Test for primes and twin primes

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ABSTRACT: the article provides a test for twin primes and new tests for primes, which are radically different from the known tests.
KEY NUMBERS: primes, twin primes, sieve, test.

1. INTRODUCTION

Prime numbers are the basis of many open questions in mathematics, so simple ways to find primes and efficient algorithms for checking natural numbers for simplicity are urgent problems.

The Sieve of Eratosthenes, the Sieve of Sundaram and the Sieve of Atkin allow you to find an initial list of primes up to a certain value [1]. However, in practice it is often necessary to check if a given number is prime, instead of getting a list of primes. Algorithms that solve this problem are called simplicity tests.

The existing algorithms for checking a number for simplicity are conventionally divided into two categories: true tests of simplicity and probabilistic tests of simplicity. The result of calculations of true tests is always the fact that the number is simple or composable. A probabilistic test shows whether a number is prime with some probability. Numbers that satisfy the probabilistic test of simplicity, but are composite, are called pseudosimple. One of the examples of such numbers is the Carmichael numbers [2].

One example of true tests of simplicity is the Luc-Lehmer Test for Mersenne numbers. The disadvantage of this test is that it only applies to numbers of a certain kind. Other examples include those based on Fermat's little theorem [3]: Pepin's test for Fermat numbers, Proth's theorem for Proth's numbers, Agrawal-Kayal-Saxena test (which is considered the first universal, polynomial, deterministic and unconditional test of simplicity), Lucas - Lehmer - Riesel test and others. Probabilistic tests of simplicity include: Fermat's test, Miller – Rabin primality test, etc.

2 TEST FOR PRIME NUMBERS

Based on the results of the study of the laws of prime numbers, carried out by the author, the following theorems were formulated.

Theorem 2.1. A natural number of the form $k^{-} = 6n - 1$ is a prime number only if, for any natural numbers n, m, t the following inequality is obtained:

(2.1) $n \neq (6m \mp 1)t \pm m.$

Note 1: in formula (2.1), the signs inside the bracket and outside the bracket must be different.

Theorem 2.2. A natural number of the form $k^+ = 6n + 1$ is a prime number only if, for any natural numbers n, m, t, the following inequality is obtained:

(2.2) $\quad n \neq (6m \mp 1)t \mp m.$

Note 2: in formula (2.2), the signs inside and outside the brackets must be the same.

Note 3: The upper signs (-) and (+) in the notation of the numbers k^- and k^+ correspond to the signs of the formulas of these numbers.

The above theorems can be used as a test to establish the simplicity of natural numbers. To check whether a given natural number a is a prime number, first, using the formulas $k^- = 6n - 1$ and $k^+ = 6n + 1$, the form of the number is determined. Let the given number be of the form $a = 6n - 1$, then if a is *a* prime number, then *n* must satisfy the following two inequalities:

1) $n \neq (6m - 1) + m$; 2) $n \neq (6m + 1) - m$; $n, m \in \mathbb{N}$.

If *n* does not satisfy even one of the two conditions, then the number a will be *a* composite number.

3 TEST FOR TWIN PRIMES

Theorem 3.1. Natural numbers of the form $k^- = 6n - 1$; $k^+ = 6n + 1$ are simple twins only if the following inequalities are obtained for any natural numbers n, m, t:

(3.1) $\quad n \neq (6m \mp 1)t \pm m,$

(3.2) $\quad n \neq (6m \mp 1)t \mp m.$

Note 4: In formula (3.1), the signs inside and outside the brackets must be different, and in formula (3.2), the signs inside and outside the brackets must be the same.

Theorem 3.1, which combines Theorems 2.1 and 2.2, can be used as a test for twin primes. Theorem 3.1 can also be used to prove that there are infinitely many twin primes.

The proofs of the above Theorems 2.1, 2.2 and 3.1 are elementary, however, due to the fact that their proofs reveal important points of unpublished works, the author decided not to include them in this article.

Reference

1. David Gries, Jayadev Misra. A Linear Sieve Algorithm for Finding Prime Numbers. — 1978.
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