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An universe age of 13.807 billion years and a proton radius of 0.8403 fm would fit perfectly Dirac/Eddington's Large Number Hypothesis

Content

Abstract

Like Hermann Weyl and Arthur Eddington before, Paul Dirac also noticed that the ratio between the dimensions of the universe (visible size and age) and the protons (diameter, duration of the passage of light through the proton) constitutes a large number of about 10^40. And that the ratio of the gravitational force and the electromagnetic force between an electron and a proton is roughly the same [1]. From this he deduced a time-variation of at least one fundamental „constant“ (he preferred the gravitational constant for this). He could only make a rough estimate, because of course he did not yet know the more precise value of 13.8 billion years world age assumed today. Especially the evaluation of the data from the Planck space telescope has produced this value in the last 10 years [2] [3].

So today we are in a position for a more detailed evaluation.
We will show that gravity could result from an universal time-energy uncertainty relation if we assume that the universe's age is 13.8 billion years. We will give by this approach a precise and straightforward formula for the gravitational constant G without magic factors, powers or roots.

Furthermore there has been a lot of guesswork in recent years about the size of the proton radius. It was triggered in 2010 by the measurements at the Swiss Paul Scherrer Institute, which measured approx. 0.84 femtometers, a radius value that is around 4 percent smaller than the independent measurements previously made. But in the meantime, this smaller value has been confirmed by several other independent measurements and is now considered the more accepted value[4].

We are sure that the value of 0.84 femtometers will soon be rid of any last doubts. Because in a finite-time universe there can be no frequencies smaller than the reciprocal of the universe age T_u and accordingly all energy values in the universe must be an integer multiple of $h / (2pi \cdot T_u)$. Thus all electromagnetic energy amounts must be rounded down by half of this value on average. We will show that for a proton radius of 0.84 femtometers, that energy value rounding is exactly in the same ratio as the gravitational force between proton and electron. So we will derive a simple equation for the proton radius from other

fundamental constants.

And we will show that the ratio between the Hubble radius (radius of observable universe) and the proton radius is obviously a constant that gives the appearance of being invariant in time.

Nothing in our universe is infinitely accurate

All established models in fundamental physics, including Einstein's General Relativity, assume that energy values in our universe can in principle become infinitely accurate.

The quantum mechanics itself says that energies can be exchanged only in discrete form (E=n·hf). But the range of values of these energies is regarded as continuous, because quantum mechanics is currently not interested in whether there can be frequencies f at all, which are infinitely close to 0.

However, a simple consideration of the Heisenberg energy-time uncertainty relation is enough to realize that in a universe that is not infinitely old, infinite energy precision is not possible.

Because of this relation and the obvious assumption that no particle can have a longer lifespan than the universe, we can specify a minimum possible energy uncertainty with

$$\Delta E_{min} >= \frac{h}{4\pi \cdot T_u} \qquad (1)$$

h: Planck's constant = 6.626·10^-34 J·s

T_u: age of the universe ≈ 13.8 billion years ≈ 4.35495·10^17s

The value T_u = 13.8 Gyr was confirmed very precisely by evaluating the data from the Planck telescope several times.

As we will show more detailed in the last section of this work, this universal energy uncertainty relation also follows from the consideration that in our finite-time universe there can be no frequencies smaller than the reciprocal of the universe age T_u and accordingly all energy values in the universe must be an integer multiple of h / (2pi·T_u). Because of this and because of the consideration of the speed of light as the maximum speed of transmission, all electromagnetic energy amounts must be rounded down on the average to half of this value. This way of looking at it also leads us to the same equation in (1).

If we put ΔE(min) in relation to the rest energy of the electron, then we get:

$$\frac{\Delta E_{min}}{E_e} = \frac{h}{4\pi T_u \cdot m_e c^2} \qquad (2)$$

m_e: mass of electron = 9.1·10^-31 kg

c: speed of light in vacuum = 299792458 m/s

This relation (2) provides a value of 1,47815·10^-39.
Not surprisingly: This is also a value that corresponds to the order of magnitude in Dirac's hypothesis.

