On some equations concerning Weak Gravity Conjecture and Swampland. Mathematical connections with some Ramanujan formulas, Riemann zeta function and some sectors of String Theory

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Abstract

In this research thesis, we analyze some equations concerning Weak Gravity Conjecture and Swampland. Furthermore, we obtain various mathematical connections with some Ramanujan formulas, Riemann zeta function and several sectors of String Theory

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Reply to – The number 1729 is 'dull': No, it is a very interesting number; it is the smallest number expressible as a sum of two cubes in two different ways, the two ways being $1^3 + 12^3$ and $9^3 + 10^3$. Srinivasa Ramanujan

More science quotes at Today in Science History todayinsci.com

Vesuvius landscape with gorse – Naples



https://www.pinterest.it/pin/95068242114589901/

We want to highlight that the development of the various equations was carried out according an our possible logical and original interpretation

From:

On the Zeros of the Davenport Heilbronn Function

S. A. Gritsenko - Received May 15, 2016 - ISSN 0081-5438, Proceedings of the Steklov Institute of Mathematics, 2017, Vol. 296, pp. 65–87.

We have:

Let

$$\varkappa = \frac{\sqrt{10 - 2\sqrt{5}} - 2}{\sqrt{5} - 1}$$

and χ_1 be a character modulo 5 such that $\chi_1(2) = i$.

The Davenport-Heilbronn function f(s) is defined by the equality

$$f(s) = \frac{1 - i\varkappa}{2}L(s,\chi_1) + \frac{1 + i\varkappa}{2}L(s,\overline{\chi}_1), \quad \text{where} \quad L(s,\chi) = \sum_{n=1}^{\infty} \frac{\chi(n)}{n^s}.$$

The function f(s) satisfies the Riemann-type functional equation

$$g(s) = g(1-s),$$
 where $g(s) = \left(\frac{\pi}{5}\right)^{-s/2} \Gamma\left(\frac{s+1}{2}\right) f(s),$

but there is no Euler product for it.

 $(\sqrt{10 - 2\sqrt{5}} - 2)/(\sqrt{5} - 1) = \kappa$

Input:

$$\frac{\sqrt{10 - 2\sqrt{5}} - 2}{\sqrt{5} - 1}$$

Decimal approximation:

0.2840790438404122960282918323931261690910880884457375827591626661

 $0.28407904384.... = \kappa$

Alternate forms:

$$\frac{1}{4} \left(\sqrt{10 - 2\sqrt{5}} - 2\sqrt{5} + \sqrt{5(10 - 2\sqrt{5})} - 2 \right)$$

$$\frac{1}{4}\left(1+\sqrt{5}\right)\left(\sqrt{10-2\sqrt{5}}-2\right)$$

$$\frac{1}{2}\left(-1-\sqrt{5}+\sqrt{2\left(5+\sqrt{5}\right)}\right)$$

Minimal polynomial:

$$x^4 + 2x^3 - 6x^2 - 2x + 1$$

Expanded forms:

$$\frac{\sqrt{10 - 2\sqrt{5}}}{\sqrt{5} - 1} - \frac{2}{\sqrt{5} - 1}$$

$$\frac{1}{4} \sqrt{10 - 2 \sqrt{5}} + \frac{1}{4} \sqrt{5 \left(10 - 2 \sqrt{5}\right)} + \frac{1}{2} \left(-1 - \sqrt{5}\right)$$

For $((((\sqrt{10-2\sqrt{5}})-2))/((\sqrt{5-1}))) = 8\pi G; G = 0.011303146014$

Indeed:

 $((((\sqrt{10-2\sqrt{5}})-2))/((\sqrt{5-1}))))/(8\pi)$

Input:

$$\frac{\frac{\sqrt{10-2\sqrt{5}}}{\sqrt{5}-2}}{8\pi}$$

Result:

$$\frac{\sqrt{10 - 2\sqrt{5}} - 2}{8(\sqrt{5} - 1)\pi}$$

Decimal approximation:

0.0113031460140052147973750129442035744685760313920017808594909667

0.01130314.... = g (gravitational coupling constant)

Property:

 $\frac{-2+\sqrt{10-2\sqrt{5}}}{8\left(-1+\sqrt{5}\right)\pi}$ is a transcendental number

Alternate forms:

$$\frac{\sqrt{10 - 2\sqrt{5}} - 2\sqrt{5} + \sqrt{5(10 - 2\sqrt{5})} - 2}{32\pi}$$

$$-\frac{1+\sqrt{5}-\sqrt{2\left(5+\sqrt{5}\right)}}{16\,\pi}$$

$$\frac{-1-\sqrt{5}+\sqrt{2\left(5+\sqrt{5}\right)}}{16\pi}$$

Expanded forms:

$$-\frac{1}{16\pi} - \frac{\sqrt{5}}{16\pi} + \frac{\sqrt{10 - 2\sqrt{5}}}{32\pi} + \frac{\sqrt{5(10 - 2\sqrt{5})}}{32\pi}$$

$$\frac{\sqrt{10 - 2\sqrt{5}}}{8(\sqrt{5} - 1)\pi} - \frac{1}{4(\sqrt{5} - 1)\pi}$$

Series representations:

$$\frac{\sqrt{10-2\sqrt{5}}-2}{(8\pi)(\sqrt{5}-1)} = \frac{-2+\sqrt{9-2\sqrt{5}}}{8\pi\left(-1+\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)}(9-2\sqrt{5})^{-k}}{8\pi\left(-1+\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)}$$

$$\frac{\sqrt{10-2\sqrt{5}}-2}{(8\pi)\left(\sqrt{5}-1\right)} = \frac{-2+\sqrt{9-2\sqrt{5}}}{8\pi\left(-1+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-1\right)^{k}\left(-\frac{1}{2}\right)_{k}\left(9-2\sqrt{5}\right)^{-k}}{k!}\right)}$$

$$\frac{\sqrt{10 - 2\sqrt{5}} - 2}{(8\pi)(\sqrt{5} - 1)} = \frac{-2 + \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (10 - 2\sqrt{5} - z_0)^k z_0^{-k}}{k!}}{8\pi \left(-1 + \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5 - z_0)^k z_0^{-k}}{k!}\right)}$$
for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \le 0$))

We note that:

$(((\sqrt{10-2\sqrt{5}})-2))((\sqrt{5-1}))*((2 i (sqrt(5) - 1) t + sqrt(5) - 1)/(2 (sqrt(2 (5 - sqrt(5))) - 2)))$

Input:

$$\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1} \times \frac{2i(\sqrt{5}-1)t+\sqrt{5}-1}{2(\sqrt{2(5-\sqrt{5})}-2)}$$

i is the imaginary unit

Exact result:

$$\frac{\left(\sqrt{10-2\sqrt{5}} - 2\right)\left(2i\left(\sqrt{5} - 1\right)t + \sqrt{5} - 1\right)}{2\left(\sqrt{5} - 1\right)\left(\sqrt{2(5-\sqrt{5})} - 2\right)}$$

Plot:



Alternate form assuming t>0:

$$\frac{i\sqrt{10-2\sqrt{5}}t}{\sqrt{2(5-\sqrt{5})}-2} - \frac{2it}{\sqrt{2(5-\sqrt{5})}} + \frac{\sqrt{2(5-\sqrt{5})}-2}{\sqrt{5(10-2\sqrt{5})}} - \frac{\sqrt{10-2\sqrt{5}}}{2(\sqrt{5}-1)\left(\sqrt{2(5-\sqrt{5})}-2\right)} - \frac{\sqrt{10-2\sqrt{5}}}{2(\sqrt{5}-1)\left(\sqrt{2(5-\sqrt{5})}-2\right)} - \frac{\sqrt{5}}{2(\sqrt{5}-1)\left(\sqrt{2(5-\sqrt{5})}-2\right)} - \frac{\sqrt{5}}{2(\sqrt{5}-1)\left(\sqrt{5}-\sqrt{5}\right)} - \frac{\sqrt{5}}{2(\sqrt{5}-1)\left(\sqrt{5}-\sqrt{5}\right)} - \frac{\sqrt{5}}{2(\sqrt{5}-1)\left(\sqrt{5}-\sqrt{5}\right)} - \frac{\sqrt{5}}{2(\sqrt{5}-\sqrt{5})} - \frac{\sqrt{5}}{2(\sqrt{5}-\sqrt{5}$$

Alternate forms:

$$\frac{1}{8}\left(1+\sqrt{5}\right)\left(2\,i\,\sqrt{2\left(3-\sqrt{5}\right)}\,t+\sqrt{5}\,-1\right)$$

 $\frac{1}{2}\,(1+2\,i\,t)$

$\frac{1}{2} + it$

1/2+*it* = real part of every nontrivial zero of the Riemann zeta function

Derivative:

$$\frac{d}{dt} \left(\frac{\left(\sqrt{10 - 2\sqrt{5}} - 2\right) \left(2i\left(\sqrt{5} - 1\right)t + \sqrt{5} - 1\right)}{\left(\sqrt{5} - 1\right) \left(2\left(\sqrt{2(5 - \sqrt{5})} - 2\right)\right)} \right) = i$$

Indefinite integral:

$$\int \frac{\left(\sqrt{10 - 2\sqrt{5}} - 2\right)\left(2i\left(\sqrt{5} - 1\right)t + \sqrt{5} - 1\right)}{\left(\sqrt{5} - 1\right)\left(2\left(\sqrt{2(5 - \sqrt{5})} - 2\right)\right)} dt = \frac{t}{2} + \frac{it^2}{2} + \text{constant}$$

And again:

 $(((\sqrt{(10-2\sqrt{5})}-2))((2x)))*((2 i (sqrt(5) - 1) t + sqrt(5) - 1)/(2 (sqrt(2 (5 - sqrt(5))) - 2)))) = (1/2+it)$

Input:

$$\frac{\sqrt{10-2\sqrt{5}}-2}{2x} \times \frac{2i(\sqrt{5}-1)t+\sqrt{5}-1}{2\left(\sqrt{2(5-\sqrt{5})}-2\right)} = \frac{1}{2}+it$$

i is the imaginary unit

Exact result:

$$\frac{\left(\sqrt{10-2\sqrt{5}}-2\right)\left(2i\left(\sqrt{5}-1\right)t+\sqrt{5}-1\right)}{4\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)x} = \frac{1}{2}+it$$

Alternate form assuming t and x are real:

$$\frac{\sqrt{5}-1}{x} = 2$$

Alternate form:

$$\frac{\left(\sqrt{5} - 1\right)\left(1 + 2\,i\,t\right)}{4\,x} = \frac{1}{2} + i\,t$$

Alternate form assuming t and x are positive: $2x + 1 = \sqrt{5}$

Expanded forms:

$$\frac{i\sqrt{5(10-2\sqrt{5})}t}{2(\sqrt{2(5-\sqrt{5})}-2)x} - \frac{i\sqrt{10-2\sqrt{5}}t}{2(\sqrt{2(5-\sqrt{5})}-2)x} - \frac{i\sqrt{5}t}{(\sqrt{2(5-\sqrt{5})}-2)x} + \frac{i\sqrt{5}t}{2(\sqrt{2(5-\sqrt{5})}-2)x} + \frac{\sqrt{5(10-2\sqrt{5})}}{4(\sqrt{2(5-\sqrt{5})}-2)x} - \frac{\sqrt{10-2\sqrt{5}}}{4(\sqrt{2(5-\sqrt{5})}-2)x} - \frac{\sqrt{5}t}{4(\sqrt{2(5-\sqrt{5})}-2)x} - \frac{\sqrt{5}t}{4(\sqrt{2(5-\sqrt{5})}-2)x} - \frac{\sqrt{5}t}{2(\sqrt{2(5-\sqrt{5})}-2)x} + \frac{1}{2(\sqrt{2(5-\sqrt{5})}-2)x} = \frac{1}{2} + it$$

$$\frac{i\sqrt{5}t}{2x} - \frac{it}{2x} + \frac{\sqrt{5}}{4x} - \frac{1}{4x} = \frac{1}{2} + it$$

Solutions:

$$t = \frac{i}{2}$$
, $x \neq 0$

$$x = \frac{\sqrt{5}}{2} - \frac{1}{2}$$

Input:

$$\frac{\sqrt{5}}{2} - \frac{1}{2}$$

Decimal approximation: 0.6180339887498948482045868343656381177203091798057628621354486227 ...

 $0.6180339887....=\frac{1}{\phi}$

Solution for the variable x:

$$x = \frac{-2i\sqrt{5}t + 2it - \sqrt{5} + 1}{-2 - 4it}$$

Implicit derivatives:

$$\frac{\partial x(t)}{\partial t} = \frac{2\left(-1+\sqrt{5}-2x\right)x}{\left(-1+\sqrt{5}\right)\left(-i+2t\right)}$$

$$\frac{\partial t(x)}{\partial x} = \frac{\left(-1 + \sqrt{5}\right)\left(-i + 2t\right)}{2\left(-1 + \sqrt{5} - 2x\right)x}$$

From:

AdS-phobia, the WGC, the Standard Model and Supersymmetry

Eduardo Gonzalo, Alvaro Herraez and Luis E. Ibanez - arXiv:1803.08455v2 [hep-th] 4 May 2018

We have that:



Figure 2: Effective potential with the Wilson lines fixed to zero, as a function of the Radion and the Higgs. The tree level potential dominates and the Higgs is not displaced from its tree level minimum by the one-loop corrections. This behavior is independent of the particular value of the Wilson lines Although not very visible in the plot, the Higgs minimum remains at the same location as R^{-1} increases.

From:

$$\begin{aligned} V_{\mathcal{C}}\left[a,0\right] &= \frac{1}{\left(2\pi a\right)^{2}} \left[\frac{1}{\pi^{2}} \sum_{p=1}^{\infty} \frac{1}{p^{4}} + \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} \left\{2\pi \left|n\right| \operatorname{Li}_{2}(e^{-2\pi \left|n\right|}) + \operatorname{Li}_{3}(e^{-2\pi \left|n\right|})\right\}\right] \\ &= \frac{1}{\left(2\pi a\right)^{2}} \left[\frac{1}{\pi^{2}} \operatorname{Li}_{4}(1) + \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} \sum_{p=1}^{\infty} \left\{\frac{2\pi}{p^{2}} \left|n\right| \left(e^{-2\pi p}\right)^{\left|n\right|} + \frac{1}{p^{3}} \left(e^{-2\pi p}\right)^{\left|n\right|}\right\}\right] \\ &= \frac{1}{\left(2\pi a\right)^{2}} \left[\frac{\pi^{2}}{90} + \frac{1}{2\pi} \sum_{p=1}^{\infty} \left\{2\pi \frac{1}{2p^{2} \sinh^{2} \pi p} + \frac{\coth \pi p}{p^{3}}\right\}\right] \\ &= \frac{1}{\left(2\pi a\right)^{2}} \left[\frac{\pi^{2}}{90} + \frac{1}{\pi} \sum_{p=1}^{\infty} \left\{2\pi \frac{1}{p^{2} \left(\cosh \pi p - 1\right)}\right\} + \frac{7\pi^{2}}{360}\right] \\ &= \frac{1}{\left(2\pi a\right)^{2}} \frac{\mathcal{G}}{3}, \end{aligned} \tag{B.12}$$

where $\mathcal{G} \simeq 0.915966$ is Catalan's constant. For the case of antiperiodic boundary conditions the Casimir energy reads:

$$V_{\mathcal{C}}\left[a,\frac{1}{2}\right] = \frac{1}{(2\pi a)^{2}} \left[\frac{1}{\pi^{2}} \sum_{p=1}^{\infty} \frac{(-1)^{p}}{p^{4}} + \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} \left\{\pi \left|2n+1\right| \operatorname{Li}_{2}(-e^{-\pi\left|2n+1\right|}) + \operatorname{Li}_{3}(-e^{-\pi\left|2n+1\right|})\right\}\right]$$
$$= \frac{1}{(2\pi a)^{2}} \left[\frac{1}{\pi^{2}} \operatorname{Li}_{4}(-1) + \frac{1}{2\pi} \sum_{n=1}^{\infty} \left\{\frac{(-1)^{p}\pi}{p^{2}} \left|2n+1\right| \left(e^{-\pi p}\right)^{\left|2n+1\right|} + \frac{(-1)^{p}}{p^{3}} \left(e^{-\pi p}\right)^{\left|2n+1\right|}\right\}\right]$$
$$= \frac{1}{(2\pi a)^{2}} \left[-\frac{7}{8} \frac{\pi^{2}}{90} - \frac{1}{2\pi} \sum_{p=1}^{\infty} \left\{2\pi \frac{(-1)^{p}\pi}{4} \operatorname{csch}^{2} \pi p + \frac{(-1)^{p}\pi}{4} \operatorname{sech}^{2} \pi p}{2p^{2}} + \frac{(-1)^{p}\operatorname{csch}^{2} \pi p}{p^{3}}\right\}\right]$$
$$= \frac{-1}{(2\pi a)^{2}} \frac{\mathcal{G}}{6} = -\frac{1}{2} V_{\mathcal{C}} \left[a, 0\right]. \tag{B.13}$$

where $\mathcal{G} \simeq 0.915966$ is Catalan's constant.

$$V_{\mathcal{C}}[a,0] = \frac{1}{(2\pi a)^2} \left[\frac{1}{\pi^2} \sum_{p=1}^{\infty} \frac{1}{p^4} + \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} \left\{ 2\pi |n| \operatorname{Li}_2(e^{-2\pi |n|}) + \operatorname{Li}_3(e^{-2\pi |n|}) \right\} \right]$$
$$= \frac{1}{(2\pi a)^2} \frac{\mathcal{G}}{3},$$

1/(2Pi*a)^2*1/3*0.915966

Input interpretation:

 $\frac{1}{(2\pi a)^2} \times \frac{1}{3} \times 0.915966$

Result:

 $\frac{0.0077339}{a^2}$

Plots:





Alternate form assuming a is real:

 $\frac{0.0077339}{a^2} + 0$

Roots:

(no roots exist)

Property as a function: Parity

even

Derivative:

$$\frac{d}{da} \left(\frac{0.0077339}{a^2} \right) = -\frac{0.0154678}{a^3}$$

Indefinite integral:

 $\int \frac{0.915966}{(2\pi a)^2 3} \, da = -\frac{0.0077339}{a} + \text{constant}$

Limit:

 $\lim_{a\to\pm\infty}\frac{0.0077339}{a^2}=0\approx 0$

Alternative representations:

 $\frac{0.915966}{3\left(2\,\pi\,a\right)^2} = \frac{0.305322}{\left(360\,a^{\,\circ}\right)^2}$

 $\frac{0.915966}{3(2\pi a)^2} = \frac{0.305322}{(-2a\,i\log(-1))^2}$

 $\frac{0.915966}{3(2\pi a)^2} = \frac{0.305322}{\left(2 \, a \cos^{-1}(-1)\right)^2}$

Series representations:

$$\frac{0.915966}{3(2\pi a)^2} = \frac{0.00477066}{a^2 \left(\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}\right)^2}$$

$$\frac{0.915966}{3(2\pi a)^2} = \frac{0.0190826}{a^2 \left(-1 + \sum_{k=1}^{\infty} \frac{2^k}{\binom{2k}{k}}\right)^2}$$

$$\frac{0.915966}{3(2\pi a)^2} = \frac{0.0763305}{a^2 \left(\sum_{k=0}^{\infty} \frac{2^{-k}(-6+50k)}{\binom{3k}{k}}\right)^2}$$

Integral representations:

0.915966		0.0190826
$3(2\pi a)^2$	=	$\overline{a^2 \left(\int_0^\infty \frac{1}{1+t^2} dt\right)^2}$

$$\frac{0.915966}{3(2\pi a)^2} = \frac{0.00477066}{a^2 \left(\int_0^1 \sqrt{1-t^2} dt\right)^2}$$

0.915966		0.019082	26
$3(2\pi a)^2$	=	$a^2 \left(\int_0^\infty \frac{\sin(t)}{t} \right)$	$dt)^2$

For a = 1:

1/(2Pi)^2*1/3*0.915966

Input interpretation:

 $\frac{1}{(2\pi)^2} \times \frac{1}{3} \times 0.915966$

Result:

0.00773390...

0.00773390....

Alternative representations:

0.915966	0.305322
$3(2\pi)^2$	$=$ ${(360^{\circ})^2}$
0.915966	0.305322
$3(2\pi)^2$	$= \frac{1}{(-2i\log(-1))^2}$

0.915966	=	0.305322
$3(2\pi)^2$		$(2\cos^{-1}(-1))^2$

Series representations:

0.915966	0.00477066
$3(2\pi)^2$	$= \frac{1}{\left(\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}\right)^2}$

$$\frac{0.915966}{3(2\pi)^2} = \frac{0.0190826}{\left(-1 + \sum_{k=1}^{\infty} \frac{2^k}{\binom{2k}{k}}\right)^2}$$

$$\frac{0.915966}{3(2\pi)^2} = \frac{0.0763305}{\left(\sum_{k=0}^{\infty} \frac{2^{-k}(-6+50k)}{\binom{3k}{k}}\right)^2}$$

Integral representations:

$$\frac{0.915966}{3(2\pi)^2} = \frac{0.0190826}{\left(\int_0^\infty \frac{1}{1+t^2} dt\right)^2}$$

$$\frac{0.915966}{3(2\pi)^2} = \frac{0.00477066}{\left(\int_0^1 \sqrt{1-t^2} dt\right)^2}$$

$$\frac{0.915966}{3(2\pi)^2} = \frac{0.0190826}{\left(\int_0^\infty \frac{\sin(t)}{t} dt\right)^2}$$

For a = 5.1, after some calculations:

$((0.00773390))1/(((1/(2Pi*5.1)^2*1/3*0.915966)))$

Input interpretation:

$$0.00773390 \times \frac{1}{\frac{1}{(2\pi \times 5.1)^2} \times \frac{1}{3} \times 0.915966}$$

Result:

26.0100...

26.01....

Alternative representations:

0.0077339	0.0077339
0.915966	0.305322
$(2\pi 5.1)^2$ 3	(1836.°) ²

0.0077339	0.0077339
0.915966	0.305322
$(2\pi 5.1)^2 3$	$(-10.2 i \log(-1))^2$

0.0077339	0.0077339
0.915966	0.305322
$(2\pi 5.1)^2 3$	$(10.2 \cos^{-1}(-1))^2$

Series representations:

$$\frac{0.0077339}{\frac{0.915966}{(2\pi\,5.1)^2\,3}} = 42.1658 \left(\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2\,k}\right)^2$$

$$\frac{0.0077339}{\frac{0.915966}{(2\pi\,5.1)^2\,3}} = 10.5415 \left(-1 + \sum_{k=1}^{\infty} \frac{2^k}{\binom{2\,k}{k}} \right)^2$$

$$\frac{0.0077339}{\frac{0.915966}{(2\pi\,5.1)^2\,3}} = 2.63537 \left(\sum_{k=0}^{\infty} \frac{2^{-k}\,(-6+50\,k)}{\binom{3\,k}{k}} \right)^2$$

Integral representations:

$$\frac{0.0077339}{\frac{0.915966}{(2\pi\,5.1)^2\,3}} = 10.5415 \left(\int_0^\infty \frac{1}{1+t^2}\,dt\right)^2$$

$$\frac{0.0077339}{\frac{0.915966}{(2\pi\,5.1)^2\,3}} = 42.1658 \left(\int_0^1 \sqrt{1-t^2} \,dt\right)^2$$

$$\frac{0.0077339}{\frac{0.915966}{(2\pi\,5.1)^2\,3}} = 10.5415 \left(\int_0^\infty \frac{\sin(t)}{t}\,dt\right)^2$$

From:

$$V_{\mathcal{C}}\left[a,\frac{1}{2}\right] = \frac{1}{\left(2\pi a\right)^{2}} \left[\frac{1}{\pi^{2}} \sum_{p=1}^{\infty} \frac{(-1)^{p}}{p^{4}} + \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} \left\{\pi \left|2n+1\right| \operatorname{Li}_{2}(-e^{-\pi \left|2n+1\right|}) + \operatorname{Li}_{3}(-e^{-\pi \left|2n+1\right|})\right\}\right]$$
$$= \frac{-1}{\left(2\pi a\right)^{2}} \frac{\mathcal{G}}{6} = -\frac{1}{2} V_{\mathcal{C}}\left[a,0\right].$$

-1/(2Pi*a)^2*1/6*0.915966

Input interpretation:

$$-\frac{\frac{1}{6} \times 0.915966}{(2 \pi a)^2}$$

Result:

 $-\frac{0.00386695}{a^2}$

Plots:

$$\begin{array}{c|c} & y \\ 15 \\ 10 \\ 5 \\ \hline -1.0 \\ -1.0 \\ -15 \\ \end{array} \begin{array}{c} 0.5 \\ 0.5 \\ 1.0 \\ a \end{array} (a \text{ from -1 to 1}) \\ a \\ \end{array}$$



Alternate form assuming a is real:

 $0 - {0.00386695\over a^2}$

Roots:

(no roots exist)

Property as a function: Parity

even

Derivative:

 $\frac{d}{da} \left(-\frac{0.00386695}{a^2} \right) = \frac{0.0077339}{a^3}$

Indefinite integral:

$$\int -\frac{0.915966}{(2\pi a)^2 6} \, da = \frac{0.00386695}{a} + \text{constant}$$

Limit:

 $\lim_{a\to\pm\infty}-\frac{0.00386695}{a^2}=0\approx 0$

Alternative representations:

 $\frac{0.915966(-1)}{6(2\pi a)^2} = -\frac{0.915966}{6(360 a^\circ)^2}$

 $\frac{0.915966(-1)}{6(2\pi a)^2} = -\frac{0.915966}{6(-2ai\log(-1))^2}$

 $\frac{0.915966\,(-1)}{6\,(2\,\pi\,a)^2} = -\frac{0.915966}{6\,\big(2\,a\cos^{-1}(-1)\big)^2}$

Series representations:

$$\frac{0.915966(-1)}{6(2\pi a)^2} = -\frac{0.00238533}{a^2 \left(\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}\right)^2}$$

$$\frac{0.915966(-1)}{6(2\pi a)^2} = -\frac{0.00954131}{a^2 \left(-1 + \sum_{k=1}^{\infty} \frac{2^k}{\binom{2k}{k}}\right)^2}$$

$$\frac{0.915966(-1)}{6(2\pi a)^2} = -\frac{0.0381653}{a^2 \left(\sum_{k=0}^{\infty} \frac{2^{-k}(-6+50k)}{\binom{3k}{k}}\right)^2}$$

Integral representations:

$$\frac{0.915966(-1)}{6(2\pi a)^2} = -\frac{0.00954131}{a^2 \left(\int_0^\infty \frac{1}{1+t^2} dt\right)^2}$$

 $\frac{0.915966(-1)}{6(2\pi a)^2} = -\frac{0.00238533}{a^2 \left(\int_0^1 \sqrt{1-t^2} dt\right)^2}$

$$\frac{0.915966(-1)}{6(2\pi a)^2} = -\frac{0.00954131}{a^2 \left(\int_0^\infty \frac{\sin(t)}{t} dt\right)^2}$$

-1/(2Pi)^2*1/6*0.915966

Input interpretation:

$$-\frac{\frac{1}{6} \times 0.915966}{(2 \pi)^2}$$

Result:

-0.00386695...

-0.00386695....

