OUR COLLAPSING FRIEDMANN UNIVERSE

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ABSTRACT

In 1907 using special relativity, Albert Einstein proved that vacuum permittivity, ε, changes in accelerating coordinate reference frames. ε is the scalar in Maxwell's equations that determines the speed of light and the strength of electrical fields. In 1952, Møller confirmed Einstein's discovery by proving that ε is a function of the curvature of static spacetimes. In 1994, Sumner proved that ε changes with the curvature of Friedmann spacetime. Photon energies are proportional to ε, but the energies of photons emitted by atoms are proportional to ε^2. This difference reverses the interpretation of Hubble redshifts. Hubble redshifts only result when a Friedmann universe is collapsing. This is confirmed by the Pantheon redshift data fit of 1048 supernovas with a negative Hubble constant H_o = -72.10 $\pm$ 0.75 km s^{-1} Mpc^{-1} and a deceleration parameter $1/2 < q_o < 0.51$. The velocity of light in Friedmann geometry is inversely proportional to the radius of the universe. The velocity of light was infinite at the Big Bang and decreased to zero at maximum size when the universe began to collapse. The velocity of light is now accelerating towards infinity. Its current value is c. Collapse will be complete in 9.05 billion years. The current age of the universe is estimated to be $1.54 \, x \, 10^4$ billion years.

1. INTRODUCTION

In Friedmann spacetime, vacuum permittivity, ε, is directly proportional to the Friedmann radius so ε changes with time. As the size of the Friedmann universe evolves, the changing strength of the electrical force between charges shifts atomic energy levels, changing the wavelengths of emitted light. The wavelengths of photons also change but only half as much. This difference in evolution of atomic emissions and photons reverses the interpretation of Hubble redshift Sumner (1994). Today the Friedmann universe is closed and collapsing, Figure 1.

In Maxwell's equations, vacuum permittivity is the scalar that determines the speed of light and the strength of electrical fields. Einstein's discovery means that both the wavelengths of photons and the wavelengths of photons emitted by atoms change with spacetime curvature in general relativity.
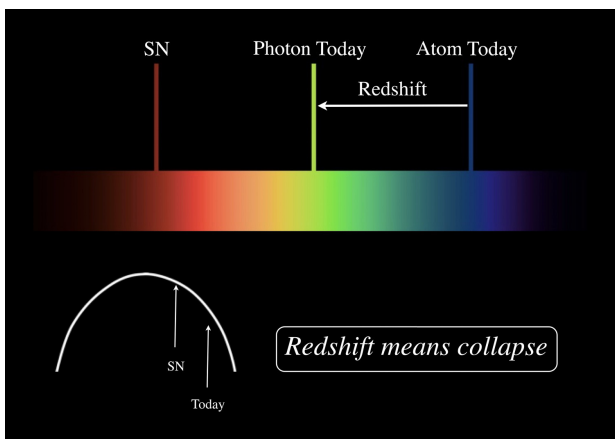

Figure 1. Photons blueshift in a collapsing universe. Atomic spectra blueshift more. Hubble redshifts of photons from the supernova (SN) are observed.

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2. MATHEMATICAL MODELS

2.1. *Einstein's Solution*

In his study of Maxwell's equations in an uniformly accelerated coordinate system, Einstein (1907)[1] concluded that the velocity of light in special relativity, c, is reduced to c^*, the local coordinate velocity of light in the accelerated system. Einstein (1989, p 310) found in an accelerated system which corresponds locally to the gravitational field of a point mass

$$c^* = c \left(1 + \frac{\Phi}{c^2} \right), \tag{1}$$

where Φ is the Newtonian gravitational potential

$$\Phi = -\frac{k \, m}{r}. \tag{2}$$

m is the mass of the object creating the gravitational field at a distance r. k is the gravitational constant.

