Motion From a Particle's Point of View: Relativistic Mechanics

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Abstract

Motion has traditionally been defined *extrinsically*, as the change in the position of an object, such as a particle, over time. In previous work titled "Motion From a Particle's Point of View: An Interpretation of the Double Slit Experiment" we argued that this definition of motion is incomplete and that it is using it which leads to the "weirdness" of quantum mechanics, as exemplified by the double slit experiment. We proposed an alternative *intrinsic* definition of motion, using which we claimed quantum mechanics can be understood without weirdness. In this paper, we extend this previous work, which only considered constant-speed motion of a single particle, to the case of: first, multiple particles in constant-speed motion; and, second, the case of multiple accelerating, that is, interacting, particles. We show we can in this way recover the key equations and phenomena of both special and general relativity. By showing that the intrinsic definition of motion underlies the theories of special and general relativity, as well as quantum mechanics, we propose it may be of use in developing a theory of quantum gravity.

1. Introduction

This paper is a direct sequel to our previous work [1], which we assume the reader is familiar with. We do not repeat the discussions, definitions, equations, postulates and lemmas of that paper here, but refer to them as needed.

In [1], we defined motion intrinsically, with the speed v along a "road" given by the reciprocal of the period of the repetition of "scenery" (such as lampposts, or graduations of a coordinate scale) along the road seen from the point of view of the object moving along the road:

$$v = \frac{1}{\tau} \tag{1}$$

As discussed in [1], roads can join and split at junctions. We can thus describe the entire road network through which a particle can move as a *road graph*, composed of a set of *vertices*, the junctions, and a set of *edges*, the roads connecting the junctions. We label each vertex by a lower case letter a, b, c ... Each road R_{ij} is identified by a pair of vertices i and j such that motion along the road is possible from i to j. We consider all the roads to be one-way, so that if motion is possible in both directions between two vertices i and j, we must have two roads connecting them, R_{ij} and R_{ji}.

Given a road R_{ij}, we denote the period of repetition of scenery along it τ_{ij} and the intrinsic speed along it as v_{ij}. These are related as before by (1):

$$v_{ij} = \frac{1}{\tau_{ij}} \tag{2}$$

We denote the total time it takes to travel from i to j by t_{ij}.

In [1], we only considered the case of a single particle moving through a road graph. In section 2 below we consider the case of multiple particles moving through the road graph, and how we can reason about the relative motion of any pair of particles. The resulting theory is identical to special relativity. In section 3 we examine the case where the multiple particles in the road graph can accelerate by interacting with each other, and show that from this we can recover the key equations and phenomena of general relativity. We thus show that using the intrinsic definition of motion allows for a framework in which both quantum and gravitational phenomena can be analyzed.

2. Special Relativity from a Road Graph

In physics, we normally think of spacetime as having a primary existence, as really being "out there," and as particles simply moving "in" this spacetime. This assumption is hidden in the traditional definition of motion as the change in position over a change in time. In the intrinsic view we turn this worldview "inside-out." We start with particles which can move. We think of a particle as akin to a driver in a car driving along a road, and gauging its speed by the period of repetition of scenery. If the road is part of a road graph, then spacetime is, in our view, just the best model or map that can be made of the junctions of the roads based only on information available to particles travelling through it. With this "flipped" worldview in mind, we now seek to reconstruct the mathematics of relativity.

A basic property of a road graph is that, in general, the total time to travel between two given vertices depends on the route taken, that is, on what other vertices are visited along the way. Let's study the simplest such case. Let's say Alice travels through our road graph from vertex a to vertex b along a road R_{ab} in a time t_{ab} and with intrinsic speed v_{ab}. Let's say Bob also starts at vertex a, but then travels to a vertex c (in time t_{ac} and with intrinsic speed v_{ac}), before continuing to vertex b (in time t_{cb} and with intrinsic speed v_{cb}). We imagine that during their travels, both Alice and Bob (our *observers*) write down the durations and intrinsic speeds of every road they traverse, and then they both stop at vertex b and meet up to compare notes, and try to build a map of the journeys they took.