Alternative representations:

$$\frac{0.915966\,(-1)}{6\,(2\,\pi)^2} = -\frac{0.915966}{6\,(360\,^\circ)^2}$$

 $\frac{0.915966(-1)}{6(2\pi)^2} = -\frac{0.915966}{6(-2i\log(-1))^2}$

 $\frac{0.915966\,(-1)}{6\,(2\,\pi)^2} = -\frac{0.915966}{6\,\big(2\cos^{-1}(-1)\big)^2}$

Series representations:

 $\frac{0.915966(-1)}{6(2\pi)^2} = -\frac{0.00238533}{\left(\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}\right)^2}$

$$\frac{0.915966(-1)}{6(2\pi)^2} = -\frac{0.00954131}{\left(-1 + \sum_{k=1}^{\infty} \frac{2^k}{\binom{2^k}{k}}\right)^2}$$

$$\frac{0.915966(-1)}{6(2\pi)^2} = -\frac{0.0381653}{\left(\sum_{k=0}^{\infty} \frac{2^{-k}(-6+50k)}{\binom{3k}{k}}\right)^2}$$

Integral representations:

$$\frac{0.915966(-1)}{6(2\pi)^2} = -\frac{0.00954131}{\left(\int_0^\infty \frac{1}{1+t^2} dt\right)^2}$$

$$\frac{0.915966(-1)}{6(2\pi)^2} = -\frac{0.00238533}{\left(\int_0^1 \sqrt{1-t^2} dt\right)^2}$$

$$\frac{0.915966(-1)}{6(2\pi)^2} = -\frac{0.00954131}{\left(\int_0^\infty \frac{\sin(t)}{t} dt\right)^2}$$

From which:

$(-0.00386695)1/(((-1/(2Pi^{*}5.1)^{2}^{*}1/6^{*}0.915966)))$

Input interpretation:

$$-0.00386695 \left(-\frac{1}{\frac{\frac{1}{6} \times 0.915966}{(2\pi \times 5.1)^2}}\right)$$

Result:

26.0100...

26.01....

Alternative representations:

-0.00386695		-0.00386695
$-\frac{0.915966}{(2 \pi 5.1)^2 6}$	=	$-\frac{0.915966}{6(1836.^{\circ})^2}$

-0.00386695		-0.00386695
$-\frac{0.915966}{(2 - 5 - 1)^2}$	=	$-\frac{0.915966}{6(-10.2 i \log(-1))^2}$
$(2\pi 5.1)$ 0		6(-10.21 log(-1))

$$\frac{-0.00386695}{-\frac{0.915966}{(2\pi\,5.1)^2\,6}} = \frac{-0.00386695}{-\frac{0.915966}{6(10.2\,\cos^{-1}(-1))^2}}$$

Series representations:

$$\frac{-0.00386695}{-\frac{0.915966}{(2\pi\,5.1)^2\,6}} = 42.1658 \left(\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2\,k}\right)^2$$

$$\frac{-0.00386695}{-\frac{0.915966}{(2\pi\,5.1)^2\,6}} = 10.5415 \left(-1 + \sum_{k=1}^{\infty} \frac{2^k}{\binom{2\,k}{k}}\right)^2$$

$$\frac{-0.00386695}{-\frac{0.915966}{(2\pi\,5.1)^2\,6}} = 2.63537 \left(\sum_{k=0}^{\infty} \frac{2^{-k}\,(-6+50\,k)}{\binom{3\,k}{k}}\right)^2$$

Integral representations:

$$\frac{-0.00386695}{-\frac{0.915966}{(2\pi\,5.1)^2\,6}} = 10.5415 \left(\int_0^\infty \frac{1}{1+t^2}\,dt\right)^2$$

$$\frac{-0.00386695}{-\frac{0.915966}{(2\pi\,5.1)^2\,6}} = 42.1658 \left(\int_0^1 \sqrt{1-t^2} \,dt\right)^2$$

$$\frac{-0.00386695}{-\frac{0.915966}{(2\pi\,5.1)^2\,6}} = 10.5415 \left(\int_0^\infty \frac{\sin(t)}{t}\,dt\right)^2$$

From the sum between the two above expression, after some calculations:

 $\begin{array}{l} Pi^{*}((([((0.00773390))1/(((1/(2Pi^{*}5.1)^{2}^{*}1/3^{*}0.915966)))]^{2} + [(-0.00386695)1/(((-1/(2Pi^{*}5.1)^{2}^{*}1/6^{*}0.915966)))]^{2})))^{-}89^{-}55^{-}8^{-}e \end{array}$

Input interpretation:

$$\pi \left(\left(0.00773390 \times \frac{1}{\frac{1}{(2\pi \times 5.1)^2} \times \frac{1}{3} \times 0.915966} \right)^2 + \left(-0.00386695 \left(-\frac{1}{\frac{1}{\frac{1}{6} \times 0.915966}} \right)^2 \right)^2 \right) - 89 - 55 - 8 - e$$

Result:

4095.99...

 $4095.99.... \approx 4096 = 64^2$

Alternative representations:

$$\pi \left(\left(\frac{0.0077339}{\frac{0.915966}{(2\pi 5.1)^2 3}} \right)^2 + \left(\frac{-0.00386695}{-\frac{0.915966}{(2\pi 5.1)^2 6}} \right)^2 \right) - 89 - 55 - 8 - e = -152 - e + 180^{\circ} \left(\left(\frac{0.0077339}{\frac{0.305322}{(1836.^{\circ})^2}} \right)^2 + \left(\frac{-0.00386695}{-\frac{0.915966}{6(1836.^{\circ})^2}} \right)^2 \right)$$

$$\pi \left(\left(\frac{0.0077339}{\frac{0.915966}{(2\pi 5.1)^2 3}} \right)^2 + \left(\frac{-0.00386695}{-\frac{0.915966}{(2\pi 5.1)^2 6}} \right)^2 \right) - 89 - 55 - 8 - e = -152 - e - i \left(\log(-1) \left(\left(\frac{0.0077339}{\frac{0.305322}{(-10.2i \log(-1))^2}} \right)^2 + \left(\frac{-0.00386695}{-\frac{0.915966}{6(-10.2i \log(-1))^2}} \right)^2 \right) \right)$$

$$\pi \left(\left(\frac{0.0077339}{\frac{0.915966}{(2\pi\,5.1)^2\,3}} \right)^2 + \left(\frac{-0.00386695}{-\frac{0.915966}{(2\pi\,5.1)^2\,6}} \right)^2 \right) - 89 - 55 - 8 - e = \\\pi \left(\left(\frac{0.0077339}{\frac{0.915966}{(2\pi\,5.1)^2\,3}} \right)^2 + \left(\frac{-0.00386695}{-\frac{0.915966}{(2\pi\,5.1)^2\,6}} \right)^2 \right) - 89 - 55 - 8 - \exp(z) \text{ for } z = 1$$

Series representations:

$$\pi \left(\left(\frac{0.0077339}{\frac{0.915966}{(2\pi 5.1)^2 3}} \right)^2 + \left(\frac{-0.00386695}{-\frac{0.915966}{(2\pi 5.1)^2 6}} \right)^2 \right) - 89 - 55 - 8 - e = 14223.7 \left(-0.0106864 + \left(\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2 k} \right)^5 - 0.0000703054 \sum_{k=0}^{\infty} \frac{1}{k!} \right)^2 \right)$$

$$\pi \left(\left(\frac{0.0077339}{\frac{0.915966}{(2\pi 5.1)^2 3}} \right)^2 + \left(\frac{-0.00386695}{-\frac{0.915966}{(2\pi 5.1)^2 6}} \right)^2 \right) - 89 - 55 - 8 - e = 14223.7 \left(-0.0106864 + \left(\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2 k} \right)^5 - 0.0000703054 \sum_{k=0}^{\infty} \frac{(-1+k)^2}{k!} \right)^{k-1} \right)$$

$$\pi \left(\left(\frac{0.0077339}{\frac{0.915966}{(2\pi 5.1)^2 \, 3}} \right)^2 + \left(\frac{-0.00386695}{-\frac{0.915966}{(2\pi 5.1)^2 \, 6}} \right)^2 \right) - 89 - 55 - 8 - e = -\left(152 + \sum_{k=0}^{\infty} \frac{1}{k!} - 13.8903 \left(\sum_{k=1}^{\infty} 4^{-k} \left(-1 + 3^k \right) \zeta(1+k) \right)^5 \right)$$

Integral representations:

$$\pi \left(\left(\frac{0.0077339}{\frac{0.915966}{(2\pi 5.1)^2 3}} \right)^2 + \left(\frac{-0.00386695}{-\frac{0.915966}{(2\pi 5.1)^2 6}} \right)^2 \right) - 89 - 55 - 8 - e = -152 - e + 444.49 \left(\int_0^\infty \frac{1}{1+t^2} dt \right)^5$$

$$\pi \left(\left(\frac{0.0077339}{\frac{0.915966}{(2\pi 5.1)^2 3}} \right)^2 + \left(\frac{-0.00386695}{-\frac{0.915966}{(2\pi 5.1)^2 6}} \right)^2 \right) - 89 - 55 - 8 - e = -152 - e + 14223.7 \left(\int_0^1 \sqrt{1 - t^2} dt \right)^5$$

$$\pi \left(\left(\frac{0.0077339}{\frac{0.915966}{(2\pi 5.1)^2 \, 3}} \right)^2 + \left(\frac{-0.00386695}{-\frac{0.915966}{(2\pi 5.1)^2 \, 6}} \right)^2 \right) - 89 - 55 - 8 - e = -152 - e + 444.49 \left(\int_0^\infty \frac{\sin(t)}{t} \, dt \right)^5$$

 $\begin{array}{l} Pi*((([((0.00773390))1/(((1/(2Pi*5.1)^{2*1/3*0.915966})))]^{2} + [(-0.00386695)1/(((-1/(2Pi*5.1)^{2*1/6*0.915966})))]^{2}))) + 123-sqrt3 \end{array}$

Input interpretation:

$$\pi \left(\left(0.00773390 \times \frac{1}{\frac{1}{(2\pi \times 5.1)^2} \times \frac{1}{3} \times 0.915966} \right)^2 + \left(-0.00386695 \left(-\frac{1}{\frac{1}{\frac{6}{5} \times 0.915966}}{\frac{1}{(2\pi \times 5.1)^2}} \right) \right)^2 \right) + 123 - \sqrt{3}$$

Result:

4371.97...

$4371.97\ldots\approx 4372$

Where 4372 is a value indicated in the fundamental Ramanujan paper "Modular equations and Approximations to π "

Hence

$$64g_{22}^{24} = e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \cdots,$$

$$64g_{22}^{-24} = 4096e^{-\pi\sqrt{22}} + \cdots,$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \dots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\dots$$

Series representations:

$$\pi \left(\left(\frac{0.0077339}{\frac{0.915966}{(2\pi 5.1)^2 3}} \right)^2 + \left(\frac{-0.00386695}{-\frac{0.915966}{(2\pi 5.1)^2 6}} \right)^2 \right) + 123 - \sqrt{3} = 123 + 13.8903 \pi^5 - \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \left(\frac{1}{2} \atop k \right)$$

$$\pi \left(\left(\frac{0.0077339}{\frac{0.915966}{(2\pi 5.1)^2 3}} \right)^2 + \left(\frac{-0.00386695}{-\frac{0.915966}{(2\pi 5.1)^2 6}} \right)^2 \right) + 123 - \sqrt{3} = 123 + 13.8903 \pi^5 - \sqrt{2} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2} \right)^k \left(-\frac{1}{2} \right)_k}{k!}$$

$$\pi \left(\left(\frac{0.0077339}{\frac{0.915966}{(2\pi 5.1)^2 3}} \right)^2 + \left(\frac{-0.00386695}{-\frac{0.915966}{(2\pi 5.1)^2 6}} \right)^2 \right) + 123 - \sqrt{3} = 123 + 13.8903 \pi^5 - \frac{\sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 2^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{2\sqrt{\pi}}$$

 $27*sqrt((Pi*((([((0.00773390))1/(((1/(2Pi*5.1)^2*1/3*0.915966)))]^2 + [(-0.00386695)1/(((-1/(2Pi*5.1)^2*1/6*0.915966)))]^2)))-89-55-8-e))+1$

Input interpretation:

$$27 \sqrt{\left(\pi \left(\left(0.00773390 \times \frac{1}{\frac{1}{(2\pi \times 5.1)^2} \times \frac{1}{3} \times 0.915966}\right)^2 + \left(-0.00386695 \left(-\frac{1}{\frac{1}{\frac{6}{5} \times 0.915966}}\right)^2\right) - 89 - 55 - 8 - e\right) + 1}\right)$$

Result:

1729.00...

1729

This result is very near to the mass of candidate glueball $f_0(1710)$ scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. (1728 = $8^2 * 3^3$) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Series representations:

$$27\sqrt{\pi\left(\left(\frac{0.0077339}{\frac{0.915966}{(2\pi\,5.1)^2\,3}}\right)^2 + \left(\frac{-0.00386695}{-\frac{0.915966}{(2\pi\,5.1)^2\,6}}\right)^2\right) - 89 - 55 - 8 - e} + 1 = 1 + 27\sqrt{-153 - e} + 13.8903\,\pi^5 \sum_{k=0}^{\infty} (-153 - e + 13.8903\,\pi^5)^{-k} \left(\frac{1}{2}, \frac{1}{k}\right)^{-k}\right)^{-k}$$

$$27\sqrt{\pi\left(\left(\frac{0.0077339}{\frac{0.915966}{(2\pi\,5.1)^2\,3}}\right)^2 + \left(\frac{-0.00386695}{-\frac{0.915966}{(2\pi\,5.1)^2\,6}}\right)^2\right) - 89 - 55 - 8 - e + 1 = 1 + 27\sqrt{-153 - e + 13.8903\,\pi^5} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-153 - e + 13.8903\,\pi^5\right)^{-k} \left(-\frac{1}{2}\right)_k}{k!}$$

$$27\sqrt{\pi\left(\left(\frac{0.0077339}{\frac{0.915966}{(2\pi 5.1)^2 3}}\right)^2 + \left(\frac{-0.00386695}{-\frac{0.915966}{(2\pi 5.1)^2 6}}\right)^2\right) - 89 - 55 - 8 - e} + 1 = 1 + 27\sqrt{z_0}\sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(-152 - e + 13.8903 \pi^5 - z_0\right)^k z_0^{-k}}{k!}}{k!}$$
for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \le 0$))

 $((27*sqrt((Pi*((([((0.00773390))1/(((1/(2Pi*5.1)^2*1/3*0.915966))))]^2 + [(-0.00386695)1/(((-1/(2Pi*5.1)^2*1/6*0.915966)))]^2)))-89-55-8-e))+1))^{1/15}$

Input interpretation:

$$\left(27\sqrt{\left(\left(\left(0.00773390\times\frac{1}{\frac{1}{(2\pi\times5.1)^2}\times\frac{1}{3}\times0.915966}\right)^2+\right.\right)^2} + \left(-0.00386695\left(-\frac{1}{\frac{1}{\frac{1}{6}\times0.915966}{(2\pi\times5.1)^2}}\right)\right)^2\right) - 89 - 55 - 8 - e\left(+1\right)^{(1/15)}$$

Result:

1.6438150495830386047005199331688482686077512380746076302517997346

...
1.643815049....
$$\approx \zeta(2) = \frac{\pi^2}{6} = 1.644934...$$

We have that:

$$V_{\mathcal{C}}^{(2)}[a,m,0] = (am)^2 \left\{ 2\pi \left(\log \left(2\pi ma\right) - \frac{1}{4} \right) - \text{Li}_2(1) + \sum_{n=1}^{\infty} 2\pi \log \left(1 - e^{-2\pi n}\right) \right\}$$
$$= (am)^2 \left\{ \pi \log \left(2\pi ma\right) - \frac{\pi^2}{6} - \frac{5\pi}{2} + \log \frac{\Gamma\left(\frac{1}{4}\right)}{2\pi^{3/4}} \right\}$$
(B.14)

Finally, for the antiperiodic case we find:

$$V_{\mathcal{C}}^{(2)}\left[a,m,\frac{1}{2}\right] = (am)^2 \left\{-\text{Li}_2(-1) + \sum_{n=-\infty}^{\infty} \pi \log\left(1 + e^{-\pi|2n+1|}\right)\right\}$$
$$= (am)^2 \left\{-\frac{\pi^2}{12} + \frac{3\pi}{4}\log 2\right\}$$
(B.15)

From:

$$(am)^{2} \left\{ \pi \log \left(2\pi ma \right) - \frac{\pi^{2}}{6} - \frac{5\pi}{2} + \log \frac{\Gamma\left(\frac{1}{4}\right)}{2\pi^{3/4}} \right\}$$

$$x^2 [Pi^{1n}(2Pi^{x})-(Pi^{2})/6 - (5Pi)/2 + \ln((gamma(1/4))/(2Pi^{3/4}))]$$

Input:

$$x^{2}\left(\pi \log(2 \, \pi \, x) - \frac{\pi^{2}}{6} - \frac{5 \, \pi}{2} + \log \left(\frac{\Gamma\left(\frac{1}{4}\right)}{2 \, \pi^{3/4}}\right)\right)$$

log(x) is the natural logarithm

 $\Gamma(x)$ is the gamma function





Alternate forms:

$$x^2\left(\pi\log(2\,\pi\,x)-\frac{1}{6}\,\pi\,(15+\pi)+\log\!\left(\frac{2\,\Gamma\!\left(\frac{5}{4}\right)}{\pi^{3/4}}\right)\right)$$

$$x^{2} \log \left(\frac{\Gamma\left(\frac{1}{4}\right)}{2\pi^{3/4}}\right) - \frac{1}{6}\pi x^{2} \left(-6 \log(2\pi x) + \pi + 15\right)$$

$$-\frac{1}{6}x^{2}\left(-6\pi\log(2\pi x)+\pi^{2}+15\pi-6\log\left(\frac{\Gamma\left(\frac{1}{4}\right)}{2\pi^{3/4}}\right)\right)$$

Expanded form:

$$-\frac{\pi^2 x^2}{6} - \frac{5\pi x^2}{2} + \pi x^2 \log(2\pi x) + x^2 \log\left(\frac{\Gamma\left(\frac{1}{4}\right)}{2\pi^{3/4}}\right)$$

Alternate form assuming x>0:

$$-\frac{1}{12} x^2 \left(-12 \pi \log(2 \pi x) + 2 \pi^2 + 30 \pi + 3 \left(\log(16) + 3 \log(\pi) - 4 \log\left(\Gamma\left(\frac{1}{4}\right)\right)\right)\right)$$

Root:

 $x \approx 3.55961$

Series expansion at x=0:

$$x^{2}\left(\pi \log(x) - \frac{\pi^{2}}{6} + \pi \left(\log(2\pi) - \frac{5}{2}\right) + \log\left(\frac{\Gamma\left(\frac{1}{4}\right)}{2\pi^{3/4}}\right)\right) + O(x^{4})$$

(Puiseux series)

Series expansion at $x=\infty$:

$$x^{2}\left(\pi \log(x) - \frac{\pi^{2}}{6} + \pi \left(\log(2\pi) - \frac{5}{2}\right) + \log\left(\frac{\Gamma\left(\frac{1}{4}\right)}{2\pi^{3/4}}\right)\right) + O\left(\left(\frac{1}{x}\right)^{4}\right)$$

(Puiseux series)

Derivative:

$$\begin{split} & \frac{d}{dx} \left(x^2 \left(\pi \log(2\pi x) - \frac{\pi^2}{6} - \frac{5\pi}{2} + \log\left(\frac{\Gamma\left(\frac{1}{4}\right)}{2\pi^{3/4}}\right) \right) \right) = \\ & -\frac{1}{3} x \left(-6\pi \log(2\pi x) + \pi^2 + 12\pi + \frac{9\log(\pi)}{2} + \log(64) - 6\log\left(\Gamma\left(\frac{1}{4}\right)\right) \right) \end{split}$$

Indefinite integral:

$$\int x^2 \left(\pi \log(2\pi x) - \frac{\pi^2}{6} - \frac{5\pi}{2} + \log\left(\frac{\Gamma\left(\frac{1}{4}\right)}{2\pi^{3/4}}\right) \right) dx = -\frac{1}{18} x^3 \left(-6\pi \log(2\pi x) + \pi^2 + 17\pi + \frac{9\log(\pi)}{2} + \log(64) - 6\log(\Gamma\left(\frac{1}{4}\right)) \right) + \text{constant}$$

(assuming a complex-valued logarithm)

Local minimum:

$$\min\left\{x^{2}\left(\pi\log(2\pi x) - \frac{\pi^{2}}{6} - \frac{5\pi}{2} + \log\left(\frac{\Gamma\left(\frac{1}{4}\right)}{2\pi^{3/4}}\right)\right\} = -2^{2/\pi-3} e^{4+\pi/3} \pi^{3/(2\pi)-1} \Gamma\left(\frac{1}{4}\right)^{-2/\pi} \text{ at } x = 2^{1/\pi-1} e^{2+\pi/6} \pi^{3/(4\pi)-1} \Gamma\left(\frac{1}{4}\right)^{-1/\pi}$$

For x = 0.5:

 $0.5^{2} (-1/6 \pi (15 + \pi) + \pi \log(2 \pi * 0.5) + \log((2 \Gamma(5/4))/\pi^{(3/4)}))$

Input:

$$0.5^{2} \left(-\frac{1}{6} \pi (15 + \pi) + \pi \log(2 \pi \times 0.5) + \log\left(\frac{2 \Gamma\left(\frac{5}{4}\right)}{\pi^{3/4}}\right) \right)$$

log(x) is the natural logarithm

 $\Gamma(x)$ is the gamma function

Result:

-1.54158...

-1.54158....

Alternative representations:

$$0.5^{2} \left(\frac{1}{6} \left(\pi \left(15 + \pi \right) \right) \left(-1 \right) + \pi \log(2 \pi \ 0.5) + \log \left(\frac{2 \Gamma\left(\frac{5}{4}\right)}{\pi^{3/4}} \right) \right) = 0.5^{2} \left(\pi \log(\pi) + \log \left(\frac{2 G\left(1 + \frac{5}{4}\right)}{G\left(\frac{5}{4}\right) \pi^{3/4}} \right) - \frac{1}{6} \pi \left(15 + \pi \right) \right)$$
$$0.5^{2} \left(\frac{1}{6} \left(\pi \left(15 + \pi \right) \right) \left(-1 \right) + \pi \log(2 \pi \, 0.5) + \log \left(\frac{2 \, \Gamma\left(\frac{5}{4}\right)}{\pi^{3/4}} \right) \right) = \\ 0.5^{2} \left(\pi \log(\pi) + \log \left(\frac{2 \, e^{-\log G(5/4) + \log G(1+5/4)}}{\pi^{3/4}} \right) - \frac{1}{6} \, \pi \left(15 + \pi \right) \right)$$

$$0.5^{2} \left(\frac{1}{6} \left(\pi \left(15 + \pi \right) \right) \left(-1 \right) + \pi \log(2 \pi \, 0.5) + \log\left(\frac{2 \, \Gamma\left(\frac{5}{4}\right)}{\pi^{3/4}} \right) \right) = 0.5^{2} \left(\pi \log(\pi) + \log\left(\frac{2 \left(-1 + \frac{5}{4} \right)!}{\pi^{3/4}} \right) - \frac{1}{6} \, \pi \left(15 + \pi \right) \right)$$

$$0.5^{2} \left(\frac{1}{6} \left(\pi \left(15 + \pi \right) \right) \left(-1 \right) + \pi \log(2 \pi 0.5) + \log \left(\frac{2 \Gamma\left(\frac{5}{4}\right)}{\pi^{3/4}} \right) \right) = -0.625 \pi - 0.0416667 \pi^{2} + 0.25 \pi \log(\pi) + 0.25 \log \left(\frac{2 \sum_{k=0}^{\infty} \frac{\left(\frac{5}{4} - z_{0}\right)^{k} \Gamma^{(k)}(z_{0})}{k!}}{\pi^{3/4}} \right)$$
for $(z_{0} \notin \mathbb{Z} \text{ or } z_{0} > 0)$

$$0.5^{2} \left(\frac{1}{6} \left(\pi \left(15 + \pi \right) \right) \left(-1 \right) + \pi \log(2 \pi 0.5) + \log \left(\frac{2 \Gamma\left(\frac{5}{4}\right)}{\pi^{3/4}} \right) \right) = \\ -0.625 \pi - 0.0416667 \pi^{2} + 0.5 i \pi^{2} \left\lfloor \frac{\arg(\pi - x)}{2 \pi} \right\rfloor + \\ 0.5 i \pi \left\lfloor \frac{\arg\left(-x + \frac{2 \Gamma\left(\frac{5}{4}\right)}{\pi^{3/4}} \right)}{2 \pi} \right\rfloor + 0.25 \log(x) + 0.25 \pi \log(x) + \\ \sum_{k=1}^{\infty} \frac{\left(-1 \right)^{k} x^{-k} \left(-0.25 \pi \left(\pi - x \right)^{k} - 0.25 \left(-x + \frac{2 \Gamma\left(\frac{5}{4}\right)}{\pi^{3/4}} \right)^{k} \right)}{k} \text{ for } x < 0$$

$$\begin{split} 0.5^2 \Biggl(\frac{1}{6} \left(\pi \left(15 + \pi \right) \right) \left(-1 \right) + \pi \log(2 \pi \, 0.5) + \log \Biggl(\frac{2 \Gamma \left(\frac{5}{4} \right)}{\pi^{3/4}} \Biggr) \Biggr) = \\ -0.625 \, \pi - 0.0416667 \, \pi^2 + 0.5 \, i \, \pi^2 \Biggl[- \frac{-\pi + \arg \left(\frac{\pi}{z_0} \right) + \arg(z_0)}{2 \, \pi} \Biggr] + \\ 0.5 \, i \, \pi \Biggl[- \frac{-\pi + \arg \left(\frac{2 \Gamma \left(\frac{5}{4} \right)}{\pi^{3/4} \, z_0} \right) + \arg(z_0)}{2 \, \pi} \Biggr] + 0.25 \log(z_0) + \\ 0.25 \, \pi \log(z_0) + \sum_{k=1}^{\infty} \frac{\left(-1 \right)^k \left(-0.25 \, \pi \left(\pi - z_0 \right)^k - 0.25 \left(\frac{2 \Gamma \left(\frac{5}{4} \right)}{\pi^{3/4}} - z_0 \right)^k \right) z_0^{-k}}{k} \end{split}$$

$$0.5^{2} \left(\frac{1}{6} \left(\pi \left(15 + \pi \right) \right) \left(-1 \right) + \pi \log(2 \pi \, 0.5) + \log \left(\frac{2 \, \Gamma\left(\frac{5}{4}\right)}{\pi^{3/4}} \right) \right) = -0.625 \, \pi - 0.0416667 \, \pi^{2} + 0.25 \, \pi \log(\pi) + 0.25 \, \log \left(\frac{2}{\pi^{3/4}} \int_{0}^{\infty} e^{-t} \sqrt[4]{t} \, dt \right)$$

$$0.5^{2} \left(\frac{1}{6} \left(\pi \left(15 + \pi \right) \right) \left(-1 \right) + \pi \log(2 \pi \, 0.5) + \log \left(\frac{2 \, \Gamma\left(\frac{5}{4}\right)}{\pi^{3/4}} \right) \right) = -0.625 \, \pi - 0.0416667 \, \pi^{2} + 0.25 \, \pi \log(\pi) + 0.25 \log \left(\frac{2}{\pi^{3/4}} \int_{0}^{1} \sqrt[4]{\log\left(\frac{1}{t}\right)} \, dt \right)$$

$$0.5^{2} \left(\frac{1}{6} \left(\pi \left(15 + \pi \right) \right) \left(-1 \right) + \pi \log(2 \pi \, 0.5) + \log \left(\frac{2 \, \Gamma\left(\frac{5}{4}\right)}{\pi^{3/4}} \right) \right) = \\ -0.625 \, \pi - 0.0416667 \, \pi^{2} + 0.25 \, \log \left(\frac{2 \exp\left(\int_{0}^{1} \frac{\frac{1}{4} - \frac{5 \, x}{4} + x^{5/4}}{(-1 + x) \log(x)} \, dx \right)}{\pi^{3/4}} \right) + 0.25 \, \pi \log(\pi)$$

For x = 2.9:
2.9^2 (-1/6
$$\pi$$
 (15 + π) + π log(2 π * 2.9) + log((2 Γ (5/4))/ π ^(3/4)))

Input:

$$2.9^{2} \left(-\frac{1}{6} \pi (15 + \pi) + \pi \log(2\pi \times 2.9) + \log\left(\frac{2\Gamma\left(\frac{5}{4}\right)}{\pi^{3/4}}\right) \right)$$

 $\log(x)$ is the natural logarithm

 $\Gamma(x)$ is the gamma function

Result:

-5.41469...

