On some equations concerning Weak Gravity Conjecture and Swampland. Mathematical connections with some Ramanujan formulas, Riemann zeta function and some sectors of String Theory

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#### Abstract

In this research thesis, we analyze some equations concerning Weak Gravity Conjecture and Swampland. Furthermore, we obtain various mathematical connections with some Ramanujan formulas, Riemann zeta function and several sectors of String Theory


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Vesuvius landscape with gorse - Naples

https://www.pinterest.it/pin/95068242114589901/

We want to highlight that the development of the various equations was carried out according an our possible logical and original interpretation

From:

## On the Zeros of the Davenport Heilbronn Function

S. A. Gritsenko - Received May 15, 2016 - ISSN 0081-5438, Proceedings of the Steklov Institute of Mathematics, 2017, Vol. 296, pp. 65-87.

We have:

Let

$$
\varkappa=\frac{\sqrt{10-2 \sqrt{5}}-2}{\sqrt{5}-1}
$$

and $\chi_{1}$ be a character modulo 5 such that $\chi_{1}(2)=i$.
The Davenport-Heilbronn function $f(s)$ is defined by the equality

$$
f(s)=\frac{1-i \varkappa}{2} L\left(s, \chi_{1}\right)+\frac{1+i \varkappa}{2} L\left(s, \bar{\chi}_{1}\right), \quad \text { where } \quad L(s, \chi)=\sum_{n=1}^{\infty} \frac{\chi(n)}{n^{s}} .
$$

The function $f(s)$ satisfies the Riemann-type functional equation

$$
g(s)=g(1-s), \quad \text { where } \quad g(s)=\left(\frac{\pi}{5}\right)^{-s / 2} \Gamma\left(\frac{s+1}{2}\right) f(s),
$$

but there is no Euler product for it.
$(\sqrt{10-2 \sqrt{5}}-2) /(\sqrt{5}-1)=\kappa$

## Input:

$$
\frac{\sqrt{10-2 \sqrt{5}}-2}{\sqrt{5}-1}
$$

Decimal approximation:
0.2840790438404122960282918323931261690910880884457375827591626661
$0.28407904384 \ldots=\kappa$

## Alternate forms:

$\frac{1}{4}(\sqrt{10-2 \sqrt{5}}-2 \sqrt{5}+\sqrt{5(10-2 \sqrt{5})}-2)$
$\frac{1}{4}(1+\sqrt{5})(\sqrt{10-2 \sqrt{5}}-2)$
$\frac{1}{2}(-1-\sqrt{5}+\sqrt{2(5+\sqrt{5})})$

Minimal polynomial:

$$
x^{4}+2 x^{3}-6 x^{2}-2 x+1
$$

## Expanded forms:

$\frac{\sqrt{10-2 \sqrt{5}}}{\sqrt{5}-1}-\frac{2}{\sqrt{5}-1}$
$\frac{1}{4} \sqrt{10-2 \sqrt{5}}+\frac{1}{4} \sqrt{5(10-2 \sqrt{5})}+\frac{1}{2}(-1-\sqrt{5})$

For $\quad((((\sqrt{ }(10-2 \sqrt{ } 5)-2))(((\sqrt{ } 5-1))))=8 \pi G ; \quad G=0.011303146014$

Indeed:
$((((\sqrt{ }(10-2 \sqrt{ } 5)-2))(((\sqrt{ } 5-1)))) /(8 \pi)$

## Input:

$\frac{\frac{\sqrt{10-2 \sqrt{5}}-2}{\sqrt{5}-1}}{8 \pi}$

## Result:

$$
\frac{\sqrt{10-2 \sqrt{5}}-2}{8(\sqrt{5}-1) \pi}
$$

## Decimal approximation:

0.0113031460140052147973750129442035744685760313920017808594909667
$0.01130314 \ldots=\mathrm{g}($ gravitational coupling constant $)$

## Property:

$\frac{-2+\sqrt{10-2 \sqrt{5}}}{8(-1+\sqrt{5}) \pi}$ is a transcendental number

## Alternate forms:

$\frac{\sqrt{10-2 \sqrt{5}}-2 \sqrt{5}+\sqrt{5(10-2 \sqrt{5})}-2}{32 \pi}$

$$
-\frac{1+\sqrt{5}-\sqrt{2(5+\sqrt{5})}}{16 \pi}
$$

$\frac{-1-\sqrt{5}+\sqrt{2(5+\sqrt{5})}}{16 \pi}$

## Expanded forms:

$-\frac{1}{16 \pi}-\frac{\sqrt{5}}{16 \pi}+\frac{\sqrt{10-2 \sqrt{5}}}{32 \pi}+\frac{\sqrt{5(10-2 \sqrt{5})}}{32 \pi}$

$$
\frac{\sqrt{10-2 \sqrt{5}}}{8(\sqrt{5}-1) \pi}-\frac{1}{4(\sqrt{5}-1) \pi}
$$

## Series representations:

$$
\frac{\sqrt{10-2 \sqrt{5}}-2}{(8 \pi)(\sqrt{5}-1)}=\frac{-2+\sqrt{9-2 \sqrt{5}} \sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}(9-2 \sqrt{5})^{-k}}{8 \pi\left(-1+\sqrt{4} \sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k}\right)}
$$

$$
\frac{\sqrt{10-2 \sqrt{5}}-2}{(8 \pi)(\sqrt{5}-1)}=\frac{-2+\sqrt{9-2 \sqrt{5}} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}(9-2 \sqrt{5})^{-k}}{k!}}{8 \pi\left(-1+\sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}\right)}
$$

$$
\frac{\sqrt{10-2 \sqrt{5}}-2}{(8 \pi)(\sqrt{5}-1)}=\frac{-2+\sqrt{z_{0}} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(10-2 \sqrt{5}-z_{0}\right)^{k} z_{0}^{-k}}{k!}}{8 \pi\left(-1+\sqrt{z_{0}} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(5-z_{0}\right)^{k} z_{0}^{-k}}{k!}\right)}
$$

for $\left(\operatorname{not}\left(z_{0} \in \mathbb{R}\right.\right.$ and $\left.-\infty<z_{0} \leq 0\right)$ )

We note that:
$(((\sqrt{ }(10-2 \sqrt{ } 5)-2))((\sqrt{ } 5-1))) *((2 \mathrm{i}(\operatorname{sqrt}(5)-1) t+\operatorname{sqrt}(5)-1) /(2(\operatorname{sqrt}(2(5-\operatorname{sqrt}(5)))-$ 2)))

## Input:

$\frac{\sqrt{10-2 \sqrt{5}}-2}{\sqrt{5}-1} \times \frac{2 i(\sqrt{5}-1) t+\sqrt{5}-1}{2(\sqrt{2(5-\sqrt{5})}-2)}$
$i$ is the imaginary unit

## Exact result:

$$
\frac{(\sqrt{10-2 \sqrt{5}}-2)(2 i(\sqrt{5}-1) t+\sqrt{5}-1)}{2(\sqrt{5}-1)(\sqrt{2(5-\sqrt{5})}-2)}
$$

## Plot:



Alternate form assuming $\mathbf{t > 0}$ :

$$
\begin{aligned}
& \frac{i \sqrt{10-2 \sqrt{5}} t}{\sqrt{2(5-\sqrt{5})}-2}-\frac{2 i t}{\sqrt{2(5-\sqrt{5})}-2}+ \\
& \frac{\sqrt{5(10-2 \sqrt{5})}}{2(\sqrt{5}-1)(\sqrt{2(5-\sqrt{5})}-2)}-\frac{\sqrt{10-2 \sqrt{5}}}{2(\sqrt{5}-1)(\sqrt{2(5-\sqrt{5})}-2)}- \\
& \frac{\sqrt{5}}{(\sqrt{5}-1)(\sqrt{2(5-\sqrt{5})}-2)}+\frac{1}{(\sqrt{5}-1)(\sqrt{2(5-\sqrt{5})}-2)}
\end{aligned}
$$

## Alternate forms:

$$
\frac{1}{8}(1+\sqrt{5})(2 i \sqrt{2(3-\sqrt{5})} t+\sqrt{5}-1)
$$

$$
\frac{1}{2}(1+2 i t)
$$

$$
\frac{1}{2}+i t
$$

$1 / 2+$ it $=$ real part of every nontrivial zero of the Riemann zeta function

## Derivative:

$$
\frac{d}{d t}\left(\frac{(\sqrt{10-2 \sqrt{5}}-2)(2 i(\sqrt{5}-1) t+\sqrt{5}-1)}{(\sqrt{5}-1)(2(\sqrt{2(5-\sqrt{5})}-2))}\right)=i
$$

## Indefinite integral:

$$
\int \frac{(\sqrt{10-2 \sqrt{5}}-2)(2 i(\sqrt{5}-1) t+\sqrt{5}-1)}{(\sqrt{5}-1)(2(\sqrt{2(5-\sqrt{5})}-2))} d t=\frac{t}{2}+\frac{i t^{2}}{2}+\text { constant }
$$

And again:
$(((\sqrt{ }(10-2 \sqrt{ } 5)-2))((2 \mathrm{x}))) *((2 \mathrm{i}(\operatorname{sqrt}(5)-1) \mathrm{t}+\operatorname{sqrt}(5)-1) /(2(\operatorname{sqrt}(2(5-\operatorname{sqrt}(5)))-2)))$ $=(1 / 2+i t)$

## Input:

$$
\frac{\sqrt{10-2 \sqrt{5}}-2}{2 x} \times \frac{2 i(\sqrt{5}-1) t+\sqrt{5}-1}{2(\sqrt{2(5-\sqrt{5})}-2)}=\frac{1}{2}+i t
$$

## Exact result:

$$
\frac{(\sqrt{10-2 \sqrt{5}}-2)(2 i(\sqrt{5}-1) t+\sqrt{5}-1)}{4(\sqrt{2(5-\sqrt{5})}-2) x}=\frac{1}{2}+i t
$$

## Alternate form assuming $t$ and $x$ are real:

$\frac{\sqrt{5}-1}{x}=2$

Alternate form:
$\frac{(\sqrt{5}-1)(1+2 i t)}{4 x}=\frac{1}{2}+i t$

Alternate form assuming $\mathbf{t}$ and $\mathbf{x}$ are positive: $2 x+1=\sqrt{5}$

## Expanded forms:

$$
\begin{aligned}
& \frac{i \sqrt{5(10-2 \sqrt{5})} t}{2(\sqrt{2(5-\sqrt{5})}-2) x}-\frac{i \sqrt{10-2 \sqrt{5}} t}{2(\sqrt{2(5-\sqrt{5})}-2) x}-\frac{i \sqrt{5} t}{(\sqrt{2(5-\sqrt{5})}-2) x}+ \\
& \frac{i t}{(\sqrt{2(5-\sqrt{5})}-2) x}+\frac{\sqrt{5(10-2 \sqrt{5})}}{4(\sqrt{2(5-\sqrt{5})}-2) x}-\frac{\sqrt{10-2 \sqrt{5}}}{4(\sqrt{2(5-\sqrt{5})}-2) x}- \\
& \frac{\sqrt{5}}{2(\sqrt{2(5-\sqrt{5})}-2) x}+\frac{1}{2(\sqrt{2(5-\sqrt{5})}-2) x}=\frac{1}{2}+i t
\end{aligned}
$$

$\frac{i \sqrt{5} t}{2 x}-\frac{i t}{2 x}+\frac{\sqrt{5}}{4 x}-\frac{1}{4 x}=\frac{1}{2}+i t$

## Solutions:

$t=\frac{i}{2}, \quad x \neq 0$
$x=\frac{\sqrt{5}}{2}-\frac{1}{2}$

## Input:

$\frac{\sqrt{5}}{2}-\frac{1}{2}$

Decimal approximation:
0.6180339887498948482045868343656381177203091798057628621354486227
$0.6180339887 \ldots=\frac{1}{\phi}$

Solution for the variable $x$ :
$x=\frac{-2 i \sqrt{5} t+2 i t-\sqrt{5}+1}{-2-4 i t}$

## Implicit derivatives:

$\frac{\partial x(t)}{\partial t}=\frac{2(-1+\sqrt{5}-2 x) x}{(-1+\sqrt{5})(-i+2 t)}$
$\frac{\partial t(x)}{\partial x}=\frac{(-1+\sqrt{5})(-i+2 t)}{2(-1+\sqrt{5}-2 x) x}$

## From:

## AdS-phobia, the WGC, the Standard Model and Supersymmetry Eduardo Gonzalo, Alvaro Herraez and Luis E. Ibanez - arXiv:1803.08455v2 [hep-th] 4 May 2018

We have that:


Figure 2: Effective potential with the Wilson lines fixed to zero, as a function of the Radion and the Higgs. The tree level potential dominates and the Higgs is not displaced from its tree level minimum by the one-loop corrections. This behavior is independent of the particular value of the Wilson lines Although not very visible in the plot, the Higgs minimum remains at the same location as $R^{-1}$ increases.

## From:

$$
\begin{align*}
V_{\mathcal{C}}[a, 0] & =\frac{1}{(2 \pi a)^{2}}\left[\frac{1}{\pi^{2}} \sum_{p=1}^{\infty} \frac{1}{p^{4}}+\frac{1}{2 \pi} \sum_{n=-\infty}^{\infty}\left\{2 \pi|n| \operatorname{Li}_{2}\left(e^{-2 \pi|n|}\right)+\mathrm{Li}_{3}\left(e^{-2 \pi|n|}\right)\right\}\right] \\
& =\frac{1}{(2 \pi a)^{2}}\left[\frac{1}{\pi^{2}} \mathrm{Li}_{4}(1)+\frac{1}{2 \pi} \sum_{n=-\infty}^{\infty} \sum_{p=1}^{\infty}\left\{\frac{2 \pi}{p^{2}}|n|\left(e^{-2 \pi p}\right)^{|n|}+\frac{1}{p^{3}}\left(e^{-2 \pi p}\right)^{|n|}\right\}\right] \\
& =\frac{1}{(2 \pi a)^{2}}\left[\frac{\pi^{2}}{90}+\frac{1}{2 \pi} \sum_{p=1}^{\infty}\left\{2 \pi \frac{1}{2 p^{2} \sinh ^{2} \pi p}+\frac{\operatorname{coth} \pi p}{p^{3}}\right\}\right] \\
& =\frac{1}{(2 \pi a)^{2}}\left[\frac{\pi^{2}}{90}+\frac{1}{\pi} \sum_{p=1}^{\infty}\left\{2 \pi \frac{1}{p^{2}(\cosh \pi p-1)}\right\}+\frac{7 \pi^{2}}{360}\right] \\
& =\frac{1}{(2 \pi a)^{2}} \frac{\mathcal{G}}{3}, \tag{B.12}
\end{align*}
$$

where $\mathcal{G} \simeq 0.915966$ is Catalan's constant. For the case of antiperiodic boundary conditions the Casimir energy reads:

$$
\begin{align*}
V_{\mathcal{C}}\left[a, \frac{1}{2}\right] & =\frac{1}{(2 \pi a)^{2}}\left[\frac{1}{\pi^{2}} \sum_{p=1}^{\infty} \frac{(-1)^{p}}{p^{4}}+\frac{1}{2 \pi} \sum_{n=-\infty}^{\infty}\left\{\pi|2 n+1| \operatorname{Li}_{2}\left(-e^{-\pi|2 n+1|}\right)+\mathrm{Li}_{3}\left(-e^{-\pi|2 n+1|}\right)\right\}\right] \\
& =\frac{1}{(2 \pi a)^{2}}\left[\frac{1}{\pi^{2}} \operatorname{Li}_{4}(-1)+\frac{1}{2 \pi} \sum_{n=1}^{\infty}\left\{\frac{(-1)^{p} \pi}{p^{2}}|2 n+1|\left(e^{-\pi p}\right)^{|2 n+1|}+\frac{(-1)^{p}}{p^{3}}\left(e^{-\pi p}\right)^{|2 n+1|}\right\}\right] \\
& =\frac{1}{(2 \pi a)^{2}}\left[-\frac{7}{8} \frac{\pi^{2}}{90}-\frac{1}{2 \pi} \sum_{p=1}^{\infty}\left\{2 \pi \frac{\frac{(-1)^{p}}{4} \operatorname{csch}^{2} \pi p+\frac{(-1)^{p}}{4} \operatorname{sech}^{2} \pi p}{2 p^{2}}+\frac{(-1)^{p} \operatorname{csch} 2 \pi p}{p^{3}}\right\}\right] \\
& =\frac{-1}{(2 \pi a)^{2}} \frac{\mathcal{G}}{6}=-\frac{1}{2} V_{\mathcal{C}}[a, 0] . \tag{B.13}
\end{align*}
$$

where $\mathcal{G} \simeq 0.915966$ is Catalan's constant.

$$
\begin{aligned}
V_{\mathcal{C}}[a, 0] & =\frac{1}{(2 \pi a)^{2}}\left[\frac{1}{\pi^{2}} \sum_{p=1}^{\infty} \frac{1}{p^{4}}+\frac{1}{2 \pi} \sum_{n=-\infty}^{\infty}\left\{2 \pi|n| \mathrm{Li}_{2}\left(e^{-2 \pi|n|}\right)+\mathrm{Li}_{3}\left(e^{-2 \pi|n|}\right)\right\}\right] \\
& =\frac{1}{(2 \pi a)^{2}} \frac{\mathcal{G}}{3}
\end{aligned}
$$

## $1 /\left(2 \mathrm{Pi}^{*} \mathrm{a}\right)^{\wedge} 2^{*} 1 / 3 * 0.915966$

## Input interpretation:

$$
\frac{1}{(2 \pi a)^{2}} \times \frac{1}{3} \times 0.915966
$$

## Result:

$\frac{0.0077339}{a^{2}}$

## Plots:




Alternate form assuming a is real:
$\frac{0.0077339}{a^{2}}+0$

## Roots:

(no roots exist)

## Property as a function:

## Parity

even

## Derivative:

$\frac{d}{d a}\left(\frac{0.0077339}{a^{2}}\right)=-\frac{0.0154678}{a^{3}}$

Indefinite integral:

$$
\int \frac{0.915966}{(2 \pi a)^{2} 3} d a=-\frac{0.0077339}{a}+\text { constant }
$$

## Limit:

$$
\lim _{a \rightarrow \pm \infty} \frac{0.0077339}{a^{2}}=0 \approx 0
$$

Alternative representations:
$\frac{0.915966}{3(2 \pi a)^{2}}=\frac{0.305322}{\left(360 a^{\circ}\right)^{2}}$
$\frac{0.915966}{3(2 \pi a)^{2}}=\frac{0.305322}{(-2 a i \log (-1))^{2}}$
$\frac{0.915966}{3(2 \pi a)^{2}}=\frac{0.305322}{\left(2 a \cos ^{-1}(-1)\right)^{2}}$

## Series representations:

$\frac{0.915966}{3(2 \pi a)^{2}}=\frac{0.00477066}{a^{2}\left(\sum_{k=0}^{\infty} \frac{(-1)^{k}}{1+2 k}\right)^{2}}$
$\frac{0.915966}{3(2 \pi a)^{2}}=\frac{0.0190826}{a^{2}\left(-1+\sum_{k=1}^{\infty} \frac{2^{k}}{\binom{2 k}{k}}\right)^{2}}$
$\frac{0.915966}{3(2 \pi a)^{2}}=\frac{0.0763305}{a^{2}\left(\sum_{k=0}^{\infty} \frac{2^{-k}(-6+50 k)}{\binom{3 k}{k}}\right)^{2}}$

## Integral representations:

$\frac{0.915966}{3(2 \pi a)^{2}}=\frac{0.0190826}{a^{2}\left(\int_{0}^{\infty} \frac{1}{1+t^{2}} d t\right)^{2}}$
$\frac{0.915966}{3(2 \pi a)^{2}}=\frac{0.00477066}{a^{2}\left(\int_{0}^{1} \sqrt{1-t^{2}} d t\right)^{2}}$
$\frac{0.915966}{3(2 \pi a)^{2}}=\frac{0.0190826}{a^{2}\left(\int_{0}^{\infty} \frac{\sin (t)}{t} d t\right)^{2}}$

For $\mathrm{a}=1$ :
$1 /(2 \mathrm{Pi})^{\wedge} 2 * 1 / 3 * 0.915966$

## Input interpretation:

$\frac{1}{(2 \pi)^{2}} \times \frac{1}{3} \times 0.915966$

## Result:

0.00773390...
0.00773390....

## Alternative representations:

$\frac{0.915966}{3(2 \pi)^{2}}=\frac{0.305322}{\left(360^{\circ}\right)^{2}}$
$\frac{0.915966}{3(2 \pi)^{2}}=\frac{0.305322}{(-2 i \log (-1))^{2}}$
$\frac{0.915966}{3(2 \pi)^{2}}=\frac{0.305322}{\left(2 \cos ^{-1}(-1)\right)^{2}}$

## Series representations:

$$
\frac{0.915966}{3(2 \pi)^{2}}=\frac{0.00477066}{\left(\sum_{k=0}^{\infty} \frac{(-1)^{k}}{1+2 k}\right)^{2}}
$$

$$
\frac{0.915966}{3(2 \pi)^{2}}=\frac{0.0190826}{\left(-1+\sum_{k=1}^{\infty} \frac{2^{k}}{\binom{2 k}{k}}\right)^{2}}
$$

$$
\frac{0.915966}{3(2 \pi)^{2}}=\frac{0.0763305}{\left(\sum_{k=0}^{\infty} \frac{2^{-k}(-6+50 k)}{\binom{3 k}{k}}\right)^{2}}
$$

Integral representations:

$$
\frac{0.915966}{3(2 \pi)^{2}}=\frac{0.0190826}{\left(\int_{0}^{\infty} \frac{1}{1+t^{2}} d t\right)^{2}}
$$

$$
\frac{0.915966}{3(2 \pi)^{2}}=\frac{0.00477066}{\left(\int_{0}^{1} \sqrt{1-t^{2}} d t\right)^{2}}
$$

$\frac{0.915966}{3(2 \pi)^{2}}=\frac{0.0190826}{\left(\int_{0}^{\infty} \frac{\sin (t)}{t} d t\right)^{2}}$

For $\mathrm{a}=5.1$, after some calculations:
$((0.00773390)) 1 /\left(\left(\left(1 /\left(2 \mathrm{Pi}^{*} 5.1\right)^{\wedge} 2^{*} 1 / 3 * 0.915966\right)\right)\right)$

## Input interpretation:

$0.00773390 \times \frac{1}{\frac{1}{(2 \pi \times 5.1)^{2}} \times \frac{1}{3} \times 0.915966}$

## Result:

26.0100...
26.01....

## Alternative representations:

$\frac{0.0077339}{\frac{0.915966}{(2 \pi 5.1)^{2} 3}}=\frac{0.0077339}{\frac{0.305322}{(1836 .)^{2}}}$

$$
\frac{0.0077339}{\frac{0.915966}{(2 \pi 5.1)^{2} 3}}=\frac{0.0077339}{\frac{0.305322}{(-10.2 i \log (-1))^{2}}}
$$

$\frac{0.0077339}{\frac{0.915966}{(2 \pi 5.1)^{2} 3}}=\frac{0.0077339}{\frac{0.305322}{\left(10.2 \cos ^{-1}(-1)\right)^{2}}}$

## Series representations:

$$
\frac{0.0077339}{\frac{0.915966}{(2 \pi 5.1)^{2} 3}}=42.1658\left(\sum_{k=0}^{\infty} \frac{(-1)^{k}}{1+2 k}\right)^{2}
$$

$$
\frac{0.0077339}{\frac{0.915966}{(2 \pi 5.1)^{2} 3}}=10.5415\left(-1+\sum_{k=1}^{\infty} \frac{2^{k}}{\binom{2 k}{k}}\right)^{2}
$$

$$
\frac{0.0077339}{\frac{0.915966}{(2 \pi 5.1)^{2} 3}}=2.63537\left(\sum_{k=0}^{\infty} \frac{2^{-k}(-6+50 k)}{\binom{3 k}{k}}\right)^{2}
$$

## Integral representations:

$$
\frac{0.0077339}{\frac{0.915966}{(2 \pi 5.1)^{2} 3}}=10.5415\left(\int_{0}^{\infty} \frac{1}{1+t^{2}} d t\right)^{2}
$$

$\frac{0.0077339}{\frac{0.915966}{(2 \pi 5.1)^{2} 3}}=42.1658\left(\int_{0}^{1} \sqrt{1-t^{2}} d t\right)^{2}$
$\frac{0.0077339}{\frac{0.915966}{(2 \pi 5.1)^{2} 3}}=10.5415\left(\int_{0}^{\infty} \frac{\sin (t)}{t} d t\right)^{2}$

From:

$$
\begin{aligned}
V_{\mathcal{C}}\left[a, \frac{1}{2}\right] & =\frac{1}{(2 \pi a)^{2}}\left[\frac{1}{\pi^{2}} \sum_{p=1}^{\infty} \frac{(-1)^{p}}{p^{4}}+\frac{1}{2 \pi} \sum_{n=-\infty}^{\infty}\left\{\pi|2 n+1| \operatorname{Li}_{2}\left(-e^{-\pi|2 n+1|}\right)+\operatorname{Li}_{3}\left(-e^{-\pi|2 n+1|}\right)\right\}\right] \\
& =\frac{-1}{(2 \pi a)^{2}} \frac{\mathcal{G}}{6}=-\frac{1}{2} V_{\mathcal{C}}[a, 0] .
\end{aligned}
$$

$-1 /(2 \mathrm{Pi} * \mathrm{a})^{\wedge} 2^{*} 1 / 6 * 0.915966$

## Input interpretation:

$-\frac{\frac{1}{6} \times 0.915966}{(2 \pi a)^{2}}$

## Result:

$-\frac{0.00386695}{a^{2}}$

## Plots:



(a from -5.1 to 5.1 )

Alternate form assuming a is real:
$0-\frac{0.00386695}{a^{2}}$

## Roots:

(no roots exist)

Property as a function:
Parity
even

## Derivative:

$\frac{d}{d a}\left(-\frac{0.00386695}{a^{2}}\right)=\frac{0.0077339}{a^{3}}$

Indefinite integral:

$$
\int-\frac{0.915966}{(2 \pi a)^{2} 6} d a=\frac{0.00386695}{a}+\text { constant }
$$

## Limit:

$$
\lim _{a \rightarrow \pm \infty}-\frac{0.00386695}{a^{2}}=0 \approx 0
$$

## Alternative representations:

$$
\frac{0.915966(-1)}{6(2 \pi a)^{2}}=-\frac{0.915966}{6\left(360 a^{\circ}\right)^{2}}
$$

$$
\frac{0.915966(-1)}{6(2 \pi a)^{2}}=-\frac{0.915966}{6(-2 a i \log (-1))^{2}}
$$

$$
\frac{0.915966(-1)}{6(2 \pi a)^{2}}=-\frac{0.915966}{6\left(2 a \cos ^{-1}(-1)\right)^{2}}
$$

## Series representations:

$$
\frac{0.915966(-1)}{6(2 \pi a)^{2}}=-\frac{0.00238533}{a^{2}\left(\sum_{k=0}^{\infty} \frac{(-1)^{k}}{1+2 k}\right)^{2}}
$$

$$
\frac{0.915966(-1)}{6(2 \pi a)^{2}}=-\frac{0.00954131}{a^{2}\left(-1+\sum_{k=1}^{\infty} \frac{2^{k}}{\binom{2 k}{k}}\right)^{2}}
$$

$$
\frac{0.915966(-1)}{6(2 \pi a)^{2}}=-\frac{0.0381653}{a^{2}\left(\sum_{k=0}^{\infty} \frac{2^{-k}(-6+50 k)}{\binom{3 k}{k}}\right)^{2}}
$$

## Integral representations:

$$
\frac{0.915966(-1)}{6(2 \pi a)^{2}}=-\frac{0.00954131}{a^{2}\left(\int_{0}^{\infty} \frac{1}{1+t^{2}} d t\right)^{2}}
$$

$$
\frac{0.915966(-1)}{6(2 \pi a)^{2}}=-\frac{0.00238533}{a^{2}\left(\int_{0}^{1} \sqrt{1-t^{2}} d t\right)^{2}}
$$

$$
\frac{0.915966(-1)}{6(2 \pi a)^{2}}=-\frac{0.00954131}{a^{2}\left(\int_{0}^{\infty} \frac{\sin (t)}{t} d t\right)^{2}}
$$

$-1 /(2 \mathrm{Pi})^{\wedge} 2^{*} 1 / 6^{*} 0.915966$

## Input interpretation:

$-\frac{\frac{1}{6} \times 0.915966}{(2 \pi)^{2}}$

## Result:

-0.00386695...
$-0.00386695 \ldots$.

Alternative representations:

$$
\frac{0.915966(-1)}{6(2 \pi)^{2}}=-\frac{0.915966}{6\left(360^{\circ}\right)^{2}}
$$

$$
\frac{0.915966(-1)}{6(2 \pi)^{2}}=-\frac{0.915966}{6(-2 i \log (-1))^{2}}
$$

$$
\frac{0.915966(-1)}{6(2 \pi)^{2}}=-\frac{0.915966}{6\left(2 \cos ^{-1}(-1)\right)^{2}}
$$

## Series representations:

$$
\frac{0.915966(-1)}{6(2 \pi)^{2}}=-\frac{0.00238533}{\left(\sum_{k=0}^{\infty} \frac{(-1)^{k}}{1+2 k}\right)^{2}}
$$

$$
\frac{0.915966(-1)}{6(2 \pi)^{2}}=-\frac{0.00954131}{\left(-1+\sum_{k=1}^{\infty} \frac{2^{k}}{\binom{2 k}{k}}\right)^{2}}
$$

$$
\frac{0.915966(-1)}{6(2 \pi)^{2}}=-\frac{0.0381653}{\left(\sum_{k=0}^{\infty} \frac{2^{-k}(-6+50 k)}{\binom{3 k}{k}}\right)^{2}}
$$

## Integral representations:

$$
\frac{0.915966(-1)}{6(2 \pi)^{2}}=-\frac{0.00954131}{\left(\int_{0}^{\infty} \frac{1}{1+t^{2}} d t\right)^{2}}
$$

$$
\frac{0.915966(-1)}{6(2 \pi)^{2}}=-\frac{0.00238533}{\left(\int_{0}^{1} \sqrt{1-t^{2}} d t\right)^{2}}
$$

$$
\frac{0.915966(-1)}{6(2 \pi)^{2}}=-\frac{0.00954131}{\left(\int_{0}^{\infty} \frac{\sin (t)}{t} d t\right)^{2}}
$$

From which:
$(-0.00386695) 1 /\left(\left(\left(-1 /(2 \mathrm{Pi} * 5.1)^{\wedge} 2 * 1 / 6 * 0.915966\right)\right)\right)$

## Input interpretation:

$-0.00386695\left(-\frac{1}{\frac{\frac{1}{6} \times 0.915966}{(2 \pi \times 5.1)^{2}}}\right)$

## Result:

26.0100...
26.01....