The connection between Einstein's result, equation (1), and the strength of the electrical field comes from the definition of relative vacuum permittivity ε,

$$\varepsilon = \frac{c}{c^*}. \tag{3}$$

Combining equations (1), (2), and (3) gives Einstein's value for ε,

$$\varepsilon(r) = \frac{1}{\left(1 - \frac{km}{rc^2} \right)}. \tag{4}$$

Møller (1952, p 308) and Landau & Lifshitz (1975, p 258) studied the effects of curved spacetime on Maxwell's equations. Both proved that in a static gravitational field the electromagnetic field equations take the form of

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[1] English translation (Einstein 1989, p 252)

Maxwell's phenomenological equations in a medium at rest with

$$\varepsilon(r) = \frac{1}{\sqrt{g_{00}}}. \tag{5}$$

g_{00} is the time component of the metric tensor $g_{\mu\nu}$.

Einstein's pre-general relativity result, equation (4), is the first approximation to the exact relativistic equation (5),

$$\varepsilon(r) = \frac{1}{\sqrt{g_{00}}} = \frac{1}{\sqrt{\left(1 - \frac{2km}{rc^2}\right)}} \approx \frac{1}{\left(1 - \frac{km}{rc^2}\right)}. \tag{6}$$

2.2. *Friedmann Solution*

Friedmann (1922) published a closed universe solution to Einstein's theory of general relativity without a cosmological constant. The Friedmann universe rapidly expands from a singularity, slowing until it reaches a maximum size before accelerating back to a singularity.
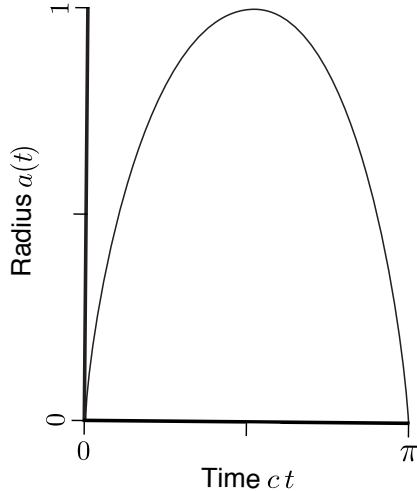

Figure 2. Friedmann's solution for a closed universe with $\alpha = 1$ in equation 8.

Friedmann assumed the metric,

$$ds^2 = c^2 dt^2 -$$
$$a^2(t)\left[\frac{dr^2}{(1-r^2)} + r^2\left(d\theta^2 + sin^2\theta\, d\phi^2\right)\right], \tag{7}$$

homogeneous, incoherent matter, conserved in amount and exerting negligible pressure. His solution is the cycloid shown in Figure 2,

$$a = \frac{\alpha}{2}(1 - \cos\psi) \qquad ct = \frac{\alpha}{2}(\psi - \sin\psi), \tag{8}$$

where α is a constant and $0 \leq \psi \leq 2\pi$ (Tolman 1934).

Sumner (1994) examined Maxwell's equations in Friedmann geometry and found that ε changes in time along with spacetime curvature,

$$\varepsilon(t) = a(t). \tag{9}$$

$a(t)$ is the radius of the Friedmann universe defined above with $\alpha = 1$.

2.3. *Local Mathematical Coordinates*

The effects of spacetime curvature in nature are often explained as gravitational forces or simply ignored. The extraordinary success of the special theory of relativity confirms this approach. But spacetime is never precisely flat as ubiquitous gravity clearly shows. Flat spacetime exists only in mathematical models. Every spacetime in nature is curved.

To understand the effects of spacetime curvature on atoms and photons, a coordinate system that includes spacetime curvature is necessary. The method used by Einstein, Møller, Landau, Lifshitz, and Sumner is adopted where a local pseudo-Cartesian coordinate system is used with the vacuum permittivity $\varepsilon(x^\mu)$ determined by the general relativistic geometry at that spacetime point. Specifically,

$$ds^2 = \frac{c^2}{\varepsilon^2(x^\mu)}dt^2 - \left[dr^2 + r^2\left(d\theta^2 + sin^2\theta\, d\phi^2\right)\right]. \tag{10}$$

If the variation in $\varepsilon(x^\mu)$ in the region of interest is ignored, equation (10) is just the metric of special relativity with a velocity of light $c/\varepsilon(x^\mu)$. If $\varepsilon(x^\mu) = 1$ the result is special relativity with spacetime curvature ignored.