-5.41469....

$$2.9^{2} \left(\frac{1}{6} \left(\pi \left(15 + \pi \right) \right) \left(-1 \right) + \pi \log(2\pi 2.9) + \log\left(\frac{2\Gamma\left(\frac{5}{4}\right)}{\pi^{3/4}}\right) \right) = 2.9^{2} \left(\pi \log(5.8\pi) + \log\left(\frac{2G\left(1 + \frac{5}{4}\right)}{G\left(\frac{5}{4}\right)\pi^{3/4}}\right) - \frac{1}{6}\pi \left(15 + \pi \right) \right)$$

$$2.9^{2} \left(\frac{1}{6} \left(\pi \left(15 + \pi \right) \right) \left(-1 \right) + \pi \log(2 \pi 2.9) + \log \left(\frac{2 \Gamma\left(\frac{5}{4}\right)}{\pi^{3/4}} \right) \right) = 2.9^{2} \left(\pi \log(5.8 \pi) + \log \left(\frac{2 e^{-\log G(5/4) + \log G(1+5/4)}}{\pi^{3/4}} \right) - \frac{1}{6} \pi \left(15 + \pi \right) \right)$$

$$2.9^{2} \left(\frac{1}{6} \left(\pi \left(15 + \pi \right) \right) \left(-1 \right) + \pi \log(2 \pi 2.9) + \log \left(\frac{2 \Gamma\left(\frac{5}{4}\right)}{\pi^{3/4}} \right) \right) = 2.9^{2} \left(\pi \log(5.8 \pi) + \log \left(\frac{2 \left(-1 + \frac{5}{4} \right)!}{\pi^{3/4}} \right) - \frac{1}{6} \pi \left(15 + \pi \right) \right)$$

$$2.9^{2} \left(\frac{1}{6} \left(\pi \left(15 + \pi \right) \right) \left(-1 \right) + \pi \log(2 \pi 2.9) + \log \left(\frac{2 \Gamma\left(\frac{5}{4}\right)}{\pi^{3/4}} \right) \right) = -21.025 \pi - 1.40167 \pi^{2} + 8.41 \pi \log(5.8 \pi) + 8.41 \log \left(\frac{2 \sum_{k=0}^{\infty} \frac{\left(\frac{5}{4} - z_{0}\right)^{k} \Gamma^{(k)}(z_{0})}{k!}}{\pi^{3/4}} \right)$$
for $(z_{0} \notin \mathbb{Z} \text{ or } z_{0} > 0)$

$$2.9^{2} \left(\frac{1}{6} \left(\pi \left(15 + \pi \right) \right) \left(-1 \right) + \pi \log(2 \pi 2.9) + \log \left(\frac{2 \Gamma\left(\frac{5}{4}\right)}{\pi^{3/4}} \right) \right) = -21.025 \pi - 1.40167 \pi^{2} + 16.82 i \pi^{2} \left\lfloor \frac{\arg(5.8 \pi - x)}{2 \pi} \right\rfloor + 16.82 i \pi \left\lfloor \frac{\arg\left(-x + \frac{2 \Gamma\left(\frac{5}{4}\right)}{\pi^{3/4}}\right)}{2 \pi} \right\rfloor + 8.41 \log(x) + 8.41 \pi \log(x) + \frac{2 \pi \left(-1\right)^{k} x^{-k} \left(-8.41 \pi \left(5.8 \pi - x\right)^{k} - 8.41 \left(-x + \frac{2 \Gamma\left(\frac{5}{4}\right)}{\pi^{3/4}}\right)^{k}\right)}{k} \text{ for } x < 0$$

$$2.9^{2} \left(\frac{1}{6} \left(\pi \left(15 + \pi \right) \right) \left(-1 \right) + \pi \log(2 \pi 2.9) + \log \left(\frac{2 \Gamma\left(\frac{5}{4}\right)}{\pi^{3/4}} \right) \right) = \\ -21.025 \pi - 1.40167 \pi^{2} + 16.82 i \pi^{2} \left[-\frac{-\pi + \arg\left(\frac{5.8\pi}{z_{0}}\right) + \arg(z_{0})}{2 \pi} \right] + \\ 16.82 i \pi \left[-\frac{-\pi + \arg\left(\frac{2 \Gamma\left(\frac{5}{4}\right)}{\pi^{3/4} z_{0}}\right) + \arg(z_{0})}{2 \pi} \right] + 8.41 \log(z_{0}) + 8.41 \pi \log(z_{0}) + \\ \sum_{k=1}^{\infty} \frac{\left(-1 \right)^{k} \left(-8.41 \pi \left(5.8 \pi - z_{0} \right)^{k} - 8.41 \left(\frac{2 \Gamma\left(\frac{5}{4}\right)}{\pi^{3/4}} - z_{0} \right)^{k} \right) z_{0}^{-k}}{k}$$

$$2.9^{2} \left(\frac{1}{6} \left(\pi \left(15 + \pi \right) \right) \left(-1 \right) + \pi \log(2 \pi 2.9) + \log \left(\frac{2 \Gamma\left(\frac{5}{4}\right)}{\pi^{3/4}} \right) \right) = -21.025 \pi - 1.40167 \pi^{2} + 8.41 \pi \log(5.8 \pi) + 8.41 \log\left(\frac{2}{\pi^{3/4}} \int_{0}^{\infty} e^{-t} \sqrt[4]{t} dt \right)$$

$$2.9^{2} \left(\frac{1}{6} \left(\pi \left(15 + \pi \right) \right) \left(-1 \right) + \pi \log(2 \pi 2.9) + \log \left(\frac{2 \Gamma\left(\frac{5}{4}\right)}{\pi^{3/4}} \right) \right) = -21.025 \pi - 1.40167 \pi^{2} + 8.41 \pi \log(5.8 \pi) + 8.41 \log \left(\frac{2}{\pi^{3/4}} \int_{0}^{1} \sqrt[4]{\log\left(\frac{1}{t}\right)} dt \right)$$

$$2.9^{2} \left(\frac{1}{6} \left(\pi \left(15 + \pi \right) \right) \left(-1 \right) + \pi \log(2 \pi 2.9) + \log \left(\frac{2 \Gamma\left(\frac{5}{4}\right)}{\pi^{3/4}} \right) \right) =$$

-21.025 \pi - 1.40167 \pi^{2} + 8.41 \log $\left(\frac{2 \exp\left(\int_{0}^{1} \frac{\frac{1}{4} - \frac{5 x}{4} + x^{5/4}}{(-1 + x) \log(x)} \, dx \right)}{\pi^{3/4}} \right) + 8.41 \pi \log(5.8 \pi)$

From which, after some calculations:

sqrt(sqrt[-(((0.5^2 (-1/6 π (15 + π) + π log(2 π * 0.5) + log((2 Γ (5/4))/ π ^(3/4))))))-(((2.9^2 (-1/6 π (15 + π) + π log(2 π * 2.9) + log((2 Γ (5/4))/ π ^(3/4))))))])

Input:

$$\sqrt{\left(\sqrt{\left(-\left(0.5^{2}\left(-\frac{1}{6}\pi\left(15+\pi\right)+\pi\log(2\,\pi\times0.5)+\log\left(\frac{2\,\Gamma\left(\frac{5}{4}\right)}{\pi^{3/4}}\right)\right)\right)-2.9^{2}\left(-\frac{1}{6}\pi\left(15+\pi\right)+\pi\log(2\,\pi\times2.9)+\log\left(\frac{2\,\Gamma\left(\frac{5}{4}\right)}{\pi^{3/4}}\right)\right)\right)-2.9^{2}\left(-\frac{1}{6}\pi\left(15+\pi\right)+\pi\log(2\,\pi\times2.9)+\log\left(\frac{2\,\Gamma\left(\frac{5}{4}\right)}{\pi^{3/4}}\right)\right)\right)}$$

log(x) is the natural logarithm

 $\Gamma(x)$ is the gamma function

Result:

1.6240300085530374986265700615390767078372322612322030846419002893

1.62403.... result quite near to the value of the golden ratio 1.618033988749...

All 2nd roots of 2.63747:

 $1.62403 e^0 \approx 1.6240$ (real, principal root)

 $1.62403 \; e^{i \, \pi} \approx -\, 1.6240 \; \; (\text{real root})$

$$\begin{split} \sqrt{\left(\sqrt{\left(-0.5^2 \left(-\frac{1}{6} \pi \left(15 + \pi\right) + \pi \log(2 \pi \, 0.5\right) + \log\left(\frac{2 \Gamma\left(\frac{5}{4}\right)}{\pi^{3/4}}\right)\right)} - \\ 2.9^2 \left(-\frac{1}{6} \pi \left(15 + \pi\right) + \pi \log(2 \pi \, 2.9) + \log\left(\frac{2 \Gamma\left(\frac{5}{4}\right)}{\pi^{3/4}}\right)\right)\right)\right) = \\ \sqrt{\left(\sqrt{\left(-0.5^2 \left(\pi \log(\pi) + \log\left(\frac{2 G\left(1 + \frac{5}{4}\right)}{G\left(\frac{5}{4}\right) \pi^{3/4}}\right) - \frac{1}{6} \pi \left(15 + \pi\right)\right)} - \\ 2.9^2 \left(\pi \log(5.8 \pi) + \log\left(\frac{2 G\left(1 + \frac{5}{4}\right)}{G\left(\frac{5}{4}\right) \pi^{3/4}}\right) - \frac{1}{6} \pi \left(15 + \pi\right)\right)\right)} \end{split}$$

$$\begin{split} \sqrt{\left(\sqrt{\left(-0.5^2 \left(-\frac{1}{6} \pi \left(15 + \pi\right) + \pi \log(2 \pi \, 0.5\right) + \log\left(\frac{2 \, \Gamma\left(\frac{5}{4}\right)}{\pi^{3/4}}\right)\right)} - \\ & 2.9^2 \left(-\frac{1}{6} \pi \left(15 + \pi\right) + \pi \log(2 \pi \, 2.9) + \log\left(\frac{2 \, \Gamma\left(\frac{5}{4}\right)}{\pi^{3/4}}\right)\right)\right)}{\sqrt{\left(\sqrt{\left(-0.5^2 \left(\pi \log(\pi) + \log\left(\frac{2 \, e^{-\log G(5/4) + \log G(1 + 5/4)}}{\pi^{3/4}}\right) - \frac{1}{6} \pi \left(15 + \pi\right)\right)} - \\ & 2.9^2 \left(\pi \log(5.8 \, \pi) + \log\left(\frac{2 \, e^{-\log G(5/4) + \log G(1 + 5/4)}}{\pi^{3/4}}\right) - \frac{1}{6} \pi \left(15 + \pi\right)\right)\right)} \end{split}$$

$$\begin{split} \sqrt{\left(\sqrt{\left(-0.5^2 \left(-\frac{1}{6} \pi \left(15 + \pi\right) + \pi \log(2 \pi \ 0.5\right) + \log\left(\frac{2 \Gamma\left(\frac{5}{4}\right)}{\pi^{3/4}}\right)\right)} - \\ 2.9^2 \left(-\frac{1}{6} \pi \left(15 + \pi\right) + \pi \log(2 \pi \ 2.9\right) + \log\left(\frac{2 \Gamma\left(\frac{5}{4}\right)}{\pi^{3/4}}\right)\right)\right)\right) = \\ \sqrt{\left(\sqrt{\left(-0.5^2 \left(\pi \log(\pi) + \log\left(\frac{2 \left(1\right)_{-1 + \frac{5}{4}}}{\pi^{3/4}}\right) - \frac{1}{6} \pi \left(15 + \pi\right)\right) - \right.} \\ 2.9^2 \left(\pi \log(5.8 \ \pi) + \log\left(\frac{2 \left(1\right)_{-1 + \frac{5}{4}}}{\pi^{3/4}}\right) - \frac{1}{6} \pi \left(15 + \pi\right)\right)\right)\right)} \end{split}$$

$$\begin{split} &\sqrt{\left(\sqrt{\left(-0.5^{2}\left(-\frac{1}{6}\pi\left(15+\pi\right)+\pi\log(2\pi\,0.5)+\log\left(\frac{2\,\Gamma\left(\frac{5}{4}\right)}{\pi^{3/4}}\right)\right)-2.9^{2}\left(-\frac{1}{6}\pi\left(15+\pi\right)+\pi\log(2\pi\,2.9)+\log\left(\frac{2\,\Gamma\left(\frac{5}{4}\right)}{\pi^{3/4}}\right)\right)\right)\right)}=\\ &\exp\left(i\,\pi\left\lfloor\frac{1}{2\,\pi}\arg\left(-x+\sqrt{\left(21.65\,\pi+1.44333\,\pi^{2}-0.25\,\pi\log(\pi)-8.41\,\pi\log(5.8\,\pi)-8.66\log\left(\frac{2\,\Gamma\left(\frac{5}{4}\right)}{\pi^{3/4}}\right)\right)\right)\right]\right)\sqrt{x}}\\ &\sum_{k=0}^{\infty}\frac{1}{k!}\left(-1\right)^{k}x^{-k}\left(-\frac{1}{2}\right)_{k}\left(-x+\sqrt{\left(21.65\,\pi+1.44333\,\pi^{2}-0.25\,\pi\log(\pi)-8.41\,\pi\log(5.8\,\pi)-8.66\log\left(\frac{2\,\Gamma\left(\frac{5}{4}\right)}{\pi^{3/4}}\right)\right)\right)^{k}} \text{ for } (x\in\mathbb{R} \text{ and } x<0) \end{split}$$

$$\begin{split} &\sqrt{\left(\sqrt{\left(-0.5^{2}\left(-\frac{1}{6}\pi\left(15+\pi\right)+\pi\log(2\pi\,0.5\right)+\log\left(\frac{2\,\Gamma\left(\frac{5}{4}\right)}{\pi^{3/4}}\right)\right)}-2.9^{2}\left(-\frac{1}{6}\pi\left(15+\pi\right)+\pi\log(2\pi\,2.9\right)+\log\left(\frac{2\,\Gamma\left(\frac{5}{4}\right)}{\pi^{3/4}}\right)\right)\right)}=\\ &\left(\frac{1}{z_{0}}\right)^{1/2}\left[\arg\left(\sqrt{21.65\,\pi+1.44333\,\pi^{2}-0.25\,\pi\log(\pi)-8.41\,\pi\log(5.8\,\pi)-8.66\log\left(\frac{2\,\Gamma\left(\frac{5}{4}\right)}{\pi^{3/4}}\right)-z_{0}}\right)/(2\pi)\right]}\\ &\left(\frac{1}{z_{0}}\right)^{1/2}\left[1+\left|\arg\left(\sqrt{21.65\,\pi+1.44333\,\pi^{2}-0.25\,\pi\log(\pi)-8.41\,\pi\log(5.8\,\pi)-8.66\log\left(\frac{2\,\Gamma\left(\frac{5}{4}\right)}{\pi^{3/4}}\right)-z_{0}}\right)/(2\pi)\right]}\\ &\sum_{k=0}^{\infty}\frac{1}{k!}\left(-1\right)^{k}\left(-\frac{1}{2}\right)_{k}\left(\sqrt{\left(21.65\,\pi+1.44333\,\pi^{2}-0.25\,\pi\log(\pi)-8.41\,\pi\log(5.8\,\pi)-8.66\log\left(\frac{2\,\Gamma\left(\frac{5}{4}\right)}{\pi^{3/4}}\right)-z_{0}}\right)^{k}z_{0}^{-k}\\ &8.41\,\pi\log(5.8\,\pi)-8.66\log\left(\frac{2\,\Gamma\left(\frac{5}{4}\right)}{\pi^{3/4}}\right)\right)-z_{0}\right)^{k}z_{0}^{-k} \end{split}$$

$$\begin{split} \sqrt{\left(\sqrt{\left(-0.5^2 \left(-\frac{1}{6} \pi \left(15 + \pi\right) + \pi \log(2 \pi \ 0.5\right) + \log\left(\frac{2 \Gamma\left(\frac{5}{4}\right)}{\pi^{3/4}}\right)\right)} - \\ & 2.9^2 \left(-\frac{1}{6} \pi \left(15 + \pi\right) + \pi \log(2 \pi \ 2.9) + \log\left(\frac{2 \Gamma\left(\frac{5}{4}\right)}{\pi^{3/4}}\right)\right)\right)\right) = \\ & \sqrt{\left(\sqrt{\left(21.65 \pi + 1.44333 \ \pi^2 - 0.25 \ \pi \log(\pi) - 8.41 \ \pi \log(5.8 \ \pi) - 8.66 \log\left(\frac{2}{\pi^{3/4}} \int_0^\infty e^{-t} \sqrt[4]{t} \ dt\right)\right)\right)} \end{split}$$

$$\begin{split} \sqrt{\left(\sqrt{\left(-0.5^2 \left(-\frac{1}{6} \pi \left(15 + \pi\right) + \pi \log(2 \pi \ 0.5\right) + \log\left(\frac{2 \Gamma\left(\frac{5}{4}\right)}{\pi^{3/4}}\right)\right)} - \\ & 2.9^2 \left(-\frac{1}{6} \pi \left(15 + \pi\right) + \pi \log(2 \pi \ 2.9) + \log\left(\frac{2 \Gamma\left(\frac{5}{4}\right)}{\pi^{3/4}}\right)\right)\right)\right) = \\ & \sqrt{\left(\sqrt{\left(21.65 \pi + 1.44333 \ \pi^2 - 0.25 \ \pi \log(\pi) - 8.41 \ \pi \log(5.8 \ \pi) - 8.66 \log\left(\frac{2}{\pi^{3/4}} \int_0^1 \sqrt[4]{\log\left(\frac{1}{t}\right)} \ dt\right)\right)\right)} \end{split}$$

$$\begin{split} \sqrt{\left(\sqrt{\left(-0.5^{2}\left(-\frac{1}{6}\pi\left(15+\pi\right)+\pi\log(2\pi\,0.5)+\log\left(\frac{2\,\Gamma\left(\frac{5}{4}\right)}{\pi^{3/4}}\right)\right)-2.9^{2}\left(-\frac{1}{6}\pi\left(15+\pi\right)+\pi\log(2\pi\,2.9)+\log\left(\frac{2\,\Gamma\left(\frac{5}{4}\right)}{\pi^{3/4}}\right)\right)\right)\right)} = \\ \sqrt{\left(\sqrt{\left(21.65\,\pi+1.44333\,\pi^{2}-0.25\,\pi\log(\pi)-8.41\,\pi\log(5.8\,\pi)-3.41\,\pi\log(5.8\,\pi)-2.41\,\pi\log(5.8\,\pi)-3.41\,\pi\log(5$$

We have that:

$$(am)^2\left\{-\frac{\pi^2}{12}+\frac{3\pi}{4}\log 2\right\}$$

0.5^2 [(-Pi^2)/12+(3Pi)/4 ln(2)]

Input:

$$0.5^2 \left(-\frac{\pi^2}{12} + \frac{3\pi}{4} \log(2) \right)$$

log(x) is the natural logarithm

Result:

0.202681...

0.202681....

$$0.5^{2} \left(-\frac{\pi^{2}}{12} + \frac{1}{4} \log(2) (3\pi) \right) = 0.5^{2} \left(\frac{3\pi \log_{e}(2)}{4} - \frac{\pi^{2}}{12} \right)$$

$$0.5^{2} \left(-\frac{\pi^{2}}{12} + \frac{1}{4} \log(2) (3\pi) \right) = 0.5^{2} \left(\frac{3}{4} \pi \log(a) \log_{a}(2) - \frac{\pi^{2}}{12} \right)$$

$$0.5^2 \left(-\frac{\pi^2}{12} + \frac{1}{4} \log(2) (3\pi) \right) = 0.5^2 \left(\frac{6}{4} \pi \coth^{-1}(3) - \frac{\pi^2}{12} \right)$$

$$0.5^{2} \left(-\frac{\pi^{2}}{12} + \frac{1}{4} \log(2) (3\pi) \right) = -0.0208333 \pi^{2} + 0.375 i \pi^{2} \left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor + 0.1875 \pi \log(x) - 0.1875 \pi \sum_{k=1}^{\infty} \frac{(-1)^{k} (2-x)^{k} x^{-k}}{k} \quad \text{for } x < 0$$

$$0.5^{2} \left(-\frac{\pi^{2}}{12} + \frac{1}{4} \log(2) (3\pi) \right) = -0.0208333 \pi^{2} + 0.375 i \pi^{2} \left[-\frac{-\pi + \arg\left(\frac{2}{z_{0}}\right) + \arg(z_{0})}{2\pi} \right] + 0.1875 \pi \log(z_{0}) - 0.1875 \pi \sum_{k=1}^{\infty} \frac{(-1)^{k} (2 - z_{0})^{k} z_{0}^{-k}}{k} \right]$$

$$0.5^{2} \left(-\frac{\pi^{2}}{12} + \frac{1}{4} \log(2) (3\pi) \right) = -0.0208333 \pi^{2} + 0.1875 \pi \left[\frac{\arg(2 - z_{0})}{2\pi} \right] \log\left(\frac{1}{z_{0}}\right) + 0.1875 \pi \log(z_{0}) + 0.1875 \pi \left[\frac{\arg(2 - z_{0})}{2\pi} \right] \log(z_{0}) - 0.1875 \pi \sum_{k=1}^{\infty} \frac{(-1)^{k} (2 - z_{0})^{k} z_{0}^{-k}}{k}$$

$$0.5^{2} \left(-\frac{\pi^{2}}{12} + \frac{1}{4} \log(2) (3\pi) \right) = -0.0208333 \pi^{2} + 0.1875 \pi \int_{1}^{2} \frac{1}{t} dt$$

$$0.5^{2} \left(-\frac{\pi^{2}}{12} + \frac{1}{4} \log(2) (3\pi) \right) = -0.0208333 \pi^{2} + \frac{0.09375}{i} \int_{-i \, \infty + \gamma}^{i \, \infty + \gamma} \frac{\Gamma(-s)^{2} \, \Gamma(1+s)}{\Gamma(1-s)} \, ds$$

for $-1 < \gamma < 0$

2.9^2 [(-Pi^2)/12+(3Pi)/4 ln(2)]

Input:

$$2.9^2 \left(-\frac{\pi^2}{12} + \frac{3\pi}{4} \log(2) \right)$$

log(x) is the natural logarithm

Result:

6.81818...

6.81818....

$$2.9^{2} \left(-\frac{\pi^{2}}{12} + \frac{1}{4} \log(2) (3\pi) \right) = 2.9^{2} \left(\frac{3\pi \log_{e}(2)}{4} - \frac{\pi^{2}}{12} \right)$$

$$2.9^{2} \left(-\frac{\pi^{2}}{12} + \frac{1}{4} \log(2) (3\pi) \right) = 2.9^{2} \left(\frac{3}{4} \pi \log(a) \log_{a}(2) - \frac{\pi^{2}}{12} \right)$$

$$2.9^{2} \left(-\frac{\pi^{2}}{12} + \frac{1}{4} \log(2) (3\pi) \right) = 2.9^{2} \left(\frac{6}{4} \pi \coth^{-1}(3) - \frac{\pi^{2}}{12} \right)$$

$$2.9^{2} \left(-\frac{\pi^{2}}{12} + \frac{1}{4} \log(2) (3\pi) \right) = -0.700833 \pi^{2} + 12.615 i \pi^{2} \left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor + 6.3075 \pi \log(x) - 6.3075 \pi \sum_{k=1}^{\infty} \frac{(-1)^{k} (2-x)^{k} x^{-k}}{k} \quad \text{for } x < 0$$

$$2.9^{2} \left(-\frac{\pi^{2}}{12} + \frac{1}{4} \log(2) (3 \pi) \right) = -0.700833 \pi^{2} + 12.615 i \pi^{2} \left[-\frac{-\pi + \arg\left(\frac{2}{z_{0}}\right) + \arg(z_{0})}{2 \pi} \right] + 6.3075 \pi \log(z_{0}) - 6.3075 \pi \sum_{k=1}^{\infty} \frac{(-1)^{k} (2 - z_{0})^{k} z_{0}^{-k}}{k} \right]$$

$$2.9^{2} \left(-\frac{\pi^{2}}{12} + \frac{1}{4} \log(2) (3 \pi) \right) = -0.700833 \pi^{2} + 6.3075 \pi \left[\frac{\arg(2 - z_{0})}{2 \pi} \right] \log\left(\frac{1}{z_{0}}\right) + 6.3075 \pi \log(z_{0}) + 6.3075 \pi \left[\frac{\arg(2 - z_{0})}{2 \pi} \right] \log(z_{0}) - 6.3075 \pi \sum_{k=1}^{\infty} \frac{(-1)^{k} (2 - z_{0})^{k} z_{0}^{-k}}{k}$$

Integral representations:

$$2.9^{2} \left(-\frac{\pi^{2}}{12} + \frac{1}{4} \log(2) \left(3 \pi \right) \right) = -0.700833 \,\pi^{2} + 6.3075 \,\pi \int_{1}^{2} \frac{1}{t} \, dt$$

$$2.9^{2} \left(-\frac{\pi^{2}}{12} + \frac{1}{4} \log(2) (3\pi) \right) = -0.700833 \pi^{2} + \frac{3.15375}{i} \int_{-i \, \infty + \gamma}^{i \, \infty + \gamma} \frac{\Gamma(-s)^{2} \, \Gamma(1+s)}{\Gamma(1-s)} \, ds$$

for $-1 < \gamma < 0$

From which:

sqrt((((1+0.9568666373)[(((2.9^2 [(-Pi^2)/12+(3Pi)/4 ln(2)]))) * (((0.5^2 [(-Pi^2)/12+(3Pi)/4 ln(2)]))))))

where 0.9568666373 is the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}} - \varphi + 1} = 1 - \frac{e^{-\pi}}{1 + \frac{e^{-2\pi}}{1 + \frac{e^{-2\pi}}{1 + \frac{e^{-3\pi}}{1 + \frac{e^{-4\pi}}{1 + \frac{e^{-4\pi}}{1 + \dots}}}}} \approx 0.9568666373$$

Input interpretation:

$$\sqrt{(1+0.9568666373)\left(\left(2.9^2\left(-\frac{\pi^2}{12}+\frac{3\pi}{4}\log(2)\right)\right)\left(0.5^2\left(-\frac{\pi^2}{12}+\frac{3\pi}{4}\log(2)\right)\right)\right)}$$

log(x) is the natural logarithm

Result:

1.64445...