That is, they will try to build a "diagram" or "map" in an Euclidean n-dimensional space which is the best possible representation of their journeys given only the intrinsic measurements they possess. As in [1], they will draw each section of road in between adjacent junctions as a straight line segment in this space. Unlike [1], which considered non-relativistic mechanics and therefore used a "pseudoextrinsic" definition of distance, in the present paper our observers will use the intrinsic definition of distance (equation (3) of [1] and (3) below) to fix the length of a road given its duration and intrinsic speed. In the case of multiple interconnected roads, Alice and Bob can use the intrinsic distances between the junctions to trilaterate the relative locations of the junctions in their diagram or map.

For every road R_{ij} traversed by Alice and Bob they multiply the duration t_{ij} by the intrinsic speed v_{ij} to compute the total intrinsic distance x_{ij} travelled from their point of view:

$$x_{ij} = v_{ij}\, t_{ij} \tag{3}$$

So when Alice and Bob meet up at vertex b they can draw a map of their journeys as a triangle in an Euclidean space, as shown in figure 1.

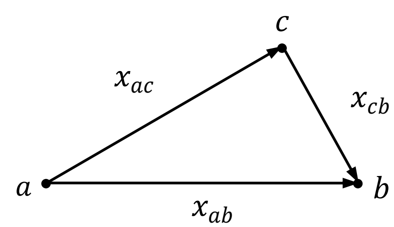

Fig. 1: Alice's and Bob's map of their journeys.

Let's say Alice wants to figure out at what point during her journey along R_{ab} she was the closest to vertex c on their map. We call this the *perigee point* of R_{ab} with respect to c. We denote the time it occurs T_{ab} as measured by Alice's clock, and let $X_{ab} = v_{ab} T_{ab}$. The perigee point can be found by drawing a line segment from, and perpendicular to, the line segment ab and ending at c in figure 1. We call the length of this new line segment the *perigee distance* h. This is shown in figure 2.

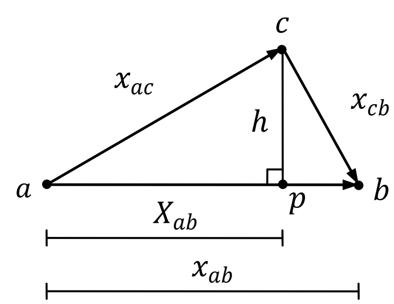

Fig. 2: p is the perigee point and h the perigee distance of road R_{ab} with respect to c.

The perigee distance is a purely artificial construct, and it cannot be measured during Alice's and Bob's journeys, only inferred after they meet up at b and compare their notes regarding the roads they traversed. Nevertheless, the distance is useful to have if we wish to build a map of a road graph, and it has some useful mathematical properties which we develop below.

From the right angle triangle apc in figure 2 we have:

$${x_{ac}}^2 = {X_{ab}}^2 + h^2 \tag{4}$$

that is,

$$\frac{{X_{ab}}^2}{{x_{ac}}^2} = 1 - \frac{h^2}{{x_{ac}}^2} \tag{5}$$

Let's define t as follows:

$$t \stackrel{\text{def}}{=} \frac{1}{v_{ac}} h \tag{6}$$

which is the time Bob would need to travel at the same speed he travelled from a to c to cover the perigee distance from c to p. Then we can write:

$$\frac{(v_{ab} T_{ab})^2}{(v_{ac} t_{ac})^2} = 1 - \frac{t^2}{{t_{ac}}^2} \tag{7}$$

or

$$\frac{{t_{ac}}^2}{{T_{ab}}^2} = \frac{{v_{ab}}^2}{{v_{ac}}^2} \frac{1}{1 - \frac{t^2}{{t_{ac}}^2}} \tag{8}$$