2.4. *Changes in Atoms and Photons*

The Bohr radius a_o of a hydrogen atom in its ground state at t is [2]

$$a_o(t) = \frac{4\pi\varepsilon_o\,\varepsilon(t)\,\hbar^2}{me^2}. \tag{11}$$

$\varepsilon_o = 8.854187817\ldots x10^{-12}$ F/m (farads per meter) is the defined value of ε_o. m is the mass of the electron, e is the charge of the electron, and $\hbar$ is Planck's constant h divided by 2π. These are assumed to remain constant as spacetime curvature changes.

The change in Bohr radius a_o as t changes is

$$\frac{a_o(t_1)}{a_o(t_2)} = \frac{\varepsilon(t_1)}{\varepsilon(t_2)}. \tag{12}$$

The characteristic wavelength λ_e emitted by a hydrogen atom during the transition between the principle quantum numbers n_2 and n_1 is

$$\lambda_e(t) = \frac{8\,\varepsilon_o^2\,\varepsilon^2(t)\,\hbar^3\,c}{me^4}\left(\frac{n_1^2\,n_2^2}{n_2^2 - n_1^2}\right). \tag{13}$$

c in equation (13) comes from the defining relationship between λ and ν, $\lambda\nu = c$.

The change in $\lambda_e(t)$ as t changes is

$$\frac{\lambda_e(t_1)}{\lambda_e(t_2)} = \frac{\varepsilon^2(t_1)}{\varepsilon^2(t_2)}. \tag{14}$$

Consider the Compton wavelength, λ_c, of a particle with mass m_p,

$$\lambda_c(t) = \frac{h}{m_p\,c^*(t)} = \frac{h\,\varepsilon(t)}{m_p\,c}. \tag{15}$$

[2] See standard texts, e.g. Leighton (1959).

The change in $\lambda_c(t)$ as t changes is

$$\frac{\lambda_c(t_1)}{\lambda_c(t_2)} = \frac{\varepsilon(t_1)}{\varepsilon(t_2)}. \tag{16}$$

The Compton wavelength of a particle is equivalent to the wavelength of a photon of the same energy as the particle. Compton and photon wavelengths have the same $\varepsilon(t)$ dependency that the Bohr radius has. The wavelength change for a photon is

$$\frac{\lambda(t_1)}{\lambda(t_2)} = \frac{\varepsilon(t_1)}{\varepsilon(t_2)}. \tag{17}$$

2.5. Gravitational Redshift

The following notation is used. The wavelength of a photon λ emitted at t_1 and examined at t_1 will be written $\lambda(t_1, t_1)$. The wavelength of a photon λ emitted at t_1 and examined at t_2 will be written $\lambda(t_1, t_2)$.

The traditional redshift z formula assumes that atomic emissions do not evolve, $\lambda(t_2, t_2) = \lambda(t_1, t_1)$, but that photons do (Equation (17)).

$$z = \frac{\lambda(t_1, t_2) - \lambda(t_1, t_1)}{\lambda(t_1, t_1)} = \frac{\varepsilon(t_2)}{\varepsilon(t_1)} - 1. \tag{18}$$

t_2 is the observer's location and t_1 is the location at the time of emission.

Since atomic emissions do evolve with spacetime geometry, a new redshift variable ζ (the Greek letter zeta) is defined to match what is done experimentally,

$$\zeta = \frac{\lambda(t_1, t_2) - \lambda(t_2, t_2)}{\lambda(t_2, t_2)}. \tag{19}$$

t_2 is the observer's location and t_1 is the location at the time of emission.

$$\zeta = \frac{\varepsilon(t_1)}{\varepsilon(t_2)} - 1. \tag{20}$$

Substituting $\varepsilon(t) = a(t)$ into equation (20) gives the redshift ζ for Friedmann geometry,

$$\zeta = \frac{a(t_1)}{a(t_2)} - 1. \tag{21}$$

Hubble redshift ($\zeta > 0$) implies $a(t_1) > a(t_2)$. The universe was larger in the past, $a(t_1)$, than it is now, $a(t_2)$. This puts us somewhere on the collapsing half of the curve in Figure 2. The logic is simple. Since Hubble shifts are red ($\zeta > 0$), the Friedmann universe is collapsing. If Hubble shifts were blue ($\zeta < 0$), the Friedmann universe would be expanding.

