

Extending QED to the Full Eight-component Vacuum Wavefunction in the Geometric Representation of Clifford Algebra

Peter Cameron

Abstract

Clifford algebra is the math language of quantum mechanics, known to most physicists in the matrix representations of Pauli and Dirac. Less familiar (but far more intuitive) is the original geometric intent of Clifford, the algebra of interactions of fundamental geometric objects - point, line, plane, and volume elements. In geometric representation, the 3D vacuum wavefunction is comprised of one scalar point, three vector line elements (orientational degrees of freedom), three bivector area elements, and one trivector volume element. Various combinations of the four fundamental constants that define the dimensionless electromagnetic coupling constant alpha (speed of light, permittivity of space, electric charge quantum and angular momentum quantum) permit assigning geometrically and topologically appropriate electric and magnetic flux quanta to the eight wavefunction components, increasing 'dimensionality' of the model to the ten degrees of freedom of string theory. Time (quantum phase) emerges from wavefunction interactions, in the dimension-increasing property of Clifford algebra wedge products. Such a 6D Yang-Mills model is naturally gauge invariant, finite, confined, asymptotically free, background independent, and contains the four forces [1,2].

[1]https://www.researchgate.net/publication/335240613_Naturalness_begets_Naturalness_An_Emergent_Definition

[2]https://www.researchgate.net/publication/335976209_Naturalness_Revisited_Space-time_Spacephase

Extending QED to Interactions of the Full Eight-Component Vacuum Wavefunction of the Geometric Representation of Clifford Algebra

DPF annual meeting
<https://indico.cern.ch/event/1034469/>

Jul 14, 2021, 4:45 PM **this talk**

talk

Beyond Standard M...

Beyond Standard Model

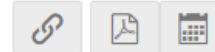
15m

youtube link to dry run

Track L (Zoom) <https://www.youtube.com/watch?v=Q1bNkXmfUq8>

- Clifford algebra in the geometric representation – vacuum wavefunction and geometric quantization
- wavefunction interactions – the geometric product
- the ‘geometric S-matrix’
- physical manifestation – coupling constant and electromagnetic quantization
- the ‘electromagnetic S-matrix’

QED Model of Massless Neutrino Oscillation in the Geometric Representation of Clifford Algebra



Jul 14, 2021, 5:00 PM **next talk**

talk

Beyond Standard M...

Beyond Standard Model

15m

Track L (Zoom)

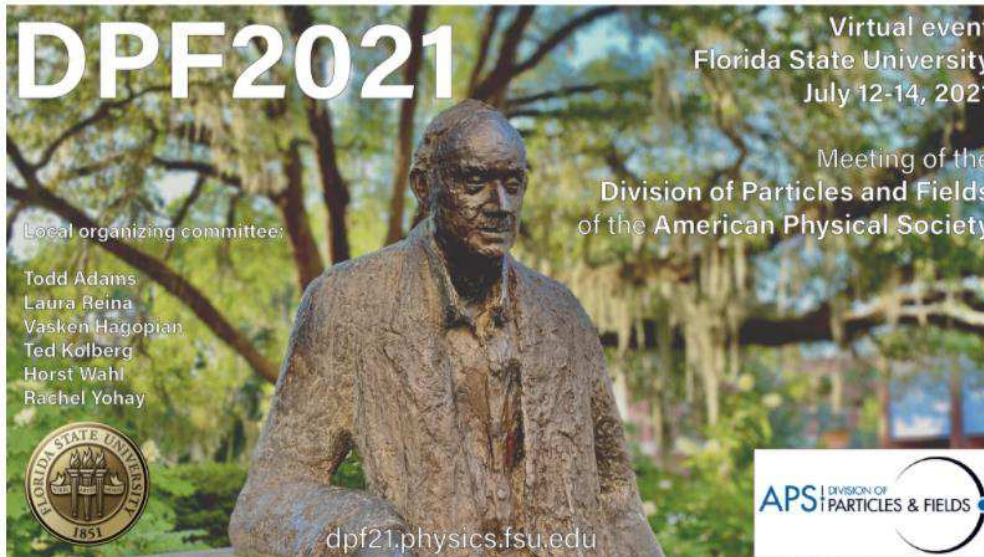
- quantized impedance networks of wavefunction interactions – the connection to physical reality
- examples – H atom, unstable particle spectrum, Planck length, cosmology, chiral anomaly,...
- massless neutrino oscillation and the **muon collider**

[Naturalness begets Naturalness: An Emergent Definition](#)

[Naturalness Revisited: not Spacetime, Spacephase](#)

