dubovik and Tugushev [19] suggested the following relation between the magnetic field B_{tor} of a toroidal solenoid and a toroidal moment T

$$T = B_{tor}\,\pi r_1^2 \pi r_2^2. \tag{4.11}$$

Introduction of $T = T_z$ from (4.7) into (4.11) then yields for the field $B(T_z)$, defined by

$$B(T_z) \equiv \frac{T_z}{\pi^2 r_1^2 r_2^2} = \frac{1}{4\pi}\frac{2i}{c\,r_1} = \frac{1}{4\pi} B_{tor} = 0.080\, B_{tor}. \qquad (4.12)$$

Alternatively, introduction of $T = T_y$ from (4.8) into (4.11) gives for the field $B(T_y)$, defined by

$$B(T_y) \equiv \frac{T_y}{\pi^2 r_1^2 r_2^2} = -\frac{1}{4\pi}\frac{r_1}{r_2}\frac{2i}{c\,r_1} = -\frac{1}{4\pi}\sqrt{\frac{\pi}{\alpha}}\, B_{tor} = -1.7 B_{tor}, \qquad (4.13)$$

where (2.19) has been substituted. In the last case the fields $B(T_y)$ and B_{tor} are of comparable strength.

According to the gravitomagnetic approach [10, 11, 16–18], an electrically neutral mass m_i may also be considered as a mass current i_m that generates a gravitomagnetic toroidal moment $\mathbf{T}(i)$ for neutrino i (i = 1, 2, 3). Analogously to the calculation of the electromagnetic components $\mathbf{T}(l)$ of (4.4), the following components of the toroidal moment $\mathbf{T}(1)$ of neutrino 1 can be obtained for $N = 1$

$$T_x = 0,$$

$$T_y = -\frac{i_m \pi r_1^2 r_2}{2c} = -\frac{G^{1/2} m_1}{4c}\frac{2\pi r_1}{T}\frac{r_2}{r_1} r_1^2 \approx -\frac{G^{1/2}\hbar}{4c}\sqrt{\frac{\alpha_W}{\pi}}\frac{\hbar}{m_1 c} = -\sqrt{\frac{\alpha_W}{4\pi}}\,\mu_z(1)\frac{\hbar}{m_1 c}, \qquad (4.14)$$

$$T_z = +\frac{i_m \pi r_1 r_2^2}{2c} = +\frac{G^{1/2} m_1}{4c}\frac{2\pi r_1}{T} r_2^2 \approx +\frac{G^{1/2}\hbar}{4c}\frac{\alpha_W}{\pi}\frac{\hbar}{m_1 c} = +\frac{\alpha_W}{2\pi}\,\mu_z(1)\frac{\hbar}{m_1 c},$$

where (3.5) has been inserted.

In addition, the gravitomagnetic approach may also be applied to the neutrinos 2 and 3. One then obtains for the T_z-component of $\mathbf{T}(i)$ (i = 2, 3) for $N = N$

$$T_z = +N\frac{i_m \pi r_1 r_2^2}{2c} = N\frac{G^{1/2} m_i}{4c}\frac{2\pi r_1}{T} r_2^2 = N\frac{G^{1/2} m_i}{4c}\frac{c}{g(i)}\frac{r_2^2}{r_1^2} r_1^2 = N\frac{G^{1/2}\hbar}{2c}\left(g_i' - 1\right) g(i)\frac{\hbar}{m_i c}. \qquad (4.15)$$

The last term on the right-hand side of (4.15) has been evaluated by combination of (3.8), (3.9) and (3.10). Calculation from (4.15) shows that the T_z-component of neutrino 2 for N = 4.9 is about two orders of magnitude smaller than the T_z-value of neutrino 3 for N = 32. In addition, it follows from (4.14) that the T_z-value of neutrino 1 for N = 1 is roughly four orders of magnitude smaller than the T_z-value of neutrino 2. Furthermore, calculation also shows that the T_x- and T_y-component are zero for the integer value N = 5. For symmetry reasons these components are also probably zero for N = 32.