Alternative representations:
$\frac{-0.00386695}{-\frac{0.915966}{(2 \pi 5.1)^{2} 6}}=\frac{-0.00386695}{-\frac{0.915966}{\left.6(1836 .)^{\circ}\right)^{2}}}$
$\frac{-0.00386695}{-\frac{0.915966}{(2 \pi 5.1)^{2} 6}}=\frac{-0.00386695}{-\frac{0.915966}{6(-10.2 i \log (-1))^{2}}}$

$$
\frac{-0.00386695}{-\frac{0.915966}{(2 \pi 5.1)^{2} 6}}=\frac{-0.00386695}{-\frac{0.915966}{6\left(10.2 \cos ^{-1}(-1)\right)^{2}}}
$$

## Series representations:

$$
\frac{-0.00386695}{-\frac{0.915966}{(2 \pi 5.1)^{2} 6}}=42.1658\left(\sum_{k=0}^{\infty} \frac{(-1)^{k}}{1+2 k}\right)^{2}
$$

$$
\frac{-0.00386695}{-\frac{0.915966}{(2 \pi 5.1)^{2} 6}}=10.5415\left(-1+\sum_{k=1}^{\infty} \frac{2^{k}}{\binom{2 k}{k}}\right)^{2}
$$

$$
\frac{-0.00386695}{-\frac{0.915966}{(2 \pi 5.1)^{2} 6}}=2.63537\left(\sum_{k=0}^{\infty} \frac{2^{-k}(-6+50 k)}{\binom{3 k}{k}}\right)^{2}
$$

## Integral representations:

$$
\frac{-0.00386695}{-\frac{0.915966}{(2 \pi 5.1)^{2} 6}}=10.5415\left(\int_{0}^{\infty} \frac{1}{1+t^{2}} d t\right)^{2}
$$

$$
\frac{-0.00386695}{-\frac{0.915966}{(2 \pi 5.1)^{2} 6}}=42.1658\left(\int_{0}^{1} \sqrt{1-t^{2}} d t\right)^{2}
$$

$\frac{-0.00386695}{-\frac{0.915966}{(2 \pi 5.1)^{2} 6}}=10.5415\left(\int_{0}^{\infty} \frac{\sin (t)}{t} d t\right)^{2}$

From the sum between the two above expression, after some calculations:
$\mathrm{Pi}^{*}\left(\left(\left(\left[((0.00773390)) 1 /\left(\left(\left(1 /\left(2 \mathrm{Pi}^{*} 5.1\right)^{\wedge} 2^{*} 1 / 3^{*} 0.915966\right)\right)\right)\right]^{\wedge} 2+[(-0.00386695) 1 /(((-\right.\right.\right.$ $\left.\left.\left.\left.\left.\left.\left.1 /\left(2 \mathrm{Pi}^{*} 5.1\right)^{\wedge} 2 * 1 / 6 * 0.915966\right)\right)\right)\right]^{\wedge} 2\right)\right)\right)-89-55-8-\mathrm{e}$

## Input interpretation:

$\pi\left(\left(0.00773390 \times \frac{1}{\frac{1}{(2 \pi \times 5.1)^{2}} \times \frac{1}{3} \times 0.915966}\right)^{2}+\left(-0.00386695\left(-\frac{1}{-\frac{\frac{1}{6} \times 0.915966}{(2 \pi \times 5.1)^{2}}}\right)\right)^{2}\right)-$
$89-55-8-e$

## Result:

4095.99...
$4095.99 \ldots \approx 4096=64^{2}$

## Alternative representations:

$$
\begin{aligned}
& \pi\left(\left(\frac{0.0077339}{\frac{0.915966}{(2 \pi 5.1)^{2} 3}}\right)^{2}+\left(\frac{-0.00386695}{\left(-\frac{0.915966}{(2 \pi 5.1)^{2} 6}\right.}\right)^{2}\right)-89-55-8-e= \\
& -152-e+180^{\circ}\left(\left(\frac{0.0077339}{\frac{0.305322}{(1836 .)^{2}}}\right)^{2}+\left(\frac{-0.00386695}{-\frac{0.915966}{6(1836 .)^{2}}}\right)^{2}\right)
\end{aligned}
$$

$$
\begin{aligned}
& \pi\left(\left(\frac{0.0077339}{\frac{0.915966}{(2 \pi 5.1)^{2} 3}}\right)^{2}+\left(\frac{-0.00386695}{-\frac{0.915966}{(2 \pi 5.1)^{2} 6}}\right)^{2}\right)-89-55-8-e= \\
& -152-e-i\left(\log (-1)\left(\left(\frac{0.0077339}{\frac{0.305322}{(-10.2 i \log (-1))^{2}}}\right)^{2}+\left(\frac{-0.00386695}{-\frac{0.915966}{6(-10.2 i \log (-1))^{2}}}\right)^{2}\right)\right)
\end{aligned}
$$

$$
\begin{aligned}
& \pi\left(\left(\frac{0.0077339}{\frac{0.915966}{(2 \pi 5.1)^{2} 3}}\right)^{2}+\left(\frac{-0.00386695}{-\frac{0.915966}{(2 \pi 5.1)^{2} 6}}\right)^{2}\right)-89-55-8-e= \\
& \quad \pi\left(\left(\frac{0.0077339}{\frac{0.915966}{(2 \pi 5.1)^{2} 3}}\right)^{2}+\left(\frac{-0.00386695}{-\frac{0.915966}{(2 \pi 5.1)^{2} 6}}\right)^{2}\right)-89-55-8-\exp (z) \text { for } z=1
\end{aligned}
$$

## Series representations:

$$
\begin{aligned}
& \pi\left(\left(\frac{0.0077339}{\frac{0.915966}{(2 \pi 5.1)^{2} 3}}\right)^{2}+\left(\frac{-0.00386695}{-\frac{0.915966}{(2 \pi 5.1)^{2} 6}}\right)^{2}\right)-89-55-8-e= \\
& \quad 14223.7\left(-0.0106864+\left(\sum_{k=0}^{\infty} \frac{(-1)^{k}}{1+2 k}\right)^{5}-0.0000703054 \sum_{k=0}^{\infty} \frac{1}{k!}\right)
\end{aligned}
$$

$$
\begin{aligned}
& \pi\left(\left(\frac{0.0077339}{\frac{0.915966}{(2 \pi 5.1)^{2} 3}}\right)^{2}+\left(\frac{-0.00386695}{-\frac{0.915966}{(2 \pi 5.1)^{2} 6}}\right)^{2}\right)-89-55-8-e= \\
& \quad 14223.7\left(-0.0106864+\left(\sum_{k=0}^{\infty} \frac{(-1)^{k}}{1+2 k}\right)^{5}-0.0000703054 \sum_{k=0}^{\infty} \frac{(-1+k)^{2}}{k!}\right)
\end{aligned}
$$

$$
\begin{aligned}
& \pi\left(\left(\frac{0.0077339}{\frac{0.915966}{(2 \pi 5.1)^{2} 3}}\right)^{2}+\left(\frac{-0.00386695}{-\frac{0.915966}{(2 \pi 5.1)^{2} 6}}\right)^{2}\right)-89-55-8-e= \\
& -\left(152+\sum_{k=0}^{\infty} \frac{1}{k!}-13.8903\left(\sum_{k=1}^{\infty} 4^{-k}\left(-1+3^{k}\right) \zeta(1+k)\right)^{5}\right)
\end{aligned}
$$

## Integral representations:

$$
\begin{aligned}
& \pi\left(\left(\frac{0.0077339}{\frac{0.915966}{(2 \pi 5.1)^{2} 3}}\right)^{2}+\left(\frac{-0.00386695}{-\frac{0.915966}{(2 \pi 5.1)^{2} 6}}\right)^{2}\right)-89-55-8-e= \\
& \quad-152-e+444.49\left(\int_{0}^{\infty} \frac{1}{1+t^{2}} d t\right)^{5}
\end{aligned}
$$

$$
\begin{aligned}
& \pi\left(\left(\frac{0.0077339}{\frac{0.915966}{(2 \pi 5.1)^{2} 3}}\right)^{2}+\left(\frac{-0.00386695}{-\frac{0.915966}{(2 \pi 5.1)^{2} 6}}\right)^{2}\right)-89-55-8-e= \\
& -152-e+14223.7\left(\int_{0}^{1} \sqrt{1-t^{2}} d t\right)^{5}
\end{aligned}
$$

$$
\begin{aligned}
& \pi\left(\left(\frac{0.0077339}{\frac{0.915966}{(2 \pi 5.1)^{2} 3}}\right)^{2}+\left(\frac{-0.00386695}{-\frac{0.915966}{(2 \pi 5.1)^{2} 6}}\right)^{2}\right)-89-55-8-e= \\
& \quad-152-e+444.49\left(\int_{0}^{\infty} \frac{\sin (t)}{t} d t\right)^{5}
\end{aligned}
$$

$\mathrm{Pi}^{*}\left(\left(\left(\left[((0.00773390)) 1 /\left(\left(\left(1 /\left(2 \mathrm{Pi}^{*} 5.1\right)^{\wedge} 2^{*} 1 / 3^{*} 0.915966\right)\right)\right)\right]^{\wedge} 2+[(-0.00386695) 1 /(((-\right.\right.\right.$ $\left.\left.\left.\left.\left.\left.\left.1 /\left(2 \mathrm{Pi}^{*} 5.1\right)^{\wedge} 2 * 1 / 6^{*} 0.915966\right)\right)\right)\right]^{\wedge} 2\right)\right)\right)+123-$ sqrt3

## Input interpretation:

$\pi\left(\left(0.00773390 \times \frac{1}{\frac{1}{(2 \pi \times 5.1)^{2}} \times \frac{1}{3} \times 0.915966}\right)^{2}+\left(-0.00386695\left(-\frac{1}{\frac{\frac{1}{6} \times 0.915966}{(2 \pi \times 5.1)^{2}}}\right)\right)^{2}\right)+$

$$
123-\sqrt{3}
$$

## Result:

4371.97...
4371.97.... $\approx 4372$

Where 4372 is a value indicated in the fundamental Ramanujan paper "Modular equations and Approximations to $\pi$ "

Hence

$$
\begin{array}{rlr}
64 g_{22}^{24} & = & e^{\pi \sqrt{22}}-24+276 e^{-\pi \sqrt{22}}-\cdots \\
64 g_{22}^{-24} & = & 4096 e^{-\pi \sqrt{22}}+\cdots
\end{array}
$$

so that

$$
64\left(g_{22}^{24}+g_{22}^{-24}\right)=e^{\pi \sqrt{22}}-24+4372 e^{-\pi \sqrt{22}}+\cdots=64\left\{(1+\sqrt{2})^{12}+(1-\sqrt{2})^{12}\right\} .
$$

Hence

$$
e^{\pi \sqrt{22}}=2508951.9982 \ldots
$$

## Series representations:

$$
\begin{aligned}
\pi & \left.\pi\left(\frac{0.0077339}{\frac{0.915966}{(2 \pi 5.1)^{2} 3}}\right)^{2}+\left(\frac{-0.00386695}{-\frac{0.915966}{(2 \pi 5.1)^{2} 6}}\right)^{2}\right)+123-\sqrt{3}= \\
& 123+13.8903 \pi^{5}-\sqrt{2} \sum_{k=0}^{\infty} 2^{-k}\binom{\frac{1}{2}}{k}
\end{aligned}
$$

$$
\begin{aligned}
& \pi\left(\left(\frac{0.0077339}{\frac{0.915966}{(2 \pi 5.1)^{2} 3}}\right)^{2}+\left(\frac{-0.00386695}{-\frac{0.915966}{(2 \pi 5.1)^{2} 6}}\right)^{2}\right)+123-\sqrt{3}= \\
& 123+13.8903 \pi^{5}-\sqrt{2} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}
\end{aligned}
$$

$$
\begin{aligned}
& \pi\left(\left(\frac{0.0077339}{\frac{0.915966}{(2 \pi 5.1)^{2} 3}}\right)^{2}+\left(\frac{-0.00386695}{-\frac{0.915966}{(2 \pi 5.1)^{2}}}\right)^{2}\right)+123-\sqrt{3}= \\
& 123+13.8903 \pi^{5}-\frac{\sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 2^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{2 \sqrt{\pi}}
\end{aligned}
$$

$27 * \operatorname{sqrt}\left(\left(\mathrm{Pi}^{*}\left(\left(\left(\left[((0.00773390)) 1 /\left(\left(\left(1 /\left(2 \mathrm{Pi}^{*} 5.1\right)^{\wedge} 2 * 1 / 3 * 0.915966\right)\right)\right)\right]^{\wedge} 2+[(-\right.\right.\right.\right.\right.$ $\left.\left.\left.\left.\left.\left.0.00386695) 1 /\left(\left(\left(-1 /\left(2 \mathrm{Pi}^{*} 5.1\right)^{\wedge} 2 * 1 / 6 * 0.915966\right)\right)\right)\right]^{\wedge} 2\right)\right)\right)-89-55-8-\mathrm{e}\right)\right)+1$

## Input interpretation:

$$
\left.\left.\left.\left.\left.\begin{array}{rl}
27 & \sqrt{((\pi}\left(\left(0.00773390 \times \frac{1}{\frac{1}{(2 \pi \times 5.1)^{2}} \times \frac{1}{3} \times 0.915966}\right)^{2}+\right. \\
\left(-0.00386695\left(-\frac{1}{\frac{1}{6} \times 0.915966}(2 \pi \times 5.1)^{2}\right.\right.
\end{array}\right)\right)\right)^{2}\right)-89-55-8-e\right)+1
$$

## Result:

1729.00...

## 1729

This result is very near to the mass of candidate glueball $\mathbf{f}_{\mathbf{0}}(\mathbf{1 7 1 0})$ scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. $\left(1728=8^{2} * 3^{3}\right.$ ) The number 1728 is one less than the Hardy-Ramanujan number 1729 (taxicab number)

## Series representations:

$$
\begin{aligned}
& 27 \sqrt{\pi\left(\left(\frac{0.0077339}{\frac{0.915966}{(2 \pi 5.1)^{2} 3}}\right)^{2}+\left(\frac{-0.00386695}{-\frac{0.915966}{(2 \pi 5.1)^{2} 6}}\right)^{2}\right)-89-55-8-e+1}= \\
& 1+27 \sqrt{-153-e+13.8903 \pi^{5}} \sum_{k=0}^{\infty}\left(-153-e+13.8903 \pi^{5}\right)^{-k}\binom{\frac{1}{2}}{k}
\end{aligned}
$$

$$
\begin{aligned}
& 27 \sqrt{\pi\left(\left(\frac{0.0077339}{\frac{0.915966}{(2 \pi 5.1)^{2} 3}}\right)^{2}+\left(\frac{-0.00386695}{-\frac{0.915966}{(2 \pi 5.1)^{2} 6}}\right)^{2}\right)-89-55-8-e+1}= \\
& 1+27 \sqrt{-153-e+13.8903 \pi^{5}} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-153-e+13.8903 \pi^{5}\right)^{-k}\left(-\frac{1}{2}\right)_{k}}{k!}
\end{aligned}
$$

$$
\begin{aligned}
& 27 \sqrt{\pi\left(\left(\frac{0.0077339}{\frac{0.915966}{(2 \pi 5.1)^{2} 3}}\right)^{2}+\left(\frac{-0.00386695}{-\frac{0.915966}{(2 \pi 5.1)^{2} 6}}\right)^{2}\right)-89-55-8-e+1}= \\
& 1+27 \sqrt{z_{0}} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(-152-e+13.8903 \pi^{5}-z_{0}\right)^{k} z_{0}^{-k}}{k!}
\end{aligned}
$$

$\left(\left(27 * \operatorname{sqrt}\left(\left(\mathrm{Pi}^{*}\left(\left(\left(\left[((0.00773390)) 1 /\left(\left(\left(1 /\left(2 \mathrm{Pi}^{*} 5.1\right)^{\wedge} 2^{*} 1 / 3 * 0.915966\right)\right)\right)\right]^{\wedge} 2+[(-\right.\right.\right.\right.\right.\right.\right.$ $\left.\left.\left.\left.\left.\left.\left.\left.0.00386695) 1 /\left(\left(\left(-1 /\left(2 \mathrm{Pi}^{*} 5.1\right)^{\wedge} 2 * 1 / 6 * 0.915966\right)\right)\right)\right]^{\wedge} 2\right)\right)\right)-89-55-8-\mathrm{e}\right)\right)+1\right)\right)^{\wedge} 1 / 15$

## Input interpretation:

$$
\begin{array}{r}
27 \sqrt{27\left(\left(0.00773390 \times \frac{1}{\frac{1}{(2 \pi \times 5.1)^{2}} \times \frac{1}{3} \times 0.915966}\right)^{2}+\right.} \\
\left.\left(-0.00386695\left(-\frac{1}{\frac{\frac{1}{6} \times 0.915966}{(2 \pi \times 5.1)^{2}}}\right)\right)^{2}\right)- \\
89-55-8-e)+1) \wedge(1 / 15)
\end{array}
$$

## Result:

1.6438150495830386047005199331688482686077512380746076302517997346
$1.643815049 \ldots \approx \zeta(2)=\frac{\pi^{2}}{6}=1.644934 \ldots$

We have that:

$$
\begin{align*}
V_{\mathcal{C}}^{(2)}[a, m, 0] & =(a m)^{2}\left\{2 \pi\left(\log (2 \pi m a)-\frac{1}{4}\right)-\operatorname{Li}_{2}(1)+\sum_{n=1}^{\infty} 2 \pi \log \left(1-e^{-2 \pi n}\right)\right\} \\
& =(a m)^{2}\left\{\pi \log (2 \pi m a)-\frac{\pi^{2}}{6}-\frac{5 \pi}{2}+\log \frac{\Gamma\left(\frac{1}{4}\right)}{2 \pi^{3 / 4}}\right\} \tag{B.14}
\end{align*}
$$

Finally, for the antiperiodic case we find:

$$
\begin{align*}
V_{\mathcal{C}}^{(2)}\left[a, m, \frac{1}{2}\right] & =(a m)^{2}\left\{-\operatorname{Li}_{2}(-1)+\sum_{n=-\infty}^{\infty} \pi \log \left(1+e^{-\pi|2 n+1|}\right)\right\} \\
& =(a m)^{2}\left\{-\frac{\pi^{2}}{12}+\frac{3 \pi}{4} \log 2\right\} \tag{B.15}
\end{align*}
$$

## From:

$$
(a m)^{2}\left\{\pi \log (2 \pi m a)-\frac{\pi^{2}}{6}-\frac{5 \pi}{2}+\log \frac{\Gamma\left(\frac{1}{4}\right)}{2 \pi^{3 / 4}}\right\}
$$

$\mathrm{x}^{\wedge} 2\left[\mathrm{Pi}^{*} \ln \left(2 \mathrm{Pi}^{*} \mathrm{x}\right)-\left(\mathrm{Pi}^{\wedge} 2\right) / 6-(5 \mathrm{Pi}) / 2+\ln \left((\operatorname{gamma}(1 / 4)) /\left(2 \mathrm{Pi}^{\wedge}(3 / 4)\right)\right]\right.$

## Input:

$x^{2}\left(\pi \log (2 \pi x)-\frac{\pi^{2}}{6}-\frac{5 \pi}{2}+\log \left(\frac{\Gamma\left(\frac{1}{4}\right)}{2 \pi^{3 / 4}}\right)\right)$

## Plots:




## Alternate forms:

$x^{2}\left(\pi \log (2 \pi x)-\frac{1}{6} \pi(15+\pi)+\log \left(\frac{2 \Gamma\left(\frac{5}{4}\right)}{\pi^{3 / 4}}\right)\right)$
$x^{2} \log \left(\frac{\Gamma\left(\frac{1}{4}\right)}{2 \pi^{3 / 4}}\right)-\frac{1}{6} \pi x^{2}(-6 \log (2 \pi x)+\pi+15)$
$-\frac{1}{6} x^{2}\left(-6 \pi \log (2 \pi x)+\pi^{2}+15 \pi-6 \log \left(\frac{\Gamma\left(\frac{1}{4}\right)}{2 \pi^{3 / 4}}\right)\right)$

## Expanded form:

$-\frac{\pi^{2} x^{2}}{6}-\frac{5 \pi x^{2}}{2}+\pi x^{2} \log (2 \pi x)+x^{2} \log \left(\frac{\Gamma\left(\frac{1}{4}\right)}{2 \pi^{3 / 4}}\right)$

## Alternate form assuming $\mathbf{x}>0$ :

$-\frac{1}{12} x^{2}\left(-12 \pi \log (2 \pi x)+2 \pi^{2}+30 \pi+3\left(\log (16)+3 \log (\pi)-4 \log \left(\Gamma\left(\frac{1}{4}\right)\right)\right)\right)$

## Root:

```
x\approx3.55961
```


## Series expansion at $\mathrm{x}=0$ :

$x^{2}\left(\pi \log (x)-\frac{\pi^{2}}{6}+\pi\left(\log (2 \pi)-\frac{5}{2}\right)+\log \left(\frac{\Gamma\left(\frac{1}{4}\right)}{2 \pi^{3 / 4}}\right)\right)+O\left(x^{4}\right)$
(Puiseux series)

Series expansion at $\mathrm{x}=\infty$ :
$x^{2}\left(\pi \log (x)-\frac{\pi^{2}}{6}+\pi\left(\log (2 \pi)-\frac{5}{2}\right)+\log \left(\frac{\Gamma\left(\frac{1}{4}\right)}{2 \pi^{3 / 4}}\right)\right)+O\left(\left(\frac{1}{x}\right)^{4}\right)$
(Puiseux series)

## Derivative:

$$
\begin{aligned}
& \frac{d}{d x}\left(x^{2}\left(\pi \log (2 \pi x)-\frac{\pi^{2}}{6}-\frac{5 \pi}{2}+\log \left(\frac{\Gamma\left(\frac{1}{4}\right)}{2 \pi^{3 / 4}}\right)\right)\right)= \\
& -\frac{1}{3} x\left(-6 \pi \log (2 \pi x)+\pi^{2}+12 \pi+\frac{9 \log (\pi)}{2}+\log (64)-6 \log \left(\Gamma\left(\frac{1}{4}\right)\right)\right)
\end{aligned}
$$

## Indefinite integral:

$$
\begin{aligned}
& \int x^{2}\left(\pi \log (2 \pi x)-\frac{\pi^{2}}{6}-\frac{5 \pi}{2}+\log \left(\frac{\mathrm{r}\left(\frac{1}{4}\right)}{2 \pi^{3 / 4}}\right)\right) d x= \\
& \quad-\frac{1}{18} x^{3}\left(-6 \pi \log (2 \pi x)+\pi^{2}+17 \pi+\frac{9 \log (\pi)}{2}+\log (64)-6 \log \left(\Gamma\left(\frac{1}{4}\right)\right)\right)+\text { constant }
\end{aligned}
$$

(assuming a complex-valued logarithm)

## Local minimum:

$$
\begin{aligned}
& \min \left\{x^{2}\left(\pi \log (2 \pi x)-\frac{\pi^{2}}{6}-\frac{5 \pi}{2}+\log \left(\frac{\Gamma\left(\frac{1}{4}\right)}{2 \pi^{3 / 4}}\right)\right)\right\}= \\
& -2^{2 / \pi-3} e^{4+\pi / 3} \pi^{3 /(2 \pi)-1} \Gamma\left(\frac{1}{4}\right)^{-2 / \pi} \text { at } x=2^{1 / \pi-1} e^{2+\pi / 6} \pi^{3 /(4 \pi)-1} \Gamma\left(\frac{1}{4}\right)^{-1 / \pi}
\end{aligned}
$$

For $\mathrm{x}=0.5$ :
$0.5^{\wedge} 2\left(-1 / 6 \pi(15+\pi)+\pi \log (2 \pi * 0.5)+\log \left((2 \Gamma(5 / 4)) / \pi^{\wedge}(3 / 4)\right)\right)$

## Input:

$0.5^{2}\left(-\frac{1}{6} \pi(15+\pi)+\pi \log (2 \pi \times 0.5)+\log \left(\frac{2 \Gamma\left(\frac{5}{4}\right)}{\pi^{3 / 4}}\right)\right)$
$\log (x)$ is the natural logarithm
$\Gamma(x)$ is the gamma function

## Result:

-1.54158...
-1.54158....

## Alternative representations:

$0.5^{2}\left(\frac{1}{6}(\pi(15+\pi))(-1)+\pi \log (2 \pi 0.5)+\log \left(\frac{2 \Gamma\left(\frac{5}{4}\right)}{\pi^{3 / 4}}\right)\right)=$

$$
0.5^{2}\left(\pi \log (\pi)+\log \left(\frac{2 G\left(1+\frac{5}{4}\right)}{G\left(\frac{5}{4}\right) \pi^{3 / 4}}\right)-\frac{1}{6} \pi(15+\pi)\right)
$$

$$
\begin{aligned}
& 0.5^{2}\left(\frac{1}{6}(\pi(15+\pi))(-1)+\pi \log (2 \pi 0.5)+\log \left(\frac{2 \Gamma\left(\frac{5}{4}\right)}{\pi^{3 / 4}}\right)\right)= \\
& 0.5^{2}\left(\pi \log (\pi)+\log \left(\frac{2 e^{-\log \mathrm{G}(5 / 4)+\log (1+5 / 4)}}{\pi^{3 / 4}}\right)-\frac{1}{6} \pi(15+\pi)\right.
\end{aligned}
$$

$$
\begin{gathered}
0.5^{2}\left(\frac{1}{6}(\pi(15+\pi))(-1)+\pi \log (2 \pi 0.5)+\log \left(\frac{2 \Gamma\left(\frac{5}{4}\right)}{\pi^{3 / 4}}\right)\right)= \\
0.5^{2}\left(\pi \log (\pi)+\log \left(\frac{2\left(-1+\frac{5}{4}\right)!}{\pi^{3 / 4}}\right)-\frac{1}{6} \pi(15+\pi)\right)
\end{gathered}
$$

## Series representations:

$$
\begin{aligned}
& 0.5^{2}\left(\frac{1}{6}(\pi(15+\pi))(-1)+\pi \log (2 \pi 0.5)+\log \left(\frac{2 \Gamma\left(\frac{5}{4}\right)}{\pi^{3 / 4}}\right)\right)= \\
& \quad-0.625 \pi-0.0416667 \pi^{2}+0.25 \pi \log (\pi)+0.25 \log \left(\frac{2 \sum_{k=0}^{\infty} \frac{\left(\frac{5}{4}-z_{0}\right)^{k} \Gamma^{(k)}\left(z_{0}\right)}{k!}}{\pi^{3 / 4}}\right)
\end{aligned}
$$

for $\left(z_{0} \notin \mathbb{Z}\right.$ or $\left.z_{0}>0\right)$

$$
\begin{aligned}
& 0.5^{2}\left(\frac{1}{6}(\pi(15+\pi))(-1)+\pi \log (2 \pi 0.5)+\log \left(\frac{2 \Gamma\left(\frac{5}{4}\right)}{\pi^{3 / 4}}\right)\right)= \\
& -0.625 \pi-0.0416667 \pi^{2}+0.5 i \pi^{2}\left\lfloor\frac{\arg (\pi-x)}{2 \pi}\right\rfloor+ \\
& 0.5 i \pi\left|\frac{\arg \left(-x+\frac{2 \Gamma\left(\frac{5}{4}\right)}{\pi^{3 / 4}}\right)}{2 \pi}\right|+0.25 \log (x)+0.25 \pi \log (x)+ \\
& \sum_{k=1}^{\infty} \frac{(-1)^{k} x^{-k}\left(-0.25 \pi(\pi-x)^{k}-0.25\left(-x+\frac{2 \Gamma\left(\frac{5}{4}\right)}{\pi^{3 / 4}}\right)^{k}\right)}{k} \text { for } x<0
\end{aligned}
$$

$$
\begin{aligned}
& 0.5^{2}\left(\frac{1}{6}(\pi(15+\pi))(-1)+\pi \log (2 \pi 0.5)+\log \left(\frac{2 \Gamma\left(\frac{5}{4}\right)}{\pi^{3 / 4}}\right)\right)= \\
& \left.-0.625 \pi-0.0416667 \pi^{2}+0.5 i \pi^{2} \left\lvert\,-\frac{-\pi+\arg \left(\frac{\pi}{z_{0}}\right)+\arg \left(z_{0}\right)}{2 \pi}\right.\right]+ \\
& 0.5 i \pi\left[\left.-\frac{-\pi+\arg \left(\frac{2 \Gamma\left(\frac{5}{4}\right)}{\pi^{3 / 4} z_{0}}\right)+\arg \left(z_{0}\right)}{2 \pi} \right\rvert\,+0.25 \log \left(z_{0}\right)+\right. \\
& 0.25 \pi \log \left(z_{0}\right)+\sum_{k=1}^{\infty} \frac{(-1)^{k}\left(-0.25 \pi\left(\pi-z_{0}\right)^{k}-0.25\left(\frac{2 \Gamma\left(\frac{5}{4}\right)}{\pi^{3 / 4}}-z_{0}\right)^{k}\right) z_{0}^{-k}}{k}
\end{aligned}
$$

## Integral representations:

$$
\begin{aligned}
& 0.5^{2}\left(\frac{1}{6}(\pi(15+\pi))(-1)+\pi \log (2 \pi 0.5)+\log \left(\frac{2 \Gamma\left(\frac{5}{4}\right)}{\pi^{3 / 4}}\right)\right)= \\
& \quad-0.625 \pi-0.0416667 \pi^{2}+0.25 \pi \log (\pi)+0.25 \log \left(\frac{2}{\pi^{3 / 4}} \int_{0}^{\infty} e^{-t} \sqrt[4]{t} d t\right)
\end{aligned}
$$

$$
\begin{aligned}
& 0.5^{2}\left(\frac{1}{6}(\pi(15+\pi))(-1)+\pi \log (2 \pi 0.5)+\log \left(\frac{2 \Gamma\left(\frac{5}{4}\right)}{\pi^{3 / 4}}\right)\right)= \\
& \quad-0.625 \pi-0.0416667 \pi^{2}+0.25 \pi \log (\pi)+0.25 \log \left(\frac{2}{\pi^{3 / 4}} \int_{0}^{1} \sqrt[4]{\log \left(\frac{1}{t}\right)} d t\right)
\end{aligned}
$$

$0.5^{2}\left(\frac{1}{6}(\pi(15+\pi))(-1)+\pi \log (2 \pi 0.5)+\log \left(\frac{2 \Gamma\left(\frac{5}{4}\right)}{\pi^{3 / 4}}\right)\right)=$

$$
-0.625 \pi-0.0416667 \pi^{2}+0.25 \log \left(\frac{2 \exp \left(\int_{0}^{1} \frac{\frac{1}{4}-\frac{5 x}{4}+x^{5 / 4}}{(-1+x) \log (x)} d x\right)}{\pi^{3 / 4}}\right)+0.25 \pi \log (\pi)
$$

For $\mathrm{x}=2.9$ :
$2.9^{\wedge} 2\left(-1 / 6 \pi(15+\pi)+\pi \log (2 \pi * 2.9)+\log \left((2 \Gamma(5 / 4)) / \pi^{\wedge}(3 / 4)\right)\right)$

## Input:

$$
2.9^{2}\left(-\frac{1}{6} \pi(15+\pi)+\pi \log (2 \pi \times 2.9)+\log \left(\frac{2 \Gamma\left(\frac{5}{4}\right)}{\pi^{3 / 4}}\right)\right)
$$

## Result:

-5.41469...
$-5.41469 \ldots$

## Alternative representations:

$$
\begin{aligned}
& 2.9^{2}\left(\frac{1}{6}(\pi(15+\pi))(-1)+\pi \log (2 \pi 2.9)+\log \left(\frac{2 \Gamma\left(\frac{5}{4}\right)}{\pi^{3 / 4}}\right)\right)= \\
& 2.9^{2}\left(\pi \log (5.8 \pi)+\log \left(\frac{2 G\left(1+\frac{5}{4}\right)}{G\left(\frac{5}{4}\right) \pi^{3 / 4}}\right)-\frac{1}{6} \pi(15+\pi)\right)
\end{aligned}
$$

$$
\begin{aligned}
& 2.9^{2}\left(\frac{1}{6}(\pi(15+\pi))(-1)+\pi \log (2 \pi 2.9)+\log \left(\frac{2 \Gamma\left(\frac{5}{4}\right)}{\pi^{3 / 4}}\right)\right)= \\
& 2.9^{2}\left(\pi \log (5.8 \pi)+\log \left(\frac{2 e^{-\log \mathrm{G}(5 / 4)+\log (1+5 / 4)}}{\pi^{3 / 4}}\right)-\frac{1}{6} \pi(15+\pi)\right)
\end{aligned}
$$

$$
\begin{gathered}
2.9^{2}\left(\frac{1}{6}(\pi(15+\pi))(-1)+\pi \log (2 \pi 2.9)+\log \left(\frac{2 \Gamma\left(\frac{5}{4}\right)}{\pi^{3 / 4}}\right)\right)= \\
2.9^{2}\left(\pi \log (5.8 \pi)+\log \left(\frac{2\left(-1+\frac{5}{4}\right)!}{\pi^{3 / 4}}\right)-\frac{1}{6} \pi(15+\pi)\right)
\end{gathered}
$$

## Series representations:

$$
\begin{aligned}
& 2.9^{2}\left(\frac{1}{6}(\pi(15+\pi))(-1)+\pi \log (2 \pi 2.9)+\log \left(\frac{2 \Gamma\left(\frac{5}{4}\right)}{\pi^{3 / 4}}\right)\right)= \\
& \quad-21.025 \pi-1.40167 \pi^{2}+8.41 \pi \log (5.8 \pi)+8.41 \log \left(\frac{2 \sum_{k=0}^{\infty} \frac{\left(\frac{5}{4}-z_{0}\right)^{k} \Gamma^{(k)}\left(z_{0}\right)}{k!}}{\pi^{3 / 4}}\right)
\end{aligned}
$$