It is particularly noticeable here that this value is very close to the ratio between the gravitational and electromagnetic force that exists between a proton and an electron. Because with

$$\frac{F(G)_{e,p}}{F(E)_{e,p}} = \frac{G m_e m_p \cdot 4\pi\epsilon_0}{e^2} \qquad (3)$$

we get in relation with (2):

$$\frac{h \cdot e^2}{16\pi^{\,2}\, T_u \cdot m_e c^2 \cdot G m_e m_p \epsilon_0} = 3.3534 \qquad (4)$$

e: elementary charge = 1,6·10^-19 C

ϵ_0: vacuum permittivity = 8.854·10^-11 F/m

m_p: mass of proton = 1,67·10^-27 kg

G: gravitational constant = 6.6743 m³ / kg·s²

That much is already said: We will see below that this value of 3.3534 can be represented by a combination of the fine structure constant α and and the ratio of proton mass and electron mass.

Now we consider the *absolute values* of the electromagnetic and gravitational forces that exist between two hydrogen atoms. For this we add the absolute values of the repulsive and attractive e-forces (otherwise we would have the value 0 and would not need to continue calculating).

The idea is that the gravitational force may result from the energy uncertainty of the electromagnetic interaction in such a way that in a finite universe, there is an asymmetry in the repulsive component and the attractive component. Hydrogen atoms are the ideal representatives of matter in our universe and make up 90% of it. We define the mass of an hydrogen atom as the sum of proton mass and election mass: $m_H = m_p + m_e$.

So:

$$\frac{F(G)\ \ between\ \ two\ \ H-atoms}{(absolute\ \ repulsive F(E) + absolute\ \ attractive\ \ F(E))\ \ between\ \ two\ \ H-atoms}$$

=

$$\frac{F(G)_H}{\sum |F(E)_H|} = \frac{G(m_e + m_p)^2 \cdot 4\pi\epsilon_0}{4e^2} = \frac{G(m_e + m_p)^2 \pi\epsilon_0}{e^2} \qquad (5)$$

Now we do this: We are looking for the factor *x* between two ratios: the ratio of the minimum energy uncertainty to the rest energy of the electron - formula (2) - and the ratio of the gravitational force value of two hydrogen atoms and the summed absolute electromagnetic forces value between the two atoms (5). So:

$$\frac{h}{4\pi T_u \cdot m_e c^2} = x \cdot \frac{G(m_e + m_p)^2 \pi\epsilon_0}{e^2} \qquad (6)$$

If we rearrange this equation to find x and set T_u = 13.8 Gyr then we get:

x ≈ 1/137

So – this is the well known value of the fine structure constant α .
The fact that this fundamental constant comes out as the relation factor between the ratio of electron rest energy to minmal universe's energy and the ratio of gravitational force to absolute electromagnetic force between Hydrogen atoms is a pretty strong link between the electromagnetic and the gravitational interaction.

In addition this relation seems to suggest that the ratio of gravitation and electromagnetism in a very young universe (T_u smaller the reciprocal of electron's compton frequency) corresponds the ratio of the electromagnetic and the strong nuclear force.

So we formulate the conjecture:

$$\frac{h}{4\pi T_u \cdot m_e c^2} = \alpha \cdot \frac{G(m_e + m_p)^2 \pi \epsilon_0}{e^2}$$

or rearranged:

$$\alpha = \frac{he^2}{4\pi^2 T_u \cdot m_e c^2 \cdot G(m_e + m_p)^2 \epsilon_0} \qquad (7)$$

If we insert the known value of the fine structure constant α = 1/137.035999 in (7), then we can calculate an exact value for the age of universe:

$$T_u = \frac{he^2}{4\pi^2 \alpha \cdot m_e c^2 \cdot G(m_e + m_p)^2 \epsilon_0} \qquad (8)$$

By using the well known definitions

$$\alpha = \frac{e^2}{2ch\epsilon_0}$$

and

$$\hbar = \frac{h}{2 \cdot \pi}$$

this can be simplified to:

$$T_u = \frac{2 \cdot \hbar^2}{m_e c \cdot G(m_e + m_p)^2} \quad (9)$$

For this combination of known physical constants we get a value of **T_u = 13.807 billion years**.