As an illustration, the orbit of neutrino 2 with ratio r_2/r_1 = 3.1 from (3.8) and calculated value N = 4.9 is considered more in detail. For clarity reasons the toroidal model in case of the integer value N = 5 and ratio r_2/r_1 = 3 is displayed in figure 2. Twenty locations of one complete, closed orbit of neutrino 2 are distinguished for N = 5, namely the location at times $t = ½\pi n/(N\omega) = ½\pi n/(2\pi N/T) = (n/4N)\,T = (n/20)\,T$, where n possesses the values 0, 1, 2, up to 20 and the locations for n = 0 and n = 20 coincide. Moreover, the closed orbit is divided in five non-closed orbits, all starting and ending at height z = 0. The first one starts at n = 0 and ends at n = 4; the second one starts at n = 4 and ends at n = 8, and so on. The locations n = 1, 5, 9, 13, 17 at height $z = -r_2 = -3r_1$ form a regular pentagon circumscribed by a circle with radius r_1. Likewise, the locations n = 3, 7, 11, 15, 19 at height $z = +r_2 = +3r_1$ also form a regular pentagon circumscribed by a circle with radius

r_1. Two regular pentagons are lying in the same plane at height $z = 0$. The first one is formed by the locations $n = 0, 4, 8, 12, 16, 20$ and is circumscribed by a circle with radius $r_1 + r_2 = 4r_1$. The second one is given by the locations $n = 2, 6, 10, 14, 18$ and is circumscribed by a circle with radius $2r_1$. So, the model predicts a total height of about $2r_2 = 6r_1$ and a maximum equatorial diameter of $8r_1$ for neutrino 2.

Furthermore, the z-component of the magnetic dipole moment, $\boldsymbol{\mu}_z(2)$ in figure 2, is the only non-zero component of $\boldsymbol{\mu}(2)$, as has been discussed in section 2. Moreover, calculation from (4.3) shows that the x- and y-component of toroidal moment $\mathbf{T}_x(2)$ and $\mathbf{T}_y(2)$ are zero for $N = 5$, so that $\mathbf{T}_z(2)$ given in (4.15) is the only surviving component of the toroidal moment $\mathbf{T}(2)$. The latter component, $\mathbf{T}_z(2)$, is also shown in figure 2.

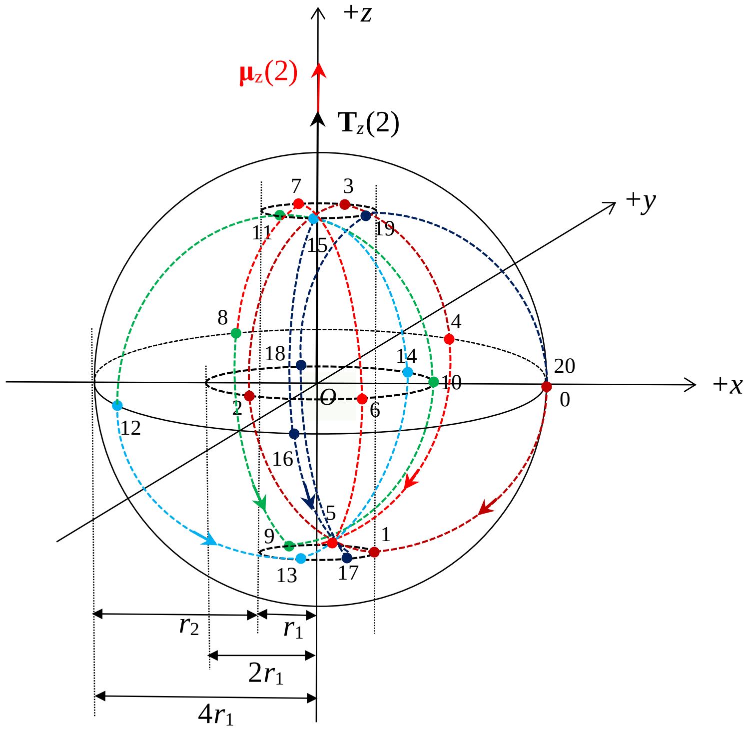

Figure 2. Toroidal model of neutrino 2 for $N = 5$ and $r_2 = 3r_1$. The coordinates x, y and z are the Cartesian coordinates from (1.1). Twenty locations of one complete closed orbit with numbers $n = 0, 1, 2, \ldots, 20$ are given. The closed orbit is divided in five non-closed orbits with different colours, all starting and ending at height $z = 0$. The coloured arrows denote the direction of the speed of the moving mass m_2. The only non-zero z-component of the magnetic dipole moment, $\boldsymbol{\mu}_z(2)$, and the only non-zero z-component of the toroidal moment, $\mathbf{T}_z(2)$, are also shown. See text for additional comment.

