Title: Primality test for Twin Prime numbers. (Argentest II).
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Abstract:
Argentest II is born, a personal research project that develops a new exclusive probabilistic primality test for Twin prime numbers. I present a test similar to Fermat's little theorem.

Twin prime numbers

A twin prime is a prime number that is either 2 less or 2 more than another prime number—for example, either member of the twin prime pair (41, 43). In other words, a twin prime is a prime that has a prime gap of two. Sometimes the term twin prime is used for a pair of twin primes; an alternative name for this is prime twin or prime pair.

Usually the pair (2, 3) is not considered to be a pair of twin primes.[2] Since 2 is the only even prime, this pair is the only pair of prime numbers that differ by one; thus twin primes are as closely spaced as possible for any other two primes.

The first few twin prime pairs are:

(3, 5), (5, 7), (11, 13), (17, 19), (29, 31), (41, 43), (59, 61), (71, 73), (101, 103), (107, 109), (137, 139), … OEIS: A077800.

Five is the only prime that belongs to two pairs, as every twin prime pair greater than (3,5) is of the form (6n+1, 6n-1) for some natural number *n.*

Probabilistic primality test for Twin prime numbers

$$\exists\, k > 0 \in \mathbb{N} / 2k + 1 = p$$

$$\frac{2^{p+2} - 8}{p} \equiv 3(Mod\ p + 2) \leftrightarrow \mathrm{p, p + 2\ are\ primes}$$

$$\therefore\ P \ \wedge\ P + 2\ are\ Twin\ primes$$

Examples

When the two numbers are prime it has congruence.	When at least one of the two numbers is not a prime number, it has no congruence.
Examples	Examples
A. Test for 3 and 5 $\frac{2^5-8}{3} \equiv 3(Mod\ 5)$	H. Test for 9 and 11 $\frac{2^{11}-8}{9} \not\equiv 3(Mod\ 11)$
B. Test for 5 and 7 $\frac{2^7-8}{5} \equiv 3(Mod\ 7)$	I. Test for 13 and 15 $\frac{2^{15}-8}{13} \not\equiv 3(Mod\ 15)$
C. Test for 11 and 13 $\frac{2^{13}-8}{11} \equiv 3(Mod\ 13)$	J. Test for 15 and 17 $\frac{2^{17}-8}{15} \not\equiv 3(Mod\ 17)$
D. Test for 17 and 19 $\frac{2^{19}-8}{17} \equiv 3(Mod\ 19)$	K. Test for 19 and 21 $\frac{2^{21}-8}{19} \not\equiv 3(Mod\ 21)$
E. Test for 29 and 31 $\frac{2^{31}-8}{29} \equiv 3(Mod\ 31)$	L. Test for 21 and 23 $\frac{2^{23}-8}{21} \not\equiv 3(Mod\ 23)$
F. Test for 41 and 43 $\frac{2^{43}-8}{41} \equiv 3(Mod\ 43)$	M. Test for 23 and 25 $\frac{2^{25}-8}{23} \not\equiv 3(Mod\ 25)$
G. Test for 59 and 61 $\frac{2^{61}-8}{59} \equiv 3(Mod\ 61)$	N. Test for 27 and 29 $\frac{2^{29}-8}{27} \not\equiv 3(Mod\ 29)$

This test is probabilistic since there are pseudo-prime numbers that pass the test like 561.

Test for 561 and 563

$$\frac{2^{563}-8}{561} \equiv 3(Mod\ 563)$$

561 is a composite number.
563 is a prime number.
Therefore these numbers are not twin prime.

Pseudo prime numbers (Psp) are a tiny portion of composite numbers that pass the test, these are known as Carmichael numbers.

$$These\ Pseudoprime\ have\ a\ prime\ partner\ P = Psp + 2$$

$Psp = \{561, 1905, 2465, 4371, 23001, 25761, 60701, 87249, 158369, \ldots\ldots\}$

$$These\ prime\ have\ a\ pseudoprime\ partner\ Psp = P + 2$$

$P = \{1103, 2699, 2819, 3643, 4679, 6599, 10259, 12799, 14489, 18719, \ldots\}$

Probabilistic primality test for Twin prime numbers

Demonstration

$$\exists\ k > 0 \in \mathbb{N}/2k + 1 = p$$

$$\boldsymbol{\frac{2^{p+2} - 8}{p} \equiv 3(Mod\ p + 2) \leftrightarrow \mathbf{p}, \mathbf{p} + 2\ \mathbf{are\ primes}}$$

Demonstration $when\ p\ =\ prime\ number\ and\ (p + 2)\ also.$

First part	Example
$\boldsymbol{\frac{2^{p+2} - 8}{p}}$ $2^{p+2} - 8 \equiv (mod\ p)$ $= 2^{p+1} - 4 \equiv (mod\ p)$ $= 2^p - 2 \equiv (mod\ p)$ $\boldsymbol{= 2^p \equiv 2(mod\ p)}$ Fermat's Little Theorem	$\boldsymbol{\frac{2^{19} - 8}{17}}$ $2^{19} - 8 \equiv (mod\ 17)$ $= 2^{18} - 4 \equiv (mod\ 17)$ $= 2^{17} - 2 \equiv (mod\ 17)$ $= 2^{17} \equiv 2\ (mod\ 17)$
Second part $\boldsymbol{\frac{2^{p+2} - 8}{p} \equiv 3(Mod\ p + 2)}$ $= \frac{2^p - 8}{p - 2} \equiv 3(Mod\ p)$ $= 2^p - 8 \equiv 3(p - 2)(Mod\ p)$ $= 2^p - 8 \equiv 3p - 6(Mod\ p)$ $\boldsymbol{Then\ \ 3p \equiv (Mod\ p)}$ $= 2^p - 8 \equiv -6(Mod\ p)$ $= 2^p \equiv -6 + 8\ (Mod\ p)$ $\boldsymbol{= 2^p \equiv 2\ (Mod\ p)}$ Fermat's Little Theorem	**Example** $\boldsymbol{\frac{2^{19} - 8}{17} \equiv 3(Mod\ 19)}$ $= 2^{19} - 8 \equiv 3 * 17(mod\ 19)$ $= 2^{19} - 8 \equiv 51(mod\ 19)$ $= 2^{19} \equiv 51 + 8(mod\ 19)$ $= 2^{19} \equiv 59(mod\ 19)$ $= 2^{19} \equiv 57 + 2(mod\ 19)$ $\boldsymbol{Then\ \ \ 57 \equiv (Mod 19)}$ $= 2^{19} \equiv 2(mod\ 19)$

Fermat's theorem

Theorem: Fermat's Little Theorem, If p is a prime number, then, for each natural number a, with a> 0

$$a^p\ \equiv\ a\ (mod\ p)$$

Program with Python 3.9

```
# Probabilistic primality test for Twin prime numbers.
# Author Gabriel M Zeolla


n = input("Enter Odd number: ")
if int(n) % 2 == 0:
    print("ERROR")
    n=   input("Enter Odd number: ")
    if int(n) % 2 == 0:
     print("ERROR")

x = ((2** (int(n)+2) - 8) // (int(n)))
r=x % (int(n)+2)

p = r == 3

if p is True:
        print(n, "and", int(n)+2, " are probable Twin prime numbers")
else:
        print(n, "and", int(n)+2,'are not Twin Prime!!')
```

Conclusion

Except for the difficulty generated by the pseudo-prime numbers, this test works correctly for all twin prime numbers without any exception.

Professor Zeolla Gabriel Martín

Other works of the author
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