$2.9^{2}\left(\frac{1}{6}(\pi(15+\pi))(-1)+\pi \log (2 \pi 2.9)+\log \left(\frac{2 \Gamma\left(\frac{5}{4}\right)}{\pi^{3 / 4}}\right)\right)=$
$-21.025 \pi-1.40167 \pi^{2}+16.82 i \pi^{2}\left\lfloor\frac{\arg (5.8 \pi-x)}{2 \pi}\right\rfloor+$
$16.82 i \pi\left\lfloor\frac{\arg \left(-x+\frac{2 \Gamma\left(\frac{5}{4}\right)}{\pi^{3 / 4}}\right)}{2 \pi}\right\rfloor+8.41 \log (x)+8.41 \pi \log (x)+$
$\sum_{k=1}^{\infty} \frac{(-1)^{k} x^{-k}\left(-8.41 \pi(5.8 \pi-x)^{k}-8.41\left(-x+\frac{2 \Gamma\left(\frac{5}{4}\right)}{\pi^{3 / 4}}\right)^{k}\right)}{k}$ for $x<0$

$$
\begin{aligned}
& 2.9^{2}\left(\frac{1}{6}(\pi(15+\pi))(-1)+\pi \log (2 \pi 2.9)+\log \left(\frac{2 \Gamma\left(\frac{5}{4}\right)}{\pi^{3 / 4}}\right)\right)= \\
& \left.-21.025 \pi-1.40167 \pi^{2}+16.82 i \pi^{2} \left\lvert\,-\frac{-\pi+\arg \left(\frac{5.8 \pi}{z_{0}}\right)+\arg \left(z_{0}\right)}{2 \pi}\right.\right]+ \\
& 16.82 i \pi\left[\left.-\frac{-\pi+\arg \left(\frac{2 \Gamma\left(\frac{5}{4}\right)}{\pi^{3 / 4} z_{0}}\right)+\arg \left(z_{0}\right)}{2 \pi} \right\rvert\,+8.41 \log \left(z_{0}\right)+8.41 \pi \log \left(z_{0}\right)+\right. \\
& \sum_{k=1}^{\infty} \frac{(-1)^{k}\left(-8.41 \pi\left(5.8 \pi-z_{0}\right)^{k}-8.41\left(\frac{2 \Gamma\left(\frac{5}{4}\right)}{\pi^{3 / 4}}-z_{0}\right)^{k}\right) z_{0}^{-k}}{k}
\end{aligned}
$$

## Integral representations:

$$
\begin{aligned}
& 2.9^{2}\left(\frac{1}{6}(\pi(15+\pi))(-1)+\pi \log (2 \pi 2.9)+\log \left(\frac{2 \Gamma\left(\frac{5}{4}\right)}{\pi^{3 / 4}}\right)\right)= \\
& \quad-21.025 \pi-1.40167 \pi^{2}+8.41 \pi \log (5.8 \pi)+8.41 \log \left(\frac{2}{\pi^{3 / 4}} \int_{0}^{\infty} e^{-t} \sqrt[4]{t} d t\right)
\end{aligned}
$$

$$
\begin{aligned}
& 2.9^{2}\left(\frac{1}{6}(\pi(15+\pi))(-1)+\pi \log (2 \pi 2.9)+\log \left(\frac{2 \Gamma\left(\frac{5}{4}\right)}{\pi^{3 / 4}}\right)\right)= \\
& -21.025 \pi-1.40167 \pi^{2}+8.41 \pi \log (5.8 \pi)+8.41 \log \left(\frac{2}{\pi^{3 / 4}} \int_{0}^{1} \sqrt[4]{\log \left(\frac{1}{t}\right)} d t\right)
\end{aligned}
$$

$2.9^{2}\left(\frac{1}{6}(\pi(15+\pi))(-1)+\pi \log (2 \pi 2.9)+\log \left(\frac{2 \Gamma\left(\frac{5}{4}\right)}{\pi^{3 / 4}}\right)\right)=$
$-21.025 \pi-1.40167 \pi^{2}+8.41 \log \left(\frac{2 \exp \left(\int_{0}^{1} \frac{\frac{1}{4}-\frac{5 x}{4}+x^{5 / 4}}{(-1+x) \log (x)} d x\right)}{\pi^{3 / 4}}\right)+8.41 \pi \log (5.8 \pi)$

From which, after some calculations:

$$
\begin{aligned}
& \operatorname{sqrt}\left(\operatorname { s q r t } \left[-\left(\left(\left(0.5^{\wedge} 2\left(-1 / 6 \pi(15+\pi)+\pi \log (2 \pi * 0.5)+\log \left((2 \Gamma(5 / 4)) / \pi^{\wedge}(3 / 4)\right)\right)\right)\right)\right)-\right.\right. \\
& \left.\left.\left(\left(\left(2.9^{\wedge} 2\left(-1 / 6 \pi(15+\pi)+\pi \log (2 \pi * 2.9)+\log \left((2 \Gamma(5 / 4)) / \pi^{\wedge}(3 / 4)\right)\right)\right)\right)\right)\right]\right)
\end{aligned}
$$

## Input:

$$
\begin{array}{r}
\sqrt{ }\left(\sqrt { } \left(-\left(0.5^{2}\left(-\frac{1}{6} \pi(15+\pi)+\pi \log (2 \pi \times 0.5)+\log \left(\frac{2 \Gamma\left(\frac{5}{4}\right)}{\pi^{3 / 4}}\right)\right)\right)-\right.\right. \\
\left.\left.\quad 2.9^{2}\left(-\frac{1}{6} \pi(15+\pi)+\pi \log (2 \pi \times 2.9)+\log \left(\frac{2 \Gamma\left(\frac{5}{4}\right)}{\pi^{3 / 4}}\right)\right)\right)\right)
\end{array}
$$

$\log (x)$ is the natural logarithm

## Result:

1.6240300085530374986265700615390767078372322612322030846419002893
$1.62403 \ldots$... result quite near to the value of the golden ratio 1.618033988749...

## All 2<sup>nd</sup> roots of 2.63747:

$1.62403 e^{0} \approx 1.6240$ (real, principal root)
$1.62403 e^{i \pi} \approx-1.6240$ (real root)

## Alternative representations:

$$
\begin{aligned}
& \sqrt{ }\left(\sqrt{\left(-0.5^{2}\left(-\frac{1}{6} \pi(15+\pi)+\pi \log (2 \pi 0.5)+\log \left(\frac{2 \Gamma\left(\frac{5}{4}\right)}{\pi^{3 / 4}}\right)\right)-\right.}\right. \\
& \sqrt{\left.\left.2.9^{2}\left(-\frac{1}{6} \pi(15+\pi)+\pi \log (2 \pi 2.9)+\log \left(\frac{2 \Gamma\left(\frac{5}{4}\right)}{\pi^{3 / 4}}\right)\right)\right)\right)}= \\
& \sqrt{ }\left(\sqrt{\left(-0.5^{2}\left(\pi \log (\pi)+\log \left(\frac{2 G\left(1+\frac{5}{4}\right)}{G\left(\frac{5}{4}\right) \pi^{3 / 4}}\right)-\frac{1}{6} \pi(15+\pi)\right)-\right.}\right. \\
& \left.2.9^{2}\left(\pi \log (5.8 \pi)+\log \left(\frac{2 G\left(1+\frac{5}{4}\right)}{G\left(\frac{5}{4}\right) \pi^{3 / 4}}\right)-\frac{1}{6} \pi(15+\pi)\right)\right)
\end{aligned}
$$

$$
\begin{aligned}
& \sqrt{ }\left(\sqrt { } \left(-0.5^{2}\left(-\frac{1}{6} \pi(15+\pi)+\pi \log (2 \pi 0.5)+\log \left(\frac{2 \Gamma\left(\frac{5}{4}\right)}{\pi^{3 / 4}}\right)\right)-\right.\right. \\
& \left.\left.2.9^{2}\left(-\frac{1}{6} \pi(15+\pi)+\pi \log (2 \pi 2.9)+\log \left(\frac{2 \Gamma\left(\frac{5}{4}\right)}{\pi^{3 / 4}}\right)\right)\right)\right)= \\
& \sqrt{ }\left(\sqrt { } \left(-0.5^{2}\left(\pi \log (\pi)+\log \left(\frac{2 e^{-\log \mathrm{G}(5 / 4)+\log \mathrm{G}(1+5 / 4)}}{\pi^{3 / 4}}\right)-\frac{1}{6} \pi(15+\pi)\right)-\right.\right. \\
& \left.\left.2.9^{2}\left(\pi \log (5.8 \pi)+\log \left(\frac{2 e^{-\log \mathrm{G}(5 / 4)+\operatorname{logG}(1+5 / 4)}}{\pi^{3 / 4}}\right)-\frac{1}{6} \pi(15+\pi)\right)\right)\right)
\end{aligned}
$$

$$
\begin{aligned}
& \sqrt{ }\left(\sqrt { } \left(-0.5^{2}\left(-\frac{1}{6} \pi(15+\pi)+\pi \log (2 \pi 0.5)+\log \left(\frac{2 \Gamma\left(\frac{5}{4}\right)}{\pi^{3 / 4}}\right)\right)-\right.\right. \\
& \left.\left.2.9^{2}\left(-\frac{1}{6} \pi(15+\pi)+\pi \log (2 \pi 2.9)+\log \left(\frac{2 \Gamma\left(\frac{5}{4}\right)}{\pi^{3 / 4}}\right)\right)\right)\right)= \\
& \sqrt{ }\left(\sqrt { } \left(-0.5^{2}\left(\pi \log (\pi)+\log \left(\frac{2(1)-1+\frac{5}{4}}{\pi^{3 / 4}}\right)-\frac{1}{6} \pi(15+\pi)\right)-\right.\right. \\
& \left.\left.2.9^{2}\left(\pi \log (5.8 \pi)+\log \left(\frac{2(1)-1+\frac{5}{4}}{\pi^{3 / 4}}\right)-\frac{1}{6} \pi(15+\pi)\right)\right)\right)
\end{aligned}
$$

## Series representations:

$$
\begin{aligned}
& \sqrt{ } \sqrt{ }\left(-0.5^{2}\left(-\frac{1}{6} \pi(15+\pi)+\pi \log (2 \pi 0.5)+\log \left(\frac{2 \Gamma\left(\frac{5}{4}\right)}{\pi^{3 / 4}}\right)\right)-\right. \\
& \left.\left.2.9^{2}\left(-\frac{1}{6} \pi(15+\pi)+\pi \log (2 \pi 2.9)+\log \left(\frac{2 \Gamma\left(\frac{5}{4}\right)}{\pi^{3 / 4}}\right)\right)\right)\right)= \\
& \exp \left(i \pi \left[\frac { 1 } { 2 \pi } \operatorname { a r g } \left(-x+\sqrt{21.65 \pi+1.44333 \pi^{2}-0.25 \pi \log (\pi)-}\right.\right.\right. \\
& \left.\left.\left.\left.8.41 \pi \log (5.8 \pi)-8.66 \log \left(\frac{2 \Gamma\left(\frac{5}{4}\right)}{\pi^{3 / 4}}\right)\right)\right)\right] \mid\right) \sqrt{x} \\
& \sum_{k=0}^{\infty} \frac{1}{k!}(-1)^{k} x^{-k}\left(-\frac{1}{2}\right)_{k}\left(-x+\sqrt{ }\left(21.65 \pi+1.44333 \pi^{2}-0.25 \pi \log (\pi)-8.41\right.\right. \\
& \left.\left.\pi \log (5.8 \pi)-8.66 \log \left(\frac{2 \Gamma\left(\frac{5}{4}\right)}{\pi^{3 / 4}}\right)\right)\right)^{k} \text { for }(x \in \mathbb{R} \text { and } x<0)
\end{aligned}
$$

$$
\begin{aligned}
& \sqrt{ } \sqrt{ }\left(-0.5^{2}\left(-\frac{1}{6} \pi(15+\pi)+\pi \log (2 \pi 0.5)+\log \left(\frac{2 \Gamma\left(\frac{5}{4}\right)}{\pi^{3 / 4}}\right)\right)-\right. \\
& \left.\left.2.9^{2}\left(-\frac{1}{6} \pi(15+\pi)+\pi \log (2 \pi 2.9)+\log \left(\frac{2 \Gamma\left(\frac{5}{4}\right)}{\pi^{3 / 4}}\right)\right)\right)\right)= \\
& \left(\frac{1}{z_{0}}\right)^{1 / 2}\left\lfloor\arg \left(\sqrt{21.65 \pi+1.44333 \pi^{2}-0.25 \pi \log (\pi)-8.41 \pi \log (5.8 \pi)-8.66 \log \left(\frac{2 \Gamma\left(\frac{5}{4}\right)}{\pi^{3 / 4}}\right)}-z_{0}\right) /(2 \pi)\right\rfloor \\
& z_{0}\left(2\left(1+\left[\arg \left(\sqrt{21.65 \pi+1.44333 \pi^{2}-0.25 \pi \log (\pi)-8.41 \pi \log (5.8 \pi)-8.66 \log \left(\frac{2 \Gamma\left(\frac{5}{4}\right)}{\pi^{3 / 4}}\right)}-z_{0}\right) /(2 \pi)\right]\right)\right. \\
& \sum_{k=0}^{\infty} \frac{1}{k!}(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(\sqrt{ } \mid 21.65 \pi+1.44333 \pi^{2}-0.25 \pi \log (\pi)-\right. \\
& \left.\left.8.41 \pi \log (5.8 \pi)-8.66 \log \left(\frac{2 \Gamma\left(\frac{5}{4}\right)}{\pi^{3 / 4}}\right)\right)-z_{0}\right)^{k} z_{0}^{-k}
\end{aligned}
$$

## Integral representations:

$$
\begin{aligned}
& \sqrt{ } \sqrt{ } \sqrt{ }\left(-0.5^{2}\left(-\frac{1}{6} \pi(15+\pi)+\pi \log (2 \pi 0.5)+\log \left(\frac{2 \Gamma\left(\frac{5}{4}\right)}{\pi^{3 / 4}}\right)\right)-\right. \\
& \left.\left.\left.2.9^{2}\left(-\frac{1}{6} \pi(15+\pi)+\pi \log (2 \pi 2.9)+\log \left(\frac{2 \Gamma\left(\frac{5}{4}\right)}{\pi^{3 / 4}}\right)\right)\right)\right)\right)= \\
& \sqrt{ }\left(\sqrt { } \left(21.65 \pi+1.44333 \pi^{2}-0.25 \pi \log (\pi)-8.41 \pi \log (5.8 \pi)-\right.\right. \\
& \left.\left.8.66 \log \left(\frac{2}{\pi^{3 / 4}} \int_{0}^{\infty} e^{-t} \sqrt[4]{t} d t\right)\right)\right)
\end{aligned}
$$

$$
\begin{aligned}
& \sqrt{ } \sqrt{ }\left(\left(-0.5^{2}\left(-\frac{1}{6} \pi(15+\pi)+\pi \log (2 \pi 0.5)+\log \left(\frac{2 \Gamma\left(\frac{5}{4}\right)}{\pi^{3 / 4}}\right)\right)-\right.\right. \\
& \left.\left.2.9^{2}\left(-\frac{1}{6} \pi(15+\pi)+\pi \log (2 \pi 2.9)+\log \left(\frac{2 \Gamma\left(\frac{5}{4}\right)}{\pi^{3 / 4}}\right)\right)\right)\right)= \\
& \sqrt{ }\left(\sqrt { } \left(21.65 \pi+1.44333 \pi^{2}-0.25 \pi \log (\pi)-8.41 \pi \log (5.8 \pi)-\right.\right. \\
& \left.\left.8.66 \log \left(\frac{2}{\pi^{3 / 4}} \int_{0}^{1} \sqrt[4]{\log \left(\frac{1}{t}\right)} d t\right)\right)\right)
\end{aligned}
$$

$$
\begin{array}{r}
\sqrt{ }\left(\sqrt { } \left(-0.5^{2}\left(-\frac{1}{6} \pi(15+\pi)+\pi \log (2 \pi 0.5)+\log \left(\frac{2 \Gamma\left(\frac{5}{4}\right)}{\pi^{3 / 4}}\right)\right)-\right.\right. \\
\left.\left.2.9^{2}\left(-\frac{1}{6} \pi(15+\pi)+\pi \log (2 \pi 2.9)+\log \left(\frac{2 \Gamma\left(\frac{5}{4}\right)}{\pi^{3 / 4}}\right)\right)\right)\right)=
\end{array}
$$

$\sqrt{ }\left|\int\right|\left(21.65 \pi+1.44333 \pi^{2}-0.25 \pi \log (\pi)-8.41 \pi \log (5.8 \pi)-\right.$

$$
\left.\left.8.66 \log \left(\frac{2\left(\int_{1}^{\infty} e^{-t} \sqrt[4]{t} d t+\sum_{k=0}^{\infty} \frac{(-1)^{k}}{\left(\frac{5}{4}+k\right) k!}\right)}{\pi^{3 / 4}}\right)\right)\right)
$$

We have that:

$$
(a m)^{2}\left\{-\frac{\pi^{2}}{12}+\frac{3 \pi}{4} \log 2\right\}
$$

$0.5^{\wedge} 2\left[\left(-\mathrm{Pi}^{\wedge} 2\right) / 12+(3 \mathrm{Pi}) / 4 \ln (2)\right]$

## Input:

$0.5^{2}\left(-\frac{\pi^{2}}{12}+\frac{3 \pi}{4} \log (2)\right)$

## Result:

0.202681...
0.202681....

## Alternative representations:

$0.5^{2}\left(-\frac{\pi^{2}}{12}+\frac{1}{4} \log (2)(3 \pi)\right)=0.5^{2}\left(\frac{3 \pi \log _{e}(2)}{4}-\frac{\pi^{2}}{12}\right)$
$0.5^{2}\left(-\frac{\pi^{2}}{12}+\frac{1}{4} \log (2)(3 \pi)\right)=0.5^{2}\left(\frac{3}{4} \pi \log (a) \log _{a}(2)-\frac{\pi^{2}}{12}\right)$
$0.5^{2}\left(-\frac{\pi^{2}}{12}+\frac{1}{4} \log (2)(3 \pi)\right)=0.5^{2}\left(\frac{6}{4} \pi \operatorname{coth}^{-1}(3)-\frac{\pi^{2}}{12}\right)$

## Series representations:

$0.5^{2}\left(-\frac{\pi^{2}}{12}+\frac{1}{4} \log (2)(3 \pi)\right)=-0.0208333 \pi^{2}+0.375 i \pi^{2}\left\lfloor\frac{\arg (2-x)}{2 \pi}\right\rfloor+$
$0.1875 \pi \log (x)-0.1875 \pi \sum_{k=1}^{\infty} \frac{(-1)^{k}(2-x)^{k} x^{-k}}{k}$ for $x<0$
$0.5^{2}\left(-\frac{\pi^{2}}{12}+\frac{1}{4} \log (2)(3 \pi)\right)=$

$$
\begin{gathered}
-0.0208333 \pi^{2}+0.375 i \pi^{2}\left[-\frac{-\pi+\arg \left(\frac{2}{z_{0}}\right)+\arg \left(z_{0}\right)}{2 \pi}\right]+ \\
0.1875 \pi \log \left(z_{0}\right)-0.1875 \pi \sum_{k=1}^{\infty} \frac{(-1)^{k}\left(2-z_{0}\right)^{k} z_{0}^{-k}}{k}
\end{gathered}
$$

$$
\begin{aligned}
& 0.5^{2}\left(-\frac{\pi^{2}}{12}+\frac{1}{4} \log (2)(3 \pi)\right)= \\
& -0.0208333 \pi^{2}+0.1875 \pi\left\lfloor\frac{\arg \left(2-z_{0}\right)}{2 \pi}\right\rfloor \log \left(\frac{1}{z_{0}}\right)+0.1875 \pi \log \left(z_{0}\right)+ \\
& 0.1875 \pi\left[\frac{\arg \left(2-z_{0}\right)}{2 \pi}\right\rfloor \log \left(z_{0}\right)-0.1875 \pi \sum_{k=1}^{\infty} \frac{(-1)^{k}\left(2-z_{0}\right)^{k} z_{0}^{-k}}{k}
\end{aligned}
$$

## Integral representations:

$0.5^{2}\left(-\frac{\pi^{2}}{12}+\frac{1}{4} \log (2)(3 \pi)\right)=-0.0208333 \pi^{2}+0.1875 \pi \int_{1}^{2} \frac{1}{t} d t$
$0.5^{2}\left(-\frac{\pi^{2}}{12}+\frac{1}{4} \log (2)(3 \pi)\right)=-0.0208333 \pi^{2}+\frac{0.09375}{i} \int_{-i \infty+\gamma}^{i \infty+\gamma} \frac{\Gamma(-s)^{2} \Gamma(1+s)}{\Gamma(1-s)} d s$ for $-1<\gamma<0$
$2.9^{\wedge} 2\left[\left(-\mathrm{Pi}^{\wedge} 2\right) / 12+(3 \mathrm{Pi}) / 4 \ln (2)\right]$

## Input:

$2.9^{2}\left(-\frac{\pi^{2}}{12}+\frac{3 \pi}{4} \log (2)\right)$
$\log (x)$ is the natural logarithm

## Result:

6.81818...
6.81818....

## Alternative representations:

$2.9^{2}\left(-\frac{\pi^{2}}{12}+\frac{1}{4} \log (2)(3 \pi)\right)=2.9^{2}\left(\frac{3 \pi \log _{e}(2)}{4}-\frac{\pi^{2}}{12}\right)$
$2.9^{2}\left(-\frac{\pi^{2}}{12}+\frac{1}{4} \log (2)(3 \pi)\right)=2.9^{2}\left(\frac{3}{4} \pi \log (a) \log _{a}(2)-\frac{\pi^{2}}{12}\right)$
$2.9^{2}\left(-\frac{\pi^{2}}{12}+\frac{1}{4} \log (2)(3 \pi)\right)=2.9^{2}\left(\frac{6}{4} \pi \operatorname{coth}^{-1}(3)-\frac{\pi^{2}}{12}\right)$

## Series representations:

$$
\begin{gathered}
2.9^{2}\left(-\frac{\pi^{2}}{12}+\frac{1}{4} \log (2)(3 \pi)\right)=-0.700833 \pi^{2}+12.615 i \pi^{2}\left\lfloor\frac{\arg (2-x)}{2 \pi}\right\rfloor+ \\
6.3075 \pi \log (x)-6.3075 \pi \sum_{k=1}^{\infty} \frac{(-1)^{k}(2-x)^{k} x^{-k}}{k} \text { for } x<0
\end{gathered}
$$

$$
\begin{aligned}
& 2.9^{2}\left(-\frac{\pi^{2}}{12}+\frac{1}{4} \log (2)(3 \pi)\right)= \\
& -0.700833 \pi^{2}+12.615 i \pi^{2}\left[-\frac{-\pi+\arg \left(\frac{2}{z_{0}}\right)+\arg \left(z_{0}\right)}{2 \pi}\right]+ \\
& 6.3075 \pi \log \left(z_{0}\right)-6.3075 \pi \sum_{k=1}^{\infty} \frac{(-1)^{k}\left(2-z_{0}\right)^{k} z_{0}^{-k}}{k}
\end{aligned}
$$

$$
\begin{aligned}
& 2.9^{2}\left(-\frac{\pi^{2}}{12}+\frac{1}{4} \log (2)(3 \pi)\right)= \\
& -0.700833 \pi^{2}+6.3075 \pi\left\lfloor\frac{\arg \left(2-z_{0}\right)}{2 \pi}\right\rfloor \log \left(\frac{1}{z_{0}}\right)+6.3075 \pi \log \left(z_{0}\right)+ \\
& \quad 6.3075 \pi\left\lfloor\frac{\arg \left(2-z_{0}\right)}{2 \pi}\right\rfloor \log \left(z_{0}\right)-6.3075 \pi \sum_{k=1}^{\infty} \frac{(-1)^{k}\left(2-z_{0}\right)^{k} z_{0}^{-k}}{k}
\end{aligned}
$$

## Integral representations:

$$
2.9^{2}\left(-\frac{\pi^{2}}{12}+\frac{1}{4} \log (2)(3 \pi)\right)=-0.700833 \pi^{2}+6.3075 \pi \int_{1}^{2} \frac{1}{t} d t
$$

$2.9^{2}\left(-\frac{\pi^{2}}{12}+\frac{1}{4} \log (2)(3 \pi)\right)=-0.700833 \pi^{2}+\frac{3.15375}{i} \int_{-i \infty+\gamma}^{i \infty+\gamma} \frac{\Gamma(-s)^{2} \Gamma(1+s)}{\Gamma(1-s)} d s$

From which:
$\operatorname{sqrt}\left(\left(\left((1+0.9568666373)\left[\left(\left(\left(2.9^{\wedge} 2\left[\left(-\mathrm{Pi}^{\wedge} 2\right) / 12+(3 \mathrm{Pi}) / 4 \ln (2)\right]\right)\right)\right) *\left(\left(\left(0.5^{\wedge} 2[(-\right.\right.\right.\right.\right.\right.\right.$ $\left.\left.\left.\left.\left.\left.\left.\left.\left.\mathrm{Pi}^{\wedge} 2\right) / 12+(3 \mathrm{Pi}) / 4 \ln (2)\right]\right)\right)\right)\right]\right)\right)\right)$
where 0.9568666373 is the value of the following Rogers-Ramanujan continued fraction:

$$
\frac{\mathrm{e}^{-\frac{\pi}{5}}}{\sqrt{(\varphi-1) \sqrt{5}}-\varphi+1}=1-\frac{\mathrm{e}^{-\pi}}{1+\frac{\mathrm{e}^{-2 \pi}}{1+\frac{\mathrm{e}^{-3 \pi}}{1+\frac{\mathrm{e}^{-4 \pi}}{1+\ldots}}}} \approx 0.9568666373
$$

## Input interpretation:

$\sqrt{(1+0.9568666373)\left(\left(2.9^{2}\left(-\frac{\pi^{2}}{12}+\frac{3 \pi}{4} \log (2)\right)\right)\left(0.5^{2}\left(-\frac{\pi^{2}}{12}+\frac{3 \pi}{4} \log (2)\right)\right)\right)}$
$\log (x)$ is the natural logarithm

## Result:

1.64445...
$1.64445 \ldots \approx \zeta(2)=\frac{\pi^{2}}{6}=1.644934 \ldots$

## All 2 <sup>nd</sup> roots of 2.70422:

$1.64445 e^{0} \approx 1.6445$ (real, principal root)
$1.64445 e^{i \pi} \approx-1.6445$ (real root)

## Alternative representations:

$$
\begin{aligned}
& \sqrt{(1+0.956867)\left(2.9^{2}\left(-\frac{\pi^{2}}{12}+\frac{1}{4}(3 \pi) \log (2)\right)\right)\left(0.5^{2}\left(-\frac{\pi^{2}}{12}+\frac{1}{4}(3 \pi) \log (2)\right)\right)}= \\
& \sqrt{1.95687 \times 0.5^{2} \times 2.9^{2}\left(\frac{3 \pi \log _{e}(2)}{4}-\frac{\pi^{2}}{12}\right)^{2}}
\end{aligned}
$$

$$
\begin{aligned}
& \sqrt{(1+0.956867)\left(2.9^{2}\left(-\frac{\pi^{2}}{12}+\frac{1}{4}(3 \pi) \log (2)\right)\right)\left(0.5^{2}\left(-\frac{\pi^{2}}{12}+\frac{1}{4}(3 \pi) \log (2)\right)\right)}= \\
& \sqrt{1.95687 \times 0.5^{2} \times 2.9^{2}\left(\frac{6}{4} \pi \operatorname{coth}^{-1}(3)-\frac{\pi^{2}}{12}\right)^{2}}
\end{aligned}
$$

$$
\begin{aligned}
& \sqrt{(1+0.956867)\left(2.9^{2}\left(-\frac{\pi^{2}}{12}+\frac{1}{4}(3 \pi) \log (2)\right)\right)\left(0.5^{2}\left(-\frac{\pi^{2}}{12}+\frac{1}{4}(3 \pi) \log (2)\right)\right)}= \\
& \sqrt{1.95687 \times 0.5^{2} \times 2.9^{2}\left(\frac{3}{4} \pi \log (a) \log _{a}(2)-\frac{\pi^{2}}{12}\right)^{2}}
\end{aligned}
$$

## Series representations:

$$
\begin{aligned}
& \sqrt{(1+0.956867)\left(2.9^{2}\left(-\frac{\pi^{2}}{12}+\frac{1}{4}(3 \pi) \log (2)\right)\right)\left(0.5^{2}\left(-\frac{\pi^{2}}{12}+\frac{1}{4}(3 \pi) \log (2)\right)\right)}= \\
& \sqrt{-1+0.0285716 \pi^{2}(\pi-9 \log (2))^{2}} \sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(-1+0.0285716 \pi^{2}(\pi-9 \log (2))^{2}\right)^{-k}
\end{aligned}
$$

$$
\begin{aligned}
& \sqrt{(1+0.956867)\left(2.9^{2}\left(-\frac{\pi^{2}}{12}+\frac{1}{4}(3 \pi) \log (2)\right)\right)\left(0.5^{2}\left(-\frac{\pi^{2}}{12}+\frac{1}{4}(3 \pi) \log (2)\right)\right)}= \\
& \sqrt{-1+0.0285716 \pi^{2}(\pi-9 \log (2))^{2}} \\
& \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-1+0.0285716 \pi^{2}(\pi-9 \log (2))^{2}\right)^{-k}\left(-\frac{1}{2}\right)_{k}}{k!}
\end{aligned}
$$

$$
\begin{array}{r}
\sqrt{(1+0.956867)\left(2.9^{2}\left(-\frac{\pi^{2}}{12}+\frac{1}{4}(3 \pi) \log (2)\right)\right)\left(0.5^{2}\left(-\frac{\pi^{2}}{12}+\frac{1}{4}(3 \pi) \log (2)\right)\right)}= \\
\sqrt{0.0285716 \pi^{2}\left(\pi-9\left(2 i \pi\left\lfloor\frac{\arg (2-x)}{2 \pi}\right\rfloor+\log (x)-\sum_{k=1}^{\infty} \frac{(-1)^{k}(2-x)^{k} x^{-k}}{k}\right)\right)^{2}}
\end{array}
$$

for $x<0$

## Integral representations:

$$
\begin{aligned}
& \sqrt{(1+0.956867)\left(2.9^{2}\left(-\frac{\pi^{2}}{12}+\frac{1}{4}(3 \pi) \log (2)\right)\right)\left(0.5^{2}\left(-\frac{\pi^{2}}{12}+\frac{1}{4}(3 \pi) \log (2)\right)\right)}= \\
& \sqrt{0.0285716 \pi^{2}\left(\pi-9 \int_{1}^{2} \frac{1}{t} d t\right)^{2}}
\end{aligned}
$$

$$
\sqrt{(1+0.956867)\left(2.9^{2}\left(-\frac{\pi^{2}}{12}+\frac{1}{4}(3 \pi) \log (2)\right)\left(0.5^{2}\left(-\frac{\pi^{2}}{12}+\frac{1}{4}(3 \pi) \log (2)\right)\right)\right.}=
$$

$$
\sqrt{\frac{0.0285716\left(i \pi^{2}-4.5 \int_{-i \infty+\gamma}^{i \infty} \frac{\Gamma(-s)^{2} \Gamma(1+s)}{\Gamma(1-s)} d s\right)^{2}}{i^{2}}} \text { for }-1<\gamma<0
$$

From:

$$
\begin{align*}
V & =-\frac{(2+2+2 \times 8)}{720 \pi R^{6}}+\sum_{P}(-1)^{2 s_{p}+1} \frac{n_{P}}{8 \pi} \times\left\{\frac{1}{90 R^{6}}-\frac{m_{p}^{2}}{6 R^{4}}\right\}+ \\
& +\sum_{A P}(-1)^{2 s_{p}+1} n_{P} \times\left\{-\frac{7}{8} \frac{1}{90 R^{6}}+\frac{1}{2} \frac{m_{p}^{2}}{6 R^{4}}\right\} \tag{2.19}
\end{align*}
$$

For:
non-interacting massive particle of spin $s_{p}$ and mass $m_{p}$
Spin $=1 / 2$,

$$
\begin{array}{c|c|c}
\text { Particle } & \text { Mass }(\mathrm{TeV}) & (-1)^{\left(2 s_{p}+1\right)} n_{p} \\
\tilde{u_{R}}, \tilde{d}_{R}, \tilde{s}_{R}, \tilde{c}_{R} & 2.76 & -24
\end{array}
$$