This value fits to the current accepted assumption made in the final report of the Planck telescope collaboration of T_u = 13.772±0.040 Gyr [3]. In their 2015 interim report [2], the match was even more accurate. There T_u was given as 13.813±0.038 Gyr .

A straightforward formula for G

Our assumption leads us into the same dilemma as Dirac: An equation with only one parameter (T_u), of which we know for sure that it changes over time and otherwise only constants, inevitably leads to at least one of these constants needs to be reinterpreted as a time-variable parameter.

And the usual suspect is still the one that Dirac had identified: The gravitational constant G. However, below we will see that there should be other options of changing assumed natural constants over time, especially since many experimental data indicate that a change in G alone over time is very unlikely [5].
If we rearrange (9) to find G we get:

$$G = \frac{2 \cdot \hbar^2}{T_u \cdot m_e c \cdot (m_e + m_p)^2} \tag{10}$$

That's a pretty straightforward equation for G. With T_u = 13.807 billion years it provides the accepted value of 6.674·10^-11 m³ / kg·s².

It is in contrast to several other works, e.g. [6] and [7], in which it is attempted to represent G only by other natural constants. These attempts relied on raising a constant to an unexplained higher power or using corrective prefactors.

The proton has actually shrunk

Now let's take a closer look at the proton. What electromagnetic energy is in the proton? No matter which substructure the proton has, from outside we can consider quite classically that the proton is a spherical charge whose energy depends on the elementary charge and its expansion, the proton radius:

$$E_p = \frac{e^2}{4\pi \cdot \epsilon_0 \cdot r_p} \tag{11}$$

e: elementary charge = 1,6·10^-19 C

ϵ_0: vacuum permittivity = 8.854·10^-11 F/m

Now we consider the relation between the minimum energy uncertainty of the universe (1) and this proton energy (11). According to the considerations in the last section of this work, we now assume that it corresponds exactly to the ratio of the Coulomb force and the gravitational force between protons and electrons. The energy uncertainty of the universe leads to the fact that the repulsive Coulomb forces between the sub-elements of the proton weaken to the same extent while the attractive forces to the electrons, which are in a completely different inertial frame, are nearly preserved:

$$\frac{\Delta E_{min}}{E_p} = \frac{F(G)_{e,p}}{F(E)_{e,p}} \qquad (12)$$

This with (1) and (11) and

$$\frac{F(G)_{e,p}}{F(E)_{e,p}} = \frac{Gm_e m_p \cdot 4\pi\epsilon_0}{e^2} \qquad (13)$$

m_e: mass of electron = 9.1·10^-31 kg

m_p: mass of proton = 1,67·10^-27 kg

G: gravitational constant = 6.6743 m³ / kg·s²

we get:

$$\frac{h \cdot 4\pi \cdot \epsilon_0 \cdot r_p}{4\pi \cdot T_u \cdot e^2} = \frac{Gm_e m_p \cdot 4\pi \cdot \epsilon_0}{e^2} \qquad (14)$$

Simplified, rearranged to r_p and using

$$\hbar = \frac{h}{2 \cdot \pi}$$

we get:

$$r_p = \frac{2 \cdot Gm_e m_p \cdot T_u}{\hbar} \qquad (15)$$

In (15) we can insert the equation (9) for T_u

$$T_u = \frac{2 \cdot \hbar^2}{m_e c \cdot G \cdot m_H^2}$$

m_H = mass of hydrogen atom = $m_p + m_e$

which we have already derived above.

Thus we get a simple, clear equation for the proton radius, which one does not see at all that we have derived it over the way of the universe's age:

$$r_p = \frac{4 \cdot \hbar \cdot m_p}{c \cdot m_H^2} \approx \frac{4 \cdot \hbar}{c \cdot m_p} \qquad (16)$$

This gives us a proton radius value of r_p = **$8.403 \cdot 10^{-16}$ m** (or $8{,}412 \cdot 10^{-16}$ m with the simplified rounding $m_H = m_p$).