 $1.64445....\approx \zeta(2)=\frac{\pi^2}{6}=1.644934...$

All 2nd roots of 2.70422:

 $1.64445 e^0 \approx 1.6445$ (real, principal root)

 $1.64445 \, e^{i\,\pi} \approx -\, 1.6445 \, \text{(real root)}$

Alternative representations:

$$\sqrt{(1+0.956867)\left(2.9^2\left(-\frac{\pi^2}{12}+\frac{1}{4}\left(3\,\pi\right)\log(2)\right)\right)\left(0.5^2\left(-\frac{\pi^2}{12}+\frac{1}{4}\left(3\,\pi\right)\log(2)\right)\right)} = \sqrt{1.95687 \times 0.5^2 \times 2.9^2\left(\frac{3\,\pi\log_e(2)}{4}-\frac{\pi^2}{12}\right)^2}$$

$$\sqrt{(1+0.956867)\left(2.9^2\left(-\frac{\pi^2}{12}+\frac{1}{4}\left(3\,\pi\right)\log(2)\right)\right)\left(0.5^2\left(-\frac{\pi^2}{12}+\frac{1}{4}\left(3\,\pi\right)\log(2)\right)\right)} = \sqrt{1.95687 \times 0.5^2 \times 2.9^2\left(\frac{6}{4}\,\pi\,\coth^{-1}(3)-\frac{\pi^2}{12}\right)^2}$$

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$$\sqrt{(1+0.956867)\left(2.9^2\left(-\frac{\pi^2}{12}+\frac{1}{4}\left(3\,\pi\right)\log(2)\right)\right)\left(0.5^2\left(-\frac{\pi^2}{12}+\frac{1}{4}\left(3\,\pi\right)\log(2)\right)\right)} = \sqrt{1.95687 \times 0.5^2 \times 2.9^2\left(\frac{3}{4}\,\pi\log(a)\log_a(2)-\frac{\pi^2}{12}\right)^2}$$

$$\sqrt{(1+0.956867)\left(2.9^2\left(-\frac{\pi^2}{12}+\frac{1}{4}\left(3\,\pi\right)\log(2)\right)\right)\left(0.5^2\left(-\frac{\pi^2}{12}+\frac{1}{4}\left(3\,\pi\right)\log(2)\right)\right)} = \sqrt{-1+0.0285716\,\pi^2\left(\pi-9\log(2)\right)^2}\sum_{k=0}^{\infty} \left(\frac{1}{2}\atop k\right)\left(-1+0.0285716\,\pi^2\left(\pi-9\log(2)\right)^2\right)^{-k}$$

$$\sqrt{ (1 + 0.956867) \left(2.9^2 \left(-\frac{\pi^2}{12} + \frac{1}{4} (3\pi) \log(2) \right) \right) \left(0.5^2 \left(-\frac{\pi^2}{12} + \frac{1}{4} (3\pi) \log(2) \right) \right)} = \sqrt{-1 + 0.0285716 \pi^2 (\pi - 9 \log(2))^2} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-1 + 0.0285716 \pi^2 (\pi - 9 \log(2))^2 \right)^{-k} \left(-\frac{1}{2} \right)_k}{k!}$$

$$\sqrt{(1+0.956867)\left(2.9^2\left(-\frac{\pi^2}{12}+\frac{1}{4}(3\pi)\log(2)\right)\right)\left(0.5^2\left(-\frac{\pi^2}{12}+\frac{1}{4}(3\pi)\log(2)\right)\right)} = \sqrt{\frac{0.0285716\pi^2\left(\pi-9\left(2\,i\,\pi\left\lfloor\frac{\arg(2-x)}{2\,\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(2-x\right)^kx^{-k}}{k}\right)\right)^2}{for \, x<0}}$$

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Integral representations:

$$\sqrt{(1+0.956867)\left(2.9^2\left(-\frac{\pi^2}{12}+\frac{1}{4}(3\pi)\log(2)\right)\right)\left(0.5^2\left(-\frac{\pi^2}{12}+\frac{1}{4}(3\pi)\log(2)\right)\right)} = \sqrt{0.0285716\pi^2\left(\pi-9\int_1^2\frac{1}{t}dt\right)^2}$$

$$\sqrt{\frac{(1+0.956867)\left(2.9^2\left(-\frac{\pi^2}{12}+\frac{1}{4}\left(3\,\pi\right)\log(2)\right)\right)\left(0.5^2\left(-\frac{\pi^2}{12}+\frac{1}{4}\left(3\,\pi\right)\log(2)\right)\right)}{\sqrt{\frac{0.0285716\left(i\,\pi^2-4.5\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,d\,s\right)^2}{i^2}} \quad \text{for } -1<\gamma<0}$$

From:

$$V = -\frac{(2+2+2\times8)}{720\pi R^6} + \sum_P (-1)^{2s_P+1} \frac{n_P}{8\pi} \times \left\{ \frac{1}{90R^6} - \frac{m_p^2}{6R^4} \right\} + \sum_{AP} (-1)^{2s_P+1} n_P \times \left\{ -\frac{7}{8} \frac{1}{90R^6} + \frac{1}{2} \frac{m_p^2}{6R^4} \right\}$$
(2.19)

For:

non-interacting massive particle of spin \boldsymbol{s}_p and mass \boldsymbol{m}_p

Spin = 1/2,

Particle Mass (TeV) $(-1)^{(2s_p+1)}n_p$ $\tilde{u_R}, \tilde{d_R}, \tilde{s_R}, \tilde{c_R}$ 2.76 -24

 $\begin{array}{l} -(2+2+2*8) / (720 Pi * x^{6}) - (24/(8 Pi)) * [(1/(90 x^{6}) - ((2.76)^{2})/(6 x^{4}))] - 24 * [(-7/(720 x^{6}) + ((2.76)^{2})/(12 x^{4}))] \end{array}$

Input:

$$-\frac{2+2+2\times 8}{720\,\pi\,x^6} - \frac{24}{8\,\pi} \left(\frac{1}{90\,x^6} - \frac{2.76^2}{6\,x^4}\right) - 24\left(-\frac{7}{720\,x^6} + \frac{2.76^2}{12\,x^4}\right)$$

Result:

$$-\frac{1}{36\pi x^6} - \frac{3\left(\frac{1}{90x^6} - \frac{1.2696}{x^4}\right)}{\pi} - 24\left(\frac{0.6348}{x^4} - \frac{7}{720x^6}\right)$$

Plots:



Alternate forms:

$$\frac{0.213881 - 14.0228 x^2}{x^6}$$

$$-\frac{4.4636 \left(3.14159 \, x^2-0.0479167\right)}{x^6}$$

$$-\frac{7929.72 x^2 - 42 \pi + 11}{180 \pi x^6}$$

Partial fraction expansion:

 $\frac{42\,\pi-11}{180\,\pi\,x^6}-\frac{14.0228}{x^4}$

Expanded form:

$$-\frac{11}{180\,\pi\,x^6} + \frac{7}{30\,x^6} - \frac{14.0228}{x^4}$$

Roots:

 $x \approx -0.1235$

 $x \approx 0.1235$

Properties as a real function: Domain

 $\{x \in \mathbb{R} : x \neq 0\}$

Range

 $\{ y \in \mathbb{R} : y \ge \left(2590\,035\,913\,944 - 31\,080\,430\,967\,328\,\pi + 124\,321\,723\,869\,312\,\pi^2 - 165\,762\,298\,492\,416\,\pi^3\right) / (1\,181\,640\,625\,\pi - 9\,023\,437\,500\,\pi^2 + 17\,226\,562\,500\,\pi^3) \}$

Parity

even

R is the set of real numbers

Derivative:

$$\frac{d}{dx} \left(-\frac{2+2+2\times 8}{720\,\pi\,x^6} - \frac{24\left(\frac{1}{90\,x^6} - \frac{2.76^2}{6\,x^4}\right)}{8\,\pi} - 24\left(-\frac{7}{720\,x^6} + \frac{2.76^2}{12\,x^4}\right) \right) = \frac{56.0913\,x^2 - 1.28329}{x^7}$$

Indefinite integral:

$$\int \left(-\frac{3\left(\frac{1}{90\,x^6} - \frac{1.2696}{x^4}\right)}{\pi} - 24\left(-\frac{7}{720\,x^6} + \frac{0.6348}{x^4} \right) - \frac{1}{36\,\pi\,x^6} \right) dx = \frac{4.67427\,x^2 - 0.0427762}{x^5} + \text{constant}$$

Global minima:

$$\min\left\{-\frac{2+2+2\times 8}{720\,\pi\,x^6} - \frac{24\left(\frac{1}{90\,x^6} - \frac{2.76^2}{6\,x^4}\right)}{8\,\pi} - 24\left(-\frac{7}{720\,x^6} + \frac{2.76^2}{12\,x^4}\right)\right\} = -\frac{2430\,850\,583\,234\,918\,645\,426\,565\,565\,575\,839\,210\,188\,800\,(4\,\pi-1)^3}{9\,165\,423\,189\,346\,859\,202\,789\,669\,808\,161\,240\,667\,(11-42\,\pi)^2\,\pi}$$
at $x = -\frac{1}{372}\sqrt{\frac{2092\,750\,755\,523\,(42\,\pi-11)}{6\,911\,957\,230\,(4\,\pi-1)}}$

$$\min\left\{-\frac{2+2+2\times 8}{720\,\pi\,x^6} - \frac{24\left(\frac{1}{90\,x^6} - \frac{2.76^2}{6\,x^4}\right)}{8\,\pi} - 24\left(-\frac{7}{720\,x^6} + \frac{2.76^2}{12\,x^4}\right)\right\} = -\frac{2430\,850\,583\,234\,918\,645\,426\,565\,565\,575\,839\,210\,188\,800\,(4\,\pi-1)^3}{9\,165\,423\,189\,346\,859\,202\,789\,669\,808\,161\,240\,667\,(11-42\,\pi)^2\,\pi}$$

at $x = \frac{1}{372}\sqrt{\frac{2092\,750\,755\,523\,(42\,\pi-11)}{6\,911\,957\,230\,(4\,\pi-1)}}$

Global minima:

$$\min\left\{-\frac{2+2+2\times 8}{720\,\pi\,x^6}-\frac{24\left(\frac{1}{90\,x^6}-\frac{2.76^2}{6\,x^4}\right)}{8\,\pi}-24\left(-\frac{7}{720\,x^6}+\frac{2.76^2}{12\,x^4}\right)\right\}\approx-8930.1$$
at $x\approx-0.15126$

$$\min\left\{-\frac{2+2+2\times 8}{720\,\pi\,x^6} - \frac{24\left(\frac{1}{90\,x^6} - \frac{2.76^2}{6\,x^4}\right)}{8\,\pi} - 24\left(-\frac{7}{720\,x^6} + \frac{2.76^2}{12\,x^4}\right)\right\} \approx -8930.1$$
at $x \approx 0.15126$

Limit:

$$\lim_{x \to \pm \infty} \left(-\frac{3\left(\frac{1}{90\,x^6} - \frac{1.2696}{x^4}\right)}{\pi} - 24\left(-\frac{7}{720\,x^6} + \frac{0.6348}{x^4} \right) - \frac{1}{36\,\pi\,x^6} \right) = 0 \approx 0$$

$$-\frac{2+2+2\times8}{720\,\pi\,x^6} - \frac{\left(\frac{1}{90\,x^6} - \frac{2.76^2}{6\,x^4}\right)24}{8\,\pi} - 24\left(-\frac{7}{720\,x^6} + \frac{2.76^2}{12\,x^4}\right) = \\-\frac{24\left(-\frac{2.76^2}{6\,x^4} + \frac{1}{90\,x^6}\right)}{1440^\circ} - 24\left(\frac{2.76^2}{12\,x^4} - \frac{7}{720\,x^6}\right) - \frac{20}{129\,600^\circ x^6}$$

$$-\frac{2+2+2\times8}{720\,\pi\,x^6} - \frac{\left(\frac{1}{90\,x^6} - \frac{2.76^2}{6\,x^4}\right)24}{8\,\pi} - 24\left(-\frac{7}{720\,x^6} + \frac{2.76^2}{12\,x^4}\right) = \frac{-24\left(-\frac{2.76^2}{6\,x^4} + \frac{1}{90\,x^6}\right)}{-8\,i\log(-1)} - 24\left(\frac{2.76^2}{12\,x^4} - \frac{7}{720\,x^6}\right) - -\frac{20}{720\,i\log(-1)\,x^6}$$

$$-\frac{2+2+2\times8}{720\,\pi\,x^6} - \frac{\left(\frac{1}{90\,x^6} - \frac{2.76^2}{6\,x^4}\right)24}{8\,\pi} - 24\left(-\frac{7}{720\,x^6} + \frac{2.76^2}{12\,x^4}\right) = \\-\frac{24\left(-\frac{2.76^2}{6\,x^4} + \frac{1}{90\,x^6}\right)}{8\cos^{-1}(-1)} - 24\left(\frac{2.76^2}{12\,x^4} - \frac{7}{720\,x^6}\right) - \frac{20}{720\cos^{-1}(-1)\,x^6}$$

$$-\frac{2+2+2\times8}{720\,\pi\,x^6} - \frac{\left(\frac{1}{90\,x^6} - \frac{2.76^2}{6\,x^4}\right)24}{8\,\pi} - 24\left(-\frac{7}{720\,x^6} + \frac{2.76^2}{12\,x^4}\right) = \\ -\frac{15.2352\left(0.00100279 - 0.0625\,x^2 - 0.0153154\,\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2\,k} + x^2\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2\,k}\right)}{x^6\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2\,k}}$$

$$-\frac{2+2+2\times 8}{720 \pi x^6} - \frac{\left(\frac{1}{90 x^6} - \frac{2.76^2}{6 x^4}\right) 24}{8 \pi} - 24 \left(-\frac{7}{720 x^6} + \frac{2.76^2}{12 x^4}\right) = \\ \left(0.9522 \left(-0.0160447 + x^2 + 1.0502\right) \right) \\ \sum_{k=0}^{\infty} \frac{1}{1+2 k} (-1)^k 1195^{-2k} \left(-0.00097629 \times 25^k + 0.186667 \times 57121^k + (0.0637456 \times 25^k - 12.1882 \times 57121^k) x^2\right) \right) \right) \\ \left(x^6 \sum_{k=0}^{\infty} \frac{1195^{-2k} \left(0.4 \left(-57121\right)^k - 0.00209205 \left(-25\right)^k\right)}{0.5+k}\right)$$

$$\begin{aligned} -\frac{2+2+2\times8}{720\,\pi\,x^6} - \frac{\left(\frac{1}{90\,x^6} - \frac{2.76^2}{6\,x^4}\right)24}{8\,\pi} - 24\left(-\frac{7}{720\,x^6} + \frac{2.76^2}{12\,x^4}\right) = \\ -\left(\left(15.2352\left(0.00401118 - 0.25\,x^2 - 0.00153154\sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2\,k} + \frac{2}{1+4\,k} + \frac{1}{3+4\,k}\right) + x^2\sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2\,k} + \frac{2}{1+4\,k} + \frac{1}{3+4\,k}\right)\right)\right) \\ \left(x^6\sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2\,k} + \frac{2}{1+4\,k} + \frac{1}{3+4\,k}\right)\right)\right)\end{aligned}$$

$$-\frac{2+2+2\times 8}{720\,\pi\,x^6} - \frac{\left(\frac{1}{90\,x^6} - \frac{2.76^2}{6\,x^4}\right)24}{8\,\pi} - 24\left(-\frac{7}{720\,x^6} + \frac{2.76^2}{12\,x^4}\right) = \\ -\frac{15.2352\left(0.00200559 - 0.125\,x^2 - 0.0153154\,\int_0^\infty \frac{1}{1+t^2}\,dt + x^2\int_0^\infty \frac{1}{1+t^2}\,dt\right)}{x^6\int_0^\infty \frac{1}{1+t^2}\,dt}$$

$$-\frac{2+2+2\times8}{720\,\pi\,x^6} - \frac{\left(\frac{1}{90\,x^6} - \frac{2.76^2}{6\,x^4}\right)24}{8\,\pi} - 24\left(-\frac{7}{720\,x^6} + \frac{2.76^2}{12\,x^4}\right) = -\frac{1}{x^6\int_0^1\sqrt{1-t^2}\,dt} 15.2352$$
$$\left(0.00100279 - 0.0625\,x^2 - 0.0153154\int_0^1\sqrt{1-t^2}\,dt + x^2\int_0^1\sqrt{1-t^2}\,dt\right)$$

$$-\frac{2+2+2\times8}{720\,\pi\,x^6} - \frac{\left(\frac{1}{90\,x^6} - \frac{2.76^2}{6\,x^4}\right)24}{8\,\pi} - 24\left(-\frac{7}{720\,x^6} + \frac{2.76^2}{12\,x^4}\right) = \\ -\frac{15.2352\left(0.00200559 - 0.125\,x^2 - 0.0153154\,\int_0^\infty \frac{\sin(t)}{t}\,dt + x^2\,\int_0^\infty \frac{\sin(t)}{t}\,dt\right)}{x^6\,\int_0^\infty \frac{\sin(t)}{t}\,dt}$$

From:

$$-\frac{1}{36\pi x^6} - \frac{3\left(\frac{1}{90x^6} - \frac{1.2696}{x^4}\right)}{\pi} - 24\left(\frac{0.6348}{x^4} - \frac{7}{720x^6}\right)$$

For x = 0.1235:

$$-(3 (1/(90 0.1235^{6}) - 1.2696/0.1235^{4}))/\pi - 24 (-7/(720 0.1235^{6}) + 0.6348/0.1235^{4}) - 1/(36 \pi 0.1235^{6})$$

Input interpretation:

$$-\frac{3\left(\frac{1}{90\times0.1235^6}-\frac{1.2696}{0.1235^4}\right)}{\pi}-24\left(-\frac{7}{720\times0.1235^6}+\frac{0.6348}{0.1235^4}\right)-\frac{1}{36\,\pi\times0.1235^6}$$

Result:

0.418882...

0.418882....

$$-\frac{3\left(\frac{1}{90\times0.1235^{6}}-\frac{1.2696}{0.1235^{4}}\right)}{\pi}-24\left(-\frac{7}{720\times0.1235^{6}}+\frac{0.6348}{0.1235^{4}}\right)-\frac{1}{36\pi0.1235^{6}}=\left(-\frac{3\left(-\frac{1.2696}{0.1235^{4}}+\frac{1}{90\times0.1235^{6}}\right)}{180^{\circ}}-24\left(\frac{0.6348}{0.1235^{4}}-\frac{7}{720\times0.1235^{6}}\right)-\frac{1}{6480^{\circ}0.1235^{6}}=271.199-\frac{4.726}{\circ}\right)$$

$$-\frac{3\left(\frac{1}{90\times0.1235^{6}}-\frac{1.2696}{0.1235^{4}}\right)}{\pi}-24\left(-\frac{7}{720\times0.1235^{6}}+\frac{0.6348}{0.1235^{4}}\right)-\frac{1}{36\,\pi\,0.1235^{6}}=\\\left(-\frac{3\left(-\frac{1.2696}{0.1235^{4}}+\frac{1}{90\times0.1235^{6}}\right)}{\cos^{-1}(-1)}-24\left(\frac{0.6348}{0.1235^{4}}-\frac{7}{720\times0.1235^{6}}\right)-\\\frac{1}{36\cos^{-1}(-1)\,0.1235^{6}}=271.199-\frac{850.681}{\cos^{-1}(-1)}\right)$$

$$-\frac{3\left(\frac{1}{90\times0.1235^{6}}-\frac{1.2696}{0.1235^{4}}\right)}{\pi}-24\left(-\frac{7}{720\times0.1235^{6}}+\frac{0.6348}{0.1235^{4}}\right)-\frac{1}{36\pi0.1235^{6}}=\\\left(-\frac{3\left(-\frac{1.2696}{0.1235^{4}}+\frac{1}{90\times0.1235^{6}}\right)}{2\,E(0)}-24\left(\frac{0.6348}{0.1235^{4}}-\frac{7}{720\times0.1235^{6}}\right)-\frac{1}{72\,E(0)\,0.1235^{6}}=\\271.199-\frac{425.34}{E(0)}\right)$$

$$-\frac{3\left(\frac{1}{90\times0.1235^{6}}-\frac{1.2696}{0.1235^{4}}\right)}{\pi}-24\left(-\frac{7}{720\times0.1235^{6}}+\frac{0.6348}{0.1235^{4}}\right)-\frac{1}{36\,\pi\,0.1235^{6}}=$$

$$271.199-\frac{212.67}{\sum_{k=0}^{\infty}\frac{(-1)^{k}}{1+2\,k}}$$

$$-\frac{3\left(\frac{1}{90\times0.1235^{6}}-\frac{1.2696}{0.1235^{4}}\right)}{\pi}-24\left(-\frac{7}{720\times0.1235^{6}}+\frac{0.6348}{0.1235^{4}}\right)-\frac{1}{36\,\pi\,0.1235^{6}}=$$

$$271.199-\frac{425.34}{-1+\sum_{k=1}^{\infty}\frac{2^{k}}{\binom{2k}{k}}}$$

$$-\frac{3\left(\frac{1}{90\times0.1235^{6}}-\frac{1.2696}{0.1235^{4}}\right)}{\pi}-24\left(-\frac{7}{720\times0.1235^{6}}+\frac{0.6348}{0.1235^{4}}\right)-\frac{1}{36\,\pi\,0.1235^{6}}=$$

$$271.199-\frac{850.681}{\sum_{k=0}^{\infty}\frac{2^{-k}\left(-6+50\,k\right)}{\binom{3\,k}{k}}}$$

$$-\frac{3\left(\frac{1}{90\times0.1235^{6}}-\frac{1.2696}{0.1235^{4}}\right)}{\pi}-24\left(-\frac{7}{720\times0.1235^{6}}+\frac{0.6348}{0.1235^{4}}\right)-\frac{1}{36\,\pi\,0.1235^{6}}=271.199-\frac{425.34}{\int_{0}^{\infty}\frac{1}{1+t^{2}}\,dt}$$

$$-\frac{3\left(\frac{1}{90\times0.1235^{6}}-\frac{1.2696}{0.1235^{4}}\right)}{\pi}-24\left(-\frac{7}{720\times0.1235^{6}}+\frac{0.6348}{0.1235^{4}}\right)-\frac{1}{36\,\pi\,0.1235^{6}}=$$

$$271.199-\frac{212.67}{\int_{0}^{1}\sqrt{1-t^{2}}\,dt}$$

$$-\frac{3\left(\frac{1}{90\times0.1235^{6}}-\frac{1.2696}{0.1235^{4}}\right)}{\pi}-24\left(-\frac{7}{720\times0.1235^{6}}+\frac{0.6348}{0.1235^{4}}\right)-\frac{1}{36\,\pi\,0.1235^{6}}=$$

$$271.199-\frac{425.34}{\int_{0}^{\infty}\frac{\sin(t)}{t}\,dt}$$

From which:

 $\begin{array}{l} (3+0.9568666373)((-(3\ (1/(90\ 0.1235^6)\ -\ 1.2696/0.1235^4))/\pi\ -\ 24\ (-7/(720\ 0.1235^6)\ +\ 0.6348/0.1235^4)\ -\ 1/(36\ \pi\ 0.1235^6)))) \end{array}$

where 0.9568666373 is the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}-\varphi+1}} = 1 - \frac{e^{-\pi}}{1+\frac{e^{-2\pi}}{1+\frac{e^{-2\pi}}{1+\frac{e^{-4\pi}}{1+\frac{e^{-4\pi}}{1+\dots}}}}} \approx 0.9568666373$$

Input interpretation:

$$\begin{pmatrix} 3+0.9568666373) \\ \left(-\frac{3\left(\frac{1}{90\times0.1235^6}-\frac{1.2696}{0.1235^4}\right)}{\pi}-24\left(-\frac{7}{720\times0.1235^6}+\frac{0.6348}{0.1235^4}\right)-\frac{1}{36\,\pi\times0.1235^6}\right)$$

Result:

1.6574587999242426068708009669496280285514866570077400318856203198 ...