$-(2+2+2 * 8) /\left(720 \mathrm{Pi}^{*} \mathrm{x}^{\wedge} 6\right)-(24 /(8 \mathrm{Pi}))^{*}\left[\left(1 /\left(90 x^{\wedge} 6\right)-\left((2.76)^{\wedge} 2\right) /\left(6 x^{\wedge} 4\right)\right)\right]-24 *[(-$ $\left.\left.7 /\left(720 x^{\wedge} 6\right)+\left((2.76)^{\wedge} 2\right) /\left(12 x^{\wedge} 4\right)\right)\right]$

## Input:

$$
-\frac{2+2+2 \times 8}{720 \pi x^{6}}-\frac{24}{8 \pi}\left(\frac{1}{90 x^{6}}-\frac{2.76^{2}}{6 x^{4}}\right)-24\left(-\frac{7}{720 x^{6}}+\frac{2.76^{2}}{12 x^{4}}\right)
$$

## Result:

$-\frac{1}{36 \pi x^{6}}-\frac{3\left(\frac{1}{90 x^{6}}-\frac{1.2696}{x^{4}}\right)}{\pi}-24\left(\frac{0.6348}{x^{4}}-\frac{7}{720 x^{6}}\right)$

## Plots:




Alternate forms:

$$
\frac{0.213881-14.0228 x^{2}}{x^{6}}
$$

$-\frac{4.4636\left(3.14159 x^{2}-0.0479167\right)}{x^{6}}$
$-\frac{7929.72 x^{2}-42 \pi+11}{180 \pi x^{6}}$

## Partial fraction expansion:

$$
\frac{42 \pi-11}{180 \pi x^{6}}-\frac{14.0228}{x^{4}}
$$

## Expanded form:

$$
-\frac{11}{180 \pi x^{6}}+\frac{7}{30 x^{6}}-\frac{14.0228}{x^{4}}
$$

## Roots:

$$
x \approx-0.1235
$$

$x \approx 0.1235$

## Properties as a real function: <br> Domain

$$
\{x \in \mathbb{R}: x \neq 0\}
$$

## Range

$$
\begin{aligned}
& \{y \in \mathbb{R}: y \geq(2590035913944-31080430967328 \pi+ \\
& \left.124321723869312 \pi^{2}-165762298492416 \pi^{3}\right) / \\
& \left.\left(1181640625 \pi-9023437500 \pi^{2}+17226562500 \pi^{3}\right)\right\}
\end{aligned}
$$

## Parity

even

Derivative:

$$
\begin{aligned}
& \frac{d}{d x}\left(-\frac{2+2+2 \times 8}{720 \pi x^{6}}-\frac{24\left(\frac{1}{90 x^{6}}-\frac{2.76^{2}}{6 x^{4}}\right)}{8 \pi}-24\left(-\frac{7}{720 x^{6}}+\frac{2.76^{2}}{12 x^{4}}\right)\right)= \\
& \frac{56.0913 x^{2}-1.28329}{x^{7}}
\end{aligned}
$$

## Indefinite integral:

$$
\begin{aligned}
& \int\left(-\frac{3\left(\frac{1}{90 x^{6}}-\frac{1.2696}{x^{4}}\right)}{\pi}-24\left(-\frac{7}{720 x^{6}}+\frac{0.6348}{x^{4}}\right)-\frac{1}{36 \pi x^{6}}\right) d x= \\
& \frac{4.67427 x^{2}-0.0427762}{x^{5}}+\text { constant }
\end{aligned}
$$

## Global minima:

$$
\begin{aligned}
& \min \left\{-\frac{2+2+2 \times 8}{720 \pi x^{6}}-\frac{24\left(\frac{1}{90 x^{6}}-\frac{2.76^{2}}{6 x^{4}}\right)}{8 \pi}-24\left(-\frac{7}{720 x^{6}}+\frac{2.76^{2}}{12 x^{4}}\right)\right\}= \\
& \quad-\frac{2430850583234918645426565565575839210188800(4 \pi-1)^{3}}{9165423189346859202789669808161240667(11-42 \pi)^{2} \pi} \\
& \text { at } x=-\frac{1}{372} \sqrt{\frac{2092750755523(42 \pi-11)}{6911957230(4 \pi-1)}}
\end{aligned}
$$

$$
\begin{aligned}
& \min \left\{-\frac{2+2+2 \times 8}{720 \pi x^{6}}-\frac{24\left(\frac{1}{90 x^{6}}-\frac{2.76^{2}}{6 x^{4}}\right)}{8 \pi}-24\left(-\frac{7}{720 x^{6}}+\frac{2.76^{2}}{12 x^{4}}\right)\right\}= \\
& \quad-\frac{2430850583234918645426565565575839210188800(4 \pi-1)^{3}}{9165423189346859202789669808161240667(11-42 \pi)^{2} \pi} \\
& \text { at } x=\frac{1}{372} \sqrt{\frac{2092750755523(42 \pi-11)}{6911957230(4 \pi-1)}}
\end{aligned}
$$

## Global minima:

$\min \left\{-\frac{2+2+2 \times 8}{720 \pi x^{6}}-\frac{24\left(\frac{1}{90 x^{6}}-\frac{2.76^{2}}{6 x^{4}}\right)}{8 \pi}-24\left(-\frac{7}{720 x^{6}}+\frac{2.76^{2}}{12 x^{4}}\right)\right\} \approx-8930.1$
at $x \approx-0.15126$

$$
\begin{aligned}
& \min \left\{-\frac{2+2+2 \times 8}{720 \pi x^{6}}-\frac{24\left(\frac{1}{90 x^{6}}-\frac{2.76^{2}}{6 x^{4}}\right)}{8 \pi}-24\left(-\frac{7}{720 x^{6}}+\frac{2.76^{2}}{12 x^{4}}\right)\right\} \approx-8930.1 \\
& \text { at } x \approx 0.15126
\end{aligned}
$$

## Limit:

$$
\lim _{x \rightarrow \pm \infty}\left(-\frac{3\left(\frac{1}{90 x^{6}}-\frac{1.2696}{x^{4}}\right)}{\pi}-24\left(-\frac{7}{720 x^{6}}+\frac{0.6348}{x^{4}}\right)-\frac{1}{36 \pi x^{6}}\right)=0 \approx 0
$$

## Alternative representations:

$$
\begin{array}{r}
-\frac{2+2+2 \times 8}{720 \pi x^{6}}-\frac{\left(\frac{1}{90 x^{6}}-\frac{2.76^{2}}{6 x^{4}}\right) 24}{8 \pi}-24\left(-\frac{7}{720 x^{6}}+\frac{2.76^{2}}{12 x^{4}}\right)= \\
-\frac{24\left(-\frac{2.76^{2}}{6 x^{4}}+\frac{1}{90 x^{6}}\right)}{1440^{\circ}}-24\left(\frac{2.76^{2}}{12 x^{4}}-\frac{7}{720 x^{6}}\right)-\frac{20}{129600^{\circ} x^{6}}
\end{array}
$$

$$
\begin{aligned}
& -\frac{2+2+2 \times 8}{720 \pi x^{6}}-\frac{\left(\frac{1}{90 x^{6}}-\frac{2.76^{2}}{6 x^{4}}\right) 24}{8 \pi}-24\left(-\frac{7}{720 x^{6}}+\frac{2.76^{2}}{12 x^{4}}\right)= \\
& \frac{-24\left(-\frac{2.76^{2}}{6 x^{4}}+\frac{1}{90 x^{6}}\right)}{-8 i \log (-1)}-24\left(\frac{2.76^{2}}{12 x^{4}}-\frac{7}{720 x^{6}}\right)--\frac{20}{720 i \log (-1) x^{6}}
\end{aligned}
$$

$$
\begin{aligned}
& -\frac{2+2+2 \times 8}{720 \pi x^{6}}-\frac{\left(\frac{1}{90 x^{6}}-\frac{2.76^{2}}{6 x^{4}}\right) 24}{8 \pi}-24\left(-\frac{7}{720 x^{6}}+\frac{2.76^{2}}{12 x^{4}}\right)= \\
& -\frac{24\left(-\frac{2.76^{2}}{6 x^{4}}+\frac{1}{90 x^{6}}\right)}{8 \cos ^{-1}(-1)}-24\left(\frac{2.76^{2}}{12 x^{4}}-\frac{7}{720 x^{6}}\right)-\frac{20}{720 \cos ^{-1}(-1) x^{6}}
\end{aligned}
$$

## Series representations:

$$
\begin{aligned}
& -\frac{2+2+2 \times 8}{720 \pi x^{6}}-\frac{\left(\frac{1}{90 x^{6}}-\frac{2.76^{2}}{6 x^{4}}\right) 24}{8 \pi}-24\left(-\frac{7}{720 x^{6}}+\frac{2.76^{2}}{12 x^{4}}\right)= \\
& -\frac{15.2352\left(0.00100279-0.0625 x^{2}-0.0153154 \sum_{k=0}^{\infty} \frac{(-1)^{k}}{1+2 k}+x^{2} \sum_{k=0}^{\infty} \frac{(-1)^{k}}{1+2 k}\right)}{x^{6} \sum_{k=0}^{\infty} \frac{(-1)^{k}}{1+2 k}}
\end{aligned}
$$

$$
\begin{aligned}
& -\frac{2+2+2 \times 8}{720 \pi x^{6}}-\frac{\left(\frac{1}{90 x^{6}}-\frac{2.76^{2}}{6 x^{4}}\right) 24}{8 \pi}-24\left(-\frac{7}{720 x^{6}}+\frac{2.76^{2}}{12 x^{4}}\right)= \\
& \left(0 . 9 5 2 2 \left(-0.0160447+x^{2}+1.0502\right.\right. \\
& \sum_{k=0}^{\infty} \frac{1}{1+2 k}(-1)^{k} 1195^{-2 k}\left(-0.00097629 \times 25^{k}+0.186667 \times 57121^{k}+\right. \\
& \left.\left.\left.\left(0.0637456 \times 25^{k}-12.1882 \times 57121^{k}\right) x^{2}\right)\right)\right) / \\
& \left(x^{6} \sum_{k=0}^{\infty} \frac{1195^{-2 k}\left(0.4(-57121)^{k}-0.00209205(-25)^{k}\right)}{0.5+k}\right)
\end{aligned}
$$

$$
\begin{gathered}
-\frac{2+2+2 \times 8}{720 \pi x^{6}}-\frac{\left(\frac{1}{90 x^{6}}-\frac{2.76^{2}}{6 x^{4}}\right) 24}{8 \pi}-24\left(-\frac{7}{720 x^{6}}+\frac{2.76^{2}}{12 x^{4}}\right)= \\
-\left(\left(1 5 . 2 3 5 2 \left(0.00401118-0.25 x^{2}-\right.\right.\right. \\
0.0153154 \sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^{k}\left(\frac{1}{1+2 k}+\frac{2}{1+4 k}+\frac{1}{3+4 k}\right)+ \\
\left.\left.x^{2} \sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^{k}\left(\frac{1}{1+2 k}+\frac{2}{1+4 k}+\frac{1}{3+4 k}\right)\right)\right) / \\
\left.\left(x^{6} \sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^{k}\left(\frac{1}{1+2 k}+\frac{2}{1+4 k}+\frac{1}{3+4 k}\right)\right)\right)
\end{gathered}
$$

## Integral representations:

$$
\begin{aligned}
& -\frac{2+2+2 \times 8}{720 \pi x^{6}}-\frac{\left(\frac{1}{90 x^{6}}-\frac{2.76^{2}}{6 x^{4}}\right) 24}{8 \pi}-24\left(-\frac{7}{720 x^{6}}+\frac{2.76^{2}}{12 x^{4}}\right)= \\
& -\frac{15.2352\left(0.00200559-0.125 x^{2}-0.0153154 \int_{0}^{\infty} \frac{1}{1+t^{2}} d t+x^{2} \int_{0}^{\infty} \frac{1}{1+t^{2}} d t\right)}{x^{6} \int_{0}^{\infty} \frac{1}{1+t^{2}} d t}
\end{aligned}
$$

$$
\begin{aligned}
- & \frac{2+2+2 \times 8}{720 \pi x^{6}}-\frac{\left(\frac{1}{90 x^{6}}-\frac{2.76^{2}}{6 x^{4}}\right) 24}{8 \pi}-24\left(-\frac{7}{720 x^{6}}+\frac{2.76^{2}}{12 x^{4}}\right)= \\
& -\frac{1}{x^{6} \int_{0}^{1} \sqrt{1-t^{2}} d t} 15.2352 \\
& \quad\left(0.00100279-0.0625 x^{2}-0.0153154 \int_{0}^{1} \sqrt{1-t^{2}} d t+x^{2} \int_{0}^{1} \sqrt{1-t^{2}} d t\right)
\end{aligned}
$$

$$
\begin{aligned}
& -\frac{2+2+2 \times 8}{720 \pi x^{6}}-\frac{\left(\frac{1}{90 x^{6}}-\frac{2.76^{2}}{6 x^{4}}\right) 24}{8 \pi}-24\left(-\frac{7}{720 x^{6}}+\frac{2.76^{2}}{12 x^{4}}\right)= \\
& -\frac{15.2352\left(0.00200559-0.125 x^{2}-0.0153154 \int_{0}^{\infty} \frac{\sin (t)}{t} d t+x^{2} \int_{0}^{\infty} \frac{\sin (t)}{t} d t\right)}{x^{6} \int_{0}^{\infty} \frac{\sin (t)}{t} d t}
\end{aligned}
$$

From:

$$
-\frac{1}{36 \pi x^{6}}-\frac{3\left(\frac{1}{90 x^{6}}-\frac{1.2696}{x^{4}}\right)}{\pi}-24\left(\frac{0.6348}{x^{4}}-\frac{7}{720 x^{6}}\right)
$$

For $x=0.1235$ :
$-\left(3\left(1 /\left(900.1235^{\wedge} 6\right)-1.2696 / 0.1235^{\wedge} 4\right)\right) / \pi-24\left(-7 /\left(7200.1235^{\wedge} 6\right)+\right.$ $\left.0.6348 / 0.1235^{\wedge} 4\right)-1 /\left(36 \pi 0.1235^{\wedge} 6\right)$

## Input interpretation:

$$
-\frac{3\left(\frac{1}{90 \times 0.1235^{6}}-\frac{1.2696}{0.1235^{4}}\right)}{\pi}-24\left(-\frac{7}{720 \times 0.1235^{6}}+\frac{0.6348}{0.1235^{4}}\right)-\frac{1}{36 \pi \times 0.1235^{6}}
$$

## Result:

0.418882...
0.418882....

## Alternative representations:

$$
\begin{aligned}
& -\frac{3\left(\frac{1}{900.1235^{6}}-\frac{1.2696}{0.1235^{4}}\right)}{\pi}-24\left(-\frac{7}{720 \times 0.1235^{6}}+\frac{0.6348}{0.1235^{4}}\right)-\frac{1}{36 \pi 0.1235^{6}}= \\
& \left(-\frac{3\left(-\frac{1.2696}{0.1235^{4}}+\frac{1}{900.1235^{6}}\right)}{180^{\circ}}-24\left(\frac{0.6348}{0.1235^{4}}-\frac{7}{720 \times 0.1235^{6}}\right)-\frac{1}{6480^{\circ} 0.1235^{6}}=\right. \\
& \left.271.199-\frac{4.726}{\circ}\right)
\end{aligned}
$$

$$
\begin{gathered}
-\frac{3\left(\frac{1}{900.1235^{6}}-\frac{1.2696}{0.1235^{4}}\right)}{\pi}-24\left(-\frac{7}{720 \times 0.1235^{6}}+\frac{0.6348}{0.1235^{4}}\right)-\frac{1}{36 \pi 0.1235^{6}}= \\
-\frac{3\left(-\frac{1.2696}{0.1235^{4}}+\frac{1}{900.1235^{6}}\right)}{\cos ^{-1}(-1)}-24\left(\frac{0.6348}{0.1235^{4}}-\frac{7}{720 \times 0.1235^{6}}\right)- \\
\left.\frac{1}{36 \cos ^{-1}(-1) 0.1235^{6}}=271.199-\frac{850.681}{\cos ^{-1}(-1)}\right)
\end{gathered}
$$

$$
\begin{aligned}
& -\frac{3\left(\frac{1}{90.0 .1235^{6}}-\frac{1.2696}{0.1235^{4}}\right)}{\pi}-24\left(-\frac{7}{720 \times 0.1235^{6}}+\frac{0.6348}{0.1235^{4}}\right)-\frac{1}{36 \pi 0.1235^{6}}= \\
& \left(-\frac{3\left(-\frac{1.2696}{0.1235^{4}}+\frac{1}{900.1235^{6}}\right)}{2 E(0)}-24\left(\frac{0.6348}{0.1235^{4}}-\frac{7}{720 \times 0.1235^{6}}\right)-\frac{1}{72 E(0) 0.1235^{6}}=\right. \\
& \left.271.199-\frac{425.34}{E(0)}\right)
\end{aligned}
$$

## Series representations:

$$
\begin{aligned}
& -\frac{3\left(\frac{1}{900.1235^{6}}-\frac{1.2696}{0.1235^{4}}\right)}{\pi}-24\left(-\frac{7}{720 \times 0.1235^{6}}+\frac{0.6348}{0.1235^{4}}\right)-\frac{1}{36 \pi 0.1235^{6}}= \\
& 271.199-\frac{212.67}{\sum_{k=0}^{\infty} \frac{(-1)^{k}}{1+2 k}}
\end{aligned}
$$

$$
\begin{aligned}
& -\frac{3\left(\frac{1}{900.1235^{6}}-\frac{1.2696}{0.1235^{4}}\right)}{\pi}-24\left(-\frac{7}{720 \times 0.1235^{6}}+\frac{0.6348}{0.1235^{4}}\right)-\frac{1}{36 \pi 0.1235^{6}}= \\
& 271.199-\frac{425.34}{-1+\sum_{k=1}^{\infty} \frac{2^{k}}{2 k}}=
\end{aligned}
$$

$$
\begin{aligned}
& -\frac{3\left(\frac{1}{900.1235^{6}}-\frac{1.2696}{0.1235^{4}}\right)}{\pi}-24\left(-\frac{7}{720 \times 0.1235^{6}}+\frac{0.6348}{0.1235^{4}}\right)-\frac{1}{36 \pi 0.1235^{6}}= \\
& 271.199-\frac{850.681}{\sum_{k=0}^{\infty} \frac{2^{-k}(-6+50 k)}{\binom{3 k}{k}}}
\end{aligned}
$$

## Integral representations:

$$
\begin{aligned}
& -\frac{3\left(\frac{1}{90.0 .1235^{6}}-\frac{1.2696}{0.1235^{4}}\right)}{\pi}-24\left(-\frac{7}{720 \times 0.1235^{6}}+\frac{0.6348}{0.1235^{4}}\right)-\frac{1}{36 \pi 0.1235^{6}}= \\
& 271.199-\frac{425.34}{\int_{0}^{\infty} \frac{1}{1+t^{2}} d t}
\end{aligned}
$$

$$
\begin{aligned}
& -\frac{3\left(\frac{1}{900.1235^{6}}-\frac{1.2696}{0.1235^{4}}\right)}{\pi}-24\left(-\frac{7}{720 \times 0.1235^{6}}+\frac{0.6348}{0.1235^{4}}\right)-\frac{1}{36 \pi 0.1235^{6}}= \\
& 271.199-\frac{212.67}{\int_{0}^{1} \sqrt{1-t^{2}} d t}
\end{aligned}
$$

$$
\begin{aligned}
& -\frac{3\left(\frac{1}{900.1235^{6}}-\frac{1.2696}{0.1235^{4}}\right)}{\pi}-24\left(-\frac{7}{720 \times 0.1235^{6}}+\frac{0.6348}{0.1235^{4}}\right)-\frac{1}{36 \pi 0.1235^{6}}= \\
& 271.199-\frac{425.34}{\int_{0}^{\infty} \frac{\sin (t)}{t} d t}
\end{aligned}
$$

From which:
$(3+0.9568666373)\left(\left(-\left(3\left(1 /\left(900.1235^{\wedge} 6\right)-1.2696 / 0.1235^{\wedge} 4\right)\right) / \pi-24(-7 /(720\right.\right.$
$\left.\left.\left.\left.\left.0.1235^{\wedge} 6\right)+0.6348 / 0.1235^{\wedge} 4\right)-1 /\left(36 \pi 0.1235^{\wedge} 6\right)\right)\right)\right)$
where 0.9568666373 is the value of the following Rogers-Ramanujan continued fraction:

$$
\frac{\mathrm{e}^{-\frac{\pi}{5}}}{\sqrt{(\varphi-1) \sqrt{5}}-\varphi+1}=1-\frac{\mathrm{e}^{-\pi}}{1+\frac{\mathrm{e}^{-2 \pi}}{1+\frac{\mathrm{e}^{-3 \pi}}{1+\frac{\mathrm{e}^{-4 \pi}}{1+\ldots}}}} \approx 0.9568666373
$$

## Input interpretation:

$$
\left(-\frac{3\left(\frac{1}{90 \times 0.1235^{6}}-\frac{1.2696}{0.1235^{4}}\right)}{\pi}-24\left(-\frac{7}{720 \times 0.1235^{6}}+\frac{0.6348}{0.1235^{4}}\right)-\frac{1}{36 \pi \times 0.1235^{6}}\right)
$$

## Result:

1.6574587999242426068708009669496280285514866570077400318856203198
$1.65745879 \ldots .$. result very near to the 14 th root of the following Ramanujan's class invariant $Q=\left(G_{505} / G_{101 / 5}\right)^{3}=1164.2696$ i.e. $1.65578 \ldots$

## Alternative representations:

$$
\begin{gathered}
\left(-\frac{3\left(\frac{1}{90.1235^{6}}-\frac{1.2696}{0.1235^{4}}\right)}{\pi}-24\left(-\frac{7}{720 \times 0.1235^{6}}+\frac{0.6348}{0.1235^{4}}\right)-\frac{1}{36 \pi 0.1235^{6}}\right)= \\
\left(3 . 9 5 6 8 7 \left(-\frac{3\left(-\frac{1.2696}{0.1235^{4}}+\frac{1}{900.1235^{6}}\right)}{180^{\circ}}-24\left(\frac{0.6348}{0.1235^{4}}-\frac{7}{720 \times 0.1235^{6}}\right)-\right.\right. \\
\left.\left(\frac{1}{6480^{\circ} 0.1235^{6}}\right)=3.95687\left(271.199-\frac{4.726}{\circ}\right)\right)
\end{gathered}
$$

$$
\begin{gathered}
\left(-\frac{3\left(\frac{1}{90.1235^{6}}-\frac{1.2696}{0.1235^{4}}\right)}{\pi}-24\left(-\frac{7}{720 \times 0.1235^{6}}+\frac{0.6348}{0.1235^{4}}\right)-\frac{1}{36 \pi 0.1235^{6}}\right)= \\
\left(3 . 9 5 6 8 7 \left(-\frac{3\left(-\frac{1.2696}{0.1235^{4}}+\frac{1}{900.1235^{6}}\right)}{\cos ^{-1}(-1)}-24\left(\frac{0.6348}{0.1235^{4}}-\frac{7}{720 \times 0.1235^{6}}\right)-\right.\right. \\
\left.\left(\frac{1}{36 \cos ^{-1}(-1) 0.1235^{6}}\right)=3.95687\left(271.199-\frac{850.681}{\cos ^{-1}(-1)}\right)\right)
\end{gathered}
$$

$$
\begin{gathered}
\left(-\frac{3\left(\frac{1}{90.0 .1235^{6}}-\frac{1.2696}{0.1235^{4}}\right)}{\pi}-24\left(-\frac{7}{720 \times 0.1235^{6}}+\frac{0.6348}{0.1235^{4}}\right)-\frac{1}{36 \pi 0.1235^{6}}\right)= \\
\left(3 . 9 5 6 8 7 \left(-\frac{3\left(-\frac{1.2696}{0.1235^{4}}+\frac{1}{900.1235^{6}}\right)}{2 E(0)}-24\left(\frac{0.6348}{0.1235^{4}}-\frac{7}{720 \times 0.1235^{6}}\right)-\right.\right. \\
\left.\left(\frac{1}{72 E(0) 0.1235^{6}}\right)=3.95687\left(271.199-\frac{425.34}{E(0)}\right)\right)
\end{gathered}
$$

## Series representations:

$$
\begin{aligned}
& \left(-\frac{3\left(\frac{1}{90.0 .1235^{6}}-\frac{1.2696}{0.1235^{4}}\right)}{\pi}-24\left(-\frac{7}{720 \times 0.1235^{6}}+\frac{0.6348}{0.1235^{4}}\right)-\frac{1}{36 \pi 0.1235^{6}}\right)= \\
& 1073.1-\frac{841.508}{\sum_{k=0}^{\infty} \frac{(-1)^{k}}{1+2 k}}
\end{aligned}
$$

$$
\begin{aligned}
& \quad\left(-\frac{3\left(\frac{1}{90.1235^{6}}-\frac{1.2696}{0.1235^{4}}\right)}{\pi}-24\left(-\frac{7}{720 \times 0.1235^{6}}+\frac{0.6348}{0.1235^{4}}\right)-\frac{1}{36 \pi 0.1235^{6}}\right)= \\
& 1073.1-\frac{1683.02}{\left.-1+\sum_{k=1}^{\infty} \frac{2^{k}}{2 k} \begin{array}{c}
2 k \\
k
\end{array}\right)}
\end{aligned}
$$

$$
\begin{aligned}
& \quad\left(-\frac{3\left(\frac{1}{90.0 .1235^{6}}-\frac{1.2696}{0.1235^{4}}\right)}{\pi}-24\left(-\frac{7}{720 \times 0.1235^{6}}+\frac{0.6348}{0.1235^{4}}\right)-\frac{1}{36 \pi 0.1235^{6}}\right)= \\
& 1073.1-\frac{3366.03}{\sum_{k=0}^{\infty} \frac{2^{-k}(-6+50 k)}{\binom{3 k}{k}}}
\end{aligned}
$$

## Integral representations:

$$
\begin{aligned}
& (3+0.956867) \\
& \left(-\frac{3\left(\frac{1}{900.1235^{6}}-\frac{1.2696}{0.1235^{4}}\right)}{\pi}-24\left(-\frac{7}{720 \times 0.1235^{6}}+\frac{0.6348}{0.1235^{4}}\right)-\frac{1}{36 \pi 0.1235^{6}}\right)= \\
& 1073.1-\frac{1683.02}{\int_{0}^{\infty} \frac{1}{1+t^{2}} d t}
\end{aligned}
$$

$$
\begin{aligned}
& \quad\left(-\frac{3\left(\frac{1}{90.1235^{6}}-\frac{1.2696}{0.1235^{4}}\right)}{\pi}-24\left(-\frac{7}{720 \times 0.1235^{6}}+\frac{0.6348}{0.1235^{4}}\right)-\frac{1}{36 \pi 0.1235^{6}}\right)= \\
& 1073.1-\frac{841.508}{\int_{0}^{1} \sqrt{1-t^{2}} d t}
\end{aligned}
$$

$$
\begin{aligned}
& \quad\left(-\frac{3\left(\frac{1}{90.0 .1235^{6}}-\frac{1.2696}{0.1235^{4}}\right)}{\pi}-24\left(-\frac{7}{720 \times 0.1235^{6}}+\frac{0.6348}{0.1235^{4}}\right)-\frac{1}{36 \pi 0.1235^{6}}\right)= \\
& 1073.1-\frac{1683.02}{\int_{0}^{\infty} \frac{\sin (t)}{t} d t}
\end{aligned}
$$

With regard the global minima:

$$
\begin{aligned}
& \min \left\{-\frac{2+2+2 \times 8}{720 \pi x^{6}}-\frac{24\left(\frac{1}{90 x^{6}}-\frac{2.76^{2}}{6 x^{4}}\right)}{8 \pi}-24\left(-\frac{7}{720 x^{6}}+\frac{2.76^{2}}{12 x^{4}}\right)\right\} \approx-8930.1 \\
& \text { at } x \approx-0.15126
\end{aligned}
$$

We have that:
$-1 / 372 \operatorname{sqrt}((2092750755523(42 \pi-11)) /(6911957230(4 \pi-1)))$

## Input:

$$
-\frac{1}{372} \sqrt{\frac{2092750755523(42 \pi-11)}{6911957230(4 \pi-1)}}
$$

## Decimal approximation:

-0.151256517156642996910451527470557054387252430352248057534918833

$$
x=-0.151256517156
$$

## Property:

$$
-\frac{1}{372} \sqrt{\frac{2092750755523(-11+42 \pi)}{6911957230(-1+4 \pi)}} \text { is a transcendental number }
$$

## Series representations:

$$
\begin{aligned}
& \frac{1}{372} \sqrt{\frac{2092750755523(42 \pi-11)}{6911957230(4 \pi-1)}(-1)}= \\
& -\frac{1}{372} \sqrt{-1+\frac{2092750755523(-11+42 \pi)}{6911957230(-1+4 \pi)}} \\
& \sum_{k=0}^{\infty}\left(-1+\frac{2092750755523(-11+42 \pi)}{6911957230(-1+4 \pi)}\right)^{-k}\binom{\frac{1}{2}}{k}
\end{aligned}
$$

$$
\begin{aligned}
& \frac{1}{372} \sqrt{\frac{2092750755523(42 \pi-11)}{6911957230(4 \pi-1)}(-1)=} \\
& -\frac{1}{372} \sqrt{-1+\frac{2092750755523(-11+42 \pi)}{6911957230(-1+4 \pi)}} \\
& \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-1+\frac{2092750755523(-11+42 \pi)}{6911957230(-1+4 \pi)}\right)^{-k}\left(-\frac{1}{2}\right)_{k}}{k!}
\end{aligned}
$$

$$
\begin{aligned}
& \frac{1}{372} \sqrt{\frac{2092750755523(42 \pi-11)}{6911957230(4 \pi-1)}(-1)=} \\
& \quad-\frac{1}{372} \sqrt{z_{0}} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(\frac{2092750755523(-11+42 \pi)}{6911957230(-1+4 \pi)}-z_{0}\right)^{k} z_{0}^{-k}}{k!} \\
& \quad \text { for }\left(\operatorname{not}\left(z_{0} \in \mathbb{R} \text { and }-\infty<z_{0} \leq 0\right)\right) \\
& \left\{-(2+2+28) /\left(720 \pi^{*}-0.151256517156^{\wedge} 6\right)-\left(2 4 \left(1 /\left(90 *_{-}-0.151256517156^{\wedge} 6\right)-\right.\right.\right. \\
& \left.\left.2.76^{\wedge} 2 /\left(6 *-0.151256517156^{\wedge 4}\right)\right)\right) /(8 \pi)-24\left(-7 /\left(720 *_{-}-0.151256517156^{\wedge} 6\right)+\right. \\
& 2.76^{\wedge} 2 /\left(12 *_{-}-0.151256517156^{\wedge 4))\}}\right.
\end{aligned}
$$

## Input interpretation:

$$
\begin{aligned}
& -\frac{2+2+2 \times 8}{720 \pi \times(-1) \times 0.151256517156^{6}}- \\
& \frac{24\left(\frac{1}{90 \times(-1) \times 0.151256517156^{6}}-\frac{2.76^{2}}{6 \times(-1) \times 0.151256517156^{4}}\right)}{8 \pi}- \\
& 24\left(-\frac{7}{720 \times(-1) \times 0.151256517156^{6}}+\frac{2.76^{2}}{12 \times(-1) \times 0.151256517156^{4}}\right)
\end{aligned}
$$

## Result:

8930.13...
8930.13...