Now we define the quantity v_r by:

$$v_r \stackrel{\text{def}}{=} \frac{t}{t_{ac}} = \frac{h}{x_{ac}} \tag{9}$$

which has a natural interpretation as the magnitude of the *relative velocity* of Alice with respect to Bob. It is a ratio of the distance Bob would have to travel from c to the perigee point p on R_{ab} to the distance he travelled along R_{ac}. It ranges from 0, in the case where the perigee distance is zero and c lies on R_{ab}, to 1, if R_{ab} and R_{ac} are perpendicular. v_r is therefore a measure of the angle between the two roads, ranging from 0 if they are parallel, to 1 if they are perpendicular, and so is also a useful quantity to define when building a map of a road graph from purely intrinsic measurements.

Let's consider the simple case when $v_{ac} = v_{ab}$, that is, when Alice and Bob see themselves to be travelling with the same intrinsic speed. We then have:

$$\frac{t_{ac}}{T_{ab}} = \frac{1}{\sqrt{1 - {v_r}^2}} \tag{10}$$

This expression has a very simple interpretation if we make one more simplifying assumption, that both roads Bob traverses are of the same duration and intrinsic speed: $t_{ac} = t_{cb}$ and $v_{ac} = v_{cb}$. Then the triangle abc in figure 2 is isosceles. When Alice and Bob meet up at b Alice will have travelled for a total time $t_{ab} = 2T_{ab}$, while Bob's total travel time would be $2t_{ac}$. Alice and Bob will interact at vertex b, that is, they will be both disturbed in the sense of P3 of [1]. But they will not agree on the time this disturbance occurred, and there is no way to assign a "correct" absolute time to when this disturbance occurred. Instead, Alice will claim the time was $2T_{ab}$, and Bob will claim it was $2t_{ac}$. Since (10) implies $t_{ac} > T_{ab}$ if $v_r > 0$, we say that Bob experiences time dilation, or that "moving clocks run slow," a key effect in the theory of special relativity.

Note that whatever v_r is (still assuming $v_{ac} = v_{ab}$) we always have:

$$T_{ab}{}^2 = t_{ac}{}^2 - t^2 \tag{11}$$

We interpret this as follows, again assuming the triangle abc is isosceles. Alice takes time $t_{ab} = 2T_{ab}$ to go from a to b. Depending on v_r, Bob will take a different time $2t_{ac}$ to go from a to b. But, independent of the magnitude of the relative velocity, the quantity $t_{ac}{}^2 - t^2$ will be constant for any given journey of Alice from a to b. We therefore call the quantity (11) an *invariant*. This is the same invariant known as Alice's proper time (time measured in Alice's rest frame) in special relativity.

When Bob is at vertex c, his clock reads t_{ac}, and the perigee distance is given by $h = v_r v_{ac} t_{ac}$. However, at the perigee point Alice's clock reads T_{ab} which is different from t_{ac} if $v_r > 0$. We can ask what the perigee distance to Alice is when Bob is partway along the road from a to c, having travelled a time equal to T_{ab}, given by $h' = v_r v_{ac} T_{ab}$. We then have:

$$\frac{h'}{h} = \frac{v_r v_{ac} T_{ab}}{v_r v_{ac} t_{ac}} = \sqrt{1 - v_r{}^2} \tag{12}$$

where we used (10). If $v_r > 0$, $h' < h$, so we say that "moving rods shrink," the phenomenon known as Lorentz contraction in special relativity.

We can summarize (10) and (12) by noting that either Alice and Bob agree on the distance h between them, but measure different times T_{ab} and t_{ac} related by (10), or they agree on the time T_{ab}, but measure different distances h and h' related by (12). These two equations form the basis of the Lorentz transform, fundamental to special relativity. We are using natural units, in which the speed of light $c = 1$. We also recognize the fact that v_r as defined in (9) cannot exceed 1 as the special relativity principle that nothing can travel faster than the speed of light.