1.65745879..... result very near to the 14th root of the following Ramanujan's class invariant $Q = (G_{505}/G_{101/5})^3 = 1164.2696$ i.e. 1.65578...

$$\begin{pmatrix} 3+0.956867)\\ \left(-\frac{3\left(\frac{1}{90\times0.1235^{6}}-\frac{1.2696}{0.1235^{4}}\right)}{\pi}-24\left(-\frac{7}{720\times0.1235^{6}}+\frac{0.6348}{0.1235^{4}}\right)-\frac{1}{36\pi0.1235^{6}}\right) = \\ \left(3.95687\left(-\frac{3\left(-\frac{1.2696}{0.1235^{4}}+\frac{1}{90\times0.1235^{6}}\right)}{180^{\circ}}-24\left(\frac{0.6348}{0.1235^{4}}-\frac{7}{720\times0.1235^{6}}\right)-\frac{1}{6480^{\circ}0.1235^{6}}\right) = 3.95687\left(271.199-\frac{4.726}{\circ}\right) \right)$$

$$(3 + 0.956867) \left(-\frac{3\left(\frac{1}{90 \times 0.1235^{6}} - \frac{1.2696}{0.1235^{4}}\right)}{\pi} - 24\left(-\frac{7}{720 \times 0.1235^{6}} + \frac{0.6348}{0.1235^{4}}\right) - \frac{1}{36 \pi 0.1235^{6}} \right) = \left(3.95687 \left(-\frac{3\left(-\frac{1.2696}{0.1235^{4}} + \frac{1}{90 \times 0.1235^{6}}\right)}{\cos^{-1}(-1)} - 24\left(\frac{0.6348}{0.1235^{4}} - \frac{7}{720 \times 0.1235^{6}}\right) - \frac{1}{36 \cos^{-1}(-1) 0.1235^{6}} \right) = 3.95687 \left(271.199 - \frac{850.681}{\cos^{-1}(-1)} \right) \right)$$

$$\begin{pmatrix} (3+0.956867) \\ \left(-\frac{3\left(\frac{1}{90\times0.1235^6} - \frac{1.2696}{0.1235^4}\right)}{\pi} - 24\left(-\frac{7}{720\times0.1235^6} + \frac{0.6348}{0.1235^4}\right) - \frac{1}{36\pi0.1235^6} \right) = \\ \left(3.95687 \left(-\frac{3\left(-\frac{1.2696}{0.1235^4} + \frac{1}{90\times0.1235^6}\right)}{2 E(0)} - 24\left(\frac{0.6348}{0.1235^4} - \frac{7}{720\times0.1235^6}\right) - \frac{1}{72 E(0) 0.1235^6} \right) = 3.95687 \left(271.199 - \frac{425.34}{E(0)} \right) \right)$$

$$\begin{pmatrix} (3+0.956867) \\ \left(-\frac{3\left(\frac{1}{90\times0.1235^6} - \frac{1.2696}{0.1235^4}\right)}{\pi} - 24\left(-\frac{7}{720\times0.1235^6} + \frac{0.6348}{0.1235^4} \right) - \frac{1}{36\pi0.1235^6} \right) = 1073.1 - \frac{841.508}{\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}}$$

$$\begin{pmatrix} 3+0.956867) \\ \left(-\frac{3\left(\frac{1}{90\times0.1235^{6}}-\frac{1.2696}{0.1235^{4}}\right)}{\pi}-24\left(-\frac{7}{720\times0.1235^{6}}+\frac{0.6348}{0.1235^{4}}\right)-\frac{1}{36\pi0.1235^{6}}\right) = \\ 1073.1-\frac{1683.02}{-1+\sum_{k=1}^{\infty}\frac{2^{k}}{\binom{2k}{k}}}$$

$$\begin{pmatrix} (3+0.956867) \\ \left(-\frac{3\left(\frac{1}{90\times0.1235^6} - \frac{1.2696}{0.1235^4}\right)}{\pi} - 24\left(-\frac{7}{720\times0.1235^6} + \frac{0.6348}{0.1235^4} \right) - \frac{1}{36\pi0.1235^6} \right) = 1073.1 - \frac{3366.03}{\sum_{k=0}^{\infty} \frac{2^{-k}(-6+50k)}{\binom{3k}{k}}}$$

$$\begin{pmatrix} 3+0.956867)\\ \left(-\frac{3\left(\frac{1}{90\times0.1235^6}-\frac{1.2696}{0.1235^4}\right)}{\pi}-24\left(-\frac{7}{720\times0.1235^6}+\frac{0.6348}{0.1235^4}\right)-\frac{1}{36\,\pi\,0.1235^6}\right) = 1073.1-\frac{1683.02}{\int_0^\infty\frac{1}{1+t^2}\,dt}$$

$$\begin{pmatrix} 3+0.956867) \\ \left(-\frac{3\left(\frac{1}{90\times0.1235^6}-\frac{1.2696}{0.1235^4}\right)}{\pi}-24\left(-\frac{7}{720\times0.1235^6}+\frac{0.6348}{0.1235^4}\right)-\frac{1}{36\,\pi\,0.1235^6}\right) = \\ 1073.1-\frac{841.508}{\int_0^1\sqrt{1-t^2}\ dt}$$

$$\begin{pmatrix} 3+0.956867) \\ \left(-\frac{3\left(\frac{1}{90\times0.1235^6}-\frac{1.2696}{0.1235^4}\right)}{\pi}-24\left(-\frac{7}{720\times0.1235^6}+\frac{0.6348}{0.1235^4}\right)-\frac{1}{36\,\pi\,0.1235^6}\right) = 1073.1 - \frac{1683.02}{\int_0^\infty \frac{\sin(t)}{t}\,dt}$$

With regard the global minima:

$$\min\left\{-\frac{2+2+2\times 8}{720\,\pi\,x^6} - \frac{24\left(\frac{1}{90\,x^6} - \frac{2.76^2}{6\,x^4}\right)}{8\,\pi} - 24\left(-\frac{7}{720\,x^6} + \frac{2.76^2}{12\,x^4}\right)\right\} \approx -8930.1$$

at $x \approx -0.15126$

We have that:

-1/372 sqrt((2092750755523 (42 π - 11))/(6911957230 (4 π - 1)))

Input:

-

$$-\frac{1}{372}\sqrt{\frac{2092750755523(42\pi-11)}{6911957230(4\pi-1)}}$$

Decimal approximation:

-0.151256517156642996910451527470557054387252430352248057534918833 ...

x = -0.151256517156

Property:

$$-\frac{1}{372}\sqrt{\frac{2092750755523(-11+42\pi)}{6911957230(-1+4\pi)}}$$
 is a transcendental number

Series representations:

$$\frac{1}{372} \sqrt{\frac{2092750755523(42\pi - 11)}{6911957230(4\pi - 1)}} (-1) = -\frac{1}{372} \sqrt{-1 + \frac{2092750755523(-11 + 42\pi)}{6911957230(-1 + 4\pi)}} \\ \sum_{k=0}^{\infty} \left(-1 + \frac{2092750755523(-11 + 42\pi)}{6911957230(-1 + 4\pi)}\right)^{-k} \left(\frac{1}{2} \atop k\right)^{-k}$$

$$\frac{1}{372} \sqrt{\frac{2092750755523(42\pi - 11)}{6911957230(4\pi - 1)}} (-1) = \\ -\frac{1}{372} \sqrt{-1 + \frac{2092750755523(-11 + 42\pi)}{6911957230(-1 + 4\pi)}} \\ \sum_{k=0}^{\infty} \frac{(-1)^k \left(-1 + \frac{2092750755523(-11 + 42\pi)}{6911957230(-1 + 4\pi)}\right)^{-k} \left(-\frac{1}{2}\right)_k}{k!}$$

$$\begin{split} &\frac{1}{372} \sqrt{\frac{2092750755523(42\pi - 11)}{6911957230(4\pi - 1)}} \quad (-1) = \\ &-\frac{1}{372} \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{2092750755523(-11+42\pi)}{6911957230(-1+4\pi)} - z_0\right)^k z_0^{-k}}{k!} \\ &\text{for (not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0)) \end{split}$$

 $\{ -(2 + 2 + 2 8)/(720 \pi^* - 0.151256517156^{\circ}6) - (24 (1/(90 * -0.151256517156^{\circ}6) - 2.76^{\circ}2/(6 * -0.151256517156^{\circ}4)))/(8 \pi) - 24 (-7/(720 * -0.151256517156^{\circ}6) + 2.76^{\circ}2/(12 * -0.151256517156^{\circ}4))) \}$

Input interpretation:

$$-\frac{2+2+2\times 8}{720 \pi \times (-1) \times 0.151256517156^{6}} - \frac{2.76^{2}}{6 \times (-1) \times 0.151256517156^{6}} - \frac{2.76^{2}}{6 \times (-1) \times 0.151256517156^{4}} - \frac{8 \pi}{24 \left(-\frac{7}{720 \times (-1) \times 0.151256517156^{6}} + \frac{2.76^{2}}{12 \times (-1) \times 0.151256517156^{4}}\right) - \frac{8 \pi}{12 \times (-1) \times 0.151256517156^{4}} - \frac{8 \pi}{12 \times (-1) \times (-1) \times 0.151256517156^{4}} - \frac{8 \pi}{12 \times (-1) \times 0.151256517156$$

Result:

8930.13...

8930.13...











$$-\frac{2+2+2\times8}{720\,\pi\,(-1)\,0.1512565171560000^6} - \frac{2.76^2}{6\,(-1)\,0.1512565171560000^4} \Big) - \frac{2.76^2}{8\,\pi} - \frac{2.76^2}{8\,\pi} - \frac{2.76^2}{720\,(-1)\,0.1512565171560000^6} + \frac{2.76^2}{12\,(-1)\,0.1512565171560000^4} \Big) = 9621.99 - \frac{2173.54}{\sum_{k=0}^{\infty} \frac{2^{-k}(-6+50\,k)}{\binom{3k}{k}}}$$

$$-\frac{2+2+2\times8}{720\,\pi\,(-1)\,0.1512565171560000^6} - \frac{2.76^2}{6\,(-1)\,0.1512565171560000^4} \Big) - \frac{2.76^2}{8\,\pi} - \frac{2.76^2}{8\,\pi} - \frac{2.76^2}{720\,(-1)\,0.1512565171560000^6} + \frac{2.76^2}{12\,(-1)\,0.1512565171560000^4} \Big) = 9621.99 - \frac{1086.77}{\int_0^\infty \frac{1}{1+t^2} dt}$$

$$-\frac{2+2+2\times8}{720\,\pi\,(-1)\,0.1512565171560000^6} -\frac{2.76^2}{6\,(-1)\,0.1512565171560000^4} \Big) \\ -\frac{24\left(\frac{1}{90\,(-1)\,0.1512565171560000^6} -\frac{2.76^2}{6\,(-1)\,0.1512565171560000^4}\right)}{8\,\pi} -\frac{2.76^2}{12\,(-1)\,0.1512565171560000^4} \Big) = 9621.99 -\frac{543.384}{\int_0^1 \sqrt{1-t^2} \,dt}$$

$$-\frac{2+2+2\times8}{720\,\pi\,(-1)\,0.1512565171560000^6} - \frac{2.76^2}{6\,(-1)\,0.1512565171560000^4} \Big) - \frac{2.76^2}{8\,\pi} - \frac{2.76^2}{8\,\pi} - \frac{2.76^2}{720\,(-1)\,0.1512565171560000^6} + \frac{2.76^2}{12\,(-1)\,0.1512565171560000^4} \Big) = 9621.99 - \frac{1086.77}{\int_0^\infty \frac{\sin(t)}{t} dt}$$

From the right-hand side of the below expression

$$\min\left\{-\frac{2+2+2\times 8}{720\,\pi\,x^6} - \frac{24\left(\frac{1}{90\,x^6} - \frac{2.76^2}{6\,x^4}\right)}{8\,\pi} - 24\left(-\frac{7}{720\,x^6} + \frac{2.76^2}{12\,x^4}\right)\right\} = \\ -\frac{2430\,850\,583\,234\,918\,645\,426\,565\,565\,575\,839\,210\,188\,800\,(4\,\pi-1)^3}{9\,165\,423\,189\,346\,859\,202\,789\,669\,808\,161\,240\,667\,(11-42\,\pi)^2\,\pi} \\ \text{at } x = -\frac{1}{372}\sqrt{\frac{2092\,750\,755\,523\,(42\,\pi-11)}{6\,911\,957\,230\,(4\,\pi-1)}}}$$

we obtain:

-(2.43085058e+43 (4 π - 1)^3)/(9.16542318e+37 (11 - 42 π)^2 π)

Input interpretation:

 $-\frac{2.43085058 \times 10^{43} \left(4 \, \pi - 1\right)^3}{9.16542318 \times 10^{37} \left(11 - 42 \, \pi\right)^2 \pi}$

Result: -8930.1296... -8930.1296... From which:

 $[-(2 + 2 + 2*8)/(720 \pi^* - 0.151256517^6) - (24(1/(90 * - 0.151256517^6) - 2.76^2/(6 * - 0.151256517^4)))/(8\pi) - 24(-7/(720 * - 0.151256517^6) + 2.76^2/(12 * - 0.151256517^4))] - (6^3 + 8^3) - 10$

Input interpretation:

$$\left(-\frac{2+2+2\times 8}{720\,\pi\times(-1)\times0.151256517^6}-\frac{24\left(\frac{1}{90\times(-1)\times0.151256517^6}-\frac{2.76^2}{6\times(-1)\times0.151256517^4}\right)}{8\,\pi}-\frac{24\left(-\frac{7}{720\times(-1)\times0.151256517^6}+\frac{2.76^2}{12\times(-1)\times0.151256517^4}\right)\right)-\left(6^3+8^3\right)-10$$

Result:

8192.13...

8192.13.... ≈ **8192**

The total amplitude vanishes for gauge group SO(8192), while the vacuum energy is negative and independent of the gauge group.

The vacuum energy and dilaton tadpole to lowest non-trivial order for the open bosonic string. While the vacuum energy is non-zero and independent of the gauge group, the dilaton tadpole is zero for a unique choice of gauge group, $SO(2^{13})$ i.e. SO(8192). (From: "Dilaton Tadpole for the Open Bosonic String " Michael R. Douglas and Benjamin Grinstein - September 2,1986)
Alternative representations:

$$\begin{pmatrix} -\frac{2+2+2\times8}{720\,\pi\,(-1)\,0.151257^6} - \frac{24\left(\frac{1}{90\,(-1)\,0.151257^6} - \frac{2.76^2}{6\,(-1)\,0.151257^4}\right)}{8\,\pi} - \\ 24\left(-\frac{7}{720\,(-1)\,0.151257^6} + \frac{2.76^2}{12\,(-1)\,0.151257^4}\right) \end{pmatrix} - \left(6^3 + 8^3\right) - 10 = \\ -10 - 6^3 - 8^3 - 24\left(-\frac{2.76^2}{12\times0.151257^4} - -\frac{7}{720\times0.151257^6}\right) - \\ \frac{24\left(\frac{-2.76^2}{-6\times0.151257^4} + -\frac{1}{90\times0.151257^6}\right)}{1440\,^\circ} - -\frac{20}{129\,600\,^\circ\,0.151257^6}$$

$$\begin{pmatrix} -\frac{2+2+2\times8}{720\,\pi\,(-1)\,0.151257^6} - \frac{24\left(\frac{1}{90\,(-1)\,0.151257^6} - \frac{2.76^2}{6\,(-1)\,0.151257^4}\right)}{8\,\pi} - \\ 24\left(-\frac{7}{720\,(-1)\,0.151257^6} + \frac{2.76^2}{12\,(-1)\,0.151257^4}\right) \end{pmatrix} - \left(6^3 + 8^3\right) - 10 = \\ -10 - 6^3 - 8^3 - 24\left(-\frac{2.76^2}{12\times0.151257^4} - -\frac{7}{720\times0.151257^6}\right) - \\ -\frac{24\left(\frac{-2.76^2}{-6\times0.151257^4} + -\frac{1}{90\times0.151257^6}\right)}{8\,i\log(-1)} - \frac{20}{720\,i\log(-1)\,0.151257^6}$$

$$\left(-\frac{2+2+2\times 8}{720\,\pi\,(-1)\,0.151257^6} - \frac{24\left(\frac{1}{90\,(-1)\,0.151257^6} - \frac{2.76^2}{6\,(-1)\,0.151257^4}\right)}{8\,\pi} - \frac{24\left(-\frac{7}{720\,(-1)\,0.151257^6} + \frac{2.76^2}{12\,(-1)\,0.151257^4}\right)\right) - \left(6^3 + 8^3\right) - 10 = -10 - 6^3 - 8^3 - 24\left(-\frac{2.76^2}{12\times0.151257^4} - -\frac{7}{720\times0.151257^6}\right) - \frac{24\left(\frac{-2.76^2}{-6\times0.151257^4} + -\frac{1}{90\times0.151257^6}\right)}{8\cos^{-1}(-1)} - \frac{20}{720\,\cos^{-1}(-1)\,0.151257^6}$$

Series representations:

$$\left(-\frac{2+2+2\times 8}{720\,\pi\,(-1)\,0.151257^6} - \frac{24\left(\frac{1}{90\,(-1)\,0.151257^6} - \frac{2.76^2}{6\,(-1)\,0.151257^4}\right)}{8\,\pi} - \frac{24\left(-\frac{7}{720\,(-1)\,0.151257^6} + \frac{2.76^2}{12\,(-1)\,0.151257^4}\right)}{12\,(-1)\,0.151257^4}\right) \right) - \left(6^3 + 8^3\right) - 10 = 8883.99 - \frac{543.384}{\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}}$$

$$\left(-\frac{2+2+2\times 8}{720\,\pi\,(-1)\,0.151257^6} - \frac{24\left(\frac{1}{90\,(-1)\,0.151257^6} - \frac{2.76^2}{6\,(-1)\,0.151257^4}\right)}{8\,\pi} - \frac{24\left(-\frac{7}{720\,(-1)\,0.151257^6} + \frac{2.76^2}{12\,(-1)\,0.151257^4}\right)\right) - \frac{22}{12\,(-1)\,0.151257^4} + \frac{2.76^2}{12\,(-1)\,0.151257^4} + \frac{2.76^$$

$$\left(-\frac{2+2+2\times 8}{720\,\pi\,(-1)\,0.151257^6} - \frac{24\left(\frac{1}{90\,(-1)\,0.151257^6} - \frac{2.76^2}{6\,(-1)\,0.151257^4}\right)}{8\,\pi} - \frac{24\left(-\frac{7}{720\,(-1)\,0.151257^6} + \frac{2.76^2}{12\,(-1)\,0.151257^4}\right)\right) - \frac{24\left(-\frac{7}{720\,(-1)\,0.151257^6} + \frac{2.76^2}{12\,(-1)\,0.151257^4}\right) - \frac{2173.54}{\sum_{k=0}^{\infty} \frac{2^{-k}(-6+50\,k)}{\binom{3k}{k}}} \right)$$

Integral representations:

$$\left(-\frac{2+2+2\times 8}{720\,\pi\,(-1)\,0.151257^6} - \frac{24\left(\frac{1}{90\,(-1)\,0.151257^6} - \frac{2.76^2}{6\,(-1)\,0.151257^4}\right)}{8\,\pi} - \frac{24\left(-\frac{7}{720\,(-1)\,0.151257^6} + \frac{2.76^2}{12\,(-1)\,0.151257^4}\right)}{12\,(-1)\,0.151257^4}\right) \right) - \left(6^3 + 8^3\right) - 10 = 8883.99 - \frac{1086.77}{\int_0^\infty \frac{1}{1+t^2}\,dt}$$

$$\left(-\frac{2+2+2\times 8}{720\,\pi\,(-1)\,0.151257^6} - \frac{24\left(\frac{1}{90\,(-1)\,0.151257^6} - \frac{2.76^2}{6\,(-1)\,0.151257^4}\right)}{8\,\pi} - \frac{24\left(-\frac{7}{720\,(-1)\,0.151257^6} + \frac{2.76^2}{12\,(-1)\,0.151257^4}\right)\right) - \frac{24\left(-\frac{7}{720\,(-1)\,0.151257^6} + \frac{2.76^2}{12\,(-1)\,0.151257^4}\right) - \frac{6^3+8^3\right) - 10 = 8883.99 - \frac{543.384}{\int_0^1 \sqrt{1-t^2} \,dt}$$

$$\left(-\frac{2+2+2\times 8}{720\,\pi\,(-1)\,0.151257^6} - \frac{24\left(\frac{1}{90\,(-1)\,0.151257^6} - \frac{2.76^2}{6\,(-1)\,0.151257^4}\right)}{8\,\pi} \right)$$

$$24\left(-\frac{7}{720\,(-1)\,0.151257^6} + \frac{2.76^2}{12\,(-1)\,0.151257^4} \right) \right) -$$

$$\left(6^3 + 8^3 \right) - 10 = 8883.99 - \frac{1086.77}{\int_0^\infty \frac{\sin(t)}{t} \,dt}$$

_

From the indefinite integral:

$$\int \left(-\frac{3\left(\frac{1}{90\,x^6} - \frac{1.2696}{x^4}\right)}{\pi} - 24\left(-\frac{7}{720\,x^6} + \frac{0.6348}{x^4} \right) - \frac{1}{36\,\pi\,x^6} \right) dx = \frac{4.67427\,x^2 - 0.0427762}{x^5} + \text{constant}$$

integral(-(3 (1/(90 x^6) - 1.2696/x^4))/ π - 24 (-7/(720 x^6) + 0.6348/x^4) - 1/(36 π x^6)) dx

Indefinite integral:



Plots of the integral:





Alternate forms of the integral:

$$\frac{4.67427 \left(x-0.095663\right) \left(x+0.095663\right)}{x^5} + \text{constant}$$

$$-\frac{0.0427762 - 4.67427 x^2}{x^5} + \text{constant}$$

Partial fraction expansion:

 $\frac{4.67427}{x^3} - \frac{0.0427762}{x^5} + \text{constant}$

Alternate form assuming x is real:

 $-\frac{0.0427762}{x^5}+\frac{4.67427}{x^3}+0+\text{constant}$

For x = 0.2:

 $(((4.67427 * 0.2^2 - 0.0427762)/0.2^{5})) + (((4.67427 * 1.2^2 - 0.0427762)/1.2^{5}))$

Input interpretation:

 $\frac{4.67427 \times 0.2^2 - 0.0427762}{0.2^5} + \frac{4.67427 \times 1.2^2 - 0.0427762}{1.2^5}$

Result:

453.29595156571502057613168724279835390946502057613168724279835390

453.2959515....

From which:

 $\frac{1}{7}(((((4.67427 * 0.2^2 - 0.0427762)/0.2^5)) + (((4.67427 * 1.2^2 - 0.0427762)/1.2^5)) -5 - ((((\sqrt{(10-2\sqrt{5})-2}))/((\sqrt{5-1}))))))$

Where $(((\sqrt{10-2\sqrt{5}})-2))/((\sqrt{5}-1))) = \kappa$

Input interpretation:

$$\frac{1}{7} \left(\frac{4.67427 \times 0.2^2 - 0.0427762}{0.2^5} + \frac{4.67427 \times 1.2^2 - 0.0427762}{1.2^5} - 5 - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{\sqrt{5} - 1} \right)$$

Result: 64.0017... $\approx 64 = 8^2$

 $\begin{array}{l} 27*1/7(((((4.67427 * 0.2^2 - 0.0427762)/0.2^5)) + (((4.67427 * 1.2^2 - 0.0427762)/1.2^5)) -5 - ((((\sqrt{(10-2}\sqrt{5})-2))/((\sqrt{5}-1)))))) + 1 \end{array}$

Input interpretation:

$$27 \times \frac{1}{7} \left(\frac{4.67427 \times 0.2^2 - 0.0427762}{0.2^5} + \frac{4.67427 \times 1.2^2 - 0.0427762}{1.2^5} - 5 - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{\sqrt{5} - 1} \right) + 1$$

Result:

1729.05... 1729.05....

This result is very near to the mass of candidate glueball $f_0(1710)$ scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. (1728 = $8^2 * 3^3$) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

$$[27*1/7(((((4.67427 * 0.2^2 - 0.0427762)/0.2^5)) + (((4.67427 * 1.2^2 - 0.0427762)/1.2^5)) -5-((((\sqrt{(10-2\sqrt{5})} - 2))/((\sqrt{5-1}))))) + 1]^{1/15}$$

Input interpretation:

$$\left(27 \times \frac{1}{7} \left(\frac{4.67427 \times 0.2^2 - 0.0427762}{0.2^5} + \frac{4.67427 \times 1.2^2 - 0.0427762}{1.2^5} - 5 - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{\sqrt{5} - 1} \right) + 1 \right) \uparrow (1/15)$$

Result:

1.64382...

$$1.64382....\approx \zeta(2)=\frac{\pi^2}{6}=1.644934...$$

From the derivative:

$$\frac{d}{dx} \left(-\frac{2+2+2\times 8}{720\,\pi\,x^6} - \frac{24\left(\frac{1}{90\,x^6} - \frac{2.76^2}{6\,x^4}\right)}{8\,\pi} - 24\left(-\frac{7}{720\,x^6} + \frac{2.76^2}{12\,x^4}\right) \right) = \frac{56.0913\,x^2 - 1.28329}{x^7}$$

 $\frac{d}{dx}(-(2+2+2\times 8)/(720 \pi x^{6}) - (24 (1/(90 x^{6}) - 2.76^{2}/(6 x^{4})))/(8 \pi) - 24 (-7/(720 x^{6}) + 2.76^{2}/(12 x^{4}))) = (56.0913 x^{2} - 1.28329)/x^{7}$

Input interpretation:

$$\frac{\partial}{\partial x} \left(-\frac{2+2+2\times 8}{720\,\pi\,x^6} - \frac{24\left(\frac{1}{90\,x^6} - \frac{2.76^2}{6\,x^4}\right)}{8\,\pi} - 24\left(-\frac{7}{720\,x^6} + \frac{2.76^2}{12\,x^4}\right) \right) = \frac{56.0913\,x^2 - 1.28329}{x^7}$$

Result:

$$\frac{56.0913 \, x^2 - 1.28329}{x^7} = \frac{56.0913 \, x^2 - 1.28329}{x^7}$$

Plot:



Alternate forms assuming x is real:

 $x = \frac{0.245293}{x}$

$$-\frac{1.28329}{x^7} + \frac{56.0913}{x^5} + 0 = -\frac{1.28329}{x^7} + \frac{56.0913}{x^5} + 0$$

Alternate form:

$$\frac{56.0913 \left(x - 0.151257\right) \left(x + 0.151257\right)}{x^7} = \frac{56.0913 \left(x - 0.151257\right) \left(x + 0.151257\right)}{x^7}$$

Alternate form assuming x is positive:

x = 0.49527

Expanded form:

$$\frac{56.0913}{x^5} - \frac{1.28329}{x^7} = \frac{56.0913}{x^5} - \frac{1.28329}{x^7}$$

Solutions:

 $x \approx -0.49527$

 $x \approx 0.49527$

0.49527

(56.0913 * 0.49527^2 - 1.28329)/0.49527^7

Input interpretation:

 $\frac{56.0913 \times 0.49527^2 - 1.28329}{0.49527^7}$

Result:

1706.7230017390806306583878395920601480117369054483945688914018818 ... 1706.723001739....

From which:

(56.0913 * 0.49527^2 - 1.28329)/0.49527^7 + 21+golden ratio - (((($\sqrt{(10-2\sqrt{5}) - 2)}$)(($\sqrt{5}$ -1))))

Input interpretation:

$$\frac{56.0913 \times 0.49527^2 - 1.28329}{0.49527^7} + 21 + \phi - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{\sqrt{5} - 1}$$

 ϕ is the golden ratio

Result:

1729.06...

1729.06....

This result is very near to the mass of candidate glueball $f_0(1710)$ scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. (1728 = $8^2 * 3^3$) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Series representations:

$$\frac{56.0913 \times 0.49527^2 - 1.28329}{0.49527^7} + 21 + \phi - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{\sqrt{5} - 1} = \left(-1725.72 - \phi + 1727.72\sqrt{4}\sum_{k=0}^{\infty} 4^{-k} \left(\frac{1}{2}\atop k\right) + \phi\sqrt{4}\sum_{k=0}^{\infty} 4^{-k} \left(\frac{1}{2}\atop k\right) - \sqrt{9 - 2\sqrt{5}}\sum_{k=0}^{\infty} \left(\frac{1}{2}\atop k\right) (9 - 2\sqrt{5})^{-k}\right) / \left(-1 + \sqrt{4}\sum_{k=0}^{\infty} 4^{-k} \left(\frac{1}{2}\atop k\right)\right)$$

$$\frac{56.0913 \times 0.49527^2 - 1.28329}{0.49527^7} + 21 + \phi - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{\sqrt{5} - 1} = \left(-1725.72 - \phi + 1727.72\sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!} + \phi\sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!} - \sqrt{9 - 2\sqrt{5}} \sum_{k=0}^{\infty} \frac{\left(-1\right)^k \left(-\frac{1}{2}\right)_k \left(9 - 2\sqrt{5}\right)^{-k}}{k!} \right) / \left(-1 + \sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right)$$

$$\frac{56.0913 \times 0.49527^2 - 1.28329}{0.49527^7} + 21 + \phi - \frac{\sqrt{10 - 2\sqrt{5} - 2}}{\sqrt{5} - 1} = \left(-1725.72 - \phi + 1727.72\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5 - z_0)^k z_0^{-k}}{k!} + \phi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5 - z_0)^k z_0^{-k}}{k!} - \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (10 - 2\sqrt{5} - z_0)^k z_0^{-k}}{k!} - \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5 - z_0)^k z_0^{-k}}{k!} - \left(-1 + \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5 - z_0)^k z_0^{-k}}{k!} \right) \right)$$
for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \le 0$))

 $[(56.0913 * 0.49527^2 - 1.28329)/0.49527^7 + 21+golden ratio - ((((\sqrt{(10-2\sqrt{5}) - 2)})((\sqrt{5-1}))))]^1/15$

Input interpretation:

$$\sqrt[15]{\frac{56.0913 \times 0.49527^2 - 1.28329}{0.49527^7} + 21 + \phi} - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{\sqrt{5} - 1}$$

 ϕ is the golden ratio

Result:

r

1.64382...