Let's now compare this value with what the experimentalists and data analysts have published in recent years:

Publisher	**Year published**	**value**	**Deviation from $8.403 \cdot 10^{-16}$ m**
Pohl et al. [8]	2010	0.84184 fm	0.18%
Antognini et al. [9]	2013	0.84087 fm	0.066%
Griffioen et al. [10]	2015	0.840 ±0.016 fm	0.036%
Bezginov et al [11]	2019	0.833 ±0.010 fm	0.88%

The first two measurements listed, which were carried out as part of the CREMA project and which triggered the proton radius puzzle, are very close to our theoretical value. The same applies to an external analysis of measurement data from the University of Mainz, which is listed in the third line. The latest measurement listed confirm the value of 0.84 fm rather than 0.88 fm and their measurement error range includes the predicted value of 0.8403.

A new fundamental constant?

We now want to compare the smallest world with the largest world and form the ratio between the proton radius and the Hubble radius r_U, the radius of the observable universe. With $r_U = c \cdot T_U$ and rearrangement of the equation (15) we get:

$$\frac{r_p}{r_u} = \frac{2 \cdot G m_e m_p}{\hbar \cdot c} \quad (17)$$

c: speed of light in vacuum = 299792458 m/s

The equation for this size ratio is reminiscent of an old acquaintance, the fine structure constant α. The following figure is intended to illustrate this:

$$\frac{r_p}{r_u} = \frac{2 \cdot G m_e m_p}{\hbar \cdot c}$$

$$\alpha = \frac{e^2}{4\pi\epsilon_0 \cdot \hbar \cdot c}$$

If this similarity should mean that also the relation between proton radius and Hubble radius is like the fine structure constant a fixed value, which is assumed not to have changed over a long period of time, then this would have serious consequences for the existing world view in physics. Above we mentioned Dirac's conjecture of a time-varying gravitational constant. However, such a single variation would now no longer be sufficient to explain a constant proton radius to Hubble radius ratio.

The mysterious 3.3534

With the knowledge of these numerical coincidences, we now try to fathom the ominous value of 3.3534, which we encountered in equation (4) when we determined the factor between $\Delta E(min)/E(e)$ and the relation F(G) / F(E) between proton and electron.

$$\frac{\Delta E_{min}}{E_e} = 3.3534 \cdot \frac{F(G)_{e,p}}{F(E)_{e,p}}$$

$E_e = m_e c^2$ – rest energy of the electron

So we repeat our numerical approach above and we are looking for a new factor x. With equation (6) we get:

$$\frac{h}{4\pi T_u \cdot m_e c^2} = \alpha \cdot \frac{G(m_e + m_p)^2 \pi\epsilon_0}{e^2} = x \cdot \alpha \cdot \frac{G m_e m_p 4\pi\epsilon_0}{e^2} \qquad (18)$$

From this we can extract:

$$x = \frac{(m_e + m_p)^2}{4 \cdot m_e m_p} \qquad (19)$$

Rearranging:

$$x = \frac{\frac{m_p}{m_e} + 2 + \frac{m_e}{m_p}}{4} \qquad (20)$$

So we've got it:

$$\frac{\alpha}{4} \cdot (\frac{m_p}{m_e} + 2 + \frac{m_e}{m_p}) = 3.3534 \qquad (21)$$

So we can see how the value is derived from two fundamental constants, namely α and the m_p/m_e ratio.

What is now striking is that this value is also the factor between the classical electron radius $r_{classic}$ and the predicted proton radius r_p :

$$\frac{r_{classic}}{r_p} = \frac{e^2}{4\pi\epsilon_0 \cdot m_e c^2 \cdot r_p} = 3.3534 \qquad (22)$$

And numerically, of course, it is noticeable that the value is close to 10/3, a factor that is quite familiar when considering a homogeneous charge distribution in a sphere. As Kritov[12] already pointed out the reciprocal value 3/10 is the product of the coefficent 3/5 that results from integrating over a sphere of constant charge density and the factor ½ which results from the Virial Theorem „that tells us that the potential energy inside a given volume is balanced by the kinetic energy of matter and equals to half of it.“. That means the mystery of the value 3.3534 can actually be reduced to the mysterious value 1.00602 ≈ 1 + 1/166.