Now that Alice and Bob know how to map out a triangle of roads, and how to define useful quantities for such triangles, they wish to map out a more complicated graph. We assume that this road graph is bidirectional, so that for every road R_{ij} there is a road R_{ji} with the same duration and intrinsic speed.

For any road R_{ij}, Alice and Bob can compute the intrinsic distance x_{ij} between i and j using (3). Let's say Bob travels through the road graph in such a way that he traverses each road R_{ij} at least once. He records all the distances x_{ij} between all pairs of vertices connected by a road. Now Bob wants to build a map of this road graph in a 3-dimensional Euclidean space. As discussed above, in this map each road between adjacent vertices i and j will be a straight line segment of length x_{ij}. Every vertex i is represented by a point with coordinates chosen so that all the distances between these points match the intrinsic distance between each pair of vertices. This process is known as trilateration. The idea is the same as reconstructing the map of a country from a table of straight-line distances between cities. A general road graph may not be able to be represented in a 3-dimensional Euclidean space, but if a specific road graph can be, we call it a *Euclidean 3-dimensional road graph*. In such a graph, each vertex i is represented by a point with coordinates given by:

$$\vec{x}_i = \begin{bmatrix} x_{1i} \\ x_{2i} \\ x_{3i} \end{bmatrix} \tag{13}$$

Such an assignment of coordinates is not unique. We use the superposed arrow on a letter to indicate 3-vectors.

Given an assignment of coordinates in a Euclidean 3-dimensional road graph, we can generalize (10) and (11) as follows. We define:

$$\vec{x}_{ij} = \vec{x}_j - \vec{x}_i \tag{14}$$

and

$$\vec{t}_{ij} = \frac{1}{v_{ij}} \vec{x}_{ij} \quad \text{and} \quad \vec{T}_{ij} = \frac{1}{v_{ij}} \vec{X}_{ij} \tag{15}$$

Referring to figure 2 and the surrounding discussion, we have:

$$\vec{x}_{ac}{}^2 = \vec{X}_{ab}{}^2 + \vec{h}^2 \tag{16}$$

where $\vec{X}_{ab}$ are the coordinates of the perigee point p minus the coordinates $\vec{x}_a$ of a, and $\vec{h}$ are the coordinates of the perigee point minus $\vec{x}_c$, those of c. It follows that (compare (5)):

$$\frac{\vec{X}_{ab}{}^2}{\vec{x}_{ac}{}^2} = 1 - \frac{\vec{h}^2}{\vec{x}_{ac}{}^2} \tag{17}$$

Similarly to (6) we define:

$$\vec{t} \stackrel{\text{def}}{=} \frac{1}{v_{ac}} \vec{h} \tag{18}$$

Using (15) and (17) we have:

$$\frac{v_{ab}{}^2 \vec{T}_{ab}{}^2}{v_{ac}{}^2 \vec{t}_{ac}{}^2} = 1 - \frac{\vec{t}^2}{\vec{t}_{ac}{}^2} \tag{19}$$

or:

$$\frac{\vec{t}_{ac}{}^2}{\vec{T}_{ab}{}^2} = \frac{v_{ab}{}^2}{v_{ac}{}^2} \frac{1}{1 - \frac{\vec{t}^2}{\vec{t}_{ac}{}^2}} \tag{20}$$

Now we can define the relative velocity vector by:

$$\vec{v}_r \stackrel{\text{def}}{=} \frac{\vec{t}}{|\vec{t}_{ac}|} \tag{21}$$

to obtain:

$$\frac{\vec{t}_{ac}{}^2}{\vec{T}_{ab}{}^2} = \frac{v_{ab}{}^2}{v_{ac}{}^2}\frac{1}{1-\vec{v}_r{}^2} \tag{22}$$

(Compare with (8)). If we assume $v_{ac} = v_{ab}$, then this is the same equation as (10), except expressed in terms of coordinates of vertices in the "map" space, not just the intrinsic distances between vertices as (10) was.