Extending two component QED Dirac Spinors: Eight component vacuum wavefunctions in the **geometric** representation of Clifford algebra

Peter Cameron
Michigan/MIT/Brookhaven (retired)



DPF2021

Virtual event
Florida State University
July 12-14, 2021

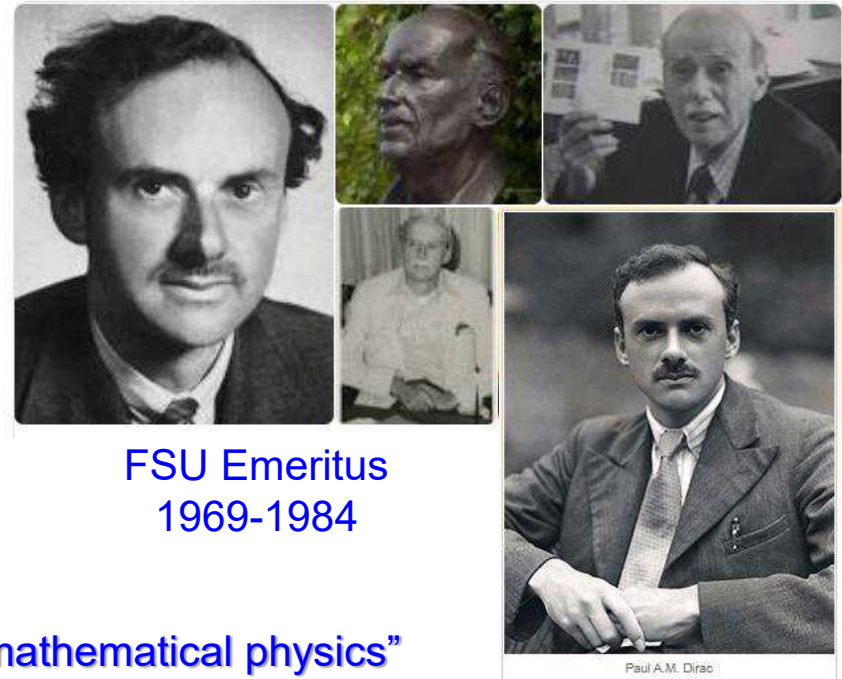
Meeting of the
Division of Particles and Fields
of the American Physical Society

Local organizing committee:
Todd Adams
Laura Reina
Vasken Hagopian
Ted Kolberg
Horst Wahl
Rachel Yohay

FLORIDA STATE UNIVERSITY
1851

aps | DIVISION OF
PARTICLES & FIELDS

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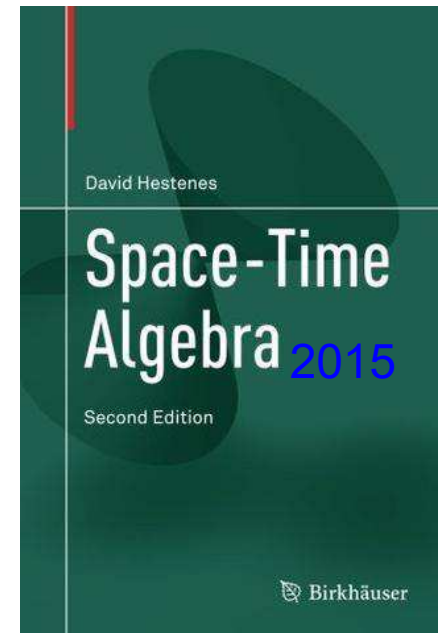
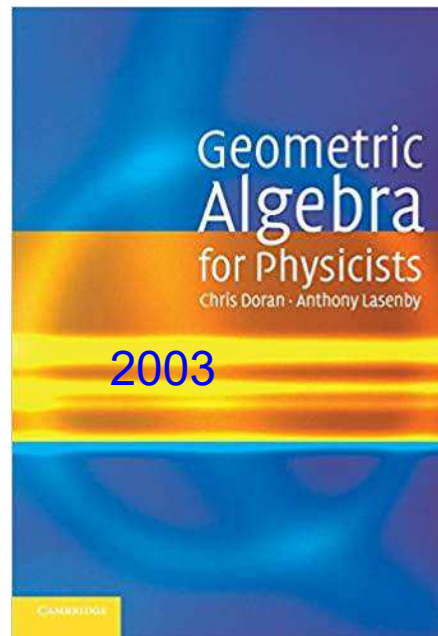
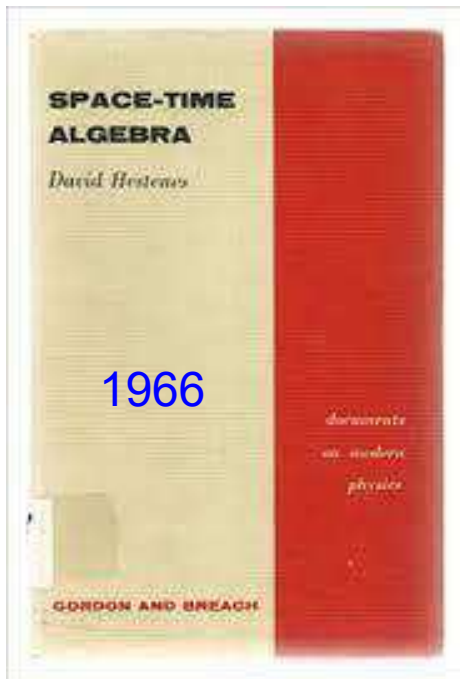