 $1.64382....\approx \zeta(2)=\frac{\pi^2}{6}=1.644934...$

Performing the double integral:

 $[-(24 (-2.76^2/(6 x^4) + 1/(90 x^6)))/(8 \cos^{(-1)(-1)}) - 24 (2.76^2/(12 x^4) - 7/(720 x^6)) - 20/(720 \cos^{(-1)(-1)} x^6)] dxdy$

Input:

$$\int \int \left(-\frac{24\left(-\frac{2.76^2}{6\,x^4} + \frac{1}{90\,x^6}\right)}{8\cos^{-1}(-1)} - 24\left(\frac{2.76^2}{12\,x^4} - \frac{7}{720\,x^6}\right) - \frac{20}{720\cos^{-1}(-1)\,x^6} \right) dx \, dy$$

 $\cos^{-1}(x)$ is the inverse cosine function

Result:

$$\frac{(4.67427 x^2 - 0.0427762) y}{x^5}$$

3D plot:



Contour plot:



Indefinite integral assuming all variables are real:

$$\Big(\frac{0.0106941}{x^4} - \frac{2.33714}{x^2}\Big)y + \text{constant}$$

From:

$$\frac{(4.67427 \, x^2 - 0.0427762) \, y}{x^5}$$

For x = 0.2 and y = 0.5:

 $(((1/4[((4.67427 * 0.2^2 - 0.0427762) 0.5)/0.2^5 + 34 - 3 - ((((\sqrt{(10-2\sqrt{5})} - 2))/((\sqrt{5-1})))))))^2 - 1/golden ratio$

Input interpretation:

$$\left(\frac{1}{4}\left(\frac{\left(4.67427\times0.2^2-0.0427762\right)\times0.5}{0.2^5}+34-3-\frac{\sqrt{10-2\,\sqrt{5}}}{\sqrt{5}\,-1}\right)\right)^2-\frac{1}{\phi}$$

 ϕ is the golden ratio

Result:

4096.02...

 $4096.02....\approx 4096 = 64^2$

Series representations:

$$\left(\frac{1}{4} \left(\frac{\left(4.67427 \times 0.2^2 - 0.0427762\right)0.5}{0.2^5} + 34 - 3 - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{\sqrt{5} - 1} \right) \right)^2 - \frac{1}{\phi} = -\frac{1}{\phi} + \frac{1}{16} \left(256.304 + \frac{2 - \sqrt{9 - 2\sqrt{5}} \sum_{k=0}^{\infty} \left(\frac{1}{2} \atop k\right) \left(9 - 2\sqrt{5}\right)^{-k}}{-1 + \sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \left(\frac{1}{2} \atop k\right)} \right)^2$$

$$\left(\frac{1}{4} \left(\frac{\left(4.67427 \times 0.2^2 - 0.0427762\right)0.5}{0.2^5} + 34 - 3 - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{\sqrt{5} - 1} \right) \right)^2 - \frac{1}{\phi} = -\frac{1}{\phi} + \frac{1}{16} \left(256.304 - \frac{-2 + \sqrt{9 - 2\sqrt{5}} \sum_{k=0}^{\infty} \frac{\left(-1\right)^k \left(-\frac{1}{2}\right)_k \left(9 - 2\sqrt{5}\right)^{-k}}{k!}}{-1 + \sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!}} \right)^2$$

$$\left(\frac{1}{4}\left(\frac{\left(4.67427\times0.2^{2}-0.0427762\right)0.5}{0.2^{5}}+34-3-\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)\right)^{2}-\frac{1}{\phi}=-\frac{1}{\phi}+\frac{1}{16}\left(256.304-\frac{-2+\sqrt{z_{0}}\sum_{k=0}^{\infty}\frac{\left(-1\right)^{k}\left(-\frac{1}{2}\right)_{k}\left(10-2\sqrt{5}-z_{0}\right)^{k}z_{0}^{-k}}{\left(-1+\sqrt{z_{0}}\sum_{k=0}^{\infty}\frac{\left(-1\right)^{k}\left(-\frac{1}{2}\right)_{k}\left(5-z_{0}\right)^{k}z_{0}^{-k}}{k!}\right)^{2}\right)$$
for (not (z_{0}\in\mathbb{R} and -m < z_{0} < 0))

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \le 0$))

 $[-(24 (-2.76^2/(6 x^4) + 1/(90 x^6)))/(8 \cos^{(-1)(-1)}) - 24 (2.76^2/(12 x^4) - 7/(720 x^6)) - 20/(720 \cos^{(-1)(-1)} x^6)] (((\sqrt{(10-2\sqrt{5})} - 2))/((\sqrt{5-1}))) dxdy$

Input:

$$\int \int \left(-\frac{24\left(-\frac{2.76^2}{6\,x^4} + \frac{1}{90\,x^6}\right)}{8\cos^{-1}(-1)} - 24\left(\frac{2.76^2}{12\,x^4} - \frac{7}{720\,x^6}\right) - \frac{20}{720\cos^{-1}(-1)\,x^6} \right) \times \frac{\sqrt{10 - 2\sqrt{5}} - 2}{\sqrt{5} - 1} \, dx \, dy$$

 $\cos^{-1}(x)$ is the inverse cosine function

Result:

$$\frac{\left(1.32786\,x^2-0.0121518\right)y}{x^5}$$

3D plot:



Contour plot:



Indefinite integral assuming all variables are real:

$$\left(\frac{0.00303796}{x^4} - \frac{0.663932}{x^2}\right)y + \text{constant}$$

For x = 0.2 and y = 0.5:

 $((-0.0121518 + 1.32786\ 0.2^2)\ 0.5)/0.2^5$

Input interpretation:

 $\frac{\left(-0.0121518+1.32786\!\times\!0.2^2\right)\!\times\!0.5}{0.2^5}$

Result: 64.0040625 $64.0040625 \approx 64 = 8^2$

From which:

 $[((-0.0121518 + 1.32786\ 0.2^{2})\ 0.5)/0.2^{5}]^{2}-1/2$

Input interpretation:

 $\left(\frac{\left(-0.0121518+1.32786\!\times\!0.2^2\right)\!\times\!0.5}{0.2^5}\right)^{\!\!2}-\frac{1}{2}$

Result: 4096.02001650390625 $4096.02001650390625 \approx 4096 = 64^2$

27*((-0.0121518 + 1.32786 0.2^2) 0.5)/0.2^5 +1

Input interpretation:

$$27 \times \frac{\left(-0.0121518 + 1.32786 \times 0.2^2\right) \times 0.5}{0.2^5} + 1$$

Result:

1729.1096875 1729.1096875

This result is very near to the mass of candidate glueball $f_0(1710)$ scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. (1728 = $8^2 * 3^3$) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Input interpretation:

$$\sqrt[15]{27 \times \frac{(-0.0121518 + 1.32786 \times 0.2^2) \times 0.5}{0.2^5}} + 1$$

Result:

1.64382...

 $1.64382....\approx \zeta(2)=\frac{\pi^2}{6}=1.644934...$

Observations

We note that, from the number 8, we obtain as follows:

8 ²
64
$8^2 \times 2 \times 8$
1024
$8^4 = 8^2 \times 2^6$
True
8 ⁴ = 4096
$8^2 \times 2^6 = 4096$
$2^{13} - 2 \times 2^4$
$z = z \times \delta$
True
$2^{13} = 8192$
$2 \times 8^4 = 8192$

We notice how from the numbers 8 and 2 we get 64, 1024, 4096 and 8192, and that 8 is the fundamental number. In fact $8^2 = 64$, $8^3 = 512$, $8^4 = 4096$. We define it "fundamental number", since 8 is a Fibonacci number, which by rule, divided by the previous one, which is 5, gives 1.6, a value that tends to the golden ratio, as for all numbers in the Fibonacci sequence

"Golden" Range



Finally we note how $8^2 = 64$, multiplied by 27, to which we add 1, is equal to 1729, the so-called "Hardy-Ramanujan number". Then taking the 15th root of 1729, we obtain a value close to $\zeta(2)$ that 1.6438 ..., which, in turn, is included in the range of what we call "golden numbers"

Furthermore for all the results very near to 1728 or 1729, adding $64 = 8^2$, one obtain values about equal to 1792 or 1793. These are values almost equal to the Planck multipole spectrum frequency 1792.35 and to the hypothetical Gluino mass

Mathematical connections with some sectors of String Theory

From:

Modular equations and approximations to π - *Srinivasa Ramanujan* Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

We have that:

Hence

$$64g_{22}^{24} = e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \cdots,$$

$$64g_{22}^{-24} = 4096e^{-\pi\sqrt{22}} + \cdots,$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \dots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\ldots$$

Again

$$G_{37} = (6 + \sqrt{37})^{\frac{1}{4}},$$

$$\begin{array}{rcl} 64G_{37}^{24} &=& e^{\pi\sqrt{37}}+24+276e^{-\pi\sqrt{37}}+\cdots,\\ 64G_{37}^{-24} &=& 4096e^{-\pi\sqrt{37}}-\cdots, \end{array}$$

so that

$$64(G_{37}^{24}+G_{37}^{-24}) = e^{\pi\sqrt{37}} + 24 + 4372e^{-\pi\sqrt{37}} - \dots = 64\{(6+\sqrt{37})^6 + (6-\sqrt{37})^6\}.$$

Hence

$$e^{\pi\sqrt{37}} = 199148647.999978\dots$$

Similarly, from

$$g_{58} = \sqrt{\left(\frac{5+\sqrt{29}}{2}\right)},$$

we obtain

$$64(g_{58}^{24} + g_{58}^{-24}) = e^{\pi\sqrt{58}} - 24 + 4372e^{-\pi\sqrt{58}} + \dots = 64\left\{\left(\frac{5+\sqrt{29}}{2}\right)^{12} + \left(\frac{5-\sqrt{29}}{2}\right)^{12}\right\}.$$

Hence

$$e^{\pi\sqrt{58}} = 24591257751.99999982\dots$$

From:

An Update on Brane Supersymmetry Breaking

J. Mourad and A. Sagnotti - arXiv:1711.11494v1 [hep-th] 30 Nov 2017

From the following vacuum equations:

$$T e^{\gamma_E \phi} = -\frac{\beta_E^{(p)} h^2}{\gamma_E} e^{-2(8-p)C + 2\beta_E^{(p)} \phi}$$
$$16 k' e^{-2C} = \frac{h^2 \left(p + 1 - \frac{2\beta_E^{(p)}}{\gamma_E}\right) e^{-2(8-p)C + 2\beta_E^{(p)} \phi}}{(7-p)}$$

$$(A')^2 = k e^{-2A} + \frac{h^2}{16(p+1)} \left(7 - p + \frac{2\beta_E^{(p)}}{\gamma_E}\right) e^{-2(8-p)C + 2\beta_E^{(p)}\phi}$$

we have obtained, from the results almost equals of the equations, putting

4096 $e^{-\pi\sqrt{18}}$ instead of

$$e^{-2(8-p)C+2\beta_E^{(p)}\phi}$$

a new possible mathematical connection between the two exponentials. Thence, also the values concerning p, C, β_E and ϕ correspond to the exponents of e (i.e. of exp). Thence we obtain for p = 5 and $\beta_E = 1/2$:

$$e^{-6C+\phi} = 4096e^{-\pi\sqrt{18}}$$

Therefore, with respect to the exponentials of the vacuum equations, the Ramanujan's exponential has a coefficient of 4096 which is equal to 642, while $-6C+\phi$ is equal to $-\pi\sqrt{18}$. From this it follows that it is possible to establish mathematically, the dilaton value.

For

exp((-Pi*sqrt(18)) we obtain:

Input:

 $\exp\left(-\pi\sqrt{18}\right)$

Exact result:

 $e^{-3\sqrt{2}\pi}$

Decimal approximation:

 $1.6272016226072509292942156739117979541838581136954016\ldots \times 10^{-6}$

1.6272016... * 10⁻⁶

Property:

 $e^{-3\sqrt{2}\ \pi}$ is a transcendental number

Series representations:

$$e^{-\pi\sqrt{18}} = e^{-\pi\sqrt{17}\sum_{k=0}^{\infty}17^{-k}\binom{1/2}{k}}$$
$$e^{-\pi\sqrt{18}} = \exp\left(-\pi\sqrt{17}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{17}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}\right)$$
$$e^{-\pi\sqrt{18}} = \exp\left(-\frac{\pi\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}17^{-s}\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}\right)$$

Now, we have the following calculations:

$$e^{-6C+\phi} = 4096e^{-\pi\sqrt{18}}$$

 $e^{-\pi\sqrt{18}} = 1.6272016...*10^{-6}$

from which:

$$\frac{1}{4096}e^{-6C+\phi} = 1.6272016\dots * 10^{-6}$$

$$0.000244140625 \ e^{-6C+\phi} = e^{-\pi\sqrt{18}} = 1.6272016... * 10^{-6}$$

Now:

$$\ln\left(e^{-\pi\sqrt{18}}\right) = -13.328648814475 = -\pi\sqrt{18}$$

And:

(1.6272016* 10^-6) *1/ (0.000244140625)

Input interpretation:

 $\frac{1.6272016}{10^6}\times\frac{1}{0.000244140625}$

Result: 0.0066650177536 0.006665017...

Thence:

$$0.000244140625 \ e^{-6C+\phi} = e^{-\pi\sqrt{18}}$$

Dividing both sides by 0.000244140625, we obtain:

 $\frac{0.000244140625}{0.000244140625} e^{-6C+\phi} = \frac{1}{0.000244140625} e^{-\pi\sqrt{18}}$

 $e^{-6C+\phi} = 0.0066650177536$

((((exp((-Pi*sqrt(18))))))*1/0.000244140625

Input interpretation:

 $\exp\Bigl(-\pi\sqrt{18}\,\Bigr)\times\frac{1}{0.000244140625}$

Result:

0.00666501785...

0.00666501785...

Series representations:

$$\frac{\exp(-\pi\sqrt{18})}{0.000244141} = 4096 \exp\left(-\pi\sqrt{17} \sum_{k=0}^{\infty} 17^{-k} {\binom{\frac{1}{2}}{\frac{2}{k}}}\right)$$
$$\frac{\exp(-\pi\sqrt{18})}{0.000244141} = 4096 \exp\left(-\pi\sqrt{17} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{17}\right)^k \left(-\frac{1}{2}\right)_k}{k!}\right)$$
$$\frac{\exp(-\pi\sqrt{18})}{0.000244141} = 4096 \exp\left(-\frac{\pi\sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 17^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{2\sqrt{\pi}}\right)$$

Now:

$$e^{-6C+\phi} = 0.0066650177536$$
$$\exp\left(-\pi\sqrt{18}\right) \times \frac{1}{0.000244140625} = e^{-\pi\sqrt{18}} \times \frac{1}{0.000244140625}$$

= 0.00666501785...

From:

 $\ln(0.00666501784619)$

Input interpretation:

log(0.00666501784619)

Result:

-5.010882647757...

-5.010882647757...

Alternative representations:

 $\log(0.006665017846190000) = \log_{\ell}(0.006665017846190000)$

 $\log(0.006665017846190000) = \log(a) \log_a(0.006665017846190000)$

 $log(0.006665017846190000) = -Li_1(0.993334982153810000)$

Series representations:

$$\begin{split} \log(0.006665017846190000) &= -\sum_{k=1}^{\infty} \frac{(-1)^k (-0.993334982153810000)^k}{k} \\ \log(0.006665017846190000) &= 2 \, i \, \pi \left[\frac{\arg(0.006665017846190000 - x)}{2 \, \pi} \right] + \\ \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (0.006665017846190000 - x)^k \, x^{-k}}{k} \quad \text{for } x < 0 \end{split}$$
$$\\ \log(0.006665017846190000) &= \left[\frac{\arg(0.006665017846190000 - z_0)}{2 \, \pi} \right] \log\left(\frac{1}{z_0}\right) + \\ \log(z_0) + \left[\frac{\arg(0.006665017846190000 - z_0)}{2 \, \pi} \right] \log(z_0) - \\ \sum_{k=1}^{\infty} \frac{(-1)^k (0.006665017846190000 - z_0)^k \, z_0^{-k}}{k} \end{split}$$

Integral representation:

 $\log(0.006665017846190000) = \int_{1}^{0.006665017846190000} \frac{1}{t} dt$

In conclusion:

$$-6C + \phi = -5.010882647757 \dots$$

and for C = 1, we obtain:

 $\phi = -5.010882647757 + 6 = 0.989117352243 = \phi$

Note that the values of n_s (spectral index) 0.965, of the average of the Omega mesons Regge slope 0.987428571 and of the dilaton 0.989117352243, are also connected to the following two Rogers-Ramanujan continued fractions:

$$\frac{e^{-\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}}-\varphi+1} = 1 - \frac{e^{-\pi}}{1+\frac{e^{-2\pi}}{1+\frac{e^{-3\pi}}{1+\frac{e^{-4\pi}}{1+\frac{e^{-4\pi}}{1+\frac{e^{-4\pi}}{1+\dots}}}}} \approx 0.9568666373$$

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

(http://www.bitman.name/math/article/102/109/)

The mean between the two results of the above Rogers-Ramanujan continued fractions is 0.97798855285, value very near to the ψ Regge slope 0.979:

 Ψ 3 $m_c = 1500$ 0.979 -0.09

Also performing the 512th root of the inverse value of the Pion meson rest mass 139.57, we obtain:

 $((1/(139.57)))^{1/512}$

Input interpretation:

$$\sqrt[512]{\frac{1}{139.57}}$$

Result:

0.990400732708644027550973755713301415460732796178555551684...

0.99040073.... result very near to the dilaton value **0**. **989117352243** = ϕ and to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

From

AdS Vacua from Dilaton Tadpoles and Form Fluxes - J. Mourad and A. Sagnotti - arXiv:1612.08566v2 [hep-th] 22 Feb 2017 - March 27, 2018

We have:

$$e^{2C} = \frac{2\xi e^{\frac{\phi}{2}}}{1 \pm \sqrt{1 - \frac{\xi T}{3} e^{2\phi}}}$$

$$\frac{h^2}{32} = \frac{\xi^7 e^{4\phi}}{\left(1 \pm \sqrt{1 - \frac{\xi T}{3} e^{2\phi}}\right)^7} \left[\frac{42}{\xi} \left(1 \pm \sqrt{1 - \frac{\xi T}{3} e^{2\phi}}\right) + 5 T e^{2\phi}\right]. \quad (2.7)$$

For

$$T = \frac{16}{\pi^2}$$
$$\xi = 1$$

we obtain:

 $(2*e^{(0.989117352243/2)) / (1+sqrt(((1-1/3*16/(Pi)^2*e^{(2*0.989117352243))))))$

Input interpretation:

$$\frac{2 \ e^{0.989117352243/2}}{1 + \sqrt{1 - \frac{1}{3} \times \frac{16}{\pi^2}} \ e^{2 \times 0.989117352243}}$$

Result: 0.83941881822... – 1.4311851867... *i*

Polar coordinates:

 $r = 1.65919106525 \text{ (radius)}, \quad \theta = -59.607521917^{\circ} \text{ (angle)}$

1.65919106525..... result very near to the 14th root of the following Ramanujan's class invariant $Q = (G_{505}/G_{101/5})^3 = 1164.2696$ i.e. 1.65578...

Series representations:

$$\frac{2 e^{0.9891173522430000/2}}{1 + \sqrt{1 - \frac{16 e^{2 \times 0.9891173522430000}}{3 \pi^{2}}}} = \frac{1}{1 + \sqrt{1 - \frac{16 e^{1.978234704486000}}{3 \pi^{2}}}} \sum_{k=0}^{\infty} \left(\frac{3}{16}\right)^{k} \left(-\frac{e^{1.978234704486000}}{\pi^{2}}\right)^{-k} \left(\frac{1}{2} \atop k\right)}$$

$$\frac{2 e^{0.9891173522430000/2}}{1 + \sqrt{1 - \frac{16 e^{2 \times 0.9891173522430000}}{3 \pi^{2}}}} = \frac{1 + \sqrt{1 - \frac{16 e^{1.978234704486000}}{3 \pi^{2}}}}{2 e^{0.4945586761215000}}$$

 $2 e^{0.9891173522430000/2}$

$$\frac{1 + \sqrt{1 - \frac{16e^{2 \times 0.9891173522430000}}{3\pi^2}}}{2e^{0.4945586761215000}} = \frac{1}{2e^{0.4945586761215000}} \frac{1 + \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k (-\frac{1}{2})_k (1 - \frac{16e^{1.978234704486000}}{3\pi^2} - z_0)^k z_0^{-k}}{k!}}{\text{for (not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0)) }$$

From

$$\frac{h^2}{32} = \frac{\xi^7 e^{4\phi}}{\left(1 \pm \sqrt{1 - \frac{\xi T}{3} e^{2\phi}}\right)^7} \left[\frac{42}{\xi} \left(1 \pm \sqrt{1 - \frac{\xi T}{3} e^{2\phi}}\right) + 5 T e^{2\phi}\right]$$

we obtain:

e^(4*0.989117352243) / (((1+sqrt(1-1/3*16/(Pi)^2*e^(2*0.989117352243)))))^7 [42(1+sqrt(1-1/3*16/(Pi)^2*e^(2*0.989117352243)))+5*16/(Pi)^2*e^(2*0.989117352243)]

Input interpretation:

$$\frac{e^{4\times0.989117352243}}{\left(1+\sqrt{1-\frac{1}{3}\times\frac{16}{\pi^2}}e^{2\times0.989117352243}}\right)^7} \\ \left(42\left(1+\sqrt{1-\frac{1}{3}\times\frac{16}{\pi^2}}e^{2\times0.989117352243}}\right)+5\times\frac{16}{\pi^2}e^{2\times0.989117352243}}\right)$$

Result: 50.84107889... – 20.34506335... *i*

Polar coordinates:

r = 54.76072411 (radius), $\theta = -21.80979492^{\circ}$ (angle)

54.76072411.....

Series representations:

$$\begin{split} & \left(\left(42 \left(1 + \sqrt{1 - \frac{16 e^{2 \times 0.9891173522430000}}{3 \pi^2}} \right) + \frac{5 \times 16 e^{2 \times 0.9891173522430000}}{\pi^2} \right)^7 = \\ & e^{4 \times 0.9891173522430000} \right) / \left(1 + \sqrt{1 - \frac{16 e^{2 \times 0.9891173522430000}}{3 \pi^2}} \right)^7 = \\ & \left(2 \left(40 e^{5.934704113458000} + 21 e^{3.956469408972000} \pi^2 + 21 e^{3.956469408972000} \pi^2 + 21 e^{3.956469408972000} \pi^2 \right)^7 \\ & \sqrt{-\frac{16 e^{1.978234704486000}}{3 \pi^2}} \sum_{k=0}^{\infty} \left(\frac{3}{16} \right)^k \left(-\frac{e^{1.978234704486000}}{\pi^2} \right)^{-k} \left(\frac{1}{2} \right) \right) \right) / \\ & \left(\pi^2 \left(1 + \sqrt{1 - \frac{16 e^{1.978234704486000}}{3 \pi^2}} \right) \sum_{k=0}^{\infty} \left(\frac{3}{16} \right)^k \left(-\frac{e^{1.978234704486000}}{\pi^2} \right)^{-k} \left(\frac{1}{2} \right) \right)^7 \right) \\ & \left(\left(42 \left(1 + \sqrt{1 - \frac{16 e^{1.978234704486000}}{3 \pi^2}} \right) + \frac{5 \times 16 e^{2 \times 0.9891173522430000}}{\pi^2} \right)^{-k} \left(\frac{1}{2} \right) \right)^7 \right) \\ & e^{4 \times 0.9891173522430000} \right) / \left(1 + \sqrt{1 - \frac{16 e^{2 \times 0.9891173522430000}}{3 \pi^2}} \right)^7 = \\ & \left(2 \left(40 e^{5.934704113458000} + 21 e^{3.956469408972000} \pi^2 + 21 e^{3.956469408972000} \pi^2 \right)^7 \\ & - \frac{16 e^{1.978234704486000}}{3 \pi^2} \right) \sum_{k=0}^{\infty} \left(-\frac{3}{16} \right)^k \left(-\frac{e^{1.978234704486000}}{\pi^2} \right)^{-k} \left(-\frac{1}{2} \right)_k \right) \right) / \\ & \left(\pi^2 \left(1 + \sqrt{-\frac{16 e^{1.978234704486000}}{3 \pi^2}} \right) \sum_{k=0}^{\infty} \left(-\frac{3}{16} \right)^k \left(-\frac{e^{1.978234704486000}}{\pi^2} \right)^{-k} \left(-\frac{1}{2} \right)_k \right) \right) / \\ & \left(\pi^2 \left(1 + \sqrt{-\frac{16 e^{1.978234704486000}}{3 \pi^2} \right) \sum_{k=0}^{\infty} \left(-\frac{3}{16} \right)^k \left(-\frac{e^{1.978234704486000}}{\pi^2} \right)^{-k} \left(-\frac{1}{2} \right)_k \right) \right) / \\ & \left(\pi^2 \left(1 + \sqrt{-\frac{16 e^{1.978234704486000}}{3 \pi^2} \right) \sum_{k=0}^{\infty} \left(-\frac{3}{16} \right)^k \left(-\frac{e^{1.978234704486000}}{\pi^2} \right)^{-k} \left(-\frac{1}{2} \right)_k \right) \right) \right) \end{pmatrix}$$

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$$\begin{pmatrix} \left(42 \left(1 + \sqrt{1 - \frac{16 \ e^{2 \times 0.9891173522430000}}{3 \ \pi^2} \right) + \frac{5 \times 16 \ e^{2 \times 0.9891173522430000}}{\pi^2} \right)^7 \\ e^{4 \times 0.9891173522430000} \\ \end{pmatrix} / \left(1 + \sqrt{1 - \frac{16 \ e^{2 \times 0.9891173522430000}}{3 \ \pi^2}} \right)^7 = \\ \left(2 \left(40 \ e^{5.934704113458000} + 21 \ e^{3.956469408972000} \ \pi^2 + 21 \ e^{3.956469408972000} \\ \\ \pi^2 \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2} \right)_k \left(1 - \frac{16 \ e^{1.978234704486000}}{3 \ \pi^2} - z_0 \right)^k \ z_0^{-k}} \\ \\ \left(\pi^2 \left(1 + \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2} \right)_k \left(1 - \frac{16 \ e^{1.978234704486000}}{3 \ \pi^2} - z_0 \right)^k \ z_0^{-k}} \\ \\ for (not (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0)) \end{cases}$$

From which:

e^(4*0.989117352243) / (((1+sqrt(1-1/3*16/(Pi)^2*e^(2*0.989117352243)))))^7 [42(1+sqrt(1-1/3*16/(Pi)^2*e^(2*0.989117352243)))+5*16/(Pi)^2*e^(2*0.989117352243)]*1/34

Input interpretation:

$$\frac{e^{4 \times 0.989117352243}}{\left(1 + \sqrt{1 - \frac{1}{3} \times \frac{16}{\pi^2}} e^{2 \times 0.989117352243}}\right)^7} \\ \left(42 \left(1 + \sqrt{1 - \frac{1}{3} \times \frac{16}{\pi^2}} e^{2 \times 0.989117352243}}\right) + 5 \times \frac{16}{\pi^2} e^{2 \times 0.989117352243}}\right) \times \frac{1}{34} + \frac{1}{3$$

Result: 1.495325850... – 0.5983842161... *i*

Polar coordinates:

r = 1.610609533 (radius), $\theta = -21.80979492^{\circ}$ (angle)

1.610609533.... result that is a good approximation to the value of the golden ratio 1.618033988749...