To make further connections with special relativity, let us define the 4-vector $\boldsymbol{x}_{ij}$ (we use bold letters for 4-vectors) by:

$$\boldsymbol{x}_{ij} \stackrel{\text{def}}{=} \begin{bmatrix} |\vec{t}_{ij}| \\ \vec{t} \end{bmatrix} \tag{23}$$

So, $\boldsymbol{x}_{ac}$ is a 4-vector where the 0-component is the reading on Bob's clock when he reaches vertex c from a, and the remaining components 1, 2, 3 are a 3-vector equal to the difference in coordinates between the perigee point of Alice and c divided by v_{ac}. The 4-vector can be thought of as the answer to the question of where Alice's perigee point is from Bob's point of view. In special relativity, this is called the coordinates of Alice in a moving frame of reference belonging to Bob. We remind ourselves, however, that the vector $\vec{t}$ is an artificial construct that can only be measured after Alice and Bob meet up. The 4-vector $\boldsymbol{x}_{ij}$ is useful to define when mapping a road graph, even if it can't be directly measured. We develop these properties below. We define the norm of a 4-vector using the Lorentz metric (compare (11)):

$$|\boldsymbol{x}_{ij}|^2 \stackrel{\text{def}}{=} |\vec{t}_{ij}|^2 - \vec{t}^2 \tag{24}$$

For the case of Alice and Bob in figure 2, using (11) we have:

$$|\boldsymbol{x}_{ac}|^2 = \vec{T}_{ab}{}^2 \tag{25}$$

which is an invariant independent of the relative velocity of Alice with respect to Bob. Further, we define the 4-vector $\boldsymbol{p}_{ijk}$:

$$\boldsymbol{p}_{ijk} \stackrel{\text{def}}{=} \frac{1}{|\vec{T}_{ij}|}\boldsymbol{x}_{ik} = \frac{1}{|\vec{T}_{ij}|}\begin{bmatrix} |\vec{t}_{ik}| \\ \vec{t} \end{bmatrix} = \frac{|\vec{t}_{ik}|}{|\vec{T}_{ij}|}\begin{bmatrix} 1 \\ \vec{v}_r \end{bmatrix} = \frac{v_{ij}}{v_{ik}}\frac{1}{\sqrt{1-\vec{v}_r{}^2}}\begin{bmatrix} 1 \\ \vec{v}_r \end{bmatrix} \tag{26}$$

and it follows from (23) and (24) that:

$$|\boldsymbol{p}_{ijk}|^2 = \left(\frac{v_{ij}}{v_{ik}}\right)^2 \frac{1}{1-\vec{v}_r{}^2}\left(1-\vec{v}_r{}^2\right) = \left(\frac{v_{ij}}{v_{ik}}\right)^2 \tag{27}$$

Comparing with special relativity, we recognize the 4-vector $\boldsymbol{p}_{ijk}$ is the energy-momentum 4-vector, and the quantity on the right hand side of (27) is the square of the rest mass μ:

$$\mu = \frac{v_{ij}}{v_{ik}} \tag{28}$$

In the case of Alice and Bob, we have:

$$|\boldsymbol{p}_{abc}| = \frac{v_{ab}}{v_{ac}} \tag{29}$$

which is the ratio of the intrinsic speeds of the traversals by Alice from a to b and Bob from a to c respectively. In our model, therefore, we interpret rest mass as the ratio of intrinsic speeds of two roads, one along which the body under study moves, and the other one along which the observer moves. This is intimately related to the role of mass in general relativity, where mass is equated with the curvature of spacetime; in our model, the higher the mass, the higher the difference in intrinsic distance travelled along the two different roads during the same length of time, as if space were "shrunk" along one road compared to the other. We develop this further in section 3.

But first, we turn to the final part of special relativity, that of conversion between mass and energy in processes where composite bodies split into parts or parts join into composite bodies, as in nuclear fission and fusion.