“Geometric Algebra is the universal language for mathematical physics”

Clifford's intent was an algebra of geometric objects. He called it 'Geometric Algebra' (1878, DoD 1879)
GA lost the math war with vector algebra (true in 3D only) of Heaviside and Gibbs (~1900)
Absent GA, the natural language of GR, Einstein adopted the calculational artifice of 'curved space' (1917)
“ “ “ “ “ “ QM, Dirac discovered the unintuitive matrix representation (1928)
GA rediscovered and extended by Hestenes, Cambridge group, many others (1965 onwards)

Division Algebras - add, subtract, multiply, **divide**

- division is essential for invertibility (... topology, singularities, dark matter, T-duality, ...)
- there exist only four division algebras – real, complex, quaternion, and octonion ← Hurwitz theorem
- these are Clifford algebras, more familiar in Pauli and Dirac matrix representations
 - Pauli matrices are basis vectors of 3D space in GA
 - Dirac matrices “ “ “ “ 4D spacetime
- eight-component 3D Pauli algebra is minimally and maximally complete
- the ‘natural’ vacuum wavefunction of quantum mechanics – the same at all scales



Outline

- • “... be bold and explicit in making **claims**...” Hestenes STA 2nd ed.
- the theoretical minimum – three assumptions
- Clifford algebra in the geometric representation – vacuum wavefunction
- wavefunction interactions – the geometric product
- the ‘geometric S-matrix’
- physical manifestation – coupling constant
- the ‘electromagnetic S-matrix’
- origin of inertial mass

- The next talk – massless neutrino oscillations

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... unstable particle lifetimes, branching ratio calculations, dark matter,
dark energy, big bang/bounce, inflation, *quantum interpretations*...