Series representations:

$$\left[\left(42 \left(1 + \sqrt{1 - \frac{16 e^{2 \times 0.9891173522430000}}{3 \pi^2}} \right) + \frac{5 \times 16 e^{2 \times 0.9891173522430000}}{\pi^2} \right)^7 \right] = e^{4 \times 0.9891173522430000} \right] / \left(34 \left(1 + \sqrt{1 - \frac{16 e^{2 \times 0.9891173522430000}}{3 \pi^2}} \right)^7 \right) = \left(40 e^{5.934704113458000} + 21 e^{3.956469408972000} \pi^2 + 21 e^{3.956469408972000} \pi^2 \right)^7 \right) = \left(\sqrt{-\frac{16 e^{1.978234704113458000}}{3 \pi^2}} \sum_{k=0}^{\infty} \left(\frac{3}{16} \right)^k \left(-\frac{e^{1.978234704486000}}{\pi^2} \right)^{-k} \left(\frac{1}{2} \right) \right) / \left(17 \pi^2 \left(1 + \sqrt{-\frac{16 e^{1.978234704486000}}{3 \pi^2}} \sum_{k=0}^{\infty} \left(\frac{3}{16} \right)^k \left(-\frac{e^{1.978234704486000}}{\pi^2} \right)^{-k} \left(\frac{1}{2} \right) \right) \right)^7$$

$$\begin{pmatrix} \left(42\left(1+\sqrt{1-\frac{16\ e^{2\times0.9891173522430000}}{3\ \pi^2}}\right)+\frac{5\times16\ e^{2\times0.9891173522430000}}{\pi^2}\right)\\ e^{4\times0.9891173522430000}\right) / \left(34\left(1+\sqrt{1-\frac{16\ e^{2\times0.9891173522430000}}{3\ \pi^2}}\right)^7\right) = \left(40\ e^{5.934704113458000}+21\ e^{3.956469408972000\ \pi^2}+21\ e^{3.956469408972000\ \pi^2}\right)^7 + 21\ e^{3.956469408972000\ \pi^2} = 12$$

$$\begin{split} \sqrt{-\frac{16 \, e^{1.978234704486000}}{3 \, \pi^2}} \sum_{k=0}^{\infty} \frac{\left(-\frac{3}{16}\right)^k \left(-\frac{e^{1.978234704486000}}{\pi^2}\right)^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) \\ \left(17 \, \pi^2 \left(1 + \sqrt{-\frac{16 \, e^{1.978234704486000}}{3 \, \pi^2}} \sum_{k=0}^{\infty} \frac{\left(-\frac{3}{16}\right)^k \left(-\frac{e^{1.978234704486000}}{\pi^2}\right)^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right)^7 \right) \\ \end{split}$$

$$\begin{split} & \left(\left(42 \left(1 + \sqrt{1 - \frac{16 e^{2 \times 0.9891173522430000}}{3 \pi^2}} \right) + \frac{5 \times 16 e^{2 \times 0.9891173522430000}}{\pi^2} \right)^7 \right) \\ & e^{4 \times 0.9891173522430000} \right) / \left(34 \left(1 + \sqrt{1 - \frac{16 e^{2 \times 0.9891173522430000}}{3 \pi^2}} \right)^7 \right) \\ & = \left(40 e^{5.934704113458000} + 21 e^{3.956469408972000} \pi^2 + 21 e^{3.956469408972000} \\ & \pi^2 \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2} \right)_k \left(1 - \frac{16 e^{1.978234704486000}}{3 \pi^2} - z_0 \right)^k z_0^{-k}}{k!} \right) / \\ & \left(17 \pi^2 \left(1 + \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2} \right)_k \left(1 - \frac{16 e^{1.978234704486000}}{3 \pi^2} - z_0 \right)^k z_0^{-k}}{k!} \right)^7 \right) \end{split}$$

for (not $(z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$

Now, we have:

$$e^{2C} = \frac{2\xi e^{-\frac{\phi}{2}}}{1 + \sqrt{1 + \frac{\xi\Lambda}{3}e^{2\phi}}}, \qquad (2.9)$$
$$\frac{\hbar^2}{32} = \frac{e^{-4\phi}}{\left[1 + \sqrt{1 + \frac{\Lambda}{3}e^{2\phi}}\right]^7} \left[42\left(1 + \sqrt{1 + \frac{\Lambda}{3}e^{2\phi}}\right) - 13\Lambda e^{2\phi}\right]. \qquad (2.10)$$

For:

 $\xi = 1$

$$\Lambda \simeq \frac{4\pi^2}{25}$$

 $\phi = 0.989117352243$

From

$$e^{2C} = \frac{2\xi e^{-\frac{\phi}{2}}}{1 + \sqrt{1 + \frac{\xi \Lambda}{3} e^{2\phi}}},$$

we obtain:

Input interpretation:

$$\frac{2 \, e^{-0.989117352243/2}}{1 + \sqrt{1 + \frac{1}{3} \left(\frac{1}{25} \left(4 \, \pi^2\right)\right) e^{2 \times 0.989117352243}}}$$

Result:

0.382082347529...

0.382082347529....

Series representations:

$$\begin{aligned} \frac{2 \ e^{-0.9891173522430000/2}}{1 + \sqrt{1 + \frac{(4 \ \pi^2) \ e^{2 \ \times 0.9891173522430000}}{3 \ \times 25}}} &= 2 \left/ \left(e^{0.4945586761215000} \right. \\ \left. \left(1 + \sqrt{\frac{4 \ e^{1.978234704486000} \ \pi^2}{75}} \right) \sum_{k=0}^{\infty} \left(\frac{75}{4} \right)^k \left(e^{1.978234704486000} \ \pi^2 \right)^{-k} \left(\frac{1}{2} \atop k \right) \right) \right) \\ \frac{2 \ e^{-0.9891173522430000/2}}{1 + \sqrt{1 + \frac{(4 \ \pi^2) \ e^{2 \ \times 0.9891173522430000}}{3 \ \times 25}}} \\ \left. \left(1 + \sqrt{\frac{4 \ e^{1.978234704486000} \ \pi^2}{75}} \right) \sum_{k=0}^{\infty} \frac{\left(-\frac{75}{4} \right)^k \left(e^{1.978234704486000} \ \pi^2 \right)^{-k} \left(-\frac{1}{2} \right)_k}{k!} \end{aligned} \end{aligned}$$

$$\frac{2 e^{-0.9891173522430000/2}}{1 + \sqrt{1 + \frac{(4 \pi^2) e^{2 \times 0.9891173522430000}}{3 \times 25}}} = \frac{2}{2}$$

$$\frac{2}{e^{0.4945586761215000} \left(1 + \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(1 + \frac{4 e^{1.978234704486000 \pi^2}}{75} - z_0\right)^k z_0^{-k}}{k!}\right)}$$
for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \le 0$))

From which:

Input interpretation:

$$\frac{1}{4 \times \frac{2 \, e^{-0.989117352243/2}}{1 + \sqrt{1 + \frac{1}{3} \left(\frac{1}{25} \left(4 \, \pi^2\right)\right) e^{2 \times 0.989117352243}}}}$$

Result:

1.65430921270...

1.6543092..... We note that, the result 1.6543092... is very near to the 14th root of the following Ramanujan's class invariant $Q = (G_{505}/G_{101/5})^3 = 1164.2696$ i.e. 1.65578...

Indeed:
$$\begin{aligned} G_{505} &= P^{-1/4} Q^{1/6} = (\sqrt{5} + 2)^{1/2} \left(\frac{\sqrt{5} + 1}{2}\right)^{1/4} (\sqrt{101} + 10)^{1/4} \\ &\times \left((130\sqrt{5} + 29\sqrt{101}) + \sqrt{169440 + 7540\sqrt{505}} \right)^{1/6}. \end{aligned}$$

Thus, it remains to show that

$$(130\sqrt{5} + 29\sqrt{101}) + \sqrt{169440 + 7540\sqrt{505}} = \left(\sqrt{\frac{113 + 5\sqrt{505}}{8}} + \sqrt{\frac{105 + 5\sqrt{505}}{8}}\right)^3,$$
which is straightforward.

which is straightforward.

$$\sqrt[14]{\left(\sqrt{\frac{113+5\sqrt{505}}{8}}+\sqrt{\frac{105+5\sqrt{505}}{8}}\right)^3} = 1,65578\dots$$

$$\begin{aligned} 1 + \frac{1}{\frac{4\left(2\,e^{-0.9891173522430000/2}\right)}{1+\sqrt{1+\frac{(4\,\pi^2)\,e^{2}\times0.9891173522430000}{3\times25}}} = \\ 1 + \frac{e^{0.4945586761215000}}{8} + \frac{1}{8}\,e^{0.4945586761215000}\,\sqrt{\frac{4\,e^{1.978234704486000}\,\pi^2}{75}} \\ &\sum_{k=0}^{\infty} \left(\frac{75}{4}\right)^k \left(e^{1.978234704486000}\,\pi^2\right)^{-k} \left(\frac{1}{2}\atop k\right) \end{aligned}$$

$$\begin{split} 1 + \frac{1}{\frac{4\left(2\,e^{-0.9891173522430000/2}\right)}{1 + \sqrt{1 + \frac{\left(4\,\pi^2\right)e^{2\,\times 0.9891173522430000}{3\,\times 25}}}}{1 + \frac{e^{0.4945586761215000}}{8} + \frac{1}{8}\,e^{0.4945586761215000}\,\sqrt{\frac{4\,e^{1.978234704486000}\,\pi^2}{75}}\\ & \sum_{k=0}^{\infty}\frac{\left(-\frac{75}{4}\right)^k\left(e^{1.978234704486000}\,\pi^2\right)^{-k}\left(-\frac{1}{2}\right)_k}{k!} \end{split}$$

$$1 + \frac{1}{\frac{4\left(2\,e^{-0.9891173522430000/2}\right)}{1+\sqrt{1+\frac{(4\,\pi^2)\,e^{2}\times0.9891173522430000}{3\times25}}} = 1 + \frac{e^{0.4945586761215000}}{8} + \frac{1}{8}\,e^{0.4945586761215000}\,\sqrt{z_0}\,\sum_{k=0}^{\infty}\,\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(1 + \frac{4\,e^{1.978234704486000\,\pi^2}}{75} - z_0\right)^k\,z_0^{-k}}{k!}$$
for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \le 0$))

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And from

$$\frac{h^2}{32} = \frac{e^{-4\phi}}{\left[1 + \sqrt{1 + \frac{\Lambda}{3}e^{2\phi}}\right]^7} \left[42\left(1 + \sqrt{1 + \frac{\Lambda}{3}e^{2\phi}}\right) - 13\Lambda e^{2\phi}\right].$$

$$\frac{h^2}{32} = \frac{e^{-4\phi}}{\left[1 + \sqrt{1 + \frac{\Lambda}{3}e^{2\phi}}\right]^7} \left[42\left(1 + \sqrt{1 + \frac{\Lambda}{3}e^{2\phi}}\right) - 13\Lambda e^{2\phi}\right].$$

Input interpretation:

 $\frac{e^{-4\times0.989117352243}}{\left(1+\sqrt{1+\frac{1}{3}\left(\frac{1}{25}\left(4\,\pi^2\right)\right)e^{2\times0.989117352243}}\right)^7} \\ \left(42\left(1+\sqrt{1+\frac{1}{3}\left(\frac{1}{25}\left(4\,\pi^2\right)\right)e^{2\times0.989117352243}}\right) - 13\left(\frac{1}{25}\left(4\,\pi^2\right)\right)e^{2\times0.989117352243}\right)\right)$

Result:

-0.034547055658...

-0.034547055658...

$$\begin{aligned} & \left(\left| 42 \left(1 + \sqrt{1 + \frac{(4\pi^2) e^{2 \times 0.9891173522430000}}{3 \times 25}} - \frac{1}{25} (4\pi^2) 13 e^{2 \times 0.9891173522430000} \right) \right) \\ & e^{-4 \times 0.9891173522430000} \right) / \left(1 + \sqrt{1 + \frac{(4\pi^2) e^{2 \times 0.9891173522430000}}{3 \times 25}} \right)^7 = \\ & - \left(\left| 42 \left(-25 e^{1.978234704486000} + 52 e^{3.956469408972000} \pi^2 - \frac{25 e^{1.978234704486000} \sqrt{\frac{4 e^{1.978234704486000} \pi^2}{75}} \right) \\ & \sum_{k=0}^{\infty} \frac{(-\frac{75}{4})^k (e^{1.978234704486000} \sqrt{\frac{4 e^{1.978234704486000} \pi^2}{75}} \frac{2}{5} e^{5.934704113458000} \left(1 + \sqrt{\frac{4 e^{1.978234704486000} \pi^2}{75}} \sum_{k=0}^{\infty} \frac{(-\frac{75}{4})^k (e^{1.978234704486000} \pi^2)^{-k} (-\frac{1}{2})_k}{k!} \right) \right) / \left(25 e^{5.934704113458000} \left(1 + \sqrt{\frac{4 e^{1.978234704486000} \pi^2}{75}} - \frac{1}{25} (4\pi^2) 13 e^{2 \times 0.9891173522430000}} \right) \right) \\ & e^{-4 \times 0.9891173522430000} \right) / \left(1 + \sqrt{1 + \frac{(4\pi^2) e^{2 \times 0.9891173522430000}}{3 \times 25}}} - \frac{1}{25} (4\pi^2) 13 e^{2 \times 0.9891173522430000}} \right)^7 = \\ & - \left(\left(42 \left(-25 e^{1.978234704486000} + 52 e^{3.956469408972000} \pi^2 - 25 e^{1.978234704486000} - 2 \pi (1)^k (-\frac{1}{2})_k (1 + \frac{4 e^{1.978234704486000} \pi^2}{75} - 2 \pi (1)^k \frac{2}{75}} - 2 \pi (1)^k \frac{2}{75} - 2 \pi (1)^k \frac{2}{75}} \right) \right) / \left(25 e^{5.934704113458000} \left(1 + \sqrt{2\pi} \sum_{k=0}^{\infty} \frac{(-1)^k (-\frac{1}{2})_k (1 + \frac{4 e^{1.978234704486000} \pi^2}{75} - 2 \pi (1)^k \frac{2}{75}} \right) \right) / \left(25 e^{5.934704113458000} \left(1 + \sqrt{2\pi} \sum_{k=0}^{\infty} \frac{(-1)^k (-\frac{1}{2})_k (1 + \frac{4 e^{1.978234704486000} \pi^2}{75} - 2 \pi (1)^k \frac{2}{75}} \right) \right) \right) / \left(25 e^{5.934704113458000} \left(1 + \sqrt{2\pi} \sum_{k=0}^{\infty} \frac{(-1)^k (-\frac{1}{2})_k (1 + \frac{4 e^{1.978234704486000} \pi^2}{75} - 2 \pi (1)^k \frac{2}{75}} \right) \right) \right) / \left(25 e^{5.934704113458000} \left(1 + \sqrt{2\pi} \sum_{k=0}^{\infty} \frac{(-1)^k (-\frac{1}{2})_k (1 + \frac{4 e^{1.978234704486000} \pi^2}{75} - 2 \pi (1)^k \frac{2}{75}} \right) \right) \right)$$

From which:

Input interpretation:

$$\begin{split} 47 \left(- \left(1 \Big/ 1 \Big/ \left(\frac{e^{-4 \times 0.989117352243}}{\left(1 + \sqrt{1 + \frac{1}{3} \left(\frac{1}{25} \left(4 \, \pi^2 \right) \right) e^{2 \times 0.989117352243}} \right)^7} \right. \\ \left. \left. \left(42 \left(1 + \sqrt{1 + \frac{1}{3} \left(\frac{1}{25} \left(4 \, \pi^2 \right) \right) e^{2 \times 0.989117352243}} \right. - \right. \\ \left. 13 \left(\frac{1}{25} \left(4 \, \pi^2 \right) \right) e^{2 \times 0.989117352243} \right) \right) \right) \right) \end{split}$$

Result:

1.6237116159...

1.6237116159.... result that is an approximation to the value of the golden ratio 1.618033988749...

$$\begin{split} - \left[\frac{47}{1} \left(e^{-4 \times 0.9891173522430000} \left(\frac{42}{1} \left(1 + \sqrt{1 + \frac{(4\pi^2) e^{2 \times 0.9891173522430000}}{3 \times 25}} \right)^{-1} \right) \right] \right] \\ - \left[\frac{1}{25} (4\pi^2) 13 e^{2 \times 0.9891173522430000}} \right] \right] \\ \left(1 + \sqrt{1 + \frac{(4\pi^2) e^{2 \times 0.9891173522430000}}{3 \times 25}} \right)^{-1} \right] = \\ \left(1974 \left(-25 e^{1.978234704486000} + 52 e^{3.956469408972000} \pi^2 - 25 e^{1.978234704486000} \sqrt{\frac{4 e^{1.978234704486000} \pi^2}{75}} \right) \\ \sum_{k=0}^{\infty} \frac{\left(-\frac{75}{4} \right)^k (e^{1.978234704486000} \pi^2)^{-k} \left(-\frac{1}{2} \right)_k}{k!} \right) \right] / \left(25 e^{5.934704113458000} \left(1 + \sqrt{\frac{4 e^{1.978234704486000} \pi^2}{75}} \sum_{k=0}^{\infty} \frac{\left(-\frac{75}{4} \right)^k (e^{1.978234704486000} \pi^2)^{-k} \left(-\frac{1}{2} \right)_k}{k!} \right)^{-1} \right) \\ - \left[\left(47 / 1 / \left(e^{-4 \times 0.9891173522430000} \left(42 \left(1 + \sqrt{1 + \frac{(4\pi^2) e^{2 \times 0.9891173522430000}}{3 \times 25}} - \frac{1}{25} (4\pi^2) 13 e^{2 \times 0.9891173522430000} \right) \right) \right] \right) / \\ \left(1 + \sqrt{1 + \frac{(4\pi^2) e^{2 \times 0.9891173522430000}}{3 \times 25}} \right)^{-1} \right) \\ = \left(1974 \left(-25 e^{1.978234704486000} + 52 e^{3.956469408972000} \pi^2 - 25 e^{1.978234704486000} \right) \right) \right)$$

 $\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(1 + \frac{4 e^{1.978234704486000 \pi^2}}{75} - z_0\right)^k z_0^{-k}}{k!} \right) \right) / \left(25 e^{5.934704113458000} \left(1 + \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(1 + \frac{4 e^{1.978234704486000 \pi^2}}{75} - z_0\right)^k z_0^{-k}}{k!} \right)^7 \right)$

for (not $(z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0)$)

And again:

32((((e^(-4*0.989117352243) / [1+sqrt(((1+1/3*(4Pi^2)/25*e^(2*0.989117352243))))]^7 * [42(1+sqrt(((1+1/3*(4Pi^2)/25*e^(2*0.989117352243))))-13*(4Pi^2)/25*e^(2*0.989117352243))])))

Input interpretation:

$$32 \left(\frac{e^{-4 \times 0.989117352243}}{\left(1 + \sqrt{1 + \frac{1}{3} \left(\frac{1}{25} \left(4 \, \pi^2 \right) \right) e^{2 \times 0.989117352243}} \right)^7} \right)^7 \left(42 \left(1 + \sqrt{1 + \frac{1}{3} \left(\frac{1}{25} \left(4 \, \pi^2 \right) \right) e^{2 \times 0.989117352243}} - 13 \left(\frac{1}{25} \left(4 \, \pi^2 \right) \right) e^{2 \times 0.989117352243} \right) \right)$$

Result:

-1.1055057810...

-1.1055057810....

We note that the result -1.1055057810... is very near to the value of Cosmological Constant, less 10^{-52} , thence 1.1056, with minus sign

$$\begin{cases} 32 \ e^{-4 \times 0.9891173522430000} \left(42 \left(1 + \sqrt{1 + \frac{(4 \ \pi^2) \ e^{2 \times 0.9891173522430000}}{3 \times 25}} - \frac{1}{25} (4 \ \pi^2) \ 13 \ e^{2 \times 0.9891173522430000} \right) \right) \\ / \\ \left(1 + \sqrt{1 + \frac{(4 \ \pi^2) \ e^{2 \times 0.9891173522430000}}{3 \times 25}} \right)^7 = -\left(\left(1344 \left(-25 \ e^{1.978234704486000} + 52 \ e^{3.956469408972000} \ \pi^2 - 25 \ e^{1.978234704486000} \sqrt{\frac{4 \ e^{1.978234704486000} \ \pi^2}{75}} \right) \\ & \sum_{k=0}^{\infty} \left(\left(\frac{75}{4} \right)^k \left(e^{1.978234704486000} \ \pi^2 \right)^{-k} \left(\frac{1}{2} \\ k \right) \right) \right) / \left(25 \ e^{5.934704113458000} \left(1 + \sqrt{\frac{4 \ e^{1.978234704486000} \ \pi^2}{75}} \right) \\ & \sum_{k=0}^{\infty} \left(\left(\frac{75}{75} \right)^k \left(e^{1.978234704486000} \ \pi^2 \right)^{-k} \left(\frac{1}{2} \\ k \right) \right) \right) / \left(25 \ e^{5.934704113458000} \left(1 + \sqrt{\frac{4 \ e^{1.978234704486000} \ \pi^2}{75}} \right) \\ & \sum_{k=0}^{\infty} \left(\left(\frac{75}{75} \right)^k \left(e^{1.978234704486000} \ \pi^2 \right)^{-k} \left(\frac{1}{2} \\ k \right) \right) \right) \right)$$

$$\begin{aligned} &\left(1+\sqrt{1+\frac{(4\pi^2)e^{2^{(-0.9891173522430000}}}{3\times25}}\right)^7 = \\ &-\left(\left(1344\left(-25e^{1.978234704486000}+52e^{3.956469408972000}\pi^2 - \\ &25e^{1.978234704486000}\sqrt{\frac{4e^{1.978234704486000}\pi^2}{75}} \\ &\sum_{k=0}^{\infty} \frac{\left(-\frac{75}{4}\right)^k (e^{1.978234704486000}\pi^2)^{-k} (-\frac{1}{2})_k}{k!}\right)\right) / \left(25e^{5.934704113458000} \\ &\left(1+\sqrt{\frac{4e^{1.978234704486000}\pi^2}{75}} \\ &\sum_{k=0}^{\infty} \frac{\left(-\frac{75}{2}\right)^k (e^{1.978234704486000}\pi^2)^{-k} (-\frac{1}{2})_k}{k!}\right)\right) / \\ &32e^{-4\times0.9891173522430000} \left(42\left(1+\sqrt{1+\frac{(4\pi^2)e^{2^{(-0.9891173522430000)}}}{3\times25}} - \\ &\frac{1}{25}(4\pi^2)13e^{2\times0.9891173522430000}}\right)\right) \\ &\left(1+\sqrt{1+\frac{(4\pi^2)e^{2^{(-0.9891173522430000)}}}{3\times25}}\right)^7 = \\ &-\left(\left(1344\left(-25e^{1.978234704486000}+52e^{3.956469408972000}\pi^2-25e^{1.978234704486000} \\ &\sqrt{z_0}\sum_{k=0}^{\infty} \frac{(-1)^k (-\frac{1}{2})_k \left(1+\frac{4e^{1.978234704486000}\pi^2}{75}-z_0\right)^k z_0^{-k}}{k!}\right)\right) / \left(25e^{5.93470411348000} \\ &\left(1+\sqrt{z_0}\sum_{k=0}^{\infty} \frac{(-1)^k (-\frac{1}{2})_k \left(1+\frac{4e^{1.978234704486000}\pi^2}{75}-z_0\right)^k z_0^{-k}}{k!}\right)\right) \right) / \\ &for (not (z_0 \in \mathbb{R} and -\infty < z_0 \le 0)) \end{aligned}$$

 $\left(32 \; e^{-4 \; \times \; 0.9891173522430000} \left(42 \left(1 + \sqrt{1 + \frac{\left(4 \; \pi^2\right) e^{2 \; \times \; 0.9891173522430000}}{3 \; \times \; 25}} \right. \right. \right. \right. \right. \\$

 $\frac{1}{25} (4 \pi^2) 13 e^{2 \times 0.9891173522430000}) \right) / (4 \pi^2) 13 e^{2 \times 0.9891173522430000}) \\ + (4 \pi^2) 12 e^{2 \times 0.991173522430000}) \\ + (4 \pi^2) 12 e^{2 \times 0.991173522430000}) \\ + (4 \pi^2) 12 e^{2 \times 0.991173522430000}) \\ + (4 \pi^2) 12 e^{2 \times 0.991173522430000}) \\ + (4 \pi^2) 12 e^{2 \times 0.991173522430000}) \\ + (4 \pi^2) 12 e^{2 \times 0.991173522430000}) \\ + (4 \pi^2) 12 e^{2 \times 0.991173522430000}) \\ + (4 \pi^2) 12 e^{2 \times 0.991173522430000}) \\ + (4 \pi^2) 12 e^{2 \times 0.991173522430000}) \\ + (4 \pi^2) 12 e^{2 \times 0.991173522430000}) \\ + (4 \pi^2) 12 e^{2 \times 0.991173522430000}) \\ + (4 \pi^2) 12 e^{2 \times 0.991173522430000}) \\ + (4 \pi^2) 12 e^{2 \times 0.991173522430000}) \\ + (4 \pi^2) 12 e^{2 \times 0.991173522430000}) \\ + (4 \pi^2) 12 e^{2 \times 0.99117352}) \\ + (4 \pi^2) 12 e^{2 \times 0.99117352}) \\ + (4 \pi^2) 12 e^{2 \times 0.99117352}) \\ + (4 \pi^2) 12 e^{2 \times 0.99117352}) \\ + (4 \pi^2) 12 e^{2 \times 0.99117352}) \\ + (4 \pi^2) 12 e^{2 \times 0.99117352}) \\ + (4 \pi^2) 12 e^{2 \times 0.99117352}) \\ + (4 \pi^2) 12 e^{2 \times 0.99117352}) \\ + (4 \pi^2) 12 e^{2 \times 0.99117352}) \\ + (4 \pi^2) 12 e^{2 \times 0.99117352}) \\ + (4 \pi^2) 12 e^{2 \times 0.9911735}) \\ + (4 \pi^2) 12 e^{2 \times 0.9911735}) \\ + (4 \pi^2) 12 e^{2 \times 0.991173}) \\ + (4 \pi^2) 12 e^{2 \times 0.991173}) \\ + (4 \pi^2) 12$

And:

```
-[32((((e^(-4*0.989117352243) /
[1+sqrt(((1+1/3*(4Pi^2)/25*e^(2*0.989117352243))))]^7 *
[42(1+sqrt(((1+1/3*(4Pi^2)/25*e^(2*0.989117352243))))-
13*(4Pi^2)/25*e^(2*0.989117352243))])))]^5
```

Input interpretation:

$$-\left(32\left(\frac{e^{-4\times0.989117352243}}{\left(1+\sqrt{1+\frac{1}{3}\left(\frac{1}{25}\left(4\,\pi^{2}\right)\right)e^{2\times0.989117352243}}\right)^{7}}\right.\\\left.\left.\left(42\left(1+\sqrt{1+\frac{1}{3}\left(\frac{1}{25}\left(4\,\pi^{2}\right)\right)e^{2\times0.989117352243}}\right.-\right.\\\left.13\left(\frac{1}{25}\left(4\,\pi^{2}\right)\right)e^{2\times0.989117352243}\right)\right)\right)\right)^{5}$$

Result:

1.651220569...