We need to slightly extend the road network model presented in [1] to encompass the splitting and joining of moving bodies. Crucially, such a splitting or joining is an interaction of bodies in the sense of P3 [1], and so is a disturbance to the motion of the bodies.

Imagine we have two cars A and B, each with a driver, and each travelling along different roads A and B. The drivers measure the times between successive lampposts, respectively τ_A and τ_B, and from them compute their intrinsic speeds v_A and v_B. So in a fixed period of time Δt, car A will pass $n_A = v_A \Delta t$ lampposts, while car B will pass $n_B = v_B \Delta t$ lampposts.

Now let the roads A and B join into one composite road AB, where the two cars A and B join into one new composite car AB. That is, from either car's point of view, it will bump into and join with the other car to form one new car AB. This process will be a disturbance to both incoming cars in the sense of P3 of [1]. We assume that the driver of the new composite car will see $n_{AB} = n_A + n_B$ lampposts in each period of time Δt. This "conservation of lampposts per unit time" is the intrinsic point of view equivalent of the extrinsic "conservation of momentum." The intrinsic speed v_{AB} of the composite car will thus be given by:

$$v_{AB} = v_A + v_B \tag{30}$$

The situation of a composite car AB splitting into two cars A and B is simply the reverse of the above scenario.

Now we can apply this expanded road graph model to discuss how Alice and Bob would deal with mapping a junction where composite roads are formed or split up. We imagine a case where two cars A and B join into one AB. So now instead of one Alice, we start with two people, Alice_A and Alice_B, which join into one Alice_{AB}. Before the junction, Bob can measure the journeys of Alice_A and Alice_B obtaining (refer (23)):

$$\boldsymbol{x}_{acA} = \begin{bmatrix} |\vec{t}_{acA}| \\ \overrightarrow{t_A} \end{bmatrix} \quad \boldsymbol{x}_{acB} = \begin{bmatrix} |\vec{t}_{acB}| \\ \overrightarrow{t_B} \end{bmatrix} \tag{31}$$

and

$$\boldsymbol{p}_{abcA} = \frac{1}{|\vec{T}_{abA}|}\boldsymbol{x}_{acA} \quad \boldsymbol{p}_{abcB} = \frac{1}{|\vec{T}_{abB}|}\boldsymbol{x}_{acB} \tag{32}$$

Now the conservation of lampposts per unit time (30) implies:

$$\boldsymbol{p}_{abcAB} = \boldsymbol{p}_{abcA} + \boldsymbol{p}_{abcB} \tag{33}$$

or,

$$\frac{1}{|\vec{T}_{abAB}|}\boldsymbol{x}_{acAB} = \frac{1}{|\vec{T}_{abA}|}\boldsymbol{x}_{acA} + \frac{1}{|\vec{T}_{abB}|}\boldsymbol{x}_{acB} \tag{34}$$

Using (26) and (28) we have:

$$\frac{\mu_{AB}}{\sqrt{1-\vec{v}_{rAB}{}^2}}\begin{bmatrix} 1 \\ \vec{v}_{rAB} \end{bmatrix} = \frac{\mu_A}{\sqrt{1-\vec{v}_{rA}{}^2}}\begin{bmatrix} 1 \\ \vec{v}_{rA} \end{bmatrix} + \frac{\mu_B}{\sqrt{1-\vec{v}_{rB}{}^2}}\begin{bmatrix} 1 \\ \vec{v}_{rB} \end{bmatrix} \tag{35}$$

where μ_A, μ_B and μ_{AB} are the rest masses of Alice_{A}, Alice_{B} and Alice_{AB} respectively.

Now let us choose a situation where $|\vec{v}_{rAB}| = 0$, that is $\vec{v}_{rA} = -\vec{v}_{rB}$. Then, looking at the 0-component of (35), we have:

$$\mu_{AB} = \frac{\mu_A + \mu_B}{\sqrt{1-\vec{v}_{rA}{}^2}} \tag{36}$$

So, as long as $|\vec{v}_{rA}| > 0$ this means:

$$\mu_{AB} > \mu_A + \mu_B \tag{37}$$

So the rest mass of the composite body is greater than the sum of the rest masses of the constituent bodies if they were in relative motion before the composite body was formed. This conversion between the relative motion of constituent parts and extra rest mass of the composite body formed of them, and vice versa, is the famous phenomenon of mass and energy equivalence in special relativity.