3. ANALYZING HUBBLE REDSHIFTS

The analysis of redshift observations must include the changes in atomic emissions in addition to the changes in photons. Astronomers measure the redshift defined by ζ, equation (21). The following derivation is similar to the one made when atomic evolution is ignored and the universe is assumed to be expanding (Narlikar 1983), but is different because ζ not z describes the observed redshift and some choices in signs are made differently when the universe is contracting (Sumner & Vityaev

2000). It is assumed that observed photons were emitted after contraction began.

The mathematical coordinate distance r to a source can be shown to be a function of the observed redshift ζ of the source and the deceleration parameter q_o in the following way.

Setting $ds = 0$ in the Friedmann metric, equation (7), gives

$$c \, dt = \frac{-a(t) \, dr}{(1 - r^2)^{1/2}}. \tag{22}$$

The source is located at the spatial coordinates $(r_1, 0, 0)$ with emission at time t_1 and the observer is at $(0, 0, 0)$ with reception at time t_2.

$$c \int_{t_1}^{t_2} \frac{dt}{a(t)} = \int_0^{r_1} \frac{dr}{(1 - r^2)^{1/2}} = \sin^{-1} r_1. \tag{23}$$

Substituting $a(t)$ and dt calculated from the Friedmann solution, equation (8), gives

$$r_1 = \sin(\psi_2 - \psi_1). \tag{24}$$

The Friedmann equation for the closed universe is (Narlikar 1983, p 113)

$$\dot{a}^2 = c^2 \left(\frac{\alpha}{a} - 1 \right). \tag{25}$$

The Hubble constant H and the deceleration parameter q are defined by

$$H(t) = \frac{\dot{a}(t)}{a(t)}, \qquad \frac{\ddot{a}(t)}{a(t)} = -q(t) H^2(t). \tag{26}$$

" $\cdot$ " indicates time derivative. H is negative and q is greater than $1/2$ for a closed, collapsing universe. Present day values are denoted by H_o and q_o.

α, the constant in equations (8), may be written (Narlikar 1983, p 114)

$$\alpha = \frac{2 q_o}{(2 q_o - 1)^{3/2}} \frac{c}{|H_o|}. \tag{27}$$

Solving for ψ_2 and ψ_1 in terms of ζ and q_o and substituting into equation (24) gives

$$r_1 = \frac{(2 q_o - 1)^{1/2}}{q_o} \left[\zeta - \frac{(1 + \zeta)(1 - q_o)}{q_o} \right] +$$
$$\frac{(1 - q_o)}{q_o} \left\{ 1 - \left[\zeta - \frac{(1 + \zeta)(1 - q_o)}{q_o} \right]^2 \right\}^{1/2}. \tag{28}$$

The flux f of photons is related to the luminosity L of the source and to its luminosity distance D_L by the equation

$$f = \frac{L}{4 \pi D_L^2}. \tag{29}$$

D_L is determined in the following way. Calculate the observed flux f by noting that L, the actual luminosity of the source, is changed by a factor of $a(t_2)/a(t_1)$ because of the apparent change of the photon's energy and changed by another factor of $a(t_2)/a(t_1)$ because of the changes in time in the local metric, equation (10). The

distance to the source is $r_1 a(t_2)$. This gives an observed flux of

$$f = \frac{L \dfrac{a^2(t_2)}{a^2(t_1)}}{4\pi r_1^2 a^2(t_2)}. \tag{30}$$

Combining equations (29) and (30) using (21) gives

$$D_L = r_1 a(t_2)(1 + \zeta). \tag{31}$$

$a(t_2)$ is (Narlikar 1983, p 114)

$$a(t_2) = \frac{-c}{H_o} \frac{1}{(2q_o - 1)^{1/2}}. \tag{32}$$

Substituting equations (28) and (32) into (31) gives

$$D_L = \frac{-c}{H_o} \frac{(1+\zeta)}{q_o} \left\{ \left[\zeta - \frac{(1+\zeta)(1-q_o)}{q_o} \right] + \frac{(1-q_o)}{(2q_o-1)^{1/2}} \left(1 - \left[\zeta - \frac{(1+\zeta)(1-q_o)}{q_o} \right]^2 \right)^{1/2} \right\}. \tag{33}$$