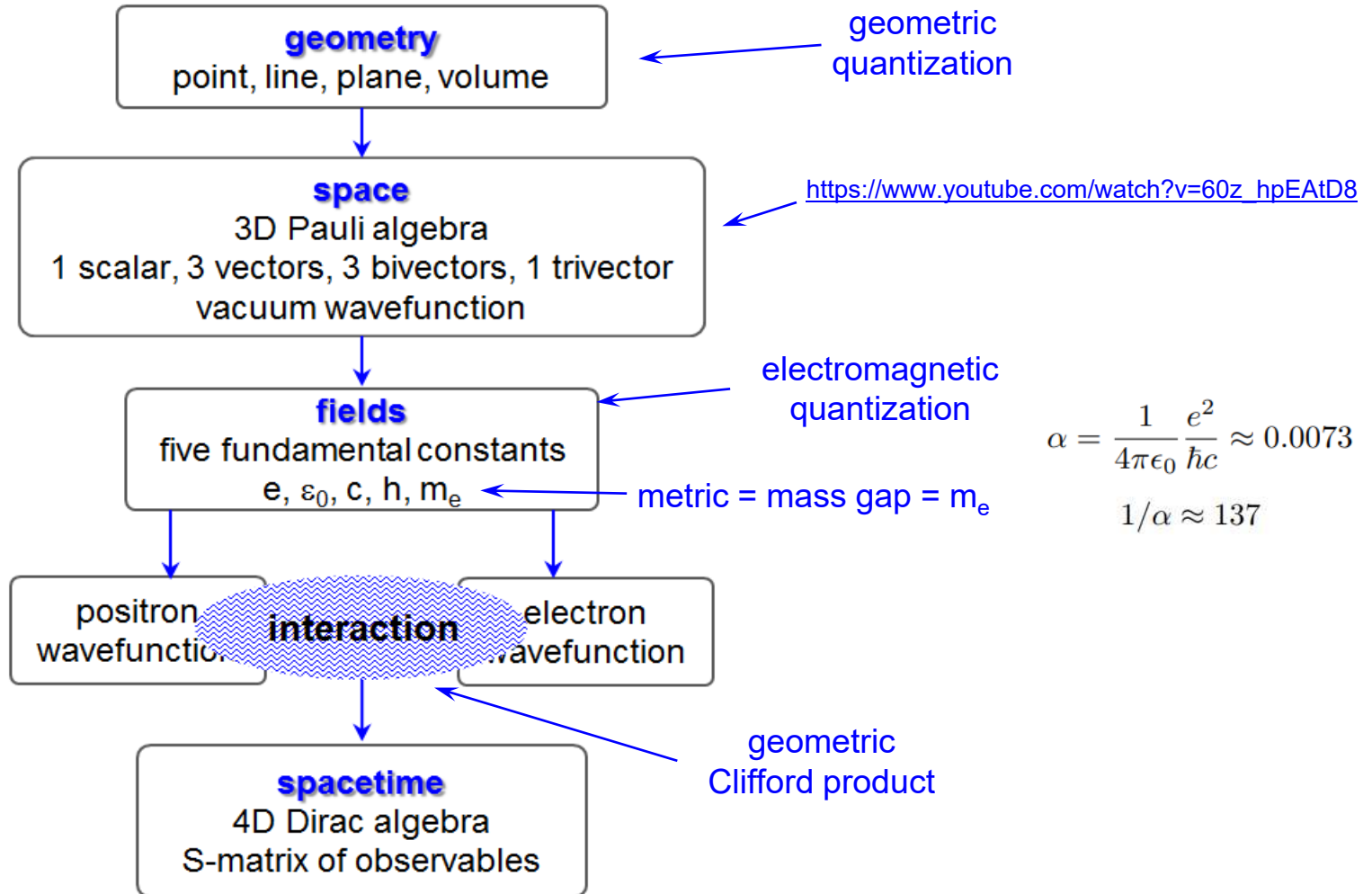
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The Theoretical Minimum

Three assumptions – geometry, fields, and ‘mass gap’



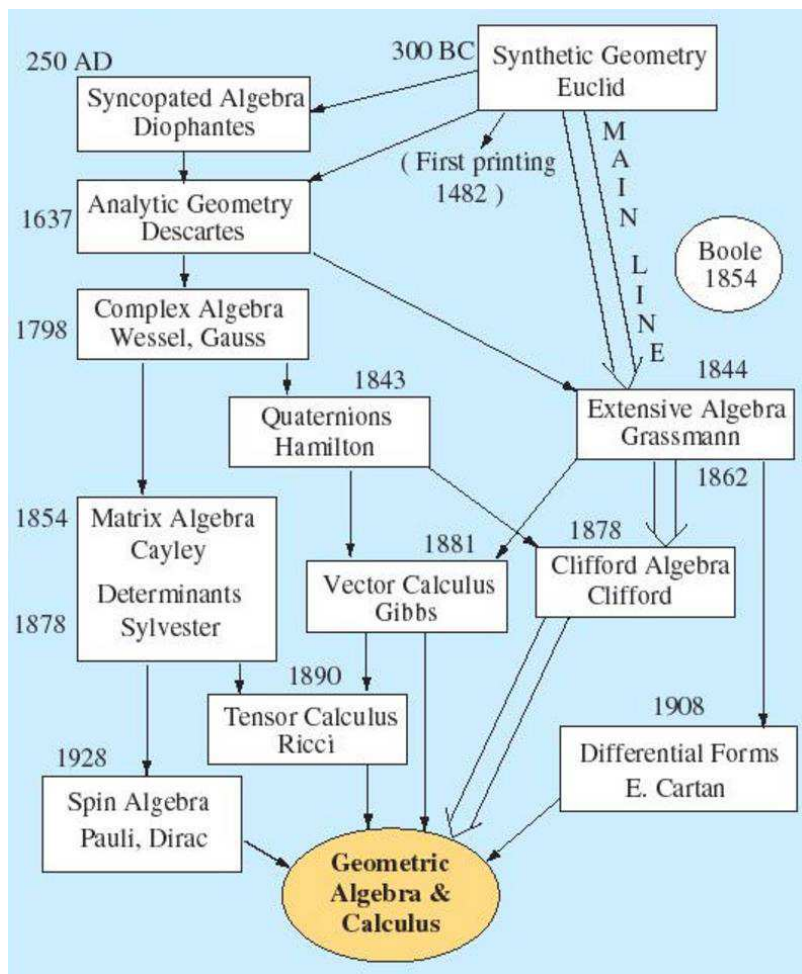
$$\alpha = \frac{1}{4\pi\epsilon_0} \frac{e^2}{\hbar c} \approx 0.0073$$
$$1/\alpha \approx 137$$