## Alternative representations:

$$
\begin{aligned}
& -\frac{2+2+2 \times 8}{720 \pi(-1) 0.1512565171560000^{6}}- \\
& \frac{24\left(\frac{1}{90(-1) 0.1512565171560000^{6}}-\frac{2.76^{2}}{6(-1) 0.1512565171560000^{4}}\right)}{8 \pi}- \\
& \\
& 24\left(-\frac{7}{720(-1) 0.1512565171560000^{6}}+\frac{2.76^{2}}{12(-1) 0.1512565171560000^{4}}\right)= \\
& \\
& -24\left(-\frac{7}{12 \times 0.1512565171560000^{4}}--\frac{7}{720 \times 0.1512565171560000^{6}}\right)- \\
& \\
& \quad \frac{24\left(\frac{-2.76^{2}}{-60.1512565171560000^{4}}+-\frac{1}{900.1512565171560000^{6}}\right)}{1440^{\circ}}- \\
& \\
& -\frac{20}{129600^{\circ} 0.1512565171560000^{6}}
\end{aligned}
$$

$$
\begin{aligned}
& -\frac{2+2+2 \times 8}{720 \pi(-1) 0.1512565171560000^{6}}- \\
& \frac{24\left(\frac{1}{90(-1) 0.1512565171560000^{6}}-\frac{2.76^{2}}{6(-1) 0.1512565171560000^{4}}\right)}{8 \pi}- \\
& 24\left(-\frac{7}{720(-1) 0.1512565171560000^{6}}+\frac{2.76^{2}}{12(-1) 0.1512565171560000^{4}}\right)= \\
& - \\
& 24\left(-\frac{7}{12 \times 0.1512565171560000^{4}}--\frac{7}{720 \times 0.1512565171560000^{6}}\right)- \\
& \\
& -\frac{24\left(\frac{-2.76^{2}}{-60.1512565171560000^{4}}+-\frac{1}{800.1512565171560000^{6}}\right)}{8 i \log (-1)}- \\
& 20
\end{aligned}
$$

$720 i \log (-1) 0.1512565171560000^{6}$

$$
\begin{aligned}
& -\frac{2+2+2 \times 8}{720 \pi(-1) 0.1512565171560000^{6}}- \\
& \frac{24\left(\frac{1}{90(-1) 0.1512565171560000^{6}}-\frac{2.76^{2}}{6(-1) 0.1512565171560000^{4}}\right)}{8 \pi}- \\
& \\
& 24\left(-\frac{7}{720(-1) 0.1512565171560000^{6}}+\frac{2.76^{2}}{12(-1) 0.1512565171560000^{4}}\right)= \\
& - \\
& 24\left(-\frac{7}{12 \times 0.1512565171560000^{4}}--\frac{720 \times 0.1512565171560000^{6}}{}{ }^{6}\right)- \\
& \\
& \\
& \\
& -\frac{24\left(\frac{-2.76^{2}}{-60.1512565171560000^{4}}+-\frac{1}{900.1512565171560000^{6}}\right)}{8 \cos ^{-1}(-1)}- \\
& 720 \cos ^{-1}(-1) 0.1512565171560000^{6}
\end{aligned}
$$

## Series representations:

$$
\begin{aligned}
& -\frac{2+2+2 \times 8}{720 \pi(-1) 0.1512565171560000^{6}}- \\
& \frac{24\left(\frac{1}{90(-1) 0.1512565171560000^{6}}-\frac{2.76^{2}}{6(-1) 0.1512565171560000^{4}}\right)}{8 \pi}- \\
& 24\left(-\frac{7}{720(-1) 0.1512565171560000^{6}}+\frac{2.76^{2}}{12(-1) 0.1512565171560000^{4}}\right)= \\
& 9621.99-\frac{543.384}{\sum_{k=0}^{\infty} \frac{(-1)^{k}}{1+2 k}}
\end{aligned}
$$

$$
\begin{aligned}
& -\frac{2+2+2 \times 8}{720 \pi(-1) 0.1512565171560000^{6}}- \\
& \frac{24\left(\frac{1}{90(-1) 0.1512565171560000^{6}}-\frac{2.76^{2}}{6(-1) 0.1512565171560000^{4}}\right)}{8 \pi}- \\
& 24\left(-\frac{7}{720(-1) 0.1512565171560000^{6}}+\frac{2.76^{2}}{12(-1) 0.1512565171560000^{4}}\right)= \\
& 9621.99-\frac{1086.77}{-1+\sum_{k=1}^{\infty} \frac{2^{k}}{\binom{k}{k}}}
\end{aligned}
$$

$$
\begin{aligned}
& -\frac{2+2+2 \times 8}{720 \pi(-1) 0.1512565171560000^{6}}- \\
& \quad \frac{24\left(\frac{1}{90(-1) 0.1512565171560000^{6}}-\frac{2.76^{2}}{6(-1) 0.1512565171560000^{4}}\right)}{8 \pi}- \\
& 24\left(-\frac{7}{720(-1) 0.1512565171560000^{6}}+\frac{2.76^{2}}{12(-1) 0.1512565171560000^{4}}\right)= \\
& 9621.99-\frac{2173.54}{\sum_{k=0}^{\infty} \frac{2^{-k}(-6+50 k)}{\binom{3 k}{k}}}
\end{aligned}
$$

## Integral representations:

$$
\begin{aligned}
& -\frac{2+2+2 \times 8}{720 \pi(-1) 0.1512565171560000^{6}}- \\
& \quad \frac{24\left(\frac{1}{90(-1) 0.1512565171560000^{6}}-\frac{2.76^{2}}{8 \pi}-1\right) 0.1512565171560000^{4}}{8 \pi}- \\
& 24\left(-\frac{7}{720(-1) 0.1512565171560000^{6}}+\frac{2.76^{2}}{12(-1) 0.1512565171560000^{4}}\right)= \\
& 9621.99-\frac{1086.77}{\int_{0}^{\infty} \frac{1}{1+t^{2}} d t}
\end{aligned}
$$

$$
\begin{aligned}
& -\frac{2+2+2 \times 8}{720 \pi(-1) 0.1512565171560000^{6}}- \\
& \left.\frac{24\left(\frac{1}{90(-1) 0.1512565171560000^{6}}-\frac{2.76^{2}}{8 \pi}-(-1) 0.1512565171560000^{4}\right.}{}\right) \\
& 24\left(-\frac{7}{720(-1) 0.1512565171560000^{6}}+\frac{2.76^{2}}{12(-1) 0.1512565171560000^{4}}\right)= \\
& 9621.99-\frac{543.384}{\int_{0}^{1} \sqrt{1-t^{2}} d t}
\end{aligned}
$$

$$
\begin{aligned}
& -\frac{2+2+2 \times 8}{720 \pi(-1) 0.1512565171560000^{6}}- \\
& \frac{24\left(\frac{1}{90(-1) 0.1512565171560000^{6}}-\frac{2.76^{2}}{8(-1) 0.1512565171560000^{4}}\right)}{8 \pi}- \\
& 24\left(-\frac{7}{720(-1) 0.1512565171560000^{6}}+\frac{2.76^{2}}{12(-1) 0.1512565171560000^{4}}\right)= \\
& 9621.99-\frac{1086.77}{\int_{0}^{\infty} \frac{\sin (t)}{t} d t}
\end{aligned}
$$

From the right-hand side of the below expression

$$
\begin{aligned}
& \min \left\{-\frac{2+2+2 \times 8}{720 \pi x^{6}}-\frac{24\left(\frac{1}{90 x^{6}}-\frac{2.76^{2}}{6 x^{4}}\right)}{8 \pi}-24\left(-\frac{7}{720 x^{6}}+\frac{2.76^{2}}{12 x^{4}}\right)\right\}= \\
& -\frac{2430850583234918645426565565575839210188800(4 \pi-1)^{3}}{9165423189346859202789669808161240667(11-42 \pi)^{2} \pi} \\
& \text { at } x=-\frac{1}{372} \sqrt{\frac{2092750755523(42 \pi-11)}{6911957230(4 \pi-1)}}
\end{aligned}
$$

we obtain:
$-\left(2.43085058 \mathrm{e}+43(4 \pi-1)^{\wedge} 3\right) /\left(9.16542318 \mathrm{e}+37(11-42 \pi)^{\wedge} 2 \pi\right)$

## Input interpretation:

$-\frac{2.43085058 \times 10^{43}(4 \pi-1)^{3}}{9.16542318 \times 10^{37}(11-42 \pi)^{2} \pi}$

## Result:

-8930.1296...
-8930.1296....

From which:

$$
\begin{aligned}
& {\left[-\left(2+2+2^{*} 8\right) /\left(720 \pi^{*}-0.151256517^{\wedge} 6\right)-\left(2 4 \left(1 /\left(90 *_{-}-0.151256517^{\wedge} 6\right)-2.76^{\wedge} 2 /(6\right.\right.\right.} \\
& \left.\left.\left.*-0.151256517^{\wedge} 4\right)\right)\right) /(8 \pi)-24\left(-7 /\left(720 *_{-}-0.151256517^{\wedge} 6\right)+2.76^{\wedge} 2 /\left(12 *_{-}\right.\right. \\
& \left.\left.\left.0.151256517^{\wedge} 4\right)\right)\right]-\left(6^{\wedge} 3+8^{\wedge} 3\right)-10
\end{aligned}
$$

## Input interpretation:

$$
\begin{aligned}
& \left(-\frac{2+2+2 \times 8}{720 \pi \times(-1) \times 0.151256517^{6}}-\frac{24\left(\frac{1}{90 \times(-1) \times 0.151256517^{6}}-\frac{2.76^{2}}{6 \times(-1) \times 0.151256517^{4}}\right)}{8 \pi}-\right. \\
& \left.24\left(-\frac{7}{720 \times(-1) \times 0.151256517^{6}}+\frac{2.76^{2}}{12 \times(-1) \times 0.151256517^{4}}\right)\right)-\left(6^{3}+\right. \\
& \left.8^{3}\right)-10
\end{aligned}
$$

## Result:

8192.13...
8192.13... $\approx 8192$

The total amplitude vanishes for gauge group $\operatorname{SO}(8192)$, while the vacuum energy is negative and independent of the gauge group.

The vacuum energy and dilaton tadpole to lowest non-trivial order for the open bosonic string. While the vacuum energy is non-zero and independent of the gauge group, the dilaton tadpole is zero for a unique choice of gauge group, $\mathrm{SO}\left(2^{13}\right)$ i.e. SO(8192). (From: "Dilaton Tadpole for the Open Bosonic String " Michael R. Douglas and Benjamin Grinstein - September 2,1986)

## Alternative representations:

$$
\begin{aligned}
& \left(-\frac{2+2+2 \times 8}{720 \pi(-1) 0.151257^{6}}-\frac{24\left(\frac{1}{90(-1) 0.151257^{6}}-\frac{2.76^{2}}{6(-1) 0.151257^{4}}\right)}{8 \pi}-\right. \\
& \left.24\left(-\frac{7}{720(-1) 0.151257^{6}}+\frac{2.76^{2}}{12(-1) 0.151257^{4}}\right)\right)-\left(6^{3}+8^{3}\right)-10= \\
& -10-6^{3}-8^{3}-24\left(-\frac{2.76^{2}}{12 \times 0.151257^{4}}--\frac{7}{720 \times 0.151257^{6}}\right)- \\
& \frac{24\left(\frac{-2.76^{2}}{-60.151257^{4}}+-\frac{1}{900.151257^{6}}\right)}{1440^{\circ}}--\frac{20}{129600^{\circ} 0.151257^{6}}
\end{aligned}
$$

$$
\begin{aligned}
& \left(-\frac{2+2+2 \times 8}{720 \pi(-1) 0.151257^{6}}-\frac{24\left(\frac{1}{90(-1) 0.151257^{6}}-\frac{2.76^{2}}{6(-1) 0.151257^{4}}\right)}{8 \pi}-\right. \\
& \left.24\left(-\frac{7}{720(-1) 0.151257^{6}}+\frac{2.76^{2}}{12(-1) 0.151257^{4}}\right)\right)-\left(6^{3}+8^{3}\right)-10= \\
& -10-6^{3}-8^{3}-24\left(-\frac{2.76^{2}}{12 \times 0.151257^{4}}--\frac{7}{720 \times 0.151257^{6}}\right)- \\
& -\frac{24\left(\frac{-2.76^{2}}{-60.151257^{4}}+-\frac{1}{900.151257^{6}}\right)}{8 i \log (-1)}-\frac{20}{720 i \log (-1) 0.151257^{6}}
\end{aligned}
$$

$$
\begin{aligned}
& \left(-\frac{2+2+2 \times 8}{720 \pi(-1) 0.151257^{6}}-\frac{24\left(\frac{1}{90(-1) 0.151257^{6}}-\frac{2.76^{2}}{6(-1) 0.151257^{4}}\right)}{8 \pi}-\right. \\
& \left.24\left(-\frac{7}{720(-1) 0.151257^{6}}+\frac{2.76^{2}}{12(-1) 0.151257^{4}}\right)\right)-\left(6^{3}+8^{3}\right)-10= \\
& -10-6^{3}-8^{3}-24\left(-\frac{2.76^{2}}{12 \times 0.151257^{4}}--\frac{7}{720 \times 0.151257^{6}}\right)- \\
& \frac{24\left(\frac{-2.76^{2}}{-60.151257^{4}}+-\frac{1}{900.151257^{6}}\right)}{8 \cos ^{-1}(-1)}--\frac{20}{720 \cos ^{-1}(-1) 0.151257^{6}}
\end{aligned}
$$

## Series representations:

$$
\begin{gathered}
\left(-\frac{2+2+2 \times 8}{720 \pi(-1) 0.151257^{6}}-\frac{24\left(\frac{1}{90(-1) 0.151257^{6}}-\frac{2.76^{2}}{6(-1) 0.151257^{4}}\right)}{8 \pi}-\right. \\
\left.24\left(-\frac{7}{720(-1) 0.151257^{6}}+\frac{2.76^{2}}{12(-1) 0.151257^{4}}\right)\right)- \\
\left(6^{3}+8^{3}\right)-10=8883.99-\frac{543.384}{\sum_{k=0}^{\infty} \frac{(-1)^{k}}{1+2 k}}
\end{gathered}
$$

$$
\begin{gathered}
\left(-\frac{2+2+2 \times 8}{720 \pi(-1) 0.151257^{6}}-\frac{24\left(\frac{1}{90(-1) 0.151257^{6}}-\frac{2.76^{2}}{6(-1) 0.151257^{4}}\right)}{8 \pi}-\right. \\
\left.24\left(-\frac{7}{720(-1) 0.151257^{6}}+\frac{2.76^{2}}{12(-1) 0.151257^{4}}\right)\right)- \\
\left(6^{3}+8^{3}\right)-10=8883.99-\frac{1086.77}{-1+\sum_{k=1}^{\infty} \frac{2^{k}}{\binom{2 k}{k}}}
\end{gathered}
$$

$$
\begin{gathered}
\left(-\frac{2+2+2 \times 8}{720 \pi(-1) 0.151257^{6}}-\frac{24\left(\frac{1}{90(-1) 0.151257^{6}}-\frac{2.76^{2}}{6(-1) 0.151257^{4}}\right)}{8 \pi}-\right. \\
\left.24\left(-\frac{7}{720(-1) 0.151257^{6}}+\frac{2.76^{2}}{12(-1) 0.151257^{4}}\right)\right)- \\
\left(6^{3}+8^{3}\right)-10=8883.99-\frac{2173.54}{\sum_{k=0}^{\infty} \frac{2^{-k}(-6+50 k)}{\binom{3 k}{k}}}
\end{gathered}
$$

## Integral representations:

$$
\begin{aligned}
& \left(-\frac{2+2+2 \times 8}{720 \pi(-1) 0.151257^{6}}-\frac{24\left(\frac{1}{90(-1) 0.151257^{6}}-\frac{2.76^{2}}{6(-1) 0.151257^{4}}\right)}{8 \pi}-\right. \\
& \left.24\left(-\frac{7}{720(-1) 0.151257^{6}}+\frac{2.76^{2}}{12(-1) 0.151257^{4}}\right)\right)- \\
& \left(6^{3}+8^{3}\right)-10=8883.99-\frac{1086.77}{\int_{0}^{\infty} \frac{1}{1+t^{2}} d t}
\end{aligned}
$$

$$
\begin{gathered}
\left(-\frac{2+2+2 \times 8}{720 \pi(-1) 0.151257^{6}}-\frac{24\left(\frac{1}{90(-1) 0.151257^{6}}-\frac{2.76^{2}}{6(-1) 0.151257^{4}}\right)}{8 \pi}-\right. \\
\left.24\left(-\frac{7}{720(-1) 0.151257^{6}}+\frac{2.76^{2}}{12(-1) 0.151257^{4}}\right)\right)- \\
\left(6^{3}+8^{3}\right)-10=8883.99-\frac{543.384}{\int_{0}^{1} \sqrt{1-t^{2}} d t}
\end{gathered}
$$

$$
\begin{aligned}
& \left(-\frac{2+2+2 \times 8}{720 \pi(-1) 0.151257^{6}}-\frac{24\left(\frac{1}{90(-1) 0.151257^{6}}-\frac{2.76^{2}}{6(-1) 0.151257^{4}}\right)}{8 \pi}-\right. \\
& \left.24\left(-\frac{7}{720(-1) 0.151257^{6}}+\frac{2.76^{2}}{12(-1) 0.151257^{4}}\right)\right)- \\
& \left(6^{3}+8^{3}\right)-10=8883.99-\frac{1086.77}{\int_{0}^{\infty} \frac{\sin (t)}{t} d t}
\end{aligned}
$$

From the indefinite integral:

$$
\begin{aligned}
& \int\left(-\frac{3\left(\frac{1}{90 x^{6}}-\frac{1.2696}{x^{4}}\right)}{\pi}-24\left(-\frac{7}{720 x^{6}}+\frac{0.6348}{x^{4}}\right)-\frac{1}{36 \pi x^{6}}\right) d x= \\
& \frac{4.67427 x^{2}-0.0427762}{x^{5}}+\text { constant }
\end{aligned}
$$

integral(-(3 $\left.\left(1 /\left(90 x^{\wedge} 6\right)-1.2696 / x^{\wedge} 4\right)\right) / \pi-24\left(-7 /\left(720 x^{\wedge} 6\right)+0.6348 / x^{\wedge} 4\right)-1 /(36 \pi$ $\left.x^{\wedge} 6\right)$ ) dx

## Indefinite integral:

$$
\begin{aligned}
& \int\left(-\frac{3\left(\frac{1}{90 x^{6}}-\frac{1.2696}{x^{4}}\right)}{\pi}-24\left(-\frac{7}{720 x^{6}}+\frac{0.6348}{x^{4}}\right)-\frac{1}{36 \pi x^{6}}\right) d x= \\
& \frac{4.67427 x^{2}-0.0427762}{x^{5}}+\text { constant }
\end{aligned}
$$

## Plots of the integral:




## Alternate forms of the integral:

$\frac{4.67427(x-0.095663)(x+0.095663)}{x^{5}}+$ constant
$-\frac{0.0427762-4.67427 x^{2}}{x^{5}}+$ constant

## Partial fraction expansion:

```
4.67427}\mp@subsup{x}{}{3}-\frac{0.0427762}{\mp@subsup{x}{}{5}}+\mathrm{ constant
```

Alternate form assuming x is real:
$-\frac{0.0427762}{x^{5}}+\frac{4.67427}{x^{3}}+0+$ constant

For $\mathrm{x}=0.2$ :
$\left(\left(\left(4.67427 * 0.2^{\wedge} 2-0.0427762\right) / 0.2^{\wedge} 5\right)\right)+\left(\left(\left(4.67427 * 1.2^{\wedge} 2-0.0427762\right) / 1.2^{\wedge} 5\right)\right)$

## Input interpretation:

$\frac{4.67427 \times 0.2^{2}-0.0427762}{0.2^{5}}+\frac{4.67427 \times 1.2^{2}-0.0427762}{1.2^{5}}$

Result:
453.29595156571502057613168724279835390946502057613168724279835390
453.2959515....

From which:
$1 / 7\left(\left(\left(\left(\left(4.67427 * 0.2^{\wedge} 2-0.0427762\right) / 0.2^{\wedge} 5\right)\right)+\left(\left(\left(4.67427 * 1.2^{\wedge} 2-\right.\right.\right.\right.\right.$
$\left.\left.\left.\left.0.0427762) / 1.2^{\wedge} 5\right)\right)-5-((((\sqrt{ }(10-2 \sqrt{ } 5)-2))((\sqrt{ } 5-1))))\right)\right)$
Where $(((\sqrt{ }(10-2 \sqrt{ } 5)-2))(((\sqrt{ } 5-1)))=\kappa$

## Input interpretation:

$$
\begin{aligned}
& \frac{1}{7}\left(\frac{4.67427 \times 0.2^{2}-0.0427762}{0.2^{5}}+\right. \\
& \left.\quad \frac{4.67427 \times 1.2^{2}-0.0427762}{1.2^{5}}-5-\frac{\sqrt{10-2 \sqrt{5}}-2}{\sqrt{5}-1}\right)
\end{aligned}
$$

## Result:

64.0017...
$64.0017 \ldots . \approx 64=8^{2}$
$27 * 1 / 7\left(\left(\left(() 4.67427 * 0.2^{\wedge} 2-0.0427762\right) / 0.2^{\wedge} 5\right)\right)+(((4.67427 * 1.2 \wedge 2-$
$\left.\left.\left.\left.0.0427762) / 1.2^{\wedge} 5\right)\right)-5-((((\sqrt{ }(10-2 \sqrt{ } 5)-2))((\sqrt{ } 5-1))))\right)\right)+1$

## Input interpretation:

$27 \times \frac{1}{7}\left(\frac{4.67427 \times 0.2^{2}-0.0427762}{0.2^{5}}+\right.$

$$
\left.\frac{4.67427 \times 1.2^{2}-0.0427762}{1.2^{5}}-5-\frac{\sqrt{10-2 \sqrt{5}}-2}{\sqrt{5}-1}\right)+1
$$

## Result:

1729.05...
1729.05....

This result is very near to the mass of candidate glueball $\mathbf{f}_{\mathbf{0}}(\mathbf{1 7 1 0})$ scalar meson. Furthermore, 1728 occurs in the algebraic formula for the $j$-invariant of an elliptic curve. $\left(1728=8^{2} * 3^{3}\right)$ The number 1728 is one less than the Hardy-Ramanujan number 1729 (taxicab number)
$\left[27 * 1 / 7\left(\left(\left(\left(\left(4.67427 * 0.2^{\wedge} 2-0.0427762\right) / 0.2^{\wedge} 5\right)\right)+\left(\left(\left(4.67427 * 1.2^{\wedge} 2-\right.\right.\right.\right.\right.\right.$ $\left.\left.\left.\left.\left.0.0427762) / 1.2^{\wedge} 5\right)\right)-5-((((\sqrt{ }(10-2 \sqrt{ } 5)-2))((\sqrt{ } 5-1))))\right)\right)+1\right]^{\wedge} 1 / 15$

## Input interpretation:

$$
\begin{aligned}
& \left(27 \times \frac{1}{7}\left(\frac{4.67427 \times 0.2^{2}-0.0427762}{0.2^{5}}+\right.\right. \\
& \left.\left.\quad \frac{4.67427 \times 1.2^{2}-0.0427762}{1.2^{5}}-5-\frac{\sqrt{10-2 \sqrt{5}}-2}{\sqrt{5}-1}\right)+1\right) \wedge(1 / 15)
\end{aligned}
$$

## Result:

1.64382...
$1.64382 \ldots \approx \zeta(2)=\frac{\pi^{2}}{6}=1.644934 \ldots$

From the derivative:

$$
\begin{aligned}
& \frac{d}{d x}\left(-\frac{2+2+2 \times 8}{720 \pi x^{6}}-\frac{24\left(\frac{1}{90 x^{6}}-\frac{2.76^{2}}{6 x^{4}}\right)}{8 \pi}-24\left(-\frac{7}{720 x^{6}}+\frac{2.76^{2}}{12 x^{4}}\right)\right)= \\
& \frac{56.0913 x^{2}-1.28329}{r^{7}}
\end{aligned}
$$

$$
\mathrm{d} / \mathrm{dx}\left(-(2+2+2 \times 8) /\left(720 \pi x^{\wedge} 6\right)-\left(24\left(1 /\left(90 x^{\wedge} 6\right)-2.76^{\wedge} 2 /\left(6 x^{\wedge} 4\right)\right)\right) /(8 \pi)-24(-\right.
$$

$$
\left.\left.7 /\left(720 x^{\wedge} 6\right)+2.76^{\wedge} 2 /\left(12 x^{\wedge} 4\right)\right)\right)=\left(56.0913 x^{\wedge} 2-1.28329\right) / x^{\wedge} 7
$$

## Input interpretation:

$$
\frac{\partial}{\partial x}\left(-\frac{2+2+2 \times 8}{720 \pi x^{6}}-\frac{24\left(\frac{1}{90 x^{6}}-\frac{2.76^{2}}{6 x^{4}}\right)}{8 \pi}-24\left(-\frac{7}{720 x^{6}}+\frac{2.76^{2}}{12 x^{4}}\right)\right)=
$$

$$
\frac{56.0913 x^{2}-1.28329}{x^{7}}
$$

## Result:

$\frac{56.0913 x^{2}-1.28329}{x^{7}}=\frac{56.0913 x^{2}-1.28329}{x^{7}}$

## Plot:



Alternate forms assuming $\mathbf{x}$ is real:
$x=\frac{0.245293}{x}$
$-\frac{1.28329}{x^{7}}+\frac{56.0913}{x^{5}}+0=-\frac{1.28329}{x^{7}}+\frac{56.0913}{x^{5}}+0$

## Alternate form:

$\frac{56.0913(x-0.151257)(x+0.151257)}{x^{7}}=\frac{56.0913(x-0.151257)(x+0.151257)}{x^{7}}$

## Alternate form assuming $\mathbf{x}$ is positive:

$x=0.49527$

## Expanded form:

$$
\frac{56.0913}{x^{5}}-\frac{1.28329}{x^{7}}=\frac{56.0913}{x^{5}}-\frac{1.28329}{x^{7}}
$$

## Solutions:

$x \approx-0.49527$
$x \approx 0.49527$
0.49527
$\left(56.0913 * 0.49527^{\wedge} 2-1.28329\right) / 0.49527 \wedge 7$

## Input interpretation:

$\frac{56.0913 \times 0.49527^{2}-1.28329}{0.49527^{7}}$

## Result:

1706.7230017390806306583878395920601480117369054483945688914018818
1706.723001739....

From which:
$\left(56.0913 * 0.49527^{\wedge} 2-1.28329\right) / 0.49527^{\wedge} 7+21+$ golden ratio $-((((\sqrt{ }(10-2 \sqrt{ } 5)-$ 2) $y((\sqrt{ } 5-1))))$

## Input interpretation:

$\frac{56.0913 \times 0.49527^{2}-1.28329}{0.49527^{7}}+21+\phi-\frac{\sqrt{10-2 \sqrt{5}}-2}{\sqrt{5}-1}$

## Result:

1729.06...
1729.06....

This result is very near to the mass of candidate glueball $\mathbf{f}_{\mathbf{0}}(\mathbf{1 7 1 0})$ scalar meson. Furthermore, 1728 occurs in the algebraic formula for the $j$-invariant of an elliptic curve. $\left(1728=8^{2} * 3^{3}\right.$ ) The number 1728 is one less than the Hardy-Ramanujan number 1729 (taxicab number)

## Series representations:

$$
\begin{aligned}
& \frac{56.0913 \times 0.49527^{2}-1.28329}{0.49527^{7}}+21+\phi-\frac{\sqrt{10-2 \sqrt{5}}-2}{\sqrt{5}-1}= \\
& \left(-1725.72-\phi+1727.72 \sqrt{4} \sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k}+\phi \sqrt{4} \sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k}-\right. \\
& \left.\sqrt{9-2 \sqrt{5}} \sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}(9-2 \sqrt{5})^{-k}\right) /\left(-1+\sqrt{4} \sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k}\right)
\end{aligned}
$$

$$
\begin{aligned}
& \frac{56.0913 \times 0.49527^{2}-1.28329}{0.49527^{7}}+21+\phi-\frac{\sqrt{10-2 \sqrt{5}}-2}{\sqrt{5}-1}= \\
& \left(-1725.72-\phi+1727.72 \sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}+\phi \sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}-\right. \\
& \left.\sqrt{9-2 \sqrt{5}} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}(9-2 \sqrt{5})^{-k}}{k!}\right) /\left(-1+\sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}\right)
\end{aligned}
$$

$$
\begin{aligned}
& \frac{56.0913 \times 0.49527^{2}-1.28329}{0.49527^{7}}+21+\phi-\frac{\sqrt{10-2 \sqrt{5}}-2}{\sqrt{5}-1}= \\
& \left(-1725.72-\phi+1727.72 \sqrt{z_{0}} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(5-z_{0}\right)^{k} z_{0}^{-k}}{k!}+\right. \\
& \phi \sqrt{z_{0}} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(5-z_{0}\right)^{k} z_{0}^{-k}}{k!}- \\
& \left.\sqrt{z_{0}} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(10-2 \sqrt{5}-z_{0}\right)^{k} z_{0}^{-k}}{k!}\right) / \\
& \left(-1+\sqrt{z_{0}} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(5-z_{0}\right)^{k} z_{0}^{-k}}{k!}\right) \\
& \text { for }\left(\operatorname{not}\left(z_{0} \in \mathbb{R} \text { and }-\infty<z_{0} \leq 0\right)\right)
\end{aligned}
$$

[(56.0913*0.49527^2-1.28329)/0.49527^7 + 21+golden ratio - ((( ( $\sqrt{ }(10-2 \sqrt{ } 5)-$ 2) $)(((\sqrt{5}-1))))]^{\wedge} 1 / 15$

## Input interpretation:

$\sqrt[15]{\frac{56.0913 \times 0.49527^{2}-1.28329}{0.49527^{7}}+21+\phi-\frac{\sqrt{10-2 \sqrt{5}}-2}{\sqrt{5}-1}}$

## Result:

1.64382...
$1.64382 \ldots \approx \zeta(2)=\frac{\pi^{2}}{6}=1.644934 \ldots$

Performing the double integral:
$\left[-\left(24\left(-2.76^{\wedge} 2 /\left(6 x^{\wedge} 4\right)+1 /\left(90 x^{\wedge} 6\right)\right)\right) /\left(8 \cos ^{\wedge}(-1)(-1)\right)-24\left(2.76^{\wedge} 2 /\left(12 x^{\wedge} 4\right)-7 /(720\right.\right.$ $\left.\left.\left.x^{\wedge} 6\right)\right)-20 /\left(720 \cos ^{\wedge}(-1)(-1) x^{\wedge} 6\right)\right] d x d y$

## Input:

$$
\iint\left(-\frac{24\left(-\frac{2.76^{2}}{6 x^{4}}+\frac{1}{90 x^{6}}\right)}{8 \cos ^{-1}(-1)}-24\left(\frac{2.76^{2}}{12 x^{4}}-\frac{7}{720 x^{6}}\right)-\frac{20}{720 \cos ^{-1}(-1) x^{6}}\right) d x d y
$$

$\cos ^{-1}(x)$ is the inverse cosine function

## Result:

$\frac{\left(4.67427 x^{2}-0.0427762\right) y}{x^{5}}$

## 3D plot:



## Contour plot:



Indefinite integral assuming all variables are real:
$\left(\frac{0.0106941}{x^{4}}-\frac{2.33714}{x^{2}}\right) y+$ constant

From:
$\frac{\left(4.67427 x^{2}-0.0427762\right) y}{x^{5}}$
For $x=0.2$ and $y=0.5$ :
$\left(\left(\left(1 / 4\left[\left(\left(4.67427 * 0.2^{\wedge} 2-0.0427762\right) 0.5\right) / 0.2^{\wedge} 5+34-3-((((\sqrt{ }(10-2 \sqrt{ } 5)-2)))((\sqrt{ } 5-\right.\right.\right.\right.$ $1))))])))^{\wedge} 2-1 /$ golden ratio

## Input interpretation:

$$
\left(\frac{1}{4}\left(\frac{\left(4.67427 \times 0.2^{2}-0.0427762\right) \times 0.5}{0.2^{5}}+34-3-\frac{\sqrt{10-2 \sqrt{5}}-2}{\sqrt{5}-1}\right)\right)^{2}-\frac{1}{\phi}
$$