The relationship between distance modulus (the difference between the apparent magnitude m and absolute magnitude M of a celestial object) and luminosity distance, D_L, is

$$m - M = 5 \log_{10} \left(\frac{D_L}{10 \, \text{parsecs}} \right). \tag{34}$$

The Hubble constant H_o (negative for the contracting half of the curve) and the deceleration parameter q_o (which must be $> 1/2$ characterizing a closed Friedmann universe) are then varied to find best least-squared fits to Hubble redshift observations of ζ and $m - M$ using equations (33) and (34).

4. FIT TO PANTHEON SN REDSHIFT DATA

The Pantheon redshift data of 1048 supernovas (Scolnic 2018) were analyzed assuming that both atoms and photons change. The Hubble constant and deceleration parameter were the only variables, see Figure 3.

Since this Friedmann universe is closed, $q_o > 1/2$. Every search conducted found a lower standard deviation when q_o was closer to $1/2$. No lower limit for $\delta = (q_o - 1/2)$ was found and the upper limit 0.51 was chosen because there is little change in the quality of fit with smaller q_o, hence $1/2 < q_o < 0.51$.

This is illustrated in Figure 4.

5. OUR UNIVERSE

$\frac{2}{3} |H_o|^{-1}$ estimates the time until collapse, t_c, of the Friedmann universe when q_o is this close to $1/2$. For $H_o = -72.10 \, km \, s^{-1} Mpc^{-1}$, $t_c = 9.05$ billion years.

The age of the universe, t_A, can be estimated from the magnitude-redshift data (Narlikar 1983, p 114) (with two signs changed to reflect contraction),

$$t_A = \frac{-1}{H_o} \left[\frac{1}{2q_o - 1} + \frac{q_o}{(2q_o - 1)^{3/2}} \cos^{-1} \frac{1 - q_o}{q_o} \right]. \tag{35}$$

A value for $\cos^{-1}$ corresponding to the fourth quadrant must be used. For $H_o = -72.10 \, km \, s^{-1} Mpc^{-1}$ and $q_o = 0.51$, $t_A = 1.54 \times 10^4$ billion years.
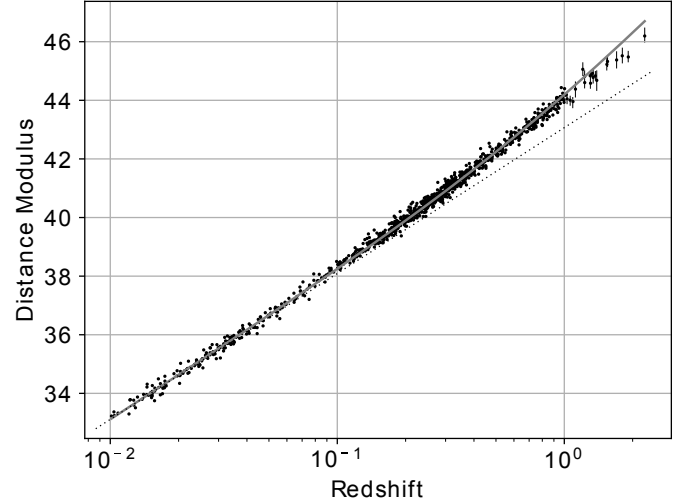