3. General Relativity from a Road Network

Combining general relativity with quantum mechanics has proven to be a challenge due to a fundamental chicken-and-egg type problem. The mass-energy in a region of spacetime depends on what interactions between particles take place there; but where these particles are, and so what interactions take place, depends on the curvature of the region of spacetime, that is, on its mass-energy. What particles do depends on where they are, and where they are depends on what they do: an unsolvable problem in any but a trivial (high-symmetry) case. In this final section we propose our expanded road network as a tool to help approach this problem from a new angle. Fist, we argue that general relativity can be recovered from the road graph model, given some simplifying assumptions, that is, our model is more general than general relativity. Then our equating of quantum mechanical measurement with the interaction of particles (see P3 of [1]) solves the chicken-and-egg problem. Which interactions occur (see P4 of [1]) and the rest masses of the product particles (see (36)) and thus the curvature of space seen by the product particles, are both decided simultaneously at every vertex at which interaction is possible, in a random but intrinsically local fashion. By looking at the interactions and changes in rest mass from the particles' points of view, we avoid the question of where in some "external" space these interactions take place, or what the curvature of that space is.

Given a vertex i in a road graph, we can look at the set of roads leaving it, consisting of roads R_{ij}, where j is the ending vertex of each outgoing road. We argued in the previous section that Alice and Bob, while maintaining their strictly intrinsic points of view, can assign to every road a 3-vector $\vec{x}_{ij} = v_{ij}\,\vec{t}_{ij}$ (see (15)). Of course, the values of v_{ij} can differ between different outgoing roads. This means that if Alice leaves vertex i and travels for a fixed amount of time $\Delta t = \left|\vec{t}_{ij}\right|$, the intrinsic distance she travels will be:

$$\Delta x_{ij} = v_{ij}\left|\vec{t}_{ij}\right| \tag{38}$$

or,

$$\left(\Delta x_{ij}\right)^2 = {v_{ij}}^2\left(\vec{t}_{ij}\right)^2 \tag{39}$$

Thus, the distance she travels in a fixed amount of time depends on the direction she travels in. So we can write the distance she travels from vertex i, namely Δx_i, as a function of $\vec{t}_{ij}$:

$$\left(\Delta x_i\left(\vec{t}_{ij}\right)\right)^2 = {v_{ij}}^2\left(\vec{t}_{ij}\right)^2 \tag{40}$$

Now we make a crucial simplifying assumption to recover the theory of general relativity. We assume that $\Delta x_i\left(\vec{t}_{ij}\right)$ varies smoothly and slowly as a function of the direction of the vector $\vec{t}_{ij}$, and that $\left|\vec{t}_{ij}\right|$ is small. This implies that the manifold in which motion is taking place is continuous, not "shredded" into interconnected ribbons as described in [1], and thus does not exhibit quantum effects such as interference. We can then approximate the function (40) by its power expansion, truncated to quadratic terms only:

$$\left(\Delta x_i\left(\vec{t}_{ij}\right)\right)^2 \approx {\vec{t}_{ij}}^{\,T} G_i\, \vec{t}_{ij} \tag{41}$$

where G_i is a 3×3 matrix of real numbers associated with the vertex i.