no free parameters

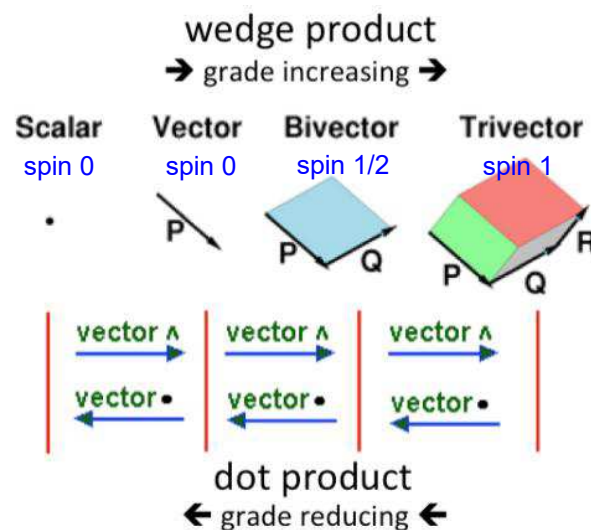
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“Geometric Algebra is the universal language for mathematical physics”



The 2002 Oersted Medal was awarded to David Hestenes by the American Physical Society for “Reforming the mathematical language of physics”



*dynamic susy!
fermions to bosons
bosons to fermions*

Given two vector bosons W and Z , the product WZ changes grades. In the product $WZ = W \cdot Z + W \wedge Z$, two grade 1 vector bosons transform to grade 0 scalar boson and grade 2 bivector fermion $WZ = \text{Higgs} + \text{top}$

Taken together, the four superheavies comprise a minimally complete 2D Clifford algebra – one scalar, two vectors, and one bivector

$$\begin{aligned} \text{sum mode } m_Z + m_W &= m_{\text{top}} \\ \text{difference mode } m_Z - m_W &= m_{\text{bottomonium}} \end{aligned}$$

no Higgs here?

the geometric S-matrix

	scalar	vector	vector	vector	bivector	bivector	bivector	trivector		
scalar	scalar	vector	vector	vector	bivector	bivector	bivector	trivector		
vector	vector	<div style="background-color: blue; color: white; padding: 5px; display: inline-block;"> <i>'geometric quantization'</i> </div>			<div style="background-color: yellow; color: black; padding: 5px; display: inline-block;"> vector + trivector </div>			<div style="background-color: blue; color: white; padding: 2px;"> bivector + quadvector </div>		
vector	vector							<div style="background-color: blue; color: white; padding: 2px;"> scalar + bivector </div>	<div style="background-color: yellow; color: black; padding: 2px;"> vector + trivector </div>	<div style="background-color: blue; color: white; padding: 2px;"> bivector + quadvector </div>
vector	vector							<div style="background-color: blue; color: white; padding: 2px;"> scalar + bivector </div>	<div style="background-color: yellow; color: black; padding: 2px;"> vector + trivector </div>	<div style="background-color: blue; color: white; padding: 2px;"> bivector + quadvector </div>
bivector	bivector	<div style="background-color: yellow; color: black; padding: 5px; display: inline-block;"> vector + trivector </div>			<div style="background-color: blue; color: white; padding: 5px; display: inline-block;"> scalar + quadvector </div>			<div style="background-color: yellow; color: black; padding: 2px;"> vector + pentavector </div>		
bivector	bivector							<div style="background-color: yellow; color: black; padding: 2px;"> vector + trivector </div>	<div style="background-color: blue; color: white; padding: 2px;"> scalar + quadvector </div>	<div style="background-color: yellow; color: black; padding: 2px;"> vector + pentavector </div>
bivector	bivector							<div style="background-color: yellow; color: black; padding: 2px;"> vector + trivector </div>	<div style="background-color: blue; color: white; padding: 2px;"> scalar + quadvector </div>	<div style="background-color: yellow; color: black; padding: 2px;"> vector + pentavector </div>
trivector	trivector	<div style="background-color: blue; color: white; padding: 5px; display: inline-block;"> bivector + quadvector </div>			<div style="background-color: yellow; color: black; padding: 5px; display: inline-block;"> vector + pentavector </div>			<div style="background-color: blue; color: white; padding: 2px;"> scalar + sextavector </div>		

blue background = even dimensions = eigenmodes ~ flavor
 yellow background = odd dimensions = transition modes ~ color