## Result:

4096.02...
$4096.02 \ldots \approx 4096=64^{2}$

Series representations:

$$
\begin{aligned}
& \left(\frac{1}{4}\left(\frac{\left(4.67427 \times 0.2^{2}-0.0427762\right) 0.5}{0.2^{5}}+34-3-\frac{\sqrt{10-2 \sqrt{5}}-2}{\sqrt{5}-1}\right)\right)^{2}-\frac{1}{\phi}= \\
& -\frac{1}{\phi}+\frac{1}{16}\left(256.304+\frac{2-\sqrt{9-2 \sqrt{5}} \sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}(9-2 \sqrt{5})^{-k}}{-1+\sqrt{4} \sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k}}\right)^{2}
\end{aligned}
$$

$$
\begin{aligned}
& \left(\frac{1}{4}\left(\frac{\left(4.67427 \times 0.2^{2}-0.0427762\right) 0.5}{0.2^{5}}+34-3-\frac{\sqrt{10-2 \sqrt{5}}-2}{\sqrt{5}-1}\right)\right)^{2}-\frac{1}{\phi}= \\
& -\frac{1}{\phi}+\frac{1}{16}\left(256.304-\frac{-2+\sqrt{9-2 \sqrt{5}} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}(9-2 \sqrt{5})^{-k}}{k!}}{\left.-1+\sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}\right)^{2}}\right.
\end{aligned}
$$

$$
\begin{aligned}
& \left(\frac{1}{4}\left(\frac{\left(4.67427 \times 0.2^{2}-0.0427762\right) 0.5}{0.2^{5}}+34-3-\frac{\sqrt{10-2 \sqrt{5}}-2}{\sqrt{5}-1}\right)\right)^{2}-\frac{1}{\phi}= \\
& \quad-\frac{1}{\phi}+\frac{1}{16}\left(256.304-\frac{-2+\sqrt{z_{0}} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(10-2 \sqrt{5}-z_{0}\right)^{k} z_{0}^{-k}}{k!}}{-1+\sqrt{z_{0}} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(5-z_{0}\right)^{k} z_{0}^{-k}}{k!}}\right)^{2}
\end{aligned}
$$

for $\left(\operatorname{not}\left(z_{0} \in \mathbb{R}\right.\right.$ and $\left.-\infty<z_{0} \leq 0\right)$ )
$\left[-\left(24\left(-2.76^{\wedge} 2 /\left(6 x^{\wedge} 4\right)+1 /\left(90 x^{\wedge} 6\right)\right)\right) /\left(8 \cos ^{\wedge}(-1)(-1)\right)-24\left(2.76^{\wedge} 2 /\left(12 x^{\wedge} 4\right)-7 /(720\right.\right.$ $\left.\left.\left.x^{\wedge} 6\right)\right)-20 /\left(720 \cos ^{\wedge}(-1)(-1) x^{\wedge} 6\right)\right](((\sqrt{ }(10-2 \sqrt{ } 5)-2))((\sqrt{ } 5-1))) d x d y$

## Input:

$$
\begin{aligned}
& \iint\left(-\frac{24\left(-\frac{2.76^{2}}{6 x^{4}}+\frac{1}{90 x^{6}}\right)}{8 \cos ^{-1}(-1)}-24\left(\frac{2.76^{2}}{12 x^{4}}-\frac{7}{720 x^{6}}\right)-\frac{20}{720 \cos ^{-1}(-1) x^{6}}\right) \times \\
& \frac{\sqrt{10-2 \sqrt{5}}-2}{\sqrt{5}-1} d x d y
\end{aligned}
$$

$\cos ^{-1}(x)$ is the inverse cosine function

## Result:

$\frac{\left(1.32786 x^{2}-0.0121518\right) y}{x^{5}}$

## 3D plot:



## Contour plot:



Indefinite integral assuming all variables are real:
$\left(\frac{0.00303796}{x^{4}}-\frac{0.663932}{x^{2}}\right) y+$ constant

For $x=0.2$ and $y=0.5$ :
$\left(\left(-0.0121518+1.327860 .2^{\wedge} 2\right) 0.5\right) / 0.2^{\wedge} 5$

## Input interpretation:

$\frac{\left(-0.0121518+1.32786 \times 0.2^{2}\right) \times 0.5}{0.2^{5}}$

## Result:

64.0040625
$64.0040625 \approx 64=8^{2}$

From which:
$\left[\left(\left(-0.0121518+1.327860 .2^{\wedge} 2\right) 0.5\right) / 0.2^{\wedge} 5\right]^{\wedge} 2-1 / 2$

## Input interpretation:

$\left(\frac{\left(-0.0121518+1.32786 \times 0.2^{2}\right) \times 0.5}{0.2^{5}}\right)^{2}-\frac{1}{2}$

## Result:

4096.02001650390625
$4096.02001650390625 \approx 4096=64^{2}$
$27^{*}\left(\left(-0.0121518+1.327860 .2^{\wedge} 2\right) 0.5\right) / 0.2^{\wedge} 5+1$

## Input interpretation:

$27 \times \frac{\left(-0.0121518+1.32786 \times 0.2^{2}\right) \times 0.5}{0.2^{5}}+1$

## Result:

1729.1096875
1729.1096875

This result is very near to the mass of candidate glueball $\mathbf{f}_{\mathbf{0}}(\mathbf{1 7 1 0})$ scalar meson. Furthermore, 1728 occurs in the algebraic formula for the $j$-invariant of an elliptic curve. $\left(1728=8^{2} * 3^{3}\right.$ ) The number 1728 is one less than the Hardy-Ramanujan number 1729 (taxicab number)
$\left[27^{*}\left(\left(-0.0121518+1.327860 .2^{\wedge} 2\right) 0.5\right) / 0.2^{\wedge} 5+1\right]^{\wedge} 1 / 15$

## Input interpretation:

$\sqrt[15]{27 \times \frac{\left(-0.0121518+1.32786 \times 0.2^{2}\right) \times 0.5}{0.2^{5}}+1}$

## Result:

1.64382...
$1.64382 \ldots \approx \zeta(2)=\frac{\pi^{2}}{6}=1.644934 \ldots$

## Observations

We note that, from the number 8 , we obtain as follows:
$8^{2}$
64
$8^{2} \times 2 \times 8$
1024
$8^{4}=8^{2} \times 2^{6}$
True
$8^{4}=4096$
$8^{2} \times 2^{6}=4096$
$2^{13}=2 \times 8^{4}$
True
$2^{13}=8192$
$2 \times 8^{4}=8192$

We notice how from the numbers 8 and 2 we get 64, 1024, 4096 and 8192 , and that 8 is the fundamental number. In fact $8^{2}=64,8^{3}=512,8^{4}=4096$. We define it "fundamental number", since 8 is a Fibonacci number, which by rule, divided by the previous one, which is 5 , gives 1.6 , a value that tends to the golden ratio, as for all numbers in the Fibonacci sequence


Finally we note how $8^{2}=64$, multiplied by 27 , to which we add 1 , is equal to 1729 , the so-called "Hardy-Ramanujan number". Then taking the 15th root of 1729 , we obtain a value close to $\zeta(2)$ that $1.6438 \ldots$, which, in turn, is included in the range of what we call "golden numbers"

Furthermore for all the results very near to 1728 or 1729 , adding $64=8^{2}$, one obtain values about equal to 1792 or 1793 . These are values almost equal to the Planck multipole spectrum frequency 1792.35 and to the hypothetical Gluino mass

## Mathematical connections with some sectors of String Theory

From:
Modular equations and approximations to $\boldsymbol{\pi}$ - Srinivasa Ramanujan
Quarterly Journal of Mathematics, XLV, 1914, 350-372
We have that:

Hence

$$
\begin{array}{rrr}
64 g_{22}^{24} & = & e^{\pi \sqrt{22}}-24+276 e^{-\pi \sqrt{22}}-\cdots \\
64 g_{22}^{-24} & = & 4096 e^{-\pi \sqrt{22}}+\cdots
\end{array}
$$

so that

$$
64\left(g_{22}^{24}+g_{22}^{-24}\right)=e^{\pi \sqrt{22}}-24+4372 e^{-\pi \sqrt{22}}+\cdots=64\left\{(1+\sqrt{2})^{12}+(1-\sqrt{2})^{12}\right\}
$$

Hence

$$
e^{\pi \sqrt{22}}=2508951.9982 \ldots
$$

Again

$$
\begin{array}{cc}
G_{37}=(6+\sqrt{37})^{\frac{1}{4}} \\
64 G_{37}^{24}= & e^{\pi \sqrt{37}}+24+276 e^{-\pi \sqrt{37}}+\cdots, \\
64 G_{37}^{-24}= & 4096 e^{-\pi \sqrt{37}}-\cdots,
\end{array}
$$

so that

$$
64\left(G_{37}^{24}+G_{37}^{-24}\right)=e^{\pi \sqrt{37}}+24+4372 e^{-\pi \sqrt{37}}-\cdots=64\left\{(6+\sqrt{37})^{6}+(6-\sqrt{37})^{6}\right\}
$$

Hence

$$
e^{\pi \sqrt{37}}=199148647.999978 \ldots
$$

Similarly, from

$$
g_{58}=\sqrt{\left(\frac{5+\sqrt{29}}{2}\right)}
$$

we obtain

$$
64\left(g_{58}^{24}+g_{58}^{-24}\right)=e^{\pi \sqrt{58}}-24+4372 e^{-\pi \sqrt{58}}+\cdots=64\left\{\left(\frac{5+\sqrt{29}}{2}\right)^{12}+\left(\frac{5-\sqrt{29}}{2}\right)^{12}\right\}
$$

Hence

$$
e^{\pi \sqrt{58}}=24591257751.99999982 \ldots
$$

## From:

## An Update on Brane Supersymmetry Breaking

J. Mourad and A. Sagnotti - arXiv:1711.11494v1 [hep-th] 30 Nov 2017

From the following vacuum equations:

$$
\begin{aligned}
T e^{\gamma_{E} \phi} & =-\frac{\beta_{E}^{(p)} h^{2}}{\gamma_{E}} e^{-2(8-p) C+2 \beta_{E}^{(p)} \phi} \\
16 k^{\prime} e^{-2 C} & =\frac{h^{2}\left(p+1-\frac{2 \beta_{E}^{(p)}}{\gamma_{E}}\right) e^{-2(8-p) C+2 \beta_{E}^{(p)} \phi}}{(7-p)} \\
\left(A^{\prime}\right)^{2} & =k e^{-2 A}+\frac{h^{2}}{16(p+1)}\left(7-p+\frac{2 \beta_{E}^{(p)}}{\gamma_{E}}\right) e^{-2(8-p) C+2 \beta_{E}^{(p)} \phi}
\end{aligned}
$$

we have obtained, from the results almost equals of the equations, putting
$4096 e^{-\pi \sqrt{18}}$ instead of

$$
e^{-2(8-p) C+2 \beta_{E}^{(p)} \phi}
$$

a new possible mathematical connection between the two exponentials. Thence, also the values concerning $p, C, \beta_{E}$ and $\phi$ correspond to the exponents of $e$ (i.e. of exp). Thence we obtain for $\mathrm{p}=5$ and $\beta_{E}=1 / 2$ :

$$
e^{-6 C+\phi}=4096 e^{-\pi \sqrt{18}}
$$

Therefore, with respect to the exponentials of the vacuum equations, the Ramanujan's exponential has a coefficient of 4096 which is equal to 642 , while $-6 \mathrm{C}+\phi$ is equal to $\pi \sqrt{18}$. From this it follows that it is possible to establish mathematically, the dilaton value.

For
$\exp \left(\left(-\mathrm{Pi}^{*} \operatorname{sqrt}(18)\right)\right.$ we obtain:

## Input:

$\exp (-\pi \sqrt{18})$

## Exact result:

$e^{-3 \sqrt{2} \pi}$

## Decimal approximation:

$1.6272016226072509292942156739117979541838581136954016 \ldots \times 10^{-6}$
$1.6272016 \ldots * 10^{-6}$

## Property:

$e^{-3 \sqrt{2} \pi}$ is a transcendental number

Series representations:
$e^{-\pi \sqrt{18}}=e^{-\pi \sqrt{17} \sum_{k=0}^{\infty} 17^{-k}\binom{1 / 2}{k}}$
$e^{-\pi \sqrt{18}}=\exp \left(-\pi \sqrt{17} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{17}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}\right)$
$e^{-\pi \sqrt{18}}=\exp \left(-\frac{\pi \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 17^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{2 \sqrt{\pi}}\right)$

Now, we have the following calculations:

$$
\begin{gathered}
e^{-6 C+\phi}=4096 e^{-\pi \sqrt{18}} \\
e^{-\pi \sqrt{18}}=1.6272016 \ldots * 10^{\wedge}-6
\end{gathered}
$$

from which:

$$
\begin{gathered}
\frac{1}{4096} e^{-6 C+\phi}=1.6272016 \ldots * 10^{\wedge}-6 \\
0.000244140625 e^{-6 C+\phi}=e^{-\pi \sqrt{18}}=1.6272016 \ldots * 10^{\wedge}-6
\end{gathered}
$$

Now:

$$
\ln \left(e^{-\pi \sqrt{18}}\right)=-13.328648814475=-\pi \sqrt{18}
$$

And:
$\left(1.6272016 * 10^{\wedge}-6\right) * 1 /(0.000244140625)$

## Input interpretation:

$\frac{1.6272016}{10^{6}} \times \frac{1}{0.000244140625}$

## Result:

0.0066650177536
0.006665017...

Thence:

$$
0.000244140625 e^{-6 C+\phi}=e^{-\pi \sqrt{18}}
$$

Dividing both sides by 0.000244140625 , we obtain:

$$
\begin{aligned}
& \frac{0.000244140625}{0.000244140625} e^{-6 C+\phi}=\frac{1}{0.000244140625} e^{-\pi \sqrt{18}} \\
& e^{-6 C+\phi}=0.0066650177536
\end{aligned}
$$

$\left(\left(\left(\left(\exp \left(\left(-\mathrm{Pi}^{*} \operatorname{sqrt}(18)\right)\right)\right)\right)\right)\right) * 1 / 0.000244140625$

## Input interpretation:

$\exp (-\pi \sqrt{18}) \times \frac{1}{0.000244140625}$

## Result:

$0.00666501785 \ldots$
$0.00666501785 \ldots$

## Series representations:

$$
\begin{aligned}
& \frac{\exp (-\pi \sqrt{18})}{0.000244141}=4096 \exp \left(-\pi \sqrt{17} \sum_{k=0}^{\infty} 17^{-k}\binom{\frac{1}{2}}{k}\right) \\
& \frac{\exp (-\pi \sqrt{18})}{0.000244141}=4096 \exp \left(-\pi \sqrt{17} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{17}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}\right) \\
& \frac{\exp (-\pi \sqrt{18})}{0.000244141}=4096 \exp \left(-\frac{\pi \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 17^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{2 \sqrt{\pi}}\right)
\end{aligned}
$$

Now:

$$
\begin{aligned}
& e^{-6 C+\phi}=0.0066650177536 \\
& \exp (-\pi \sqrt{18}) \times \frac{1}{0.000244140625}= \\
& e^{-\pi \sqrt{18}} \times \frac{1}{0.000244140625} \\
& =0.00666501785 \ldots
\end{aligned}
$$

From:
$\ln (0.00666501784619)$

## Input interpretation:

$\log (0.00666501784619)$

## Result:

-5.010882647757...
$-5.010882647757 .$.

## Alternative representations:

$\log (0.006665017846190000)=\log _{e}(0.006665017846190000)$
$\log (0.006665017846190000)=\log (a) \log _{a}(0.006665017846190000)$
$\log (0.006665017846190000)=-\operatorname{Li}_{1}(0.993334982153810000)$

## Series representations:

$$
\begin{aligned}
& \log (0.006665017846190000)=-\sum_{k=1}^{\infty} \frac{(-1)^{k}(-0.993334982153810000)^{k}}{k} \\
& \left.\log (0.006665017846190000)=2 i \pi \left\lvert\, \frac{\arg (0.006665017846190000-x)}{2 \pi}\right.\right\rfloor+ \\
& \quad \log (x)-\sum_{k=1}^{\infty} \frac{(-1)^{k}(0.006665017846190000-x)^{k} x^{-k}}{k} \text { for } x<0 \\
& \left.\log (0.006665017846190000)=\left\lvert\, \frac{\arg \left(0.006665017846190000-z_{0}\right)}{2 \pi}\right.\right\rfloor \log \left(\frac{1}{z_{0}}\right)+ \\
& \quad \log \left(z_{0}\right)+\left\lfloor\frac{\arg \left(0.006665017846190000-z_{0}\right)}{2 \pi}\right\rfloor \log \left(z_{0}\right)- \\
& \sum_{k=1}^{\infty} \frac{(-1)^{k}\left(0.006665017846190000-z_{0}\right)^{k} z_{0}^{-k}}{k}
\end{aligned}
$$

## Integral representation:

$\log (0.006665017846190000)=\int_{1}^{0.006665017846190000} \frac{1}{t} d t$

In conclusion:

$$
-6 C+\phi=-5.010882647757 \ldots
$$

and for $\mathrm{C}=1$, we obtain:
$\phi=-5.010882647757+6=\mathbf{0 . 9 8 9 1 1 7 3 5 2 2 4 3}=\boldsymbol{\phi}$

Note that the values of $\mathrm{n}_{\mathrm{s}}$ (spectral index) 0.965 , of the average of the Omega mesons Regge slope 0.987428571 and of the dilaton 0.989117352243 , are also connected to the following two Rogers-Ramanujan continued fractions:

$$
\begin{aligned}
& \frac{\mathrm{e}^{-\frac{\pi}{5}}}{\sqrt{(\varphi-1) \sqrt{5}}-\varphi+1}=1-\frac{\mathrm{e}^{-\pi}}{1+\frac{\mathrm{e}^{-2 \pi}}{1+\frac{\mathrm{e}^{-3 \pi}}{1+\frac{\mathrm{e}^{-4 \pi}}{1+\ldots}}}} \approx 0.9568666373 \\
& \frac{\mathrm{e}^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} \\
& 1+\sqrt[5]{\sqrt{\varphi^{5 \sqrt[4]{5^{3}}}}-1}
\end{aligned} \varphi+1 \quad 1-\frac{\mathrm{e}^{-\pi \sqrt{5}}}{1+\frac{\mathrm{e}^{-2 \pi \sqrt{5}}}{1+\frac{\mathrm{e}^{-3 \pi \sqrt{5}}}{1+\frac{\mathrm{e}^{-4 \pi \sqrt{5}}}{1+\ldots}}}} \approx 0.9991104684
$$

(http://www.bitman.name/math/article/102/109/)

The mean between the two results of the above Rogers-Ramanujan continued fractions is 0.97798855285 , value very near to the $\psi$ Regge slope 0.979 :
$\Psi \quad|3|$
$m_{c}=1500$
| 0.979
$-0.09$

Also performing the $512^{\text {th }}$ root of the inverse value of the Pion meson rest mass 139.57, we obtain:
$((1 /(139.57)))^{\wedge} 1 / 512$

## Input interpretation:

$\sqrt[512]{\frac{1}{139.57}}$

## Result:

$0.990400732708644027550973755713301415460732796178555551684 \ldots$
$0.99040073 \ldots$. result very near to the dilaton value $\mathbf{0 . 9 8 9 1 1 7 3 5 2 2 4 3}=\boldsymbol{\phi}$ and to the value of the following Rogers-Ramanujan continued fraction:
$\frac{\mathrm{e}^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^{5 \sqrt[4]{5^{3}}}}-1}}-\varphi+1}=1-\frac{\mathrm{e}^{-\pi \sqrt{5}}}{1+\frac{\mathrm{e}^{-2 \pi \sqrt{5}}}{1+\frac{\mathrm{e}^{-3 \pi \sqrt{5}}}{1+\frac{\mathrm{e}^{-4 \pi \sqrt{5}}}{1+\ldots}}}} \approx 0.9991104684$

## From

AdS Vacua from Dilaton Tadpoles and Form Fluxes - J. Mourad and A. Sagnotti - arXiv:1612.08566v2 [hep-th] 22 Feb 2017 - March 27, 2018

We have:

$$
\begin{align*}
e^{2 C} & =\frac{2 \xi e^{\frac{\phi}{2}}}{1 \pm \sqrt{1-\frac{\xi T}{3} e^{2 \phi}}} \\
\frac{h^{2}}{32} & =\frac{\xi^{7} e^{4 \phi}}{\left(1 \pm \sqrt{1-\frac{\xi T}{3} e^{2 \phi}}\right)^{7}}\left[\frac{42}{\xi}\left(1 \pm \sqrt{1-\frac{\xi T}{3} e^{2 \phi}}\right)+5 T e^{2 \phi}\right] . \tag{2.7}
\end{align*}
$$

For
$T=\frac{16}{\pi^{2}}$
$\xi=1$
we obtain:
$\left(2 * \mathrm{e}^{\wedge}(0.989117352243 / 2)\right) /\left(1+\operatorname{sqrt}\left(\left(\left(1-1 / 3^{*} 16 /(\mathrm{Pi})^{\wedge} 2^{*} \mathrm{e}^{\wedge}(2 * 0.989117352243)\right)\right)\right)\right)$

## Input interpretation:

$\frac{2 e^{0.989117352243 / 2}}{1+\sqrt{1-\frac{1}{3} \times \frac{16}{\pi^{2}} e^{2 \times 0.989117352243}}}$

## Result:

0.83941881822... -
1.4311851867... $i$

## Polar coordinates:

$r=1.65919106525$ (radius), $\quad \theta=-59.607521917^{\circ}$ (angle)
$1.65919106525 \ldots$. result very near to the 14th root of the following Ramanujan's class invariant $Q=\left(G_{505} / G_{101 / 5}\right)^{3}=1164.2696$ i.e. $1.65578 \ldots$

## Series representations:



```
\(\frac{2 e^{0.9891173522430000 / 2}}{1+\sqrt{1-\frac{16 e^{2 \times 0.9891173522430000}}{3 \pi^{2}}}}=\)
\(\left.\frac{2 e^{0.4945586761215000}}{k!}-z_{0}\right)^{k} z_{0}^{-k}\)
for \(\left(\operatorname{not}\left(z_{0} \in \mathbb{R}\right.\right.\) and \(\left.\left.-\infty<z_{0} \leq 0\right)\right)\)
```

From

$$
\frac{h^{2}}{32}=\frac{\xi^{7} e^{4 \phi}}{\left(1 \pm \sqrt{1-\frac{\xi T}{3} e^{2 \phi}}\right)^{7}}\left[\frac{42}{\xi}\left(1 \pm \sqrt{1-\frac{\xi T}{3} e^{2 \phi}}\right)+5 T e^{2 \phi}\right]
$$

we obtain:
$\mathrm{e}^{\wedge}(4 * 0.989117352243) /\left(\left(\left(1+\operatorname{sqrt}\left(1-1 / 3^{*} 16 /(\mathrm{Pi})^{\wedge} 2^{*} \mathrm{e}^{\wedge}(2 * 0.989117352243)\right)\right)\right)\right)^{\wedge} 7$
[42(1+sqrt(1-
$\left.\left.\left.1 / 3^{*} 16 /(\mathrm{Pi})^{\wedge} 2 * \mathrm{e}^{\wedge}(2 * 0.989117352243)\right)\right)+5^{*} 16 /(\mathrm{Pi})^{\wedge} 2^{*} \mathrm{e}^{\wedge}(2 * 0.989117352243)\right]$

## Input interpretation:

$\frac{e^{4 \times 0.989117352243}}{\left(1+\sqrt{1-\frac{1}{3} \times \frac{16}{\pi^{2}} e^{2 \times 0.989117352243}}\right)^{7}}$
$\left(42\left(1+\sqrt{1-\frac{1}{3} \times \frac{16}{\pi^{2}} e^{2 \times 0.989117352243}}\right)+5 \times \frac{16}{\pi^{2}} e^{2 \times 0.989117352243}\right)$

## Result:

50.84107889... -
20.34506335... $i$

## Polar coordinates:

$r=54.76072411$ (radius), $\theta=-21.80979492^{\circ}$ (angle)
54.76072411.....

## Series representations:

$$
\begin{aligned}
& \left(\left(42\left(1+\sqrt{1-\frac{16 e^{2 \times 0.9891173522430000}}{3 \pi^{2}}}\right)+\frac{5 \times 16 e^{2 \times 0.9891173522430000}}{\pi^{2}}\right)\right. \\
& \left.e^{4 \times 0.0891173522430000}\right) /\left(1+\sqrt{1-\frac{16 e^{2 \times 0.9891173522430000}}{3 \pi^{2}}}\right)^{7}= \\
& \left\{2 \left(40 e^{5.934704113458000}+21 e^{3.956469408972000} \pi^{2}+21 e^{3.9564694089772000} \pi^{2}\right.\right. \\
& \left.\sqrt{-\frac{16 e^{1.978234704486000}}{3 \pi^{2}}} \sum_{k=0}^{\infty}\left(\frac{3}{16}\right)^{k}\left(-\frac{e^{1.978234704486000}}{\pi^{2}}\right)^{-k}\binom{\frac{1}{2}}{k}\right) / \\
& \left(\pi^{2}\left(1+\sqrt{-\frac{16 e^{1.978234704486000}}{3 \pi^{2}}} \sum_{k=0}^{\infty}\left(\frac{3}{16}\right)^{k}\left(-\frac{e^{1.978234704486000}}{\pi^{2}}\right)^{-k}\binom{\frac{1}{2}}{k}\right)^{7}\right) \\
& \left(\int 42\left(1+\sqrt{1-\frac{16 e^{2 \times 0.9891173522430000}}{3 \pi^{2}}}\right)+\frac{5 \times 16 e^{2 \times 0.9891173522433000}}{\pi^{2}}\right) \\
& \left.e^{4 \times 0.9891173522430000}\right) /\left(1+\sqrt{1-\frac{16 e^{2 \times 0.9891173522430000}}{3 \pi^{2}}}\right)^{7}= \\
& \left\{2 \left(40 e^{5.334704113458000}+21 e^{3.956469908972000} \pi^{2}+21 e^{3.956469408972000} \pi^{2}\right.\right. \\
& \left.\sqrt{-\frac{16 e^{1.978234704486000}}{3 \pi^{2}}} \sum_{k=0}^{\infty} \frac{\left(-\frac{3}{16}\right)^{k}\left(-\frac{e^{1.978234704486000}}{\pi^{2}}\right)^{-k}\left(-\frac{1}{2}\right)_{k}}{k!}\right) / \\
& \left(\pi^{2}\left(1+\sqrt{-\frac{16 e^{1.978234704886000}}{3 \pi^{2}}} \sum_{k=0}^{\infty} \frac{\left(-\frac{3}{16}\right)^{k}\left(-\frac{e^{1.97823470488000}}{\pi^{2}}\right)^{-k}\left(-\frac{1}{2}\right)_{k}}{k!}\right)^{7}\right)
\end{aligned}
$$

$$
\begin{gathered}
\left(\begin{array}{c}
42\left(1+\sqrt{1-\frac{16 e^{2 \times 0.9891173522430000}}{3 \pi^{2}}}\right)+\frac{5 \times 16 e^{2 \times 0.9891173522430000}}{\pi^{2}}
\end{array}\right) \\
\left(2\left(40 e^{4 \times 0.9891173522430000}\right) /\left(1+\sqrt{1-\frac{16 e^{2 \times 0.9891173522430000}}{3 \pi^{2}}}\right)^{7}=\right. \\
\left(\pi^{7} \sqrt{2} \sqrt{z_{0}} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(1-\frac{16 e^{1.97823470448456000}}{3 \pi^{2}}-z_{0}\right)^{k} z_{0}^{-k}}{k!}\right) / \\
\left(\pi^{2}\left(1+\sqrt{z_{0}} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(1-\frac{16 e^{1.978234704486000}}{3 \pi^{2}}-z_{0}\right)^{k} z_{0}^{-k}}{k!}\right)^{7}\right)
\end{gathered}
$$

for ( $\operatorname{not}\left(z_{0} \in \mathbb{R}\right.$ and $\left.-\infty<z_{0} \leq 0\right)$ )

From which:
$\mathrm{e}^{\wedge}(4 * 0.989117352243) /\left(\left(\left(1+\operatorname{sqrt}\left(1-1 / 3^{*} 16 /(\mathrm{Pi})^{\wedge} 2^{*} \mathrm{e}^{\wedge}(2 * 0.989117352243)\right)\right)\right)\right)^{\wedge} 7$ [42(1+sqrt(1-
$\left.\left.\left.1 / 3 * 16 /(\mathrm{Pi})^{\wedge} 2^{*} \mathrm{e}^{\wedge}(2 * 0.989117352243)\right)\right)+5 * 16 /(\mathrm{Pi})^{\wedge} 2 * \mathrm{e}^{\wedge}(2 * 0.989117352243)\right]^{*} 1 / 34$

## Input interpretation:



## Result:

1.495325850... -
$0.5983842161 \ldots i$

## Polar coordinates:

```
r=1.610609533 (radius), 0=-21.80979492 (angle)
```

$1.610609533 \ldots$. result that is a good approximation to the value of the golden ratio 1.618033988749...

## Series representations:

$$
\begin{gathered}
\left.\left.\left(\begin{array}{l}
\left.42\left(1+\sqrt{1-\frac{16 e^{20.9891173522430000}}{3 \pi^{2}}}\right)+\frac{5 \times 16 e^{20.9891173522430000}}{\pi^{2}}\right) \\
\left.e^{40.9891173522430000}\right) /\left(34\left(1+\sqrt{1-\frac{16 e^{20.9891173522430000}}{3 \pi^{2}}}\right)\right. \\
\left(40 e^{5.934704113458000}+21 e^{3.956469408972000} \pi^{2}+21 e^{3.956469408972000} \pi^{2}\right. \\
\left(\sqrt{-\frac{16 e^{1.978234704486000}}{3 \pi^{2}}} \sum_{k=0}^{\infty}\left(\frac{3}{16}\right)^{k}\left(-\frac{e^{1.978234704486000}}{\pi^{2}}\right)^{-k}\binom{\frac{1}{2}}{k}\right) / \\
\left(17 \pi^{2}\left(1+\sqrt{-\frac{16 e^{1.978234704486000}}{3 \pi^{2}}} \sum_{k=0}^{\infty}\left(\frac{3}{16}\right)^{k}\left(-\frac{e^{1.978234704486000}}{\pi^{2}}\right)^{-k}\left(\frac{1}{2}\right)\right)^{7}\right. \\
k
\end{array}\right)\right)^{7}\right)
\end{gathered}
$$

$$
\begin{array}{r}
\left(\left(42\left(1+\sqrt{1-\frac{16 e^{20.9891173522430000}}{3 \pi^{2}}}\right)+\frac{5 \times 16 e^{20.9891173522430000}}{\pi^{2}}\right)\right. \\
\left.e^{4 \times 0.9891173522430000}\right) /\left(34\left(1+\sqrt{1-\frac{16 e^{20.9891173522430000}}{3 \pi^{2}}}\right)^{7}\right)=
\end{array}
$$

$$
\left(40 e^{5.934704113458000}+21 e^{3.956469408972000} \pi^{2}+21 e^{3.956469408972000} \pi^{2}\right.
$$

$$
\left.\sqrt{-\frac{16 e^{1.978234704486000}}{3 \pi^{2}}} \sum_{k=0}^{\infty} \frac{\left(-\frac{3}{16}\right)^{k}\left(-\frac{e^{1.978234704486000}}{\pi^{2}}\right)^{-k}\left(-\frac{1}{2}\right)_{k}}{k!}\right) /
$$

$$
\left(17 \pi^{2}\left(1+\sqrt{-\frac{16 e^{1.978234704486000}}{3 \pi^{2}}} \sum_{k=0}^{\infty} \frac{\left(-\frac{3}{16}\right)^{k}\left(-\frac{e^{1.978234704486000}}{\pi^{2}}\right)^{-k}\left(-\frac{1}{2}\right)_{k}}{k!}\right)^{7}\right)
$$

$$
\left.\begin{array}{c}
\left(\left(42\left(1+\sqrt{1-\frac{16 e^{20.9891173522430000}}{3 \pi^{2}}}\right)+\frac{5 \times 16 e^{20.9891173522430000}}{\pi^{2}}\right)\right. \\
\left.e^{40.9891173522430000}\right) /\left(34\left(1+\sqrt{1-\frac{16 e^{20.9891173522430000}}{3 \pi^{2}}}\right)^{7}\right)= \\
\left(40 e^{5.934704113458000}+21 e^{3.956469408972000} \pi^{2}+21 e^{3.956469408972000}\right.
\end{array}\right)=
$$

for ( $\operatorname{not}\left(z_{0} \in \mathbb{R}\right.$ and $-\infty<z_{0} \leq 0$ ) )

Now, we have:

$$
\begin{align*}
e^{2 C} & =\frac{2 \xi e^{-\frac{\phi}{2}}}{1+\sqrt{1+\frac{\xi \Lambda}{3} e^{2 \phi}}},  \tag{2.9}\\
\frac{h^{2}}{32} & =\frac{e^{-4 \phi}}{\left[1+\sqrt{1+\frac{\Lambda}{3} e^{2 \phi}}\right]^{7}}\left[42\left(1+\sqrt{1+\frac{\Lambda}{3} e^{2 \phi}}\right)-13 \Lambda e^{2 \phi}\right]
\end{align*}
$$

For:
$\xi=1$
$\Lambda \simeq \frac{4 \pi^{2}}{25}$
$\phi=0.989117352243$

From

$$
e^{2 C}=\frac{2 \xi e^{-\frac{\phi}{2}}}{1+\sqrt{1+\frac{\xi \Lambda}{3} e^{2 \phi}}},
$$

we obtain:
$\left(\left(2{ }^{*} \mathrm{e}^{\wedge}(-0.989117352243 / 2)\right)\right) /$
$\left(\left(\left(\left(1+\operatorname{sqrt}\left(\left(\left(1+1 / 3 *\left(4 \mathrm{Pi}^{\wedge} 2\right) / 25^{*} \mathrm{e}^{\wedge}(2 * 0.989117352243)\right)\right)\right)\right)\right)\right)\right)$

## Input interpretation:

$2 e^{-0.989117352243 / 2}$
$1+\sqrt{1+\frac{1}{3}\left(\frac{1}{25}\left(4 \pi^{2}\right)\right) e^{2 \times 0.989117352243}}$

## Result:

0.382082347529...
$0.382082347529 \ldots$

## Series representations:

$$
\begin{aligned}
\frac{2 e^{-0.9891173522430000 / 2}}{1+\sqrt{1+\frac{\left(4 \pi^{2}\right) e^{2 \times 0.9891173522430000}}{3 \times 25}}} & =2 /\left(e^{0.4945586761215000}\right. \\
\quad\left(1+\sqrt{\frac{4 e^{1.978234704486000} \pi^{2}}{75}}\right. & \left.\sum_{k=0}^{\infty}\left(\frac{75}{4}\right)^{k}\left(e^{1.978234704486000} \pi^{2}\right)^{-k}\binom{\frac{1}{2}}{k}\right)
\end{aligned}
$$

$\frac{2 e^{-0.9891173522430000 / 2}}{1+\sqrt{1+\frac{\left(4 \pi^{2}\right) e^{2 \times 0.9891173522430000}}{3 \times 25}}}=2 /\left(e^{0.4945586761215000}\right.$

$$
\left.\left(1+\sqrt{\frac{4 e^{1.978234704486000} \pi^{2}}{75}} \sum_{k=0}^{\infty} \frac{\left(-\frac{75}{4}\right)^{k}\left(e^{1.978234704486000} \pi^{2}\right)^{-k}\left(-\frac{1}{2}\right)_{k}}{k!}\right)\right)
$$

```
\(\frac{2 e^{-0.9891173522430000 / 2}}{1+\sqrt{1+\frac{\left(4 \pi^{2}\right) e^{2 \times 0.9891173522430000}}{3 \times 25}}}=\)
2
    \(e^{0.4945586761215000}\left(1+\sqrt{z_{0}} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(1+\frac{4 e^{1.978234704486000} \pi^{2}}{75}-z_{0}\right)^{k} z_{0}^{-k}}{k!}\right)\)
for \(\left(\operatorname{not}\left(z_{0} \in \mathbb{R}\right.\right.\) and \(\left.\left.-\infty<z_{0} \leq 0\right)\right)\)
```

From which:
$1+1 /\left(\left(\left(4\left(\left(2 * \mathrm{e}^{\wedge}(-0.989117352243 / 2)\right)\right) /\right.\right.\right.$
$\left.\left.\left.\left(\left(\left(\left(1+\operatorname{sqrt}\left(\left(\left(1+1 / 3^{*}\left(4 \mathrm{Pi}^{\wedge} 2\right) / 25^{*} \mathrm{e}^{\wedge}\left(2^{*} 0.989117352243\right)\right)\right)\right)\right)\right)\right)\right)\right)\right)\right)$

## Input interpretation:



## Result:

1.65430921270...
$1.6543092 \ldots$. We note that, the result $1.6543092 \ldots$ is very near to the 14 th root of the following Ramanujan's class invariant $Q=\left(G_{505} / G_{101 / 5}\right)^{3}=1164.2696$ i.e. 1.65578...