If Alice travels for a time Δt along road R_{ij}, then Bob, who is travelling from i along a different road, will describe Alice's motion by the reading of his clock and the perigee distance, that is the 4-vector $\boldsymbol{x}_{ij}$ (see (23)). So Bob can express (41) in terms of $\boldsymbol{x}_{ij}$:

$$\left(\Delta x_i\left(\boldsymbol{x}_{ij}\right)\right)^2 \approx {\boldsymbol{x}_{ij}}^{T} G'_i\, \boldsymbol{x}_{ij} \tag{42}$$

Where G'_i is a 4×4 matrix. We recognize G'_i is what is called the *metric* in general relativity, that is, (42) corresponds to the equation for distance in general relativity:

$$ds^2 = g_{\mu\nu}\, dx^\mu\, dx^\nu \tag{43}$$

where $g_{\mu\nu}$ is a function of position, just as G'_i is a function of the vertex i.

Now that, given the smoothness assumption, we have defined a metric on a road graph, and since having a metric implies we are dealing with a manifold which in general can be curved (if $g_{\mu\nu}$ and G'_i have non-zero off-diagonal components), we have recovered the curved spacetime of general relativity from the road graph model. It remains to show that this curvature is related to the mass-energy in the same way in general relativity as in the road graph model.

Feynman discussed the meaning of general relativity in a lecture [2]: "Consider a small three-dimensional sphere, of given surface area. Its actual radius exceeds the radius calculated by Euclidean geometry ($\sqrt{area/4\pi}$) by an amount proportional to the amount of matter inside the sphere ($r - \sqrt{area/4\pi} = G/3c^2\, m_{inside}$)." A similar interpretation is discussed in [3]. We wish to show that we can recover this same relation from the road graph model. Let's imagine a road R_{ab} from vertex a located in the center of a small 3-sphere to a vertex b on the surface of it. This road is defined by its intrinsic speed v_{ab} and duration t_{ab}. Let's imagine a copy of this situation, with road R_{cd} going from a vertex c at the center of another 3-sphere to vertex d on its surface, with intrinsic speed $v_{cd} = v_{ab} + \Delta v$ and duration same as R_{ab}, that is $t_{cd} = t_{ab}$. In the first case, Alice travelling along road R_{ab} will conclude the radius of the sphere is:

$$r' = v_{ab}\, t_{ab} \tag{44}$$

while in the second case, travelling from c to d:

$$r = v_{cd}\, t_{cd} = v_{cd}\, t_{ab} \tag{45}$$

Then we have:

$$r - r' = (v_{cd} - v_{ab})t_{ab} = \Delta v t_{ab} \tag{46}$$

We call the first case, with road R_{ab}, the "empty space" case, and the second case with R_{cd}, the "presence of matter" case. We see that as Δv increases, r exceeds r' by a larger and larger amount. That is, in the same amount of time, the person travelling to the surface of the second sphere from its center will cover a greater and greater distance. But (28) recognizes the ratio of the intrinsic speeds of two roads, one being the observer's, as the concept called "rest mass" in special relativity, So, if we keep the observer's intrinsic speed constant, we have:

$$\mu \propto \Delta v \tag{47}$$

Now Δv is just the extra intrinsic speed in the presence of matter case, that is the extra rest mass encountered by Alice for a total duration t_{ab}. So the total extra rest mass encountered by Alice is:

$$m_{inside} \propto \Delta v t_{ab} \tag{48}$$

So we have:

$$r - r' \propto m_{inside} \tag{49}$$

as described in the quote by Feynman above.

4. Conclusion

We have shown that the road graph model lies below the theories of special and general relativity (and in previous work [1] – quantum mechanics) and is consistent with their predictions, in the sense that these theories can be recovered from the road graph model. We propose this model can be used to develop a theory of quantum gravity by approaching it from a new, intrinsic, point of view. The metric of general relativity is recovered in the smooth-case ("not shredded" manifold) approximation to the road graph. But this simplifying approximation need not be made, and the bare road graph model can be used to model phenomena with both quantum and gravitational aspects. Our equating the interaction between particles with quantum measurement solves the chicken-and-egg problem inherent in combining quantum and gravitational theories. We propose the road graph model could be used for calculations of quantum gravity phenomena from which quantitative predictions could be made to test our model experimentally.

References

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