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$$\alpha = \frac{1}{4\pi\epsilon_0} \frac{e^2}{\hbar c} \approx 0.0073$$

$$1/\alpha \approx 137$$

physical manifestation – coupling constant

electric charge

$$e := 1.602176487 \cdot 10^{-19} \cdot \text{coul}$$

magnetic charge

$$g := \frac{\hbar}{e}$$

$$g = 4.1356673326 \times 10^{-15} \text{ tesla} \cdot \text{m}^2$$

magnetic flux quantum

$$\Phi_B := \frac{\hbar}{e}$$

$$\Phi_B = 4.1356673326 \times 10^{-15} \text{ tesla} \cdot \text{m}^2$$

large electric flux quantum
(photon)

$$\Phi_{E1} := \frac{\hbar \cdot c}{e}$$

$$\Phi_{E1} = 1.2398418751 \times 10^0 \text{ mV} \cdot \text{mm}$$

small electric flux quantum
(electron)

$$\Phi_{E2} := \frac{e}{\epsilon_0}$$

$$\Phi_{E2} = 1.809512651 \times 10^{-2} \text{ volt} \cdot \mu\text{m}$$

Bohr magneton

$$\mu_B := \frac{e \cdot \lambda_{\text{bar}} \cdot c}{2}$$

$$\mu_B = 9.2740091365 \times 10^{-24} \frac{\text{joule}}{\text{tesla}}$$

large EDM

$$d_{\text{Bohr1}} := \frac{g \cdot \hbar \text{bar}}{\mu_0 \cdot m_e \cdot c^2}$$