Indeed:

$$
\begin{aligned}
G_{505}=P^{-1 / 4} Q^{1 / 6}= & (\sqrt{5}+2)^{1 / 2}\left(\frac{\sqrt{5}+1}{2}\right)^{1 / 4}(\sqrt{101}+10)^{1 / 4} \\
& \times((130 \sqrt{5}+29 \sqrt{101})+\sqrt{169440+7540 \sqrt{505}})^{1 / 6}
\end{aligned}
$$

Thus, it remains to show that

$$
(130 \sqrt{5}+29 \sqrt{101})+\sqrt{169440+7540 \sqrt{505}}=\left(\sqrt{\frac{113+5 \sqrt{505}}{8}}+\sqrt{\frac{105+5 \sqrt{505}}{8}}\right)^{3},
$$

which is straightforward.

$$
\sqrt[14]{\left(\sqrt{\frac{113+5 \sqrt{505}}{8}}+\sqrt{\frac{105+5 \sqrt{505}}{8}}\right)^{3}}=1,65578 \ldots
$$

## Series representations:

$$
\begin{aligned}
& 1+\frac{1}{\frac{4\left(2 e^{-0.9891173522430000 / 2}\right)}{\sqrt{\left(4 \pi^{2}\right) e^{2}}}}= \\
& 1+\sqrt{1+\frac{\left(4 \pi^{2}\right) e^{2 \times 0.9891173522430000}}{3 \times 25}} \\
& 1+\frac{e^{0.4945586761215000}}{8}+\frac{1}{8} e^{0.4945586761215000} \sqrt{\frac{4 e^{1.978234704486000} \pi^{2}}{75}} \\
& \sum_{k=0}^{\infty}\left(\frac{75}{4}\right)^{k}\left(e^{1.978234704486000} \pi^{2}\right)^{-k}\binom{\frac{1}{2}}{k} \\
& \begin{array}{l}
1+\frac{1}{\frac{4\left(2 e^{-0.9891173522430000 / 2}\right)}{1+\sqrt{1+\frac{\left(4 \pi^{2}\right) e^{2 \times 0.9891173522430000}}{3 \times 25}}}}= \\
1+\frac{e^{0.4945586761215000}}{8}+\frac{1}{8} e^{0.4945586761215000} \sqrt{\frac{4 e^{1.978234704486000} \pi^{2}}{75}}
\end{array} \\
& \sum_{k=0}^{\infty} \frac{\left(-\frac{75}{4}\right)^{k}\left(e^{1.978234704486000} \pi^{2}\right)^{-k}\left(-\frac{1}{2}\right)_{k}}{k!}
\end{aligned}
$$

$$
\begin{aligned}
& 1+\frac{1}{\frac{4\left(2 e^{-0.9891173522430000 / 2}\right)}{1+\sqrt{1+\frac{\left(4 \pi^{2}\right) e^{2 \times 0.9891173522430000}}{3 \times 25}}}}=1+\frac{e^{0.4945586761215000}}{8}+ \\
& \frac{1}{8} e^{0.4945586761215000} \sqrt{z_{0}} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(1+\frac{4 e^{1.978234704486000} \pi^{2}}{75}-z_{0}\right)^{k} z_{0}^{-k}}{k!}
\end{aligned}
$$

for $\left(\operatorname{not}\left(z_{0} \in \mathbb{R}\right.\right.$ and $\left.\left.-\infty<z_{0} \leq 0\right)\right)$

And from

$$
\frac{h^{2}}{32}=\frac{e^{-4 \phi}}{\left[1+\sqrt{1+\frac{\Lambda}{3} e^{2 \phi}}\right]^{7}}\left[42\left(1+\sqrt{1+\frac{\Lambda}{3} e^{2 \phi}}\right)-13 \Lambda e^{2 \phi}\right]
$$

we obtain:
$\mathrm{e}^{\wedge}\left(-4^{*} 0.989117352243\right) /\left[1+\operatorname{sqrt}\left(\left(\left(1+1 / 3^{*}\left(4 \mathrm{Pi}^{\wedge} 2\right) / 25^{*} \mathrm{e}^{\wedge}(2 * 0.989117352243)\right)\right)\right]^{\wedge} 7^{*}\right.$ $\left[42\left(1+\operatorname{sqrt}\left(\left(\left(1+1 / 3^{*}\left(4 \mathrm{Pi}^{\wedge} 2\right) / 25^{*} \mathrm{e}^{\wedge}(2 * 0.989117352243)\right)\right)-\right.\right.\right.$ $\left.13 *\left(4 \mathrm{Pi}^{\wedge} 2\right) / 25 * \mathrm{e}^{\wedge}(2 * 0.989117352243)\right]$

## Input interpretation:

$$
\begin{aligned}
& \frac{e^{-4 \times 0.989117352243}}{\left(1+\sqrt{1+\frac{1}{3}\left(\frac{1}{25}\left(4 \pi^{2}\right)\right) e^{2 \times 0.989117352243}}\right)^{7}} \\
& \left(42\left(1+\sqrt{1+\frac{1}{3}\left(\frac{1}{25}\left(4 \pi^{2}\right)\right) e^{2 \times 0.989117352243}}-13\left(\frac{1}{25}\left(4 \pi^{2}\right)\right) e^{2 \times 0.989117352243}\right)\right)
\end{aligned}
$$

## Result:

-0.034547055658...
$-0.034547055658 \ldots$

## Series representations:

$$
\left.\left.\left.\left.\begin{array}{c}
\left(\left(42\left(1+\sqrt{1+\frac{\left(4 \pi^{2}\right) e^{2 \times 0.9891173522430000}}{3 \times 25}}-\frac{1}{25}\left(4 \pi^{2}\right) 13 e^{2 \times 0.9891173522430000}\right)\right)\right. \\
\left.e^{-4 \times 0.9891173522430000}\right) /\left(1+\sqrt{1+\frac{\left(4 \pi^{2}\right) e^{2 \times 0.9891173522430000}}{3 \times 25}}\right)^{7}= \\
-\left(\left(4 2 \left(-25 e^{1.978234704486000}+52 e^{3.956469408972000} \pi^{2}-\right.\right.\right. \\
25 e^{1.978234704486000} \sqrt{\frac{4 e^{1.978234704486000} \pi^{2}}{75}} \\
\left(1+\sqrt{\left.\left.\frac{\sum_{k=0}^{\infty}\left(\frac{75}{4}\right)^{k}\left(e^{1.978234704486000} \pi^{2}\right)^{-k}\left(\frac{1}{2}\right)}{2} k\right)\right) /\left(25 e^{5.978234704486000} \pi^{2}\right.} \frac{75}{k} \sum_{k=0}^{\infty}\left(\frac{75}{4}\right)^{k}\left(e^{1.978234704486000} \pi^{2}\right)^{-k}\left(\frac{1}{2}\right)^{2}\right. \\
k
\end{array}\right)\right)^{7}\right)\right) .
$$

$$
\begin{aligned}
& \left(\left(42\left(1+\sqrt{1+\frac{\left(4 \pi^{2}\right) e^{2 \times 0.9891173522430000}}{3 \times 25}}-\frac{1}{25}\left(4 \pi^{2}\right) 13 e^{2 \times 0.9891173522430000}\right)\right)\right. \\
& \left.e^{-4 \times 0.9891173522430000}\right) /\left(1+\sqrt{1+\frac{\left(4 \pi^{2}\right) e^{2 \times 0.9891173522430000}}{3 \times 25}}\right)^{7}= \\
& -\int\left(4 2 \left(-25 e^{1.978234704486000}+52 e^{3.956469408972000} \pi^{2}-\right.\right. \\
& 25 e^{1.978234704486000} \sqrt{\frac{4 e^{1.978234704486000} \pi^{2}}{75}} \\
& \left.\left.\sum_{k=0}^{\infty} \frac{\left(-\frac{75}{4}\right)^{k}\left(e^{1.978234704486000} \pi^{2}\right)^{-k}\left(-\frac{1}{2}\right)_{k}}{k!}\right)\right) /\left(25 e^{5.934704113458000}\right. \\
& \left.\left(1+\sqrt{\frac{4 e^{1.978234704486000} \pi^{2}}{75}} \sum_{k=0}^{\infty} \frac{\left(-\frac{75}{4}\right)^{k}\left(e^{1.978234704486000} \pi^{2}\right)^{-k}\left(-\frac{1}{2}\right)_{k}}{k!}\right)\right) \\
& \left(\left(42\left(1+\sqrt{1+\frac{\left(4 \pi^{2}\right) e^{2 \times 0.9891173522430000}}{3 \times 25}}-\frac{1}{25}\left(4 \pi^{2}\right) 13 e^{2 \times 0.9891173522430000}\right)\right)\right. \\
& \left.e^{-4 \times 0.9891173522430000}\right) /\left(1+\sqrt{1+\frac{\left(4 \pi^{2}\right) e^{2 \times 0.9891173522430000}}{3 \times 25}}\right)^{7}= \\
& -\int\left(4 2 \left(-25 e^{1.978234704486000}+52 e^{3.956469408972000} \pi^{2}-25 e^{1.978234704486000}\right.\right. \\
& \left.\sqrt{z_{0}} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(1+\frac{4 e^{1.978234704486000} \pi^{2}}{75}-z_{0}\right)^{k} z_{0}^{-k}}{k!}\right) / / 25 \\
& e^{5.934704113458000} \\
& \left.\left.\left(1+\sqrt{z_{0}} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(1+\frac{4 e^{1.978234704486000} \pi^{2}}{75}-z_{0}\right)^{k} z_{0}^{-k}}{k!}\right)^{7}\right)\right) \\
& \text { for ( } \operatorname{not}\left(z_{0} \in \mathbb{R} \text { and }-\infty<z_{0} \leq 0\right) \text { ) }
\end{aligned}
$$

## From which:

$47 * 1 /\left(\left(\left(-1 /\left(()\left(\left(\mathrm{e}^{\wedge}(-4 * 0.989117352243) /\right.\right.\right.\right.\right.\right.$
$\left[1+\operatorname{sqrt}\left(\left(\left(1+1 / 3^{*}\left(4 \mathrm{Pi}^{\wedge} 2\right) / 25^{*} \mathrm{e}^{\wedge}\left(2^{*} 0.989117352243\right)\right)\right)\right)\right]^{\wedge} 7$ * [42(1+sqrt(((1+1/3*(4Pi^2)/25* $\left.\left.\left.{ }^{\wedge}(2 * 0.989117352243)\right)\right)\right)-$ $\left.\left.\left.\left.\left.\left.\left.\left.\left.13 *\left(4 \mathrm{Pi}^{\wedge} 2\right) / 25 * \mathrm{e}^{\wedge}(2 * 0.989117352243)\right)\right]\right)\right)\right)\right)\right)\right)\right)$ )

## Input interpretation:

$$
47\left(-1 / 1 /\left(\frac{e^{-4 \times 0.989117352243}}{\left(1+\sqrt{1+\frac{1}{3}\left(\frac{1}{25}\left(4 \pi^{2}\right)\right) e^{2 \times 0.989117352243}}\right)^{7}}\right)\binom{\left(4 2 \left(1+\sqrt{1+\frac{1}{3}\left(\frac{1}{25}\left(4 \pi^{2}\right)\right) e^{2 \times 0.989117352243}}-\right.\right.}{\left.\left.\left.13\left(\frac{1}{25}\left(4 \pi^{2}\right)\right) e^{2 \times 0.989117352243}\right)\right)\right)}\right)
$$

## Result:

1.6237116159...
$1.6237116159 \ldots$. result that is an approximation to the value of the golden ratio 1.618033988749...

## Series representations:

$$
\begin{gathered}
-\left(47 / 1 /\left(e ^ { - 4 \times 0 . 9 8 9 1 1 7 3 5 2 2 4 3 0 0 0 0 } \left(4 2 \left(1+\sqrt{1+\frac{\left(4 \pi^{2}\right) e^{2 \times 0.9891173522430000}}{3 \times 25}}-\right.\right.\right.\right. \\
\left.\left.\left.\frac{1}{25}\left(4 \pi^{2}\right) 13 e^{2 \times 0.9891173522430000}\right)\right)\right) / \\
\left.\left(1+\sqrt{1+\frac{\left(4 \pi^{2}\right) e^{2 \times 0.9891173522430000}}{3 \times 25}}\right)^{7}\right)=
\end{gathered}
$$

$$
11974\left(-25 e^{1.978234704486000}+52 e^{3.956469408972000} \pi^{2}-\right.
$$

$$
25 e^{1.978234704486000} \sqrt{\frac{4 e^{1.978234704486000} \pi^{2}}{75}}
$$

$$
\left.\sum_{k=0}^{\infty}\left(\frac{75}{4}\right)^{k}\left(e^{1.978234704486000} \pi^{2}\right)^{-k}\binom{\frac{1}{2}}{k}\right) /\left(25 e^{5.934704113458000}\right.
$$

$$
\left.\left(1+\sqrt{\frac{4 e^{1.978234704486000} \pi^{2}}{75}} \sum_{k=0}^{\infty}\left(\frac{75}{4}\right)^{k}\left(e^{1.978234704486000} \pi^{2}\right)^{-k}\binom{\frac{1}{2}}{k}\right)^{7}\right)
$$

$$
\begin{aligned}
& -\left\{47 / 1 /\left(e ^ { - 4 \times 0 . 9 8 9 1 1 7 3 5 2 2 4 3 0 0 0 0 } \left(4 2 \left(1+\sqrt{1+\frac{\left(4 \pi^{2}\right) e^{2 \times 0.9891173522430000}}{3 \times 25}}-\right.\right.\right.\right. \\
& \left.\left.\left.\frac{1}{25}\left(4 \pi^{2}\right) 13 e^{2 \times 0.9891173522430000}\right)\right)\right) / \\
& \left.\left(1+\sqrt{1+\frac{\left(4 \pi^{2}\right) e^{2 \times 0.9891173522430000}}{3 \times 25}}\right)^{7}\right)= \\
& \left(1 9 7 4 \left(-25 e^{1.978234704486000}+52 e^{3.956469408972000} \pi^{2}-\right.\right. \\
& 25 e^{1.978234704486000} \sqrt{\frac{4 e^{1.978234704486000} \pi^{2}}{75}} \\
& \left.\sum_{k=0}^{\infty} \frac{\left(-\frac{75}{4}\right)^{k}\left(e^{1.978234704486000} \pi^{2}\right)^{-k}\left(-\frac{1}{2}\right)_{k}}{k!}\right) / / 25 e^{5.934704113458000} \\
& \left.\left(1+\sqrt{\frac{4 e^{1.978234704486000} \pi^{2}}{75}} \sum_{k=0}^{\infty} \frac{\left(-\frac{75}{4}\right)^{k}\left(e^{1.978234704486000} \pi^{2}\right)^{-k}\left(-\frac{1}{2}\right)_{k}}{k!}\right)^{7}\right) \\
& -\left\{47 / 1 /\left(e ^ { - 4 \times 0 . 9 8 9 1 1 7 3 5 2 2 4 3 0 0 0 0 } \left(4 2 \left(1+\sqrt{1+\frac{\left(4 \pi^{2}\right) e^{2 \times 0.9891173522430000}}{3 \times 25}}-\right.\right.\right.\right. \\
& \left.\left.\left.\frac{1}{25}\left(4 \pi^{2}\right) 13 e^{2 \times 0.9891173522430000}\right)\right)\right) / \\
& \left.\left(1+\sqrt{1+\frac{\left(4 \pi^{2}\right) e^{2 \times 0.9891173522430000}}{3 \times 25}}\right)^{7}\right)= \\
& 1974\left(-25 e^{1.978234704486000}+52 e^{3.956469408972000} \pi^{2}-25 e^{1.978234704486000}\right. \\
& \left.\sqrt{z_{0}} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(1+\frac{4 e^{1.978234704486000} \pi^{2}}{75}-z_{0}\right)^{k} z_{0}^{-k}}{k!}\right) / / 25 \\
& e^{5.934704113458000} \\
& \left.\left(1+\sqrt{z_{0}} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(1+\frac{4 e^{1.978234704486000} \pi^{2}}{75}-z_{0}\right)^{k} z_{0}^{-k}}{k!}\right)^{7}\right) \\
& \text { for ( } \operatorname{not}\left(z_{0} \in \mathbb{R} \text { and }-\infty<z_{0} \leq 0\right) \text { ) }
\end{aligned}
$$

And again:

32(()( $\mathrm{e}^{\wedge}\left(-4^{*} 0.989117352243\right) /$
$\left[1+\operatorname{sqrt}\left(\left(\left(1+1 / 3^{*}\left(4 \mathrm{Pi}^{\wedge} 2\right) / 25^{*} \mathrm{e}^{\wedge}(2 * 0.989117352243)\right)\right)\right)\right]^{\wedge} 7 *$ $\left[42\left(1+\operatorname{sqrt}\left(\left(\left(1+1 / 3^{*}\left(4 \mathrm{Pi}^{\wedge} 2\right) / 25^{*} \mathrm{e}^{\wedge}(2 * 0.989117352243)\right)\right)\right)-\right.\right.$ $\left.\left.\left.\left.\left.\left.13 *\left(4 \mathrm{Pi}^{\wedge} 2\right) / 25^{*} \mathrm{e}^{\wedge}(2 * 0.989117352243)\right)\right]\right)\right)\right)\right)$

## Input interpretation:

$$
\begin{aligned}
& 32\left(\frac{e^{-4 \times 0.989117352243}}{\left(1+\sqrt{1+\frac{1}{3}\left(\frac{1}{25}\left(4 \pi^{2}\right)\right) e^{2 \times 0.089117352243}}\right)^{7}}\right. \\
& \left.\quad\left(42\left(1+\sqrt{1+\frac{1}{3}\left(\frac{1}{25}\left(4 \pi^{2}\right)\right) e^{2 \times 0.989117352243}}-13\left(\frac{1}{25}\left(4 \pi^{2}\right)\right) e^{2 \times 0.989117352243}\right)\right)\right)
\end{aligned}
$$

## Result:

-1.1055057810...
$-1.1055057810 \ldots$.
We note that the result $-1.1055057810 \ldots$ is very near to the value of Cosmological Constant, less $10^{-52}$, thence 1.1056 , with minus sign

## Series representations:

$$
\begin{aligned}
& \left(3 2 e ^ { - 4 \times 0 . 9 8 9 1 1 7 3 5 2 2 4 3 0 0 0 0 } \left(4 2 \left(1+\sqrt{1+\frac{\left(4 \pi^{2}\right) e^{2 \times 0.9891173522430000}}{3 \times 25}}-\right.\right.\right. \\
& \left.\left.\left.\frac{1}{25}\left(4 \pi^{2}\right) 13 e^{2 \times 0.9891173522430000}\right)\right)\right) / \\
& \left(1+\sqrt{1+\frac{\left(4 \pi^{2}\right) e^{2 \times 0.9891173522430000}}{3 \times 25}}\right)^{7}= \\
& -\left(\int 1 3 4 4 \left(-25 e^{1.978234704486000}+52 e^{3.956469408972000} \pi^{2}-\right.\right. \\
& 25 e^{1.978234704486000} \sqrt{\frac{4 e^{1.978234704486000} \pi^{2}}{75}} \\
& \left.\left.\sum_{k=0}^{\infty}\left(\frac{75}{4}\right)^{k}\left(e^{1.978234704486000} \pi^{2}\right)^{-k}\binom{\frac{1}{2}}{k}\right)\right) /\left(25 e^{5.934704113458000}\right. \\
& \left.\left.\left(1+\sqrt{\frac{4 e^{1.978234704486000} \pi^{2}}{75}} \sum_{k=0}^{\infty}\left(\frac{75}{4}\right)^{k}\left(e^{1.978234704486000} \pi^{2}\right)^{-k}\binom{\frac{1}{2}}{k}\right)^{7}\right)\right)
\end{aligned}
$$

$$
\begin{aligned}
& \left(3 2 e ^ { - 4 \times 0 . 9 8 9 1 1 7 3 5 2 2 4 3 0 0 0 0 } \left(4 2 \left(1+\sqrt{1+\frac{\left(4 \pi^{2}\right) e^{2 \times 0.9891173522430000}}{3 \times 25}}-\right.\right.\right. \\
& \left.\left.\left.\frac{1}{25}\left(4 \pi^{2}\right) 13 e^{2 \times 0.9891173522430000}\right)\right)\right) / \\
& \left(1+\sqrt{1+\frac{\left(4 \pi^{2}\right) e^{2 \times 0.9891173522430000}}{3 \times 25}}\right)^{7}= \\
& -\left(\int 1 3 4 4 \left(-25 e^{1.978234704486000}+52 e^{3.956469408972000} \pi^{2}-\right.\right. \\
& 25 e^{1.978234704486000} \sqrt{\frac{4 e^{1.978234704486000} \pi^{2}}{75}} \\
& \left.\sum_{k=0}^{\infty} \frac{\left(-\frac{75}{4}\right)^{k}\left(e^{1.978234704486000} \pi^{2}\right)^{-k}\left(-\frac{1}{2}\right)_{k}}{k!}\right) / / 25 e^{5.934704113458000} \\
& \left.\left(1+\sqrt{\frac{4 e^{1.978234704486000} \pi^{2}}{75}} \sum_{k=0}^{\infty} \frac{\left(-\frac{75}{4}\right)^{k}\left(e^{1.978234704486000} \pi^{2}\right)^{-k}\left(-\frac{1}{2}\right)_{k}}{k!}\right)\right) \\
& \left(3 2 e ^ { - 4 \times 0 . 9 8 9 1 1 7 3 5 2 2 4 3 0 0 0 0 } \left(4 2 \left(1+\sqrt{1+\frac{\left(4 \pi^{2}\right) e^{2 \times 0.9891173522430000}}{3 \times 25}}-\right.\right.\right. \\
& \left.\left.\left.\frac{1}{25}\left(4 \pi^{2}\right) 13 e^{2 \times 0.9891173522430000}\right)\right)\right) / \\
& \left(1+\sqrt{1+\frac{\left(4 \pi^{2}\right) e^{2 \times 0.9891173522430000}}{3 \times 25}}\right)^{7}= \\
& -\int\left(1 3 4 4 \left(-25 e^{1.978234704486000}+52 e^{3.956469408972000} \pi^{2}-25 e^{1.978234704486000}\right.\right. \\
& \left.\sqrt{z_{0}} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(1+\frac{4 e^{1.978234704486000} \pi^{2}}{75}-z_{0}\right)^{k} z_{0}^{-k}}{k!}\right) / / 25 \\
& e^{5.934704113458000} \\
& \left.\left.\left(1+\sqrt{z_{0}} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(1+\frac{4 e^{1.978234704486000} \pi^{2}}{75}-z_{0}\right)^{k} z_{0}^{k}}{k!}\right)^{7}\right)\right) \\
& \text { for ( } \operatorname{not}\left(z_{0} \in \mathbb{R} \text { and }-\infty<z_{0} \leq 0\right) \text { ) }
\end{aligned}
$$

And:
-[32(()( $\mathrm{e}^{\wedge}\left(-4^{*} 0.989117352243\right) /$
$\left[1+\operatorname{sqrt}\left(\left(\left(1+1 / 3 *\left(4 \mathrm{Pi}^{\wedge} 2\right) / 25^{*} \mathrm{e}^{\wedge}(2 * 0.989117352243)\right)\right)\right)\right]^{\wedge} 7 *$ $\left[42\left(1+\operatorname{sqrt}\left(\left(\left(1+1 / 3^{*}\left(4 \mathrm{Pi}^{\wedge} 2\right) / 25^{*} \mathrm{e}^{\wedge}(2 * 0.989117352243)\right)\right)\right)-\right.\right.$ $\left.\left.\left.\left.\left.\left.\left.13 *\left(4 \mathrm{Pi}^{\wedge} 2\right) / 25 * \mathrm{e}^{\wedge}(2 * 0.989117352243)\right)\right]\right)\right)\right)\right)\right]^{\wedge} 5$

## Input interpretation:

$$
-\left(\begin{array}{l}
-\left(\frac{e^{-4 \times 0.089117352243}}{\left(1+\sqrt{1+\frac{1}{3}\left(\frac{1}{25}\left(4 \pi^{2}\right)\right) e^{2 \times 0.989117352243}}\right)^{7}}\right. \\
\left(4 2 \left(1+\sqrt{1+\frac{1}{3}\left(\frac{1}{25}\left(4 \pi^{2}\right)\right) e^{2 \times 0.989117352243}}-\right.\right. \\
\left.\left.\left.13\left(\frac{1}{25}\left(4 \pi^{2}\right)\right) e^{2 \times 0.989117352243}\right)\right)\right)
\end{array}\right)^{5}
$$

## Result:

1.651220569...
$1.651220569 \ldots$. result very near to the 14th root of the following Ramanujan's class invariant $Q=\left(G_{505} / G_{101 / 5}\right)^{3}=1164.2696$ i.e. $1.65578 \ldots$

## Series representations:

$$
\begin{aligned}
& -\left(\int 3 2 e ^ { - 4 \times 0 . 9 8 9 1 1 7 3 5 2 2 4 3 0 0 0 0 } \left(4 2 \left(1+\sqrt{1+\frac{\left(4 \pi^{2}\right) e^{2 \times 0.9891173522430000}}{3 \times 25}}-\right.\right.\right. \\
& \left.\left.\left.\frac{1}{25}\left(4 \pi^{2}\right) 13 e^{2 \times 0.9891173522430000}\right)\right)\right) / \\
& \left.\left(1+\sqrt{1+\frac{\left(4 \pi^{2}\right) e^{2 \times 0.9891173522430000}}{3 \times 25}}\right)^{7}\right)^{5}= \\
& \left(4 3 8 5 2 7 0 0 5 7 1 4 0 2 2 4 \left(-25+52 e^{1.978234704486000} \pi^{2}-25 \sqrt{\frac{4 e^{1.978234704486000} \pi^{2}}{75}}\right.\right. \\
& \left.\left.\sum_{k=0}^{\infty}\left(\frac{75}{4}\right)^{k}\left(e^{1.978234704486000} \pi^{2}\right)^{-k}\binom{\frac{1}{2}}{k}\right)^{5}\right) / \\
& \left(9 7 6 5 6 2 5 e ^ { 1 9 . 7 8 2 3 4 7 0 4 4 8 6 0 0 0 } \left(1+\sqrt{\frac{4 e^{1.978234704486000} \pi^{2}}{75}}\right.\right. \\
& \left.\left.\sum_{k=0}^{\infty}\left(\frac{75}{4}\right)^{k}\left(e^{1.978234704486000} \pi^{2}\right)^{-k}\binom{\frac{1}{2}}{k}\right)^{35}\right)
\end{aligned}
$$

$$
\begin{aligned}
& -\left(\left(3 2 e ^ { - 4 \times 0 . 9 8 9 1 1 7 3 5 2 2 4 3 0 0 0 0 } \left(4 2 \left(1+\sqrt{1+\frac{\left(4 \pi^{2}\right) e^{2 \times 0.9891173522430000}}{3 \times 25}}-\right.\right.\right.\right. \\
& \left.\left.\left.\frac{1}{25}\left(4 \pi^{2}\right) 13 e^{2 \times 0.9891173522430000}\right)\right)\right) / \\
& \left.\left(1+\sqrt{1+\frac{\left(4 \pi^{2}\right) e^{2 \times 0.9891173522430000}}{3 \times 25}}\right)^{7}\right)^{5}= \\
& \left(4 3 8 5 2 7 0 0 5 7 1 4 0 2 2 4 \left(-25+52 e^{1.978234704486000} \pi^{2}-25 \sqrt{\frac{4 e^{1.978234704486000} \pi^{2}}{75}}\right.\right. \\
& \left.\left.\sum_{k=0}^{\infty} \frac{\left(-\frac{75}{4}\right)^{k}\left(e^{1.978234704486000} \pi^{2}\right)^{-k}\left(-\frac{1}{2}\right)_{k}}{7}\right)\right) / \\
& k! \\
& \left(9 7 6 5 6 2 5 e ^ { 1 9 . 7 8 2 3 4 7 0 4 4 8 6 0 0 0 } \left(1+\sqrt{\frac{4 e^{1.978234704486000} \pi^{2}}{75}}\right.\right. \\
& \left.\sum_{k=0}^{\infty} \frac{\left(-\frac{75}{4}\right)^{k}\left(e^{1.978234704486000} \pi^{2}\right)^{-k}\left(-\frac{1}{2}\right)_{k}}{k!}\right) \\
& k!
\end{aligned}
$$

$$
-\left(\int 3 2 e ^ { - 4 \times 0 . 9 8 9 1 1 7 3 5 2 2 4 3 0 0 0 0 } \left(4 2 \left(1+\sqrt{1+\frac{\left(4 \pi^{2}\right) e^{2 \times 0.9891173522430000}}{3 \times 25}}-\right.\right.\right.
$$

$$
\left.\left.\left.\frac{1}{25}\left(4 \pi^{2}\right) 13 e^{2 \times 0.9891173522430000}\right)\right)\right) /
$$

$$
\left.\left(1+\sqrt{1+\frac{\left(4 \pi^{2}\right) e^{2 \times 0.9891173522430000}}{3 \times 25}}\right)^{7}\right)^{5}=
$$

$$
\left(4 3 8 5 2 7 0 0 5 7 1 4 0 2 2 4 \left(-25+52 e^{1.978234704486000} \pi^{2}-\right.\right.
$$

$$
\left.\left.25 \sqrt{z_{0}} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(1+\frac{4 e^{1.978234704486000} \pi^{2}}{75}-z_{0}\right)^{k} z_{0}^{-k}}{k!}\right)^{5}\right) /
$$

$$
\left(9765625 e^{19.78234704486000}\right.
$$

$$
\left.\left(1+\sqrt{z_{0}} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(1+\frac{4 e^{1.978234704486000} \pi^{2}}{75}-z_{0}\right)^{k} z_{0}^{-k}}{k!}\right)^{35}\right)
$$

for ( $\operatorname{not}\left(z_{0} \in \mathbb{R}\right.$ and $\left.-\infty<z_{0} \leq 0\right)$ )

We obtain also:
-[32(()(e^(-4*0.989117352243) /
$\left[1+\operatorname{sqrt}\left(\left(\left(1+1 / 3 *\left(4 \mathrm{Pi}^{\wedge} 2\right) / 25^{*} \mathrm{e}^{\wedge}(2 * 0.989117352243)\right)\right)\right)\right]^{\wedge} 7 *$ $\left[42\left(1+\operatorname{sqrt}\left(\left(\left(1+1 / 3^{*}\left(4 \mathrm{Pi}^{\wedge} 2\right) / 25^{*} \mathrm{e}^{\wedge}(2 * 0.989117352243)\right)\right)\right)-\right.\right.$ $\left.\left.\left.\left.\left.\left.\left.13 *\left(4 \mathrm{Pi}^{\wedge} 2\right) / 25^{*} \mathrm{e}^{\wedge}(2 * 0.989117352243)\right)\right]\right)\right)\right)\right)\right]^{\wedge} 1 / 2$

## Input interpretation:

$$
-\sqrt{\left(\frac{e^{-4 \times 0.989117352243}}{\left(42\left(1+\sqrt{1+\frac{1}{3}\left(\frac{1}{25}\left(4 \pi^{2}\right)\right) e^{2 \times 0.989117352243}}\right)^{7}\right.}\right.} \underset{\left(4 2 \left(1+\sqrt{1+\frac{1}{3}\left(\frac{1}{25}\left(4 \pi^{2}\right)\right) e^{2 \times 0.989117352243}}\right.\right.}{\left.\left.13\left(\frac{1}{25}\left(4 \pi^{2}\right)\right) e^{2 \times 0.989117352243}\right)\right)}-
$$

## Result:

- 0
1.0514303501...