Because:

$$\frac{r_{classic}}{r_p} : \frac{10}{3} = \frac{3 \cdot 3.3534}{10} = 1.00602 \qquad (23)$$

So if the ratio between the predicted proton radius and classical radius would be exactly 10/3, i.e. the proton radius would be larger by about 0.6%, then according to equation (17) the Hubble radius would also be 0.6% larger. For the universe's age a value of nearly 13.89 billion years would result.

For this value Kritov [12] has found an interesting numerical connection:

$$13.89\ Gyr \cdot f_c(e) = 2^{128} \tag{24}$$

where $f_c(e)$ is the reduced compton wavelength of the electron.
That means:

$$1.00602 \cdot T_u \cdot f_c(e) = 2^{128} \tag{25}$$

As already noted, we are not fans of looking for numerical relationships in magic higher powers, roots, and prefactors. Much of this reminds of Kees de Jager's „Cyclosophy“[1] and usually raises more open questions instead of solving any. But here we want to make an exception, because the magic number 2^{128} is somehow something special. Everybody who is a little bit engaged in computer science knows what we mean („digital physics“).

So to conclude this work, let's play around with this number a bit. With (22), (23), (25) and $f_c(e) = 2\pi \cdot m_e \cdot c^2 / h$ we get:

$$\frac{3 \cdot e^2 \cdot T_u}{2 \cdot 10 \cdot h \cdot \epsilon_0 \cdot r_p} = 2^{128} \tag{26}$$

And with $r_U = c \cdot T_U$ and $\alpha = e^2 / (2 \cdot \epsilon_0 \cdot h \cdot c)$ finally:

$$\boxed{\frac{3}{10} \cdot \alpha \cdot \frac{r_u}{r_p} = 2^{128}} \tag{27}$$

As mentioned above, we assume in the ratio of proton radius and Hubble radius a fundamental, time invariant constant equal to the fine structure constant. Both constants are united in (27).

1 https://en.wikipedia.org/wiki/Kees_de_Jager#Cyclosophy

If we replace r_u/r_p by the equivalent from (17) in (27) and rearrange to G, we also obtain the equation as Kritov [12] did:

$$G = \frac{3 \cdot e^2 \cdot 2^{-128}}{20 \cdot 4\pi\epsilon_0 \cdot m_p m_e} \qquad (28)$$

It yields a value of 6.67463·10^-11 m³/(kg·s²), slightly higher than the current CODATA-2018 value we calculated with in this and the previous work (6.67430·10^-11)

If this assumption with the 2^{128} approach is correct, the slightly higher G-value would also mean that the value calculated by us for the universe age would drop slightly from somewhat under 13.807 Gyr to somewhat over 13.806 Gyr., thus around a few hundred thousand years.

Approach to deriving the found numerical coincidences from a fundamental consideration

The starting point of our approach is the simple statement that in a finite universe with age T_u there can be no electromagnetic interactions with frequencies smaller than the reciprocal of T_u . So we formulate

Postulate 1: All (electromagnetic) frequencies in the universe can only be positive integer multiples of the reciprocal of the universe's age.

$$f(u) \in \frac{n}{T_u}, n \in \mathbb{N}$$

$$or$$

$$\omega(u) \in \frac{2\pi \cdot n}{T_u}, n \in \mathbb{N} \qquad (29)$$

Furthermore, we postulate that the space and thus the (electromagnetic) wavelengths can take on significantly finer values. That means there can be longer wavelengths than $c \cdot T_u$ and many possible wavelength values in between $c \cdot T_u / (n+1)$ and than $c \cdot T_u / n$. This is by the way in accordance with the inflation hyothesis of the universe, which states that the visible universe had a significantly larger diameter immediately after the Big Bang than the product of $2 \cdot c \cdot T_u$.