$$d_{\text{Bohr1}} = 4.2391764 \times 10^{-30} \text{ mCoul}$$

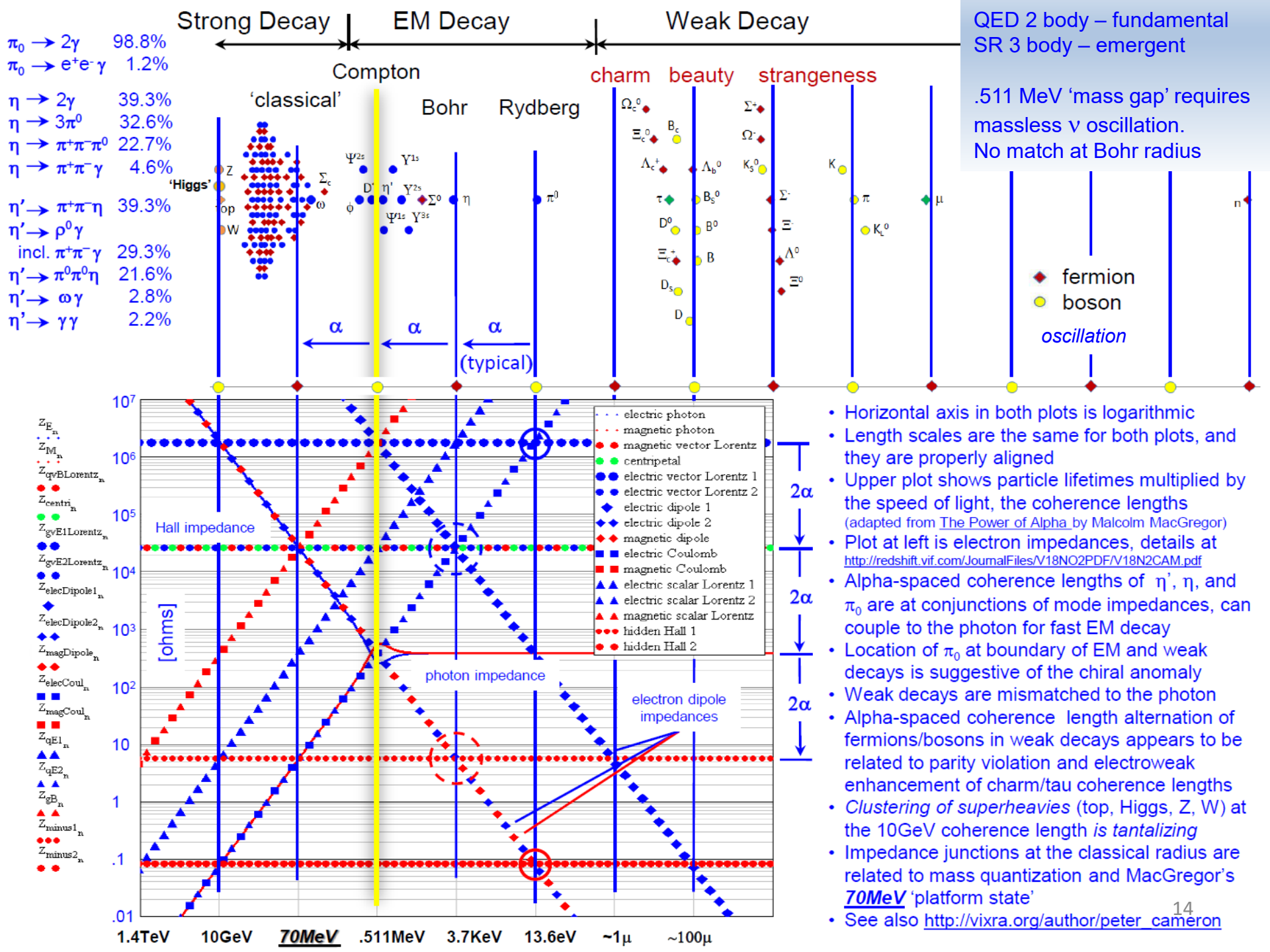
small EDM

$$d_{\text{Bohr2}} := e \cdot \lambda_{\text{bar}} \cdot e$$

$$d_{\text{Bohr2}} = 6.1869529329 \times 10^{-32} \text{ mCoul}$$

	electric charge e scalar	elec dipole moment 1 d_{E1} vector	elec dipole moment 2 d_{E2} vector	mag flux quantum ϕ_B vector	elec flux quantum 1 ϕ_{E1} bivector	elec flux quantum 2 ϕ_{E2} bivector	magnetic moment μ_{Bohr} bivector	magnetic charge g trivector
e	ee scalar	ed_{E1}	ed_{E2} vector	$e\phi_B$ ●	$e\phi_{E1}$ ▲	$e\phi_{E2}$ ▲	$e\mu_B$	eg trivector
d_{E1}	$d_{E1}e$	$d_{E1}d_{E1}$ ◆	$d_{E1}d_{E2}$	$d_{E1}\phi_B$	$d_{E1}\phi_{E1}$	$d_{E1}\phi_{E2}$	$d_{E1}\mu_B$	$d_{E1}g$
d_{E2}	$d_{E2}e$	$d_{E2}d_{E1}$	$d_{E2}d_{E2}$ ◆	$d_{E2}\phi_B$	$d_{E2}\phi_{E1}$	$d_{E2}\phi_{E2}$	$d_{E2}\mu_B$	$d_{E2}g$
ϕ_B	$\phi_B e$ ●	$\phi_B d_{E1}$	$\phi_B d_{E2}$ scalar + bivector	$\phi_B \phi_B$	$\phi_B \phi_{E1}$ γ	$\phi_B \phi_{E2}$ vector + trivector	$\phi_B \mu_B$	$\phi_B g$ bv + qv
ϕ_{E1}	$\phi_{E1} e$ ▲	$\phi_{E1} d_{E1}$	$\phi_{E1} d_{E2}$	$\phi_{E1} \phi_B$ γ	$\phi_{E1} \phi_{E1}$	$\phi_{E1} \phi_{E2}$	$\phi_{E1} \mu_B$	$\phi_{E1} g$ ●
ϕ_{E2}	$\phi_{E2} e$ ▲	$\phi_{E2} d_{E1}$	$\phi_{E2} d_{E2}$	$\phi_{E2} \phi_B$	$\phi_{E2} \phi_{E1}$	$\phi_{E2} \phi_{E2}$	$\phi_{E2} \mu_B$	$\phi_{E2} g$ ●
μ_B	$\mu_B e$ bivector	$\mu_B d_{E1}$	$\mu_B d_{E2}$ vector + trivector	$\mu_B \phi_B$	$\mu_B \phi_{E1}$	$\mu_B \phi_{E2}$ scalar + quadvector	$\mu_B \mu_B$ ◆	$\mu_B g$ vector + pv
g	ge trivector	gd_{E1}	gd_{E2} bivector + quadvector	$g\phi_B$ ▲	$g\phi_{E1}$ ●	$g\phi_{E2}$ ●	$g\mu_B$	gg scalar + sv

S-matrix of Dirac's QED, extended to the full eight-component vacuum wavefunction in the geometric representation of Clifford algebra. Symbols (triangle, diamond,...) correspond to following slides.