1.651220569.... result very near to the 14th root of the following Ramanujan's class invariant $Q = (G_{505}/G_{101/5})^3 = 1164.2696$ i.e. 1.65578...

$$-\left[\left(32\ e^{-4\times0.9891173522430000}\left(42\left(1+\sqrt{1+\frac{(4\ \pi^2)\ e^{2\times0.9891173522430000}}{3\times25}}\right)-\frac{1}{25}(4\ \pi^2)\ 13\ e^{2\times0.9891173522430000}}\right)\right]\right)/\left(1+\sqrt{1+\frac{(4\ \pi^2)\ e^{2\times0.9891173522430000}}{3\times25}}\right)^7\right)^5 = \left(4\ 385\ 270\ 057\ 140\ 224\left(-25+52\ e^{1.978234704486000}\ \pi^2-25\ \sqrt{\frac{4\ e^{1.978234704486000}\ \pi^2}{75}}\right)\right)$$
$$= \left(9\ 765\ 625\ e^{19.78234704486000}\left(1+\sqrt{\frac{4\ e^{1.978234704486000}\ \pi^2}{75}}\right)^{-k}\left(\frac{1}{2}\atop{k}\right)\right)^5\right)/\left(9\ 765\ 625\ e^{19.78234704486000}\left(1+\sqrt{\frac{4\ e^{1.978234704486000}\ \pi^2}{75}}\right)\right)^{-k}$$

$$\begin{bmatrix} 9\,765\,625\ e^{19.7823470\,448\,6000} \left[1 + \sqrt{\frac{4\ e^{1.97823470\,448\,6000}\ \pi^2}{75}} \\ \sum_{k=0}^{\infty} \frac{\left(-\frac{75}{4}\right)^k \left(e^{1.97823470\,448\,6000\ \pi^2}\right)^{-k} \left(-\frac{1}{2}\right)_k}{k!} \end{bmatrix}^{35} \end{bmatrix}$$

$$32\ e^{-4 \times 0.9891173522430000} \left[42 \left[1 + \sqrt{1 + \frac{(4\ \pi^2)\ e^{2 \times 0.9891173522430000}}{3 \times 25}} - \frac{1}{25} \left(4\ \pi^2\right) 13\ e^{2 \times 0.9891173522430000}} \right] \right] \right] / \left[1 + \sqrt{1 + \frac{(4\ \pi^2)\ e^{2 \times 0.9891173522430000}}{3 \times 25}} \right]^7 \right]^5 = \left[4\ 385\ 270\ 057\ 140\ 224 \left[-25 + 52\ e^{1.978234704486000\ \pi^2} - 2 \\ 25\ \sqrt{z_0}\ \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(1 + \frac{4\ e^{1.978234704486000\ \pi^2}}{75} - z_0\right)^k z_0^{-k}} \right]^5 \right]$$

$$\left[9\ 765\ 625\ e^{19.78234704486000} \\ \left[1 + \sqrt{z_0}\ \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(1 + \frac{4\ e^{1.978234704486000\ \pi^2}}{75} - z_0\right)^k z_0^{-k} \right]^5 \right]$$
for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \le 0$))

_

$$-\left[\left(32\ e^{-4\times0.9891173522430000}\left(42\left(1+\sqrt{1+\frac{(4\ \pi^2)\ e^{2\times0.9891173522430000}}{3\times25}}\right)-\frac{1}{25}\left(4\ \pi^2\right)13\ e^{2\times0.9891173522430000}\right)\right]\right]/\left(1+\sqrt{1+\frac{(4\ \pi^2)\ e^{2\times0.9891173522430000}}{3\times25}}\right)^7\right)^5 = \left(4\ 385\ 270\ 057\ 140\ 224\left(-25+52\ e^{1.978234704486000\ \pi^2}-25\ \sqrt{\frac{4\ e^{1.978234704486000\ \pi^2}}{75}}\right) \\ \sum_{k=0}^{\infty} \frac{\left(-\frac{75}{4}\right)^k\ (e^{1.978234704486000\ \pi^2})^{-k}\ \left(-\frac{1}{2}\right)_k}{k!}\right)^5\right)/\left(9\ 765\ 625\ e^{19.78234704486000\ }\left(1+\sqrt{\frac{4\ e^{1.978234704486000\ \pi^2}}{75}}\right)^{35}\right)$$

We obtain also:

```
-[32((((e^(-4*0.989117352243) /
[1+sqrt(((1+1/3*(4Pi^2)/25*e^(2*0.989117352243))))]^7 *
[42(1+sqrt(((1+1/3*(4Pi^2)/25*e^(2*0.989117352243))))-
13*(4Pi^2)/25*e^(2*0.989117352243))])))]^1/2
```

Input interpretation:

$$-\sqrt{\left(32\left(\frac{e^{-4\times0.989117352243}}{\left(1+\sqrt{1+\frac{1}{3}\left(\frac{1}{25}\left(4\,\pi^{2}\right)\right)e^{2\times0.989117352243}}\right)^{7}}\right.\\\left.\left.\left.\left(42\left(1+\sqrt{1+\frac{1}{3}\left(\frac{1}{25}\left(4\,\pi^{2}\right)\right)e^{2\times0.989117352243}}\right.-\right.\\\left.13\left(\frac{1}{25}\left(4\,\pi^{2}\right)\right)e^{2\times0.989117352243}\right)\right)\right)\right)\right)}$$

Result:

- 0 1.0514303501... i

Polar coordinates:

r = 1.05143035007 (radius), $\theta = -90^{\circ}$ (angle)

1.05143035007

$$-\sqrt{\left[\left(32\ e^{-4\times0.9891173522430000}\left(42\left(1+\sqrt{1+\frac{(4\ \pi^2)\ e^{2\times0.9891173522430000}}{3\times25}}\right)-\frac{1}{25}\left(4\ \pi^2\right)13\ e^{2\times0.9891173522430000}\right)\right]\right)}/\left(1+\sqrt{1+\frac{(4\ \pi^2)\ e^{2\times0.9891173522430000}}{3\times25}}\right)^7\right)=-\frac{8}{5}\ \sqrt{21}$$

$$\sqrt{\left[\left(25-52\ e^{1.978234704486000\ \pi^2}+25\ \sqrt{\frac{4\ e^{1.978234704486000\ \pi^2}}{75}}\right)-\frac{8}{5}\ \sqrt{21}\right]}$$

$$\sum_{k=0}^{\infty}\left(\frac{75}{4}\right)^k\left(e^{1.978234704486000\ \pi^2}\right)^{-k}\left(\frac{1}{2}\\k\right)\right)/\left(e^{3.956469408972000}\left(1+\sqrt{\frac{4\ e^{1.978234704486000\ \pi^2}}{75}}\right)-\frac{8}{5}\left(\frac{75}{4}\right)^k\left(e^{1.978234704486000\ \pi^2}\right)^{-k}\left(\frac{1}{2}\\k\right)\right)/\left(e^{3.956469408972000}\left(1+\sqrt{\frac{4\ e^{1.978234704486000\ \pi^2}}{75}}\right)-\frac{8}{5}\left(\frac{75}{4}\right)^k\left(e^{1.978234704486000\ \pi^2}\right)^{-k}\left(\frac{1}{2}\\k\right)\right)/\left(e^{3.956469408972000}\left(1+\sqrt{\frac{4\ e^{1.978234704486000\ \pi^2}}{75}}\right)^{-k}\left(\frac{1}{2}\\k\right)\right)/\left(e^{3.956469408972000}\left(1+\sqrt{\frac{4\ e^{1.978234704486000\ \pi^2}}{75}}\right)^{-k}\left(e^{1.978234704486000\ \pi^2}\right)^{-k}\left(\frac{1}{2}\\k\right)\right)/\left(e^{3.956469408972000}\left(1+\sqrt{\frac{4\ e^{1.978234704486000\ \pi^2}}{75}}\right)^{-k}\left(e^{1.978234704486000\ \pi^2}\right)^{-k}\left(\frac{1}{2}\\k\right)\right)/\left(e^{3.956469408972000}\left(1+\sqrt{\frac{4\ e^{1.978234704486000\ \pi^2}}{75}}\right)^{-k}\left(e^{1.978234704486000\ \pi^2}\right)^{-k}\left(\frac{1}{2}\\k\right)\right)}\right)$$

$$-\sqrt{\left(\left[32\ e^{-4\times0.9891173522430000}\left(42\left(1+\sqrt{1+\frac{(4\ \pi^2)\ e^{2\times0.9891173522430000}}{3\times25}}\right)\right)\right)\right)}\right)}$$
$$\left(1+\sqrt{1+\frac{(4\ \pi^2)\ e^{2\times0.9891173522430000}}{3\times25}}\right)^7\right)=-\frac{8}{5}\ \sqrt{21}$$
$$\sqrt{\left(\left[25-52\ e^{1.978234704486000\ \pi^2}+25\ \sqrt{\frac{4\ e^{1.978234704486000\ \pi^2}}{75}}\right]\right)}$$
$$\left(\frac{e^{3.956469408972000}\left(1+\sqrt{\frac{4\ e^{1.978234704486000\ \pi^2}}{75}}\right)\right)\right)$$
$$\left(\frac{e^{3.956469408972000}\left(1+\sqrt{\frac{4\ e^{1.978234704486000\ \pi^2}}{75}}\right)$$
$$\left(\frac{e^{3.956469408972000}\left(1+\sqrt{\frac{4\ e^{1.978234704486000\ \pi^2}}{75}}\right)\right)\right)$$

$$-\sqrt{\left|\left|\left|32\ e^{-4\times0.9891173522430000}\left(42\left(1+\sqrt{1+\frac{(4\ \pi^2)\ e^{2\times0.9891173522430000}}{3\times25}}\right)-\frac{1}{25}\ (4\ \pi^2)\ 13\ e^{2\times0.9891173522430000}\right)\right|\right|\right|}\right|$$
$$\left(1+\sqrt{1+\frac{(4\ \pi^2)\ e^{2\times0.9891173522430000}}{3\times25}}\right)^7\right)=$$
$$-\frac{8}{5}\ \sqrt{21}\ \sqrt{\left|\left|\left|25-52\ e^{1.978234704486000\ \pi^2}+\frac{25}{75}-z_0\right|^k\ z_0^{-k}\right|\right|}\right|}\right|}$$
$$\left(e^{3.956469408972000}\left(1+\sqrt{z_0}\ \sum_{k=0}^{\infty}\ \frac{(-1)^k\ (-\frac{1}{2})_k\ (1+\frac{4\ e^{1.978234704486000\ \pi^2}}{75}-z_0)^k\ z_0^{-k}}{k!}\right)\right|$$

for (not $(z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0)$)

```
\begin{array}{l} 1 \ / \ -[32((((e^{-4*0.989117352243}) \ / \\ [1+sqrt(((1+1/3*(4Pi^2)/25*e^{-2*0.989117352243}))))]^7 \ * \\ [42(1+sqrt(((1+1/3*(4Pi^2)/25*e^{-2*0.989117352243}))))^{-1} \\ 13*(4Pi^2)/25*e^{-2*0.989117352243})])))]^1/2 \end{array}
```

Input interpretation:

$$-\left(1 \left/ \left(\sqrt{\left(32 \left(\frac{e^{-4 \times 0.989117352243}}{\left(1 + \sqrt{1 + \frac{1}{3} \left(\frac{1}{25} \left(4 \pi^2 \right) \right) e^{2 \times 0.989117352243}} \right)^7} \right. \right. \right. \\ \left. \left. \left(42 \left(1 + \sqrt{1 + \frac{1}{3} \left(\frac{1}{25} \left(4 \pi^2 \right) \right) e^{2 \times 0.989117352243}} \right. - 13 \left(\frac{1}{25} \left(4 \pi^2 \right) \right) e^{2 \times 0.989117352243}} \right) \right] \right) \right)$$

Result: 0.95108534763... *i*

Polar coordinates:

r = 0.95108534763 (radius), $\theta = 90^{\circ}$ (angle)

0.95108534763

We know that the primordial fluctuations are consistent with Gaussian purely adiabatic scalar perturbations characterized by a power spectrum with a spectral index $n_s = 0.965 \pm 0.004$, consistent with the predictions of slow-roll, single-field, inflation.

Thence 0.95108534763 is a result very near to the spectral index n_s , to the mesonic Regge slope, to the inflaton value at the end of the inflation 0.9402 and to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}}-\varphi+1} = 1 - \frac{e^{-\pi}}{1+\frac{e^{-2\pi}}{1+\frac{e^{-2\pi}}{1+\frac{e^{-3\pi}}{1+\frac{e^{-4\pi}}{1+\frac{e^{-4\pi}}{1+\dots}}}}} \approx 0.9568666373$$

Series representations:

$$-\left(1 / \left(\sqrt{\left\| \left\| 32 \ e^{-4 \times 0.9891173522430000} \left\{ 42 \left(1 + \sqrt{1 + \frac{(4 \pi^2) \ e^{2 \times 0.9891173522430000}}{3 \times 25} - \frac{1}{25} (4 \pi^2) 13 \ e^{2 \times 0.9891173522430000} \right) \right\| \right) / \left(1 + \sqrt{1 + \frac{(4 \pi^2) \ e^{2 \times 0.9891173522430000}}{3 \times 25}} \right)^7 \right) \right) \right) = -\left(5 / \left\{ 8 \sqrt{21} \sqrt{\left\| \left\| 25 - 52 \ e^{1.978234704486000} \ \pi^2 + 25} \sqrt{\frac{4 \ e^{1.978234704486000} \ \pi^2}{75}} \right. \right. \right. \right) \right) \right) / \left(e^{3.956469408972000} \left(1 + \sqrt{\frac{4 \ e^{1.978234704486000} \ \pi^2}{75}} - \frac{\sum_{k=0}^{\infty} \left(\frac{75}{4}\right)^k \left(e^{1.978234704486000} \ \pi^2 \right)^{-k} \left(\frac{1}{2} \\ k \right) \right) / \left(e^{3.956469408972000} \left(1 + \sqrt{\frac{4 \ e^{1.978234704486000} \ \pi^2}{75}} - \frac{\sum_{k=0}^{\infty} \left(\frac{75}{4}\right)^k \left(e^{1.978234704486000} \ \pi^2 \right)^{-k} \left(\frac{1}{2} \\ k \right) \right) \right) \right) \right)$$

$$-\left(1 / \left(\sqrt{\left\| \left\| 32 e^{-4 \times 0.9891173522430000} \left\{ 42 \left(1 + \sqrt{1 + \frac{(4 \pi^2) e^{2 \times 0.9891173522430000}}{3 \times 25}} - \frac{1}{25} (4 \pi^2) 13 e^{2 \times 0.9891173522430000} \right) \right\| \right) / \left(1 + \sqrt{1 + \frac{(4 \pi^2) e^{2 \times 0.9891173522430000}}{3 \times 25}} \right)^7 \right) \right) = -\left(5 / \left\| 8 \sqrt{21} \sqrt{\left\| \left\| \left[25 - 52 e^{1.978234704486000} \pi^2 + 25 \sqrt{\frac{4 e^{1.978234704486000} \pi^2}{75}} - \frac{\sum_{k=0}^{\infty} \frac{(-\frac{75}{4})^k (e^{1.978234704486000} \pi^2)^{-k} (-\frac{1}{2})_k}{k!} \right) \right/ \right) \right\|$$

$$-\left(1 / \left(\sqrt{\left(\left(32 e^{-4 \times 0.9891173522430000} \left(42 \left(1 + \sqrt{1 + \frac{(4 \pi^2) e^{2 \times 0.9891173522430000}}{3 \times 25}} - \frac{1}{25} \right) \right) \right) \right) \right) \right) - \frac{1}{25} + \frac{1}{2$$

for (not $(z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0)$)

From the previous expression

$$\frac{e^{-4\times0.989117352243}}{\left(1+\sqrt{1+\frac{1}{3}\left(\frac{1}{25}\left(4\,\pi^2\right)\right)e^{2\times0.989117352243}}\right)^7} \\ \left(42\left(1+\sqrt{1+\frac{1}{3}\left(\frac{1}{25}\left(4\,\pi^2\right)\right)e^{2\times0.989117352243}}\right) - 13\left(\frac{1}{25}\left(4\,\pi^2\right)\right)e^{2\times0.989117352243}\right)\right) \\$$

= -0.034547055658...

we have also:

Input interpretation:

 $\frac{1}{4 \times \frac{2 \, e^{-0.989117352243/2}}{1 + \sqrt{1 + \frac{1}{3} \left(\frac{1}{25} \left(4 \, \pi^2\right)\right) e^{2 \times 0.989117352243}}}} - 0.034547055658$

Result:

1.61976215705...

1.61976215705..... result that is a very good approximation to the value of the golden ratio 1.618033988749...

$$\begin{split} 1 + \frac{1}{\frac{4\left(2\,e^{-0.9891173522430000/2}\right)}{1+\sqrt{1+\left(\frac{4\pi^2}{2}\right)e^{2\times0.9891173522430000}}} & -0.0345470556580000 = \\ 0.9654529443420000 + \frac{e^{0.4945586761215000}}{8} & + \frac{1}{8}\,e^{0.4945586761215000} \\ \sqrt{\frac{4\,e^{1.978234704486000}\,\pi^2}{75}} & \sum_{k=0}^{\infty} \left(\frac{75}{4}\right)^k \left(e^{1.978234704486000}\,\pi^2\right)^{-k} \left(\frac{1}{2}\\k\right) \\ 1 + \frac{1}{\frac{4\left(2\,e^{-0.9891173522430000/2}\right)}{1+\sqrt{1+\left(\frac{4\pi^2}{2}\right)e^{2\times0.9891173522430000}}}} & -0.0345470556580000 = \\ \sqrt{\frac{4\,e^{1.978234704486000}\,\pi^2}{3\times25}} \\ 0.9654529443420000 + \frac{e^{0.4945586761215000}}{8} & + \frac{1}{8}\,e^{0.4945586761215000} \\ \sqrt{\frac{4\,e^{1.978234704486000}\,\pi^2}{75}} & \sum_{k=0}^{\infty} \left(\frac{-\frac{75}{4}}{k}\right)^k \left(e^{1.978234704486000}\,\pi^2\right)^{-k} \left(-\frac{1}{2}\right)_k \end{split}$$

$$\begin{aligned} 1 + \frac{1}{\frac{4\left(2\,e^{-0.9891173522430000/2}\right)}{1+\sqrt{1+\frac{(4\,\pi^2)\,e^{2}\times0.9891173522430000}{3\times25}}} & -0.0345470556580000 = \\ 0.9654529443420000 + \frac{e^{0.4945586761215000}}{8} & + \\ \frac{1}{8}\,e^{0.4945586761215000}\,\sqrt{z_0}\,\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(1+\frac{4\,e^{1.978234704486000\,\pi^2}}{75}-z_0\right)^k\,z_0^{-k}}{k!} \\ & \text{for (not } (z_0\in\mathbb{R} \text{ and } -\infty < z_0 \le 0)) \end{aligned}$$

Appendix



From: A. Sagnotti – AstronomiAmo, 23.04.2020

In the above figure, it is said that: "why a given shape of the extra dimensions? Crucial, it determines the predictions for α ".

We propose that whatever shape the compactified dimensions are, their geometry must be based on the values of the golden ratio and $\zeta(2)$, (the latter connected to 1728 or 1729, whose fifteenth root provides an excellent approximation to the above mentioned value) which are recurrent as solutions of the equations that we are going to develop. It is important to specify that the initial conditions are **always** values belonging to a fundamental chapter of the work of S. Ramanujan "Modular equations and Appoximations to Pi" (see references). These values are some multiples of 8 (64 and 4096), 276, which added to 4096, is equal to 4372, and finally $e^{\pi\sqrt{22}}$

Proposal of geometric connections between Swampland, Landscape and Riemann zeta function



We notice a certain similarity between the figure that represents the Stringscape and that inherent in the Riemann zeta function: a case? We believe not.



Figure 2: Effective potential with the Wilson lines fixed to zero, as a function of the Radion and the Higgs. The tree level potential dominates and the Higgs is not displaced from its tree level minimum by the one-loop corrections. This behavior is independent of the particular value of the Wilson lines. Although not very visible in the plot, the Higgs minimum remains at the same location as R^{-1} increases.

From: (The Riemann hypothesis illuminated by the Newton flow of ζ - *J W Neuberger*, *C Feiler*, *H Maier and W P Schleich* - Received 29 April 2015, revised 5 August 2015. Accepted for publication 19 August 2015 Published 1 October 2015 - https://doi.org/10.1088/0031-8949/90/10/108015)



Figure 2. Lines in the complex plane where the Riemann zeta function ζ is real (green) depicted on a relief representing the positive absolute value of ζ for arguments $s \circ s + it$ where the real part of ζ is positive, and the negative absolute value of ζ where the real part of ζ is negative. This representation brings out most clearly that the lines of constant phase corresponding to phases of integer multiples of 2p run down the hills on the left-hand side, turn around on the right and terminate in the non-trivial zeros. This pattern repeats itself infinitely many times. The points of arrival and departure on the right-hand side of the picture are equally spaced and given by the following equation:

$$\tau'_k \equiv k \frac{\pi}{\ln 2},$$

with $k = ..., -2, -1, 0, 1, 2,....$

2Pi/(ln(2))

Input: $2 \times \frac{\pi}{\log(2)}$

Exact result:

 $\frac{2\pi}{\log(2)}$

Decimal approximation:

9.0647202836543876192553658914333336203437229354475911683720330958

•••

9.06472028365....

Alternative representations:

 $\frac{2\pi}{\log(2)} = \frac{2\pi}{\log_e(2)}$

 $\frac{2\pi}{\log(2)} = \frac{2\pi}{\log(a)\log_a(2)}$

 $\frac{2\pi}{\log(2)} = \frac{2\pi}{2\coth^{-1}(3)}$

Series representations:

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{2i\pi \lfloor \frac{\arg(2-x)}{2\pi} \rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}}{k}} \text{ for } x < 0$$

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\log(z_0) + \left\lfloor \frac{\arg(2-z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}}{k}}$$

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{2i\pi \left\lfloor \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right\rfloor + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}}{k}$$

Integral representations:

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\int_{1}^{2} \frac{1}{t} dt}$$

$$\frac{2\pi}{\log(2)} = \frac{4i\pi^2}{\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds} \quad \text{for } -1 < \gamma < 0$$

From which:

 $(2\text{Pi}/(\ln(2)))^*(1/12 \pi \log(2))$

Input:

$$\left(2 \times \frac{\pi}{\log(2)}\right) \left(\frac{1}{12} \pi \log(2)\right)$$

log(x) is the natural logarithm

Exact result:

 $\frac{\pi^2}{6}$

Decimal approximation:

1.6449340668482264364724151666460251892189499012067984377355582293 ...

$$1.6449340668.... = \zeta(2) = \frac{\pi^2}{6} = 1.644934...$$

Property:

 $\frac{\pi^2}{6}$ is a transcendental number

Alternative representations:

 $\frac{(\pi \log(2)) 2\pi}{12 \log(2)} = \frac{2\pi^2 \log_e(2)}{12 \log_e(2)}$

 $\frac{(\pi \log(2)) 2\pi}{12 \log(2)} = \frac{2\pi^2 \log(a) \log_a(2)}{12 (\log(a) \log_a(2))}$

 $\frac{(\pi \log(2)) 2\pi}{12 \log(2)} = \frac{4\pi^2 \coth^{-1}(3)}{12 \left(2 \coth^{-1}(3)\right)}$

Series representations:

$$\frac{(\pi \log(2)) 2\pi}{12 \log(2)} = \sum_{k=1}^{\infty} \frac{1}{k^2}$$
$$\frac{(\pi \log(2)) 2\pi}{12 \log(2)} = -2 \sum_{k=1}^{\infty} \frac{(-1)^k}{k^2}$$

$$\frac{(\pi \log(2)) 2\pi}{12 \log(2)} = \frac{4}{3} \sum_{k=0}^{\infty} \frac{1}{(1+2k)^2}$$

Integral representations:

$$\frac{(\pi \log(2)) 2\pi}{12 \log(2)} = \frac{8}{3} \left(\int_0^1 \sqrt{1 - t^2} \, dt \right)^2$$

$$\frac{(\pi \log(2)) 2\pi}{12 \log(2)} = \frac{2}{3} \left(\int_0^\infty \frac{1}{1+t^2} dt \right)^2$$

$$\frac{(\pi \log(2)) 2\pi}{12 \log(2)} = \frac{2}{3} \left(\int_0^1 \frac{1}{\sqrt{1 - t^2}} dt \right)^2$$

From:

Pi/(ln(2))

Input:

 $\frac{\pi}{\log(2)}$

log(x) is the natural logarithm

Decimal approximation:

```
4.53236014182719380962768294571666668101718614677237955841860165479
....
4.53236014.....
```

Alternative representations:

 $\frac{\pi}{\log(2)} = \frac{\pi}{\log_e(2)}$

 $\frac{\pi}{\log(2)} = \frac{\pi}{\log(a)\log_a(2)}$

 $\frac{\pi}{\log(2)} = \frac{\pi}{2\coth^{-1}(3)}$

$$\frac{\pi}{\log(2)} = \frac{\pi}{2i\pi \left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}}{k}} \text{ for } x < 0$$

$$\frac{\pi}{\log(2)} = \frac{\pi}{\log(z_0) + \left\lfloor \frac{\arg(2-z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}}{k}}$$

$$\frac{\pi}{\log(2)} = \frac{\pi}{2i\pi \left\lfloor \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right\rfloor + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}}{k}$$

Integral representations:

$$\frac{\pi}{\log(2)} = \frac{\pi}{\int_1^2 \frac{1}{t} dt}$$

$$\frac{\pi}{\log(2)} = \frac{2i\pi^2}{\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma} \frac{\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds} \quad \text{for } -1 < \gamma < 0$$

From which:

 $(Pi/(ln(2)))*(1/6 \pi \log(2))$

Input:

 $\frac{\pi}{\log(2)}\left(\frac{1}{6}\,\pi\log(2)\right)$

log(x) is the natural logarithm

Exact result:

 $\frac{\pi^2}{6}$

Decimal approximation:

1.6449340668482264364724151666460251892189499012067984377355582293 ...

$$1.6449340668.... = \zeta(2) = \frac{\pi^2}{6} = 1.644934...$$

Property:

 $\frac{\pi^2}{6}$ is a transcendental number

Alternative representations:

$(\pi \log(2)) \pi$	$\pi^2 \log_e(2)$
6 log(2)	$= \frac{1}{6 \log_e(2)}$

$(\pi \log(2)) \pi$	=	$\pi^2 \log(a) \log_a(2)$
6 log(2)		$\overline{6(\log(a)\log_a(2))}$

$(\pi \log(2)) \pi$		$2\pi^2 \operatorname{coth}^{-1}(3)$
6 log(2)	=	$\overline{6(2 \operatorname{coth}^{-1}(3))}$

$$\frac{(\pi \log(2)) \pi}{6 \log(2)} = \sum_{k=1}^{\infty} \frac{1}{k^2}$$

$$\frac{(\pi \log(2)) \pi}{6 \log(2)} = -2 \sum_{k=1}^{\infty} \frac{(-1)^k}{k^2}$$

$$\frac{(\pi \log(2)) \pi}{6 \log(2)} = \frac{4}{3} \sum_{k=0}^{\infty} \frac{1}{(1+2k)^2}$$

Integral representations:

$$\frac{(\pi \log(2))\pi}{6\log(2)} = \frac{8}{3} \left(\int_0^1 \sqrt{1-t^2} \, dt \right)^2$$

$$\frac{(\pi \log(2)) \pi}{6 \log(2)} = \frac{2}{3} \left(\int_0^\infty \frac{1}{1+t^2} dt \right)^2$$

$$\frac{(\pi \log(2)) \pi}{6 \log(2)} = \frac{2}{3} \left(\int_0^1 \frac{1}{\sqrt{1 - t^2}} dt \right)^2$$

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