## Polar coordinates:

$r=1.05143035007$ (radius), $\theta=-90^{\circ}$ (angle)
1.05143035007

## Series representations:

$$
\begin{aligned}
& -\sqrt{ } \left\lvert\,\left(3 2 e ^ { - 4 \times 0 . 9 8 9 1 1 7 3 5 2 2 4 3 0 0 0 0 } \left(4 2 \left(1+\sqrt{1+\frac{\left(4 \pi^{2}\right) e^{2 \times 0.9891173522430000}}{3 \times 25}}-\right.\right.\right.\right. \\
& \left.\left.\left.\frac{1}{25}\left(4 \pi^{2}\right) 13 e^{2 \times 0.9891173522430000}\right)\right)\right) / \\
& \left.\left(1+\sqrt{1+\frac{\left(4 \pi^{2}\right) e^{2 \times 0.9891173522430000}}{3 \times 25}}\right)^{7}\right)=-\frac{8}{5} \sqrt{21} \\
& \sqrt{ } \left\lvert\,\left(25-52 e^{1.978234704486000} \pi^{2}+25 \sqrt{\frac{4 e^{1.978234704486000} \pi^{2}}{75}}\right.\right. \\
& \left.\sum_{k=0}^{\infty}\left(\frac{75}{4}\right)^{k}\left(e^{1.978234704486000} \pi^{2}\right)^{-k}\binom{\frac{1}{2}}{k}\right) /\left(e^{3.956469408972000}\right. \\
& \left.\left.\left(1+\sqrt{\frac{4 e^{1.978234704486000} \pi^{2}}{75}} \sum_{k=0}^{\infty}\left(\frac{75}{4}\right)^{k}\left(e^{1.978234704486000} \pi^{2}\right)^{-k}\binom{\frac{1}{2}}{k}\right)^{7}\right)\right)
\end{aligned}
$$

$$
-\sqrt{ }\left(\left(3 2 e ^ { - 4 \times 0 . 9 8 9 1 1 7 3 5 2 2 4 3 0 0 0 0 } \left(4 2 \left(1+\sqrt{1+\frac{\left(4 \pi^{2}\right) e^{2 \times 0.9891173522430000}}{3 \times 25}}-\right.\right.\right.\right.
$$

$$
\left.\left.\frac{1}{25}\left(4 \pi^{2}\right) 13 e^{2 \times 0.9891173522430000}\right)\right) /
$$

$$
\left.\left(1+\sqrt{1+\frac{\left(4 \pi^{2}\right) e^{2 \times 0.9891173522430000}}{3 \times 25}}\right)^{7}\right)=
$$

$$
-\frac{8}{5} \sqrt{21} \sqrt{ } \sqrt{\left(25-52 e^{1.978234704486000} \pi^{2}+\right.}
$$

$$
\left.25 \sqrt{z_{0}} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(1+\frac{4 e^{1.978234704486000} \pi^{2}}{75}-z_{0}\right)^{k} z_{0}^{-k}}{k!}\right) /
$$

$$
\left(e^{3.956469408972000}\right.
$$

$$
\left.\left.\left(1+\sqrt{z_{0}} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(1+\frac{4 e^{1.978234704486000} \pi^{2}}{75}-z_{0}\right)^{k} z_{0}^{-k}}{k!}\right)^{7}\right)\right)
$$

for $\left(\operatorname{not}\left(z_{0} \in \mathbb{R}\right.\right.$ and $\left.\left.-\infty<z_{0} \leq 0\right)\right)$

$$
\begin{aligned}
& -\sqrt{ } \left\lvert\,\left(3 2 e ^ { - 4 \times 0 . 9 8 9 1 1 7 3 5 2 2 4 3 0 0 0 0 } \left(4 2 \left(1+\sqrt{1+\frac{\left(4 \pi^{2}\right) e^{2 \times 0.9891173522430000}}{3 \times 25}}-\right.\right.\right.\right. \\
& \left.\left.\frac{1}{25}\left(4 \pi^{2}\right) 13 e^{2 \times 0.9891173522430000}\right)\right) / \\
& \left.\left(1+\sqrt{1+\frac{\left(4 \pi^{2}\right) e^{2 \times 0.9891173522430000}}{3 \times 25}}\right)^{7}\right)=-\frac{8}{5} \sqrt{21} \\
& \sqrt{ } \left\lvert\,\left(25-52 e^{1.978234704486000} \pi^{2}+25 \sqrt{\frac{4 e^{1.978234704486000} \pi^{2}}{75}}\right.\right. \\
& \left.\sum_{k=0}^{\infty} \frac{\left(-\frac{75}{4}\right)^{k}\left(e^{1.978234704486000} \pi^{2}\right)^{-k}\left(-\frac{1}{2}\right)_{k}}{k!}\right) / \\
& \left(e ^ { 3 . 9 5 6 4 6 9 4 0 8 9 7 2 0 0 0 } \left(1+\sqrt{\frac{4 e^{1.978234704486000} \pi^{2}}{75}}\right.\right. \\
& \left.\left.\left.\sum_{k=0}^{\infty} \frac{\left(-\frac{75}{4}\right)^{k}\left(e^{1.978234704486000} \pi^{2}\right)^{-k}\left(-\frac{1}{2}\right)_{k}}{k!}\right)^{7}\right)\right)
\end{aligned}
$$

$1 /-\left[32\left(\left(() e^{\wedge}\left(-4^{*} 0.989117352243\right) /\right.\right.\right.$
$\left[1+\operatorname{sqrt}\left(\left(\left(1+1 / 3^{*}\left(4 \mathrm{Pi}^{\wedge} 2\right) / 25^{*} \mathrm{e}^{\wedge}(2 * 0.989117352243)\right)\right)\right)\right]^{\wedge} 7 *$ $\left[42\left(1+\operatorname{sqrt}\left(\left(\left(1+1 / 3 *\left(4 \mathrm{Pi}^{\wedge} 2\right) / 25^{*} \mathrm{e}^{\wedge}(2 * 0.989117352243)\right)\right)\right)-\right.\right.$ $\left.\left.\left.\left.\left.\left.\left.13 *\left(4 \mathrm{Pi}^{\wedge} 2\right) / 25^{*} \mathrm{e}^{\wedge}(2 * 0.989117352243)\right)\right]\right)\right)\right)\right)\right]^{\wedge} 1 / 2$

## Input interpretation:



$$
\begin{gathered}
\left(4 2 \left(1+\sqrt{1+\frac{1}{3}\left(\frac{1}{25}\left(4 \pi^{2}\right)\right) e^{2 \times 0.989117352243}}-\right.\right. \\
\left.\left.\left.13\left(\frac{1}{25}\left(4 \pi^{2}\right)\right) e^{2 \times 0.989117352243}\right)\right)\right)
\end{gathered}
$$

## Result:

0.95108534763... $i$

## Polar coordinates:

$r=0.95108534763$ (radius), $\theta=90^{\circ}$ (angle)
0.95108534763

We know that the primordial fluctuations are consistent with Gaussian purely adiabatic scalar perturbations characterized by a power spectrum with a spectral index $\mathrm{n}_{\mathrm{s}}=0.965 \pm 0.004$, consistent with the predictions of slow-roll, single-field, inflation.

Thence 0.95108534763 is a result very near to the spectral index $\mathrm{n}_{\mathrm{s}}$, to the mesonic Regge slope, to the inflaton value at the end of the inflation 0.9402 and to the value of the following Rogers-Ramanujan continued fraction:

$$
\frac{\mathrm{e}^{-\frac{\pi}{5}}}{\sqrt{(\varphi-1) \sqrt{5}}-\varphi+1}=1-\frac{\mathrm{e}^{-\pi}}{1+\frac{\mathrm{e}^{-2 \pi}}{1+\frac{\mathrm{e}^{-3 \pi}}{1+\frac{\mathrm{e}^{-4 \pi}}{1+\ldots}}}} \approx 0.9568666373
$$

## Series representations:

$$
\begin{aligned}
& -\left(1 / \int \sqrt{ } \left\lvert\,\left(3 2 e ^ { - 4 \times 0 . 9 8 9 1 1 7 3 5 2 2 4 3 0 0 0 0 } \left(4 2 \left(1+\sqrt{1+\frac{\left(4 \pi^{2}\right) e^{2 \times 0.9891173522430000}}{3 \times 25}}-\right.\right.\right.\right.\right. \\
& \left.\left.\left.\frac{1}{25}\left(4 \pi^{2}\right) 13 e^{2 \times 0.9891173522430000}\right)\right)\right) / \\
& \left.\left.\left.\left(1+\sqrt{1+\frac{\left(4 \pi^{2}\right) e^{2 \times 0.9891173522430000}}{3 \times 25}}\right)^{7}\right)\right)\right)= \\
& -\left(5 / \int 8 \sqrt{21} \sqrt{ } \left\lvert\,\left(25-52 e^{1.978234704486000} \pi^{2}+25 \sqrt{\frac{4 e^{1.978234704486000} \pi^{2}}{75}}\right.\right.\right. \\
& \left.\sum_{k=0}^{\infty}\left(\frac{75}{4}\right)^{k}\left(e^{1.978234704486000} \pi^{2}\right)^{-k}\binom{\frac{1}{2}}{k}\right) / \\
& \left(e ^ { 3 . 0 5 6 4 6 9 4 0 8 9 7 2 0 0 0 } \left(1+\sqrt{\frac{4 e^{1.978234704486000} \pi^{2}}{75}}\right.\right. \\
& \left.\left.\left.\left.\left.\left.\sum_{k=0}^{\infty}\left(\frac{75}{4}\right)^{k}\left(e^{1.978234704486000} \pi^{2}\right)^{-k}\binom{\frac{1}{2}}{k}\right)^{7}\right)\right)\right)\right)\right)
\end{aligned}
$$

$$
\begin{aligned}
& -\int 1 / \int \sqrt{ } \left\lvert\,\left(3 2 e ^ { - 4 \times 0 . 9 8 9 1 1 7 3 5 2 2 4 3 0 0 0 0 } \left(4 2 \left(1+\sqrt{1+\frac{\left(4 \pi^{2}\right) e^{2 \times 0.9891173522430000}}{3 \times 25}}-\right.\right.\right.\right. \\
& \left.\left.\left.\frac{1}{25}\left(4 \pi^{2}\right) 13 e^{2 \times 0.9891173522430000}\right)\right)\right) / \\
& \left.\left.\left.\left(1+\sqrt{1+\frac{\left(4 \pi^{2}\right) e^{2 \times 0.9891173522430000}}{3 \times 25}}\right)^{7}\right)\right)\right)= \\
& -\left(5 /(8 \sqrt{21}) \left\lvert\,\left(25-52 e^{1.978234704486000} \pi^{2}+25 \sqrt{\frac{4 e^{1.978234704486000} \pi^{2}}{75}}\right.\right.\right. \\
& \left.\sum_{k=0}^{\infty} \frac{\left(-\frac{75}{4}\right)^{k}\left(e^{1.978234704486000} \pi^{2}\right)^{-k}\left(-\frac{1}{2}\right)_{k}}{k!}\right) / \\
& \left(e ^ { 3 . 9 5 6 4 6 9 4 0 8 9 7 2 0 0 0 } \left(1+\sqrt{\frac{4 e^{1.978234704486000} \pi^{2}}{75}}\right.\right. \\
& \left.\left.\left.\left.\sum_{k=0}^{\infty} \frac{\left(-\frac{75}{4}\right)^{k}\left(e^{1.978234704486000} \pi^{2}\right)^{-k}\left(-\frac{1}{2}\right)_{k}}{k!}\right)^{7}\right)\right)\right) \mid
\end{aligned}
$$

$$
\begin{aligned}
& -1 / / \int \left\lvert\,\left(3 2 e ^ { - 4 \times 0 . 9 8 9 1 1 7 3 5 2 2 4 3 0 0 0 0 } \left(4 2 \left(1+\sqrt{1+\frac{\left(4 \pi^{2}\right) e^{2 \times 0.9891173522430000}}{3 \times 25}}-\frac{1}{25}\right.\right.\right.\right. \\
& \left.\left.\left.\left(4 \pi^{2}\right) 13 e^{2 \times 0.9891173522430000}\right)\right)\right) / \\
& \left.\left.\left.\left(1+\sqrt{1+\frac{\left(4 \pi^{2}\right) e^{2 \times 0.9891173522430000}}{3 \times 25}}\right)^{7}\right)\right)\right)= \\
& -\int 5 /(8 \sqrt{21}) \mid\left(\mid 25-52 e^{1.978234704486000} \pi^{2}+25 \sqrt{z_{0}}\right. \\
& \left.\sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(1+\frac{4 e^{1.978234704486000} \pi^{2}}{75}-z_{0}\right)^{k} z_{0}^{k}}{k!}\right) / \\
& \int e^{3.956469408972000} \\
& \left(1+\sqrt{z_{0}} \sum_{k=0}^{\infty}\right. \\
& \left.\left.\left.\left.\left.\frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(1+\frac{4 e^{1.978234704486000} \pi^{2}}{75}-z_{0}\right)^{k} z_{0}^{-k}}{k!}\right)^{7}\right)\right) \int\right)\right) \\
& \text { for }\left(\operatorname{not}\left(z_{0} \in \mathbb{R} \text { and }-\infty<z_{0} \leq 0\right)\right. \text { ) }
\end{aligned}
$$

From the previous expression

$$
\begin{aligned}
& \frac{e^{-4 \times 0.989117352243}}{\left(1+\sqrt{1+\frac{1}{3}\left(\frac{1}{25}\left(4 \pi^{2}\right)\right) e^{2 \times 0.989117352243}}\right)^{7}} \\
& \left(42\left(1+\sqrt{1+\frac{1}{3}\left(\frac{1}{25}\left(4 \pi^{2}\right)\right) e^{2 \times 0.989117352243}}-13\left(\frac{1}{25}\left(4 \pi^{2}\right)\right) e^{2 \times 0.989117352243}\right)\right) \\
& =-0.034547055658 \ldots
\end{aligned}
$$

we have also:
$1+1 /\left(\left(\left(4\left(\left(2 * \mathrm{e}^{\wedge}(-0.989117352243 / 2)\right)\right) /\right.\right.\right.$
$\left.\left.\left.\left(\left(\left(\left(1+\operatorname{sqrt}\left(\left(\left(1+1 / 3^{*}\left(4 \mathrm{Pi}^{\wedge} 2\right) / 25^{*} \mathrm{e}^{\wedge}(2 * 0.989117352243)\right)\right)\right)\right)\right)\right)\right)\right)\right)\right)+(-0.034547055658)$

## Input interpretation:

$$
1+\frac{1}{4 \times \frac{2 e^{-0.989117352243 / 2}}{1+\sqrt{1+\frac{1}{3}\left(\frac{1}{25}\left(4 \pi^{2}\right)\right) e^{2 \times 0.989117352243}}}}-0.034547055658
$$

## Result:

1.61976215705...
$1.61976215705 \ldots$. result that is a very good approximation to the value of the golden ratio $1.618033988749 \ldots$

## Series representations:

$$
\begin{aligned}
& 1+\frac{1}{\frac{4\left(2 e^{-0.9891173522430000 / 2}\right)}{1+\sqrt{1+\frac{\left(4 \pi^{2}\right) e^{2 \times 0.9891173522430000}}{3 \times 25}}}-0.0345470556580000=} \\
& 0.9654529443420000+\frac{e^{0.4945586761215000}}{8}+\frac{1}{8} e^{0.4945586761215000} \\
& \left.\sqrt{\frac{4 e^{1.978234704486000} \pi^{2}}{75} \sum_{k=0}^{\infty}\left(\frac{75}{4}\right)^{k}\left(e^{1.978234704486000} \pi^{2}\right)^{-k}\left(\frac{1}{2}\right.} k\right) \\
& 1+\frac{1}{\frac{4\left(2 e^{-0.9891173522430000 / 2}\right)}{1+\sqrt{1+\frac{\left(4 \pi^{2}\right) e^{2 \times 0.9891173522430000}}{3 \times 25}}}}-0.0345470556580000= \\
& 0.9654529443420000+\frac{e^{0.4945586761215000}}{8}+\frac{1}{8} e^{0.4945586761215000} \\
& \sqrt{\frac{4 e^{1.978234704486000} \pi^{2}}{75}} \sum_{k=0}^{\infty} \frac{\left(-\frac{75}{4}\right)^{k}\left(e^{1.978234704486000} \pi^{2}\right)^{-k}\left(-\frac{1}{2}\right)_{k}}{k!}
\end{aligned}
$$

$$
\begin{aligned}
1+ & \frac{1}{\frac{4\left(2 e^{-0.9891173522430000 / 2}\right)}{1+\sqrt{1+\frac{\left(4 \pi^{2}\right) e^{2 \times 0.9891173522430000}}{3 \times 25}}}-0.0345470556580000=} \\
& 0.9654529443420000+\frac{e^{0.4945586761215000}}{8}+ \\
\quad & \frac{1}{8} e^{0.4945586761215000} \sqrt{z_{0}} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(1+\frac{4 e^{1.978234704486000} \pi^{2}}{75}-z_{0}\right)^{k} z_{0}^{-k}}{k!} \\
& \text { for }\left(\operatorname{not}\left(z_{0} \in \mathbb{R} \text { and }-\infty<z_{0} \leq 0\right)\right)
\end{aligned}
$$

## Appendix



From: A. Sagnotti - AstronomiAmo, 23.04.2020

In the above figure, it is said that: "why a given shape of the extra dimensions? Crucial, it determines the predictions for $\alpha$ ".

We propose that whatever shape the compactified dimensions are, their geometry must be based on the values of the golden ratio and $\zeta(2)$, (the latter connected to 1728 or 1729 , whose fifteenth root provides an excellent approximation to the above mentioned value) which are recurrent as solutions of the equations that we are going to develop. It is important to specify that the initial conditions are always values belonging to a fundamental chapter of the work of S. Ramanujan "Modular equations and Appoximations to $\mathrm{Pi}^{\prime \prime}$ (see references). These values are some multiples of 8 (64 and 4096), 276, which added to 4096 , is equal to 4372 , and finally $\mathrm{e}^{\pi / 22}$

Proposal of geometric connections between Swampland, Landscape and Riemann zeta function


Contributions of Euler, Gauss and Riemann to the Study of Primes Numbers - Carlos Figueroa
Raul Riera, German Campoy, Rene Betancourt - Volume 1, Issue 9, December 2014, PP 73-81

We notice a certain similarity between the figure that represents the Stringscape and that inherent in the Riemann zeta function: a case? We believe not.


Figure 2: Effective potential with the Wilson lines fixed to zero, as a function of the Radion and the Higgs. The tree level potential dominates and the Higgs is not displaced from its tree level minimum by the one-loop corrections. This behavior is independent of the particular value of the Wilson lines. Although not very visible in the plot, the Higgs minimum remains at the same location as $\mathrm{R}^{-1}$ increases.

From: (The Riemann hypothesis illuminated by the Newton flow of $\zeta$ - J W Neuberger, C Feiler, H Maier and W P Schleich - Received 29 April 2015, revised 5 August 2015. Accepted for publication 19 August 2015 Published 1 October 2015 - https://doi.org/10.1088/0031-8949/90/10/108015)


Figure 2. Lines in the complex plane where the Riemann zeta function $\zeta$ is real (green) depicted on a relief representing the positive absolute value of $\zeta$ for arguments $s \circ s+i t$ where the real part of $\zeta$ is positive, and the negative absolute value of $\zeta$ where the real part of $\zeta$ is negative. This representation brings out most clearly that the lines of constant phase corresponding to phases of integer multiples of $2 p$ run down the hills on the left-hand side, turn around on the right and terminate in the non-trivial zeros. This pattern repeats itself infinitely many times. The points of arrival and departure on the right-hand side of the picture are equally spaced and given by the following equation:

$$
\tau_{k}^{\prime} \equiv k \frac{\pi}{\ln 2}
$$

with $k=\ldots,-2,-1,0,1,2, \ldots$.
$2 \mathrm{Pi} /(\ln (2))$
Input:
$2 \times \frac{\pi}{\log (2)}$

## Exact result:

$\frac{2 \pi}{\log (2)}$

## Decimal approximation:

9.0647202836543876192553658914333336203437229354475911683720330958
9.06472028365....

## Alternative representations:

$\frac{2 \pi}{\log (2)}=\frac{2 \pi}{\log _{e}(2)}$
$\frac{2 \pi}{\log (2)}=\frac{2 \pi}{\log (a) \log _{a}(2)}$
$\frac{2 \pi}{\log (2)}=\frac{2 \pi}{2 \operatorname{coth}^{-1}(3)}$

## Series representations:

$$
\frac{2 \pi}{\log (2)}=\frac{2 \pi}{2 i \pi\left\lfloor\frac{\arg (2-x)}{2 \pi}\right\rfloor+\log (x)-\sum_{k=1}^{\infty} \frac{(-1)^{k}(2-x)^{k} x^{-k}}{k}} \text { for } x<0
$$

$$
\frac{2 \pi}{\log (2)}=\frac{2 \pi}{\log \left(z_{0}\right)+\left\lfloor\frac{\arg \left(2-z_{0}\right)}{2 \pi}\right\rfloor\left(\log \left(\frac{1}{z_{0}}\right)+\log \left(z_{0}\right)\right)-\sum_{k=1}^{\infty} \frac{(-1)^{k}\left(2-z_{0}\right)^{k} z_{0}^{-k}}{k}}
$$

$$
\frac{2 \pi}{\log (2)}=\frac{2 \pi}{2 i \pi\left[\frac{\pi-\arg \left(\frac{1}{z_{0}}\right)-\arg \left(z_{0}\right)}{2 \pi} \left\lvert\,+\log \left(z_{0}\right)-\sum_{k=1}^{\infty} \frac{(-1)^{k}\left(2-z_{0}\right)^{k} z_{0}^{-k}}{k}\right.\right.}
$$

## Integral representations:

$$
\frac{2 \pi}{\log (2)}=\frac{2 \pi}{\int_{1}^{2} \frac{1}{t} d t}
$$

$\frac{2 \pi}{\log (2)}=\frac{4 i \pi^{2}}{\int_{-i \infty 0+\gamma}^{i \infty+\gamma} \frac{\Gamma(-s)^{2} \Gamma(1+s)}{\Gamma(1-s)} d s}$ for $-1<\gamma<0$

From which:
$(2 \mathrm{Pi} /(\ln (2)))^{*}(1 / 12 \pi \log (2))$

## Input:

$\left(2 \times \frac{\pi}{\log (2)}\right)\left(\frac{1}{12} \pi \log (2)\right)$
$\log (x)$ is the natural logarithm

## Exact result:

$\frac{\pi^{2}}{6}$

## Decimal approximation:

1.6449340668482264364724151666460251892189499012067984377355582293
$1.6449340668 \ldots=\zeta(2)=\frac{\pi^{2}}{6}=1.644934 \ldots$

## Property:

$\frac{\pi^{2}}{6}$ is a transcendental number

## Alternative representations:

$$
\frac{(\pi \log (2)) 2 \pi}{12 \log (2)}=\frac{2 \pi^{2} \log _{e}(2)}{12 \log _{e}(2)}
$$

$\frac{(\pi \log (2)) 2 \pi}{12 \log (2)}=\frac{2 \pi^{2} \log (a) \log _{a}(2)}{12\left(\log (a) \log _{a}(2)\right)}$

$$
\frac{(\pi \log (2)) 2 \pi}{12 \log (2)}=\frac{4 \pi^{2} \operatorname{coth}^{-1}(3)}{12\left(2 \operatorname{coth}^{-1}(3)\right)}
$$

## Series representations:

$$
\frac{(\pi \log (2)) 2 \pi}{12 \log (2)}=\sum_{k=1}^{\infty} \frac{1}{k^{2}}
$$

$$
\frac{(\pi \log (2)) 2 \pi}{12 \log (2)}=-2 \sum_{k=1}^{\infty} \frac{(-1)^{k}}{k^{2}}
$$

$$
\frac{(\pi \log (2)) 2 \pi}{12 \log (2)}=\frac{4}{3} \sum_{k=0}^{\infty} \frac{1}{(1+2 k)^{2}}
$$

## Integral representations:

$$
\frac{(\pi \log (2)) 2 \pi}{12 \log (2)}=\frac{8}{3}\left(\int_{0}^{1} \sqrt{1-t^{2}} d t\right)^{2}
$$

$$
\frac{(\pi \log (2)) 2 \pi}{12 \log (2)}=\frac{2}{3}\left(\int_{0}^{\infty} \frac{1}{1+t^{2}} d t\right)^{2}
$$

$$
\frac{(\pi \log (2)) 2 \pi}{12 \log (2)}=\frac{2}{3}\left(\int_{0}^{1} \frac{1}{\sqrt{1-t^{2}}} d t\right)^{2}
$$

From:
$\mathrm{Pi} /(\ln (2))$

## Input:

$\frac{\pi}{\log (2)}$

$\log (x)$ is the natural logarithm

## Decimal approximation:

4.5323601418271938096276829457166668101718614677237955841860165479
4.53236014.....

Alternative representations:
$\frac{\pi}{\log (2)}=\frac{\pi}{\log _{e}(2)}$
$\frac{\pi}{\log (2)}=\frac{\pi}{\log (a) \log _{a}(2)}$
$\frac{\pi}{\log (2)}=\frac{\pi}{2 \operatorname{coth}^{-1}(3)}$

Series representations:
$\frac{\pi}{\log (2)}=\frac{\pi}{2 i \pi\left\lfloor\frac{\arg (2-x)}{2 \pi}\right\rfloor+\log (x)-\sum_{k=1}^{\infty} \frac{(-1)^{k}(2-x)^{k} x^{-k}}{k}}$ for $x<0$
$\frac{\pi}{\log (2)}=\frac{\pi}{\log \left(z_{0}\right)+\left\lfloor\frac{\arg \left(2-z_{0}\right)}{2 \pi}\right\rfloor\left(\log \left(\frac{1}{z_{0}}\right)+\log \left(z_{0}\right)\right)-\sum_{k=1}^{\infty} \frac{(-1)^{k}\left(2-z_{0}\right)^{k} z_{0}^{-k}}{k}}$

$$
\frac{\pi}{\log (2)}=\frac{\pi}{2 i \pi\left\lfloor\frac{\pi-\arg \left(\frac{1}{z_{0}}\right)-\arg \left(z_{0}\right)}{2 \pi}\right\rfloor+\log \left(z_{0}\right)-\sum_{k=1}^{\infty} \frac{(-1)^{k}\left(2-z_{0}\right)^{k} z_{0}^{-k}}{k}}
$$

## Integral representations:

$$
\frac{\pi}{\log (2)}=\frac{\pi}{\int_{1}^{2} \frac{1}{t} d t}
$$

$\frac{\pi}{\log (2)}=\frac{2 i \pi^{2}}{\int_{-i \infty+\gamma}^{i \infty+\gamma} \frac{\Gamma(-s)^{2} \Gamma(1+s)}{\Gamma(1-s)} d s}$ for $-1<\gamma<0$

From which:
$(\mathrm{Pi} /(\ln (2))) *(1 / 6 \pi \log (2))$

## Input:

$\frac{\pi}{\log (2)}\left(\frac{1}{6} \pi \log (2)\right)$
$\log (x)$ is the natural logarithm

## Exact result:

$\frac{\pi^{2}}{6}$

## Decimal approximation:

1.6449340668482264364724151666460251892189499012067984377355582293
$1.6449340668 \ldots=\zeta(2)=\frac{\pi^{2}}{6}=1.644934 \ldots$

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$$
\frac{(\pi \log (2)) \pi}{6 \log (2)}=\frac{\pi^{2} \log _{e}(2)}{6 \log _{e}(2)}
$$

$$
\frac{(\pi \log (2)) \pi}{6 \log (2)}=\frac{\pi^{2} \log (a) \log _{a}(2)}{6\left(\log (a) \log _{a}(2)\right)}
$$

$$
\frac{(\pi \log (2)) \pi}{6 \log (2)}=\frac{2 \pi^{2} \operatorname{coth}^{-1}(3)}{6\left(2 \operatorname{coth}^{-1}(3)\right)}
$$

## Series representations:

$$
\frac{(\pi \log (2)) \pi}{6 \log (2)}=\sum_{k=1}^{\infty} \frac{1}{k^{2}}
$$

$$
\frac{(\pi \log (2)) \pi}{6 \log (2)}=-2 \sum_{k=1}^{\infty} \frac{(-1)^{k}}{k^{2}}
$$

$$
\frac{(\pi \log (2)) \pi}{6 \log (2)}=\frac{4}{3} \sum_{k=0}^{\infty} \frac{1}{(1+2 k)^{2}}
$$

## Integral representations:

$\frac{(\pi \log (2)) \pi}{6 \log (2)}=\frac{8}{3}\left(\int_{0}^{1} \sqrt{1-t^{2}} d t\right)^{2}$
$\frac{(\pi \log (2)) \pi}{6 \log (2)}=\frac{2}{3}\left(\int_{0}^{\infty} \frac{1}{1+t^{2}} d t\right)^{2}$

$$
\frac{(\pi \log (2)) \pi}{6 \log (2)}=\frac{2}{3}\left(\int_{0}^{1} \frac{1}{\sqrt{1-t^{2}}} d t\right)^{2}
$$

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