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'classical'

$$d_{\text{Bohr}1} \cdot E_1 = 7.0025246458 \times 10^1 \text{ MeV}$$

$$\frac{g}{\mu_0} \cdot B \cdot \lambda_{\text{bar}_e} = 7.0025246458 \times 10^1 \text{ MeV}$$

$$\pi \cdot \epsilon_0 \lambda_{\text{bar}_e}^3 \cdot E_1^2 = 7.0025246458 \times 10^1 \text{ MeV}$$

$$\pi \lambda_{\text{bar}_e}^3 \sqrt{\frac{\epsilon_0}{\mu_0}} E_1 \cdot B = 7.0025246458 \times 10^1 \text{ MeV}$$

$$\frac{\pi \lambda_{\text{bar}_e}^3}{\mu_0} \cdot B^2 = 7.0025246458 \times 10^1 \text{ MeV}$$

Compton

$$2\mu_B \cdot B = 1.02199782 \times 10^0 \text{ MeV}$$

$$d_{\text{Bohr}1} \cdot E_2 = 1.02199782 \times 10^0 \text{ MeV}$$

$$d_{\text{Bohr}2} \cdot E_1 = 1.02199782 \times 10^0 \text{ MeV}$$

$$e \cdot E_1 \cdot \lambda_{\text{bar}_e} = 1.02199782 \times 10^0 \text{ MeV}$$

$$\pi \cdot \epsilon_0 \lambda_{\text{bar}_e}^3 \cdot E_1 \cdot E_2 = 1.02199782 \times 10^0 \text{ MeV}$$

$$\pi \lambda_{\text{bar}_e}^3 \sqrt{\frac{\epsilon_0}{\mu_0}} E_2 \cdot B = 1.02199782 \times 10^0 \text{ MeV}$$

Bohr

$$d_{\text{Bohr}2} \cdot E_2 = 1.4915756772 \times 10^1 \text{ KeV}$$

$$e \cdot E_2 \cdot \lambda_{\text{bar}_e} = 1.4915756772 \times 10^1 \text{ KeV}$$

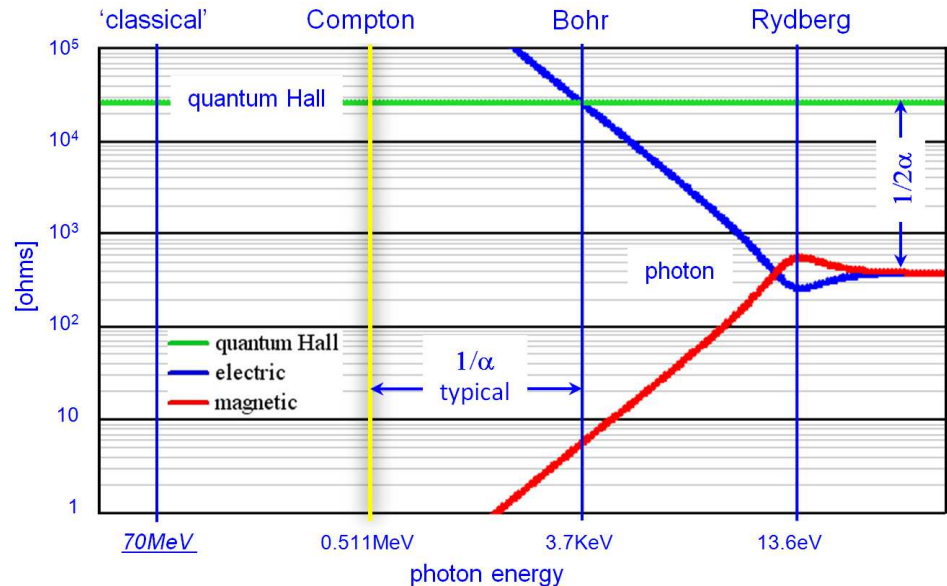
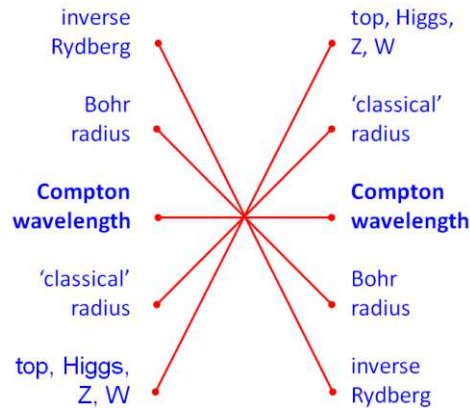
$$\pi \cdot \epsilon_0 \lambda_{\text{bar}_e}^3 \cdot E_2^2 = 1.4915756772 \times 10^1 \text{ KeV}$$

Apeiron, Vol. 18, No. 2, April 2011

topological inversion

electric charge

magnetic charge



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