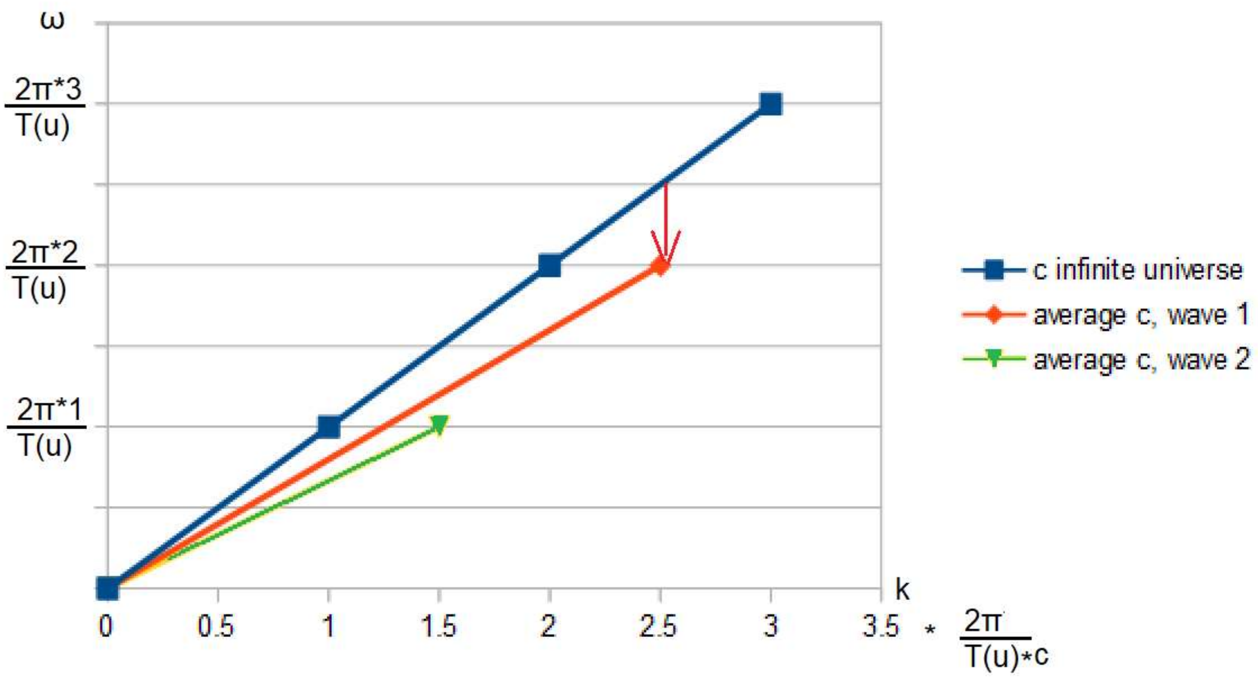
So we can define postulate 2:

$$\lambda(u) \in m * \frac{c \cdot T_u}{n}, n, m \in \mathbb{N}, m \gg 1$$

$$k(u) \in \frac{1}{m} * \frac{2\pi \cdot n}{c \cdot T_u}, n, m \in \mathbb{N}, m \gg 1 \qquad (30)$$

In accordance with the special theory of relativity, we define that no (electromagnetic) wave can travel faster than the speed of light. For all wavelengths not equal to any $n \cdot c \cdot T_u$ this means that their value for the velocity of propagation must be rounded down to a value below c.

This fact should be illustrated in the following diagram:



For the wavelength marked red with lambda = 2.5 c · T_u, the frequency 2.5 / T_u would actually be provided for a continuous frequency range. However, since only discrete values are allowed for the frequencies (n / T_u, n natural number), it must be rounded down to 2·c·T_u, because at f=3·c· T_u the wave would have faster than light speed.

That means: The speed of light c is an upper limit that can only be reached in an infinite universe for the entirety of all wavelengths. In a finite universe all electromagnetic waves with wavelength <> n·c· T_u , that means almost all waves move at a speed below the speed of light even in the purest vacuum and even without the assumption of the interactions with „virtual particles" in the vacuum. We will therefore refer to c as c_∞ from now on and $c_{max}(T_u,\lambda)$ as the expected value of the maximal speed of an electromagnetic wave with the length lambda in a universe with the age of T_u .