Figure 3. The solid line is the fit to the Pantheon redshift data with the parameters $H_o = -72.10 \pm 0.75$ km s^{-1} Mpc^{-1} and $1/2 < q_o < 0.51$. The dotted straight line is included to visually clarify the upward curve (or "acceleration") of the data and fit. The average data error is 0.1418. The standard deviation for this fit is 0.1515.
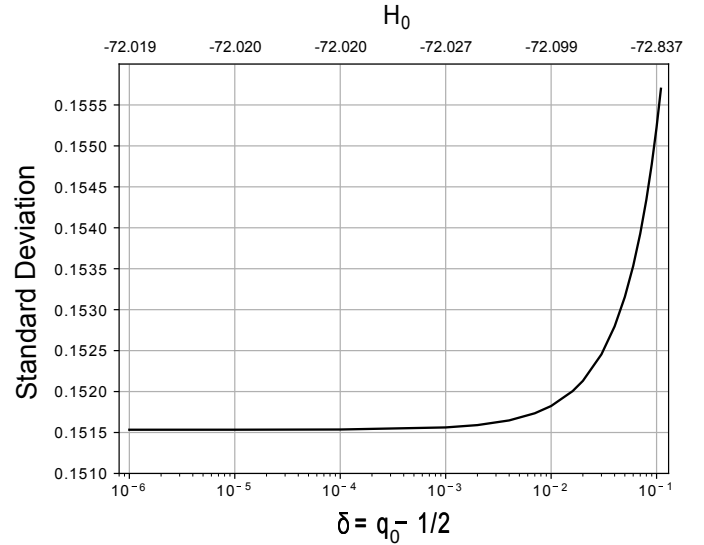

Figure 4. Standard deviation for fits at smaller values of $\delta = (q_o - 1/2)$. The values of H_o on the top axis are the best fits for the δ values on the bottom axis. No minimum for δ was found.

For $q_o = 1/2$, equation 35 gives an age of $t_A = \infty$ as it should for a flat universe. While the Pantheon data makes a persuasive case that our universe is closed, nearly flat, and very old, it does not give definitive answers to the questions "How flat is the universe?" and "How old is the universe?"

6. VELOCITY OF LIGHT

Einstein (1907)[3] concluded that the velocity of light in special relativity, c, is reduced in an uniformly accelerated coordinate system to c^*, the local coordinate velocity of light. The ratio c/c^* is relative vacuum permittivity ε, equation 3.

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[3] English translation (Einstein 1989, p 252)

For Friedmann geometry $\varepsilon(t) = a(t)$. Equation 3 gives

$$c^*(t)a(t) \; = \; c^*(t_o)a(t_o), \qquad (36)$$

where $a(t_o)$ is the radius of the universe when the velocity of light is $c^*(t_o) = c = 2.998 \times 10^{10}$ cm/sec.

At the Big Bang (when $a(t) = 0$), the local coordinate velocity $c^*(t)$ was infinite before dropping to its current value c at 9.05×10^9 years, before reaching a minimum at full expansion, and then increasing back toward infinity again at collapse.

The symmetry of the Friedmann cycloid is used to equate the velocity and radius during the expansion period to the current collapsing data for H_o, q_o, and $t_o = t_c$ derived from H_o. Equations 8 then give ψ_o, $a(t_o)$ and α.

7. MATHEMATICS AND PHYSICS

The mathematics of general relativity isn't a physical theory until mathematical concepts such as $g_{\mu\nu}$ and x^μ are linked by axioms to specific physical measurements. Albert Einstein took this step, just as he did for special relativity, by asserting that measurements made with rigid meter sticks and balance clocks are equivalent to the mathematical distances and times of general relativity. Assuming a rigid meter stick is equivalent to assuming that atoms never change. Even as he did this Einstein had qualms about his choices.

From Einstein's 1921 Nobel Lecture:

> ... it would be logically more correct to begin with the whole of the laws and ... to put the unambiguous relation to the world of experience last instead of already fulfilling it in an imperfect form for an artificially isolated part, namely the space-time metric. We are not, however, sufficiently advanced in our knowledge of Nature's elementary laws to adopt this more perfect method without going out of our depth. (Einstein 1967, p 483)

It is intriguing that it was Einstein who discovered vacuum permittivity depends on gravity. In 1907, there was no general relativity, no Bohr atom, and no clear understanding of photons. When these theories were later in place, the connection provided by vacuum permittivity between spacetime curvature and atomic structure was overlooked. Einstein (1949, p 685) knew that the "tools for measurement do not lead an independent existence alongside of the objects implicated by the field-equations." What he did not realize was that the solution was already in his 1907 paper and that there was no need of "going out of our depth" to create the more complete general relativity he wanted, where the "tools for measurement" depend on spacetime exactly as "other objects implicated by the field-equations."