5. Final remarks and conclusions

The set of Cartesian coordinates of eq. (1.1) is the basic postulate in the present investigation of toroidal models. Related sets of such equations have previously been given by Hu [1], Marinov *et al.* [2], Sbitnev [3] and Consa [5–7]. The three Cartesian coordinates contain a total of four unknown quantities: two geometric parameters, r_1 and r_2, where r_1 is the radius of the torus and r_2 the radius of the tube, an angular frequency ω and a toroidal factor N. In this work we follow the toroidal solenoid model for an electron, as described by Consa [5, 6], but this model is also applied to the muon and tau lepton. Moreover, the model is extended to the three observed neutrinos with mass m_1, m_2 and m_3, respectively. Such an attempt is possible, because *theoretical* values for the magnetic dipole moments $\mu(1)$, $\mu(2)$ and $\mu(3)$ [8, 9] and the corresponding masses [10, 11] are available.

A second general postulate in the present toroidal model is the choice of the radius r_1 in eq. (1.12). The same formal expression of this radius is applicated to the electron, the muon and the tau lepton and the three observed neutrinos. This postulate deviates from the choice used by Consa [5, 6]. The two different expressions for the radius r_1 are discussed in sections 1 and 2. In both cases the choice of r_1 reduces the number of unknown parameters by one.

Comparison of the observed magnetic dipole moments of all charged leptons, first order anomalous contributions included, with the predicted ones shows agreement for $N = 1$ and $r_2^2/r_1^2 \approx \alpha/\pi$ (α is the fine-structure constant). As a result, explicit values for r_1, r_2, ω and N are obtained for all charged leptons. For neutrinos only *theoretical* values for the magnetic moments and masses are available (see refs. [8, 9] and [10, 11], respectively), so that values for r_1, r_2, ω and N are more uncertain. For the neutrino of mass m_1 a value of $N = 1$ may also be adequate, but theoretical values for the magnetic moments $\mu(2)$ and $\mu(3)$ of neutrinos of mass m_2 and m_3 suggest higher values for N.

Furthermore, the total Einstein energy $E = mc^2$ is split in two parts E_1 and E_2, where the first part E_1 is defined by $E_1 \equiv \hbar\omega$ (see section 1). For $N = 1$ the ratio r_2/r_1 of the charged leptons and neutrino 1 is small, so that the energy E_1 dominates. For neutrinos 2 and 3 the expected values of N and ratio r_2/r_1 are both higher than unity value, so that energy E_2 dominates. A summary of the obtained results is given in table 1.

Table 1. Theoretical results for charged leptons ($l = e, \mu, \tau$) and neutrinos (i = 1, 2, 3). See text for further comment.