So we can make postulate 3:

$$c_{max}(T_u, \lambda) = c_\infty - \Delta c(T_u, \lambda)$$

$$\Delta c(T_u, \lambda) = \frac{\lambda}{4\pi \cdot T_u} \qquad (31)$$

->

$$\boxed{c_{max}(T_u, \lambda) = c_\infty - \frac{\lambda}{4\pi \cdot T_u}}$$

This equation shows that energy states with low frequencies have a higher deviation from c than energies with high frequencies.

Now we can see: the relation $\Delta c(\lambda) / c_\infty$ for a Compton wavelength λ_c corresponds to the relation for the minimum energy uncertainty of the equivalent particle, as we calculated it for the electron in (2):

$$\frac{\Delta c(T_u, \lambda_c)}{c_\infty} = \frac{\lambda_c}{4\pi \cdot T_u \cdot c_\infty} \qquad (32)$$

With the definition of the Compton wavelength of electron:

$$\lambda_c = \frac{h}{m_e \cdot c_\infty}$$

we get:

$$\frac{\Delta c(T_u, \lambda_c)}{c_\infty} = \frac{h}{4\pi T_u \cdot m_e c_\infty^2} = \frac{\Delta E_{min}}{E_e} \qquad (33)$$

Now we have everything together to be able to explain an asymmetry in the attractive and repulsive components of the electromagnetic force: Our matter is structured in such a way that elementary particles with opposite charges are in very different inertial systems, in contrast to particles with the same charges.

As a result, particles with the opposite charge have a smaller wavelength and thus, according to (31), a smaller Δc than particles with the same charge. The larger Δc for particles with the same charge lead to a higher weakening of their repulsive interaction than that of the attractive interaction with the opposite particles (with higher wavelength).

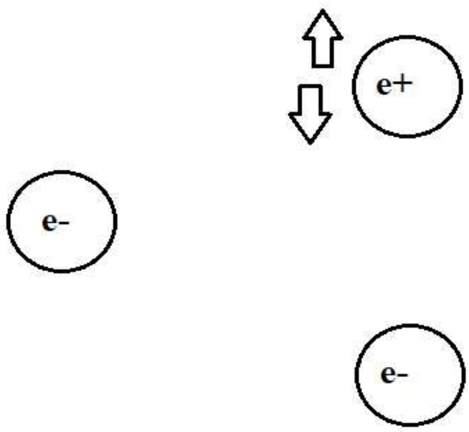


Illustration: In the intertial system of the electrons the moving positron has a higher wavelength and consequently a smaller Δc and so a higher c_max. Therefore the attractive interaction between the moving positron and an electron is higher than the repulsive interaction between the two electrons.

Discussion

We predict a value of 13.807 billion years for the exact universe's age and 0.8403 fm for the proton radius. At the moment, further precision experiments are underway to measure the latter [13].

The solution approach described at the end of this work is a clear break with the established models of fundamental physics, most notably general relativity, which assume that the speed of light in vacuum is independent of the frequency/wavelength. The question now is: who is right? To answer this, one would have to look at a very low frequency/very long wavelength signal that has traveled far and measure the transit time difference to a high frequency signal that has traveled the same distance.

The problem will be to measure just such low frequency signals from space. Even with a signal with a frequency of only 0.1 mHz or λ=3 billion km, the deviation Δ c(λ) according to (31) would be only about 5*10^-7 m/s. But if one could actually measure and compare such extremely low frequency signals without interference, one would be able to falsify or confirm our approach.

So for now let's think if the further proton radius measurements should also yield a value around 0.84 fm and T_u = 13.8 billion years remains consensus in established physics, then it would make more sense to follow the described approach instead of searching further in mathematical ivory towers (string theory et al.) for the connection between universe and elementary particles. Even if this would mean that many established models of thinking in physics, which have emerged in the last 107 years, would have to be questioned.

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