Schrödinger (1939) published his seminal discovery that every quantum wavelength expands and contracts in proportion to the radius of a closed Friedmann universe. Schrödinger argued that if spacetime is curved as general relativity requires, then its effects on quantum processes must not be dismissed without careful investigation. Using the equations of relativistic quantum mechanics, Schrödinger found that the plane-wave eigenfunctions characteristic of flat spacetimes are replaced in the curved spacetime of the closed Friedmann universe by wave functions with wavelengths that are directly proportional to the Friedmann radius.

This means that every eigenfunction changes wavelength as the radius of the universe changes. The quantum systems they describe change as well. In an expanding universe quantum systems expand. In a contracting universe they contract. The assumption is often made that small quantum systems are isolated and that their properties remain constant as the Friedmann universe evolves. This assumption is incompatible with relativistic quantum mechanics and with the curved spacetime of general relativity as Schrödinger proved (Sumner & Sumner 2000).

These changes in quantum systems may equivalently be viewed as a logical consequence of the fact that the energy and momentum of "isolated systems" are not conserved. Energy and momentum change when the spacetime curvature of the universe changes. Schrödinger (1956, p 58) wrote:

> In an expanding space *all momenta decrease* ... for bodies acted on by no other forces than gravitation ... This simple law has an even simpler interpretation in wave mechanics: all wavelengths, being inversely proportional to the momenta, simply expand with space. [4]

In a contracting space, the opposite is true. *All momenta increase* and all wavelengths, being inversely proportional to the momenta, simply contract with space.

Schrödinger had a deep understanding of both wave mechanics and general relativity. Like most physicists, Schrödinger "knew" Hubble redshift meant that the universe is expanding, a hangover from the pre-relativistic interpretations of redshifts originally made by Slipher (1917) and Hubble (1929) who tentatively assumed that all galactic redshifts are solely Doppler shifted photons. It is interesting to speculate how long it would have taken Schrödinger to correctly interpret Hubble redshift if he had asked himself the question: "Would the changes in atoms and photons that I found change my interpretation of Hubble redshift?"

Feynman (1967, p 55) was correct when he observed that "Physics is not mathematics, and mathematics is not physics ... mathematicians prepare abstract reasoning that's ready to be used if you will only have a set of axioms about the real world ..." Assuming that meter sticks are made of atoms that never change does not belong in that set, nor does assuming that the speed of light is a constant.

8. CONCLUSIONS

Vacuum permittivity, a measure of the strength of electric fields and light velocity in a vacuum, changes with the spacetime curvature of general relativity. This changes atomic energy levels, photon wavelengths, and the velocity of light. For Friedmann geometry a comparison of photons emitted long ago to those emitted today predicts that Hubble redshifts result from a universe accelerating in collapse. This is confirmed by the Pantheon redshift data where no modifications to general relativity or to Friedmann's 1922 assumptions are

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[4] Pauli (1958, p 220) made the same observation.

necessary to explain Hubble redshift. Assuming changes in both atomic emissions and photon wavelengths and varying only H_o and q_o gives $H_o = -72.10 \pm 0.75$ km s^{-1} Mpc^{-1} and $1/2 < q_o < 0.51$. The average data error is 0.1418. For these fit parameters the standard deviation is 0.1515. The changes in atoms and photons derived here agree with Schrödinger's 1939 conclusion that quantum wave functions expand and contract with the radius of a closed Friedmann universe. The velocity of light is inversely proportional to vacuum permittivity which is proportional to the radius of the Friedmann universe. Light velocity was infinite at the Big Bang, dropped to its current value c at 9.05×10^9 years on its way to a minimum at full expansion. c will be infinite again in 9.05×10^9 years. The estimated age of the universe is $t_A = 1.54 \times 10^4$ billion years.

9. ACKNOWLEDGMENTS

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Thanks for reading!