Charged leptons l	Neutrinos i		
$N = 1$ for $l = e, \mu, \tau$	$N = 1$ for $i = 1$	$N = 4.9$ for $i = 2$	$N = 32$ for $i = 3$
$g = \sqrt{1+\frac{3}{2}\frac{r_2^2}{r_1^2}}$ see eq. (1.4)	$g(1) = \sqrt{1+\frac{3}{2}\frac{r_2^2}{r_1^2}}$ see eq. (1.4)	$g(2) = 15.3$ see eqs. (1.4) and (3.8)	$g(3) = 256$ see eqs. (1.4) and (3.9)
$g_l' \equiv 1+\frac{1}{2}\frac{r_2^2}{r_1^2}$ see eq. (2.11)	$g_1' \equiv 1+\frac{1}{2}\frac{r_2^2}{r_1^2}$ see eq. (3.4)	$g_2' = 5.75$ see eqs. (3.8) and (3.10)	$g_3' = 33$ see eqs. (3.9) and (3.10)
$\mu_z(l) = \frac{e\hbar}{2m_l c}\left(1+\frac{1}{2}\frac{r_2^2}{r_1^2}\right)$ see eq. (2.9)	$\mu_z(1) = \frac{G^{1/2}\hbar}{2c}\left(1+\frac{1}{2}\frac{r_2^2}{r_1^2}\right)$ $\mu_z(1) = \frac{G^{1/2}\hbar}{2c} g_1(1) \approx \frac{G^{1/2}\hbar}{2c}$ see eq. (3.4)	$\mu_z(2) = \frac{G^{1/2}\hbar}{2c} g_2'$ $\mu_z(2) = 5.75\frac{G^{1/2}\hbar}{2c}$ see eqs. (3.6) and (3.10)	$\mu_z(3) = \frac{G^{1/2}\hbar}{2c} g_3'$ $\mu_z(3) = 33.0\frac{G^{1/2}\hbar}{2c}$ see eqs. (3.6) and (3.10)
$r_1 = g\frac{\hbar}{m_l c} \approx \frac{\hbar}{m_l c}$ see eq. (1.12)	$r_1(1) = g(1)\frac{\hbar}{m_1 c} \approx \frac{\hbar}{m_1 c}$ eq. (1.12)	$r_1(2) = g(2)\frac{\hbar}{m_2 c} \approx 2.7\frac{\hbar}{m_1 c}$ eq. (3.14)	$r_1(3) = g(3)\frac{\hbar}{m_3 c} \approx 7.8\frac{\hbar}{m_1 c}$ eq. (3.15)
$\frac{r_2}{r_1} \approx \sqrt{\frac{\alpha}{\pi}}$ see eq. (2.19)	$\frac{r_2}{r_1} \approx \sqrt{\frac{\alpha_W}{\pi}}$ see eq. (3.5)	$\frac{r_2}{r_1} = \sqrt{2(g_2'-1)} = 3.1$ see eqs. (3.8) and (3.10)	$\frac{r_2}{r_1} \approx \sqrt{2(g_3'-1)} = 8.0$ see eqs. (3.9) and (3.10)
$E_1 = \frac{m_l c^2}{g^2} \approx m_l c^2$ see eq. (1.14)	$E_1 = \frac{m_1 c^2}{g(1)^2} \approx m_1 c^2$ see eq. (1.14)	$E_1 = \frac{m_2 c^2}{g(2)^2} \approx \frac{m_2 c^2}{234}$ see eq. (3.17)	$E_1 = \frac{m_3 c^2}{g(3)^2} \approx \frac{m_3 c^2}{65600}$ see eq. (3.18)
$E_2 = \frac{3}{2}\frac{\alpha}{\pi} m_l c^2$ see eq. (2.21)	$E_2 = \frac{3}{2}\frac{\alpha_W}{\pi} m_1 c^2$ see eq. (3.16)	$E_2 = m_2 c^2\left(1-E_1\right) \approx m_2 c^2$ see eq. (3.17)	$E_2 = m_3 c^2\left(1-E_1\right) \approx m_3 c^2$ see eq. (3.18)
$T_z = \frac{i\pi r_1 r_2^2}{2c} \approx$ $\frac{\alpha}{2\pi}\mu(l)\frac{\hbar}{m_l c}$ see eq. (4.7)	$T_z = \frac{i_m \pi r_1 r_2^2}{2c} \approx$ $\frac{\alpha_W}{2\pi}\mu(1)\frac{\hbar}{m_1 c}$ see eq. (4.14)	$T_z = N\frac{i_m \pi r_1 r_2^2}{2c} \approx$ $N\frac{G^{1/2}\hbar}{2c}(g_2'-1)\, g(2)\frac{\hbar}{m_2 c}$ see eq. (4.15)	$T_z = N\frac{i_m \pi r_1 r_2^2}{2c} \approx$ $N\frac{G^{1/2}\hbar}{2c}(g_3'-1)\, g(3)\frac{\hbar}{m_3 c}$ see eq. (4.15)

Finally, the toroidal moments of all charged leptons $\mathbf{T}(l)$ ($l = e$, μ, or τ) and neutrinos $\mathbf{T}(i)$ (i = 1, 2, 3) are calculated. For electron, muon, tauon and neutrino 1 the predicted T_y-component dominates over the T_z-component, whereas the T_x-component is found to be zero for N =1. For neutrino 2 both the T_x- and T_y-component are zero for the integer value N = 5. The values of the T_z-components of the toroidal moments of neutrinos 2 and 3 with $N \approx 5$ and N = 32, respectively, are much greater than that of neutrino 1 with value N = 1.

Summing up, the proposed toroidal model is compatible with known evidence, but the validity of the proposed postulates is uncertain. Agreement is obtained with observed results for the magnetic dipole moments of the charged leptons, first order anomalous contribution included. At present, the obtained results for neutrinos are compatible with theoretical predictions from refs. [8–11]. However, additional evidence is necessary.

Acknowledgement

I thank my son Pieter for help in programming and for correction of the English text.

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