Analyzing a fundamental equation concerning the "Ramanujan's Letter to Hardy on 16.1.1913". New possible mathematical connections with the Cosmological Constant in Quantum Space-Time and with some topics of String Theory

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Abstract

In this paper, we analyze a fundamental equation concerning the "Ramanujan's Letter to Hardy on 16.1.1913". We describe the new possible mathematical connections with the Cosmological Constant in Quantum Space-Time and with some topics of String Theory.

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From:

The man who new infinity: a life of the genius Ramanujan - *Robert Kanigel* - Copyright © 1991

In this paper, we study the following equation:

$$\int_0^\infty \frac{1+\left(\frac{x}{b+1}\right)^2}{1+\left(\frac{x}{a}\right)^2} \cdot \frac{1+\left(\frac{x}{b+2}\right)^2}{1+\left(\frac{x}{a+1}\right)^2} \cdots dx = \tfrac{1}{2}\pi^{1/2} \frac{\Gamma(a+\tfrac{1}{2})\ \Gamma(b+1)\ \Gamma(b-a+\tfrac{1}{2})}{\Gamma(a)\ \Gamma(b+\tfrac{1}{2})\ \Gamma(b-a+1)}$$

We calculate the integral:

integrate((1+(x/(b+1))^2) / (1+(x/a)^2) * (1+(x/(b+2))^2) / (1+(x/(a+1))^2))dx

Indefinite integral

$$\int \frac{\left(1+\left(\frac{x}{b+1}\right)^2\right)\left(1+\left(\frac{x}{b+2}\right)^2\right)}{\left(1+\left(\frac{x}{a}\right)^2\right)\left(1+\left(\frac{x}{a+1}\right)^2\right)}\,dx =$$

$$\Bigg(a(a+1)\Big((a+1)\left(a^4-a^2\left(2b^2+6b+5\right)+\left(b^2+3b+2\right)^2\right)\tan^{-1}\left(\frac{x}{a}\right)+ a\Big(\left(2a^2+3a+1\right)x-\big(a^4+4a^3+a^2\left(-2b^2-6b+1\right)- 2a\left(2b^2+6b+3\right)+b\left(b^3+6b^2+11b+6\right)\big) \tan^{-1}\left(\frac{x}{a+1}\right)\Big)\Big)\Bigg)\Big/\left((2a+1)\left(b^2+3b+2\right)^2\right) + \text{constant}$$

$\tan^{-1}(x)$ is the inverse tangent function

The study of this function provides the following representations:

Alternate forms of the integral

$$\left(a (a+1) \left((a-b-1) (a+b+2) \left((a+1) (a-b-2) (a+b+1) \tan^{-1}\left(\frac{x}{a}\right) - a (a-b) (a+b+3) \tan^{-1}\left(\frac{x}{a+1}\right)\right) + a (a+1) (2 a+1) x\right)\right) \Big/ \left((2 a+1) (b+1)^2 (b+2)^2\right) + \text{constant}$$

$$\left(a (a+1) \left(\frac{1}{2} i (a+1) \left(a^4 - a^2 \left(2 b^2 + 6 b + 5\right) + \left(b^2 + 3 b + 2\right)^2\right) \left(\log\left(1 - \frac{i x}{a}\right) - \log\left(1 + \frac{i x}{a}\right)\right) + a \left(\left(2 a^2 + 3 a + 1\right) x - \frac{1}{2} i \left(a^4 + 4 a^3 + a^2 \left(-2 b^2 - 6 b + 1\right) - 2 a \left(2 b^2 + 6 b + 3\right) + b \left(b^3 + 6 b^2 + 11 b + 6\right)\right) \left(\log\left(1 - \frac{i x}{a+1}\right) - \log\left(1 + \frac{i x}{a+1}\right)\right)\right)\right)\right) \Big/ \left((2 a+1) \left(b^2 + 3 b + 2\right)^2\right) + \text{constant}$$

$$\left(a (a+1) \left(\left(a^5 + a^4 + \left(a^3 + a^2\right) \left(-2 b^2 - 6 b - 5\right) + a \left(b^4 + 6 b^3 + 13 b^2 + 12 b + 4\right) + b^4 + 6 b^3 + 13 b^2 + 12 b + 4\right) \tan^{-1}\left(\frac{x}{a}\right) - a \left(a^2 \left(\left(-2 b^2 - 6 b + 1\right) \tan^{-1}\left(\frac{x}{a+1}\right) - 2 x\right) + \left(a^4 + 4 a^3 + b^4 + 6 b^3 + 11 b^2 + 6 b\right) \tan^{-1}\left(\frac{x}{a+1}\right) + a \left(\left(-4 b^2 - 12 b\right) \tan^{-1}\left(\frac{x}{a+1}\right) - 3 \left(2 \tan^{-1}\left(\frac{x}{a+1}\right) + x\right)\right) - x\right)\right)\right) \Big/ \left((2 a+1) \left(b^2 + 3 b + 2\right)^2\right) + \text{constant}$$

$\log(x)$ is the natural logarithm

Expanded form of the integral

$$\frac{\tan^{-1}\left(\frac{x}{a}\right) a^7}{(2a+1)\left(b^2+3b+2\right)^2} - \frac{\tan^{-1}\left(\frac{x}{a+1}\right) a^7}{(2a+1)\left(b^2+3b+2\right)^2} + \frac{2\tan^{-1}\left(\frac{x}{a}\right) a^6}{(2a+1)\left(b^2+3b+2\right)^2} -$$

$$\frac{5\tan^{-1}\left(\frac{x}{a+1}\right) a^6}{(2a+1)\left(b^2+3b+2\right)^2} + \frac{2x a^5}{(2a+1)\left(b^2+3b+2\right)^2} - \frac{2b^2\tan^{-1}\left(\frac{x}{a}\right) a^5}{(2a+1)\left(b^2+3b+2\right)^2} -$$

$$\frac{6b\tan^{-1}\left(\frac{x}{a}\right) a^5}{(2a+1)\left(b^2+3b+2\right)^2} - \frac{4\tan^{-1}\left(\frac{x}{a}\right) a^5}{(2a+1)\left(b^2+3b+2\right)^2} + \frac{2b^2\tan^{-1}\left(\frac{x}{a+1}\right) a^5}{(2a+1)\left(b^2+3b+2\right)^2} +$$

$$\frac{6b\tan^{-1}\left(\frac{x}{a+1}\right) a^5}{(2a+1)\left(b^2+3b+2\right)^2} - \frac{5\tan^{-1}\left(\frac{x}{a+1}\right) a^5}{(2a+1)\left(b^2+3b+2\right)^2} + \frac{5x a^4}{(2a+1)\left(b^2+3b+2\right)^2} -$$

$$\frac{4b^2\tan^{-1}\left(\frac{x}{a}\right) a^4}{(2a+1)\left(b^2+3b+2\right)^2} - \frac{12b\tan^{-1}\left(\frac{x}{a}\right) a^4}{(2a+1)\left(b^2+3b+2\right)^2} - \frac{10\tan^{-1}\left(\frac{x}{a}\right) a^4}{(2a+1)\left(b^2+3b+2\right)^2} +$$

$$\frac{6b^2\tan^{-1}\left(\frac{x}{a+1}\right) a^4}{(2a+1)\left(b^2+3b+2\right)^2} + \frac{18b\tan^{-1}\left(\frac{x}{a+1}\right) a^4}{(2a+1)\left(b^2+3b+2\right)^2} + \frac{5\tan^{-1}\left(\frac{x}{a+1}\right) a^4}{(2a+1)\left(b^2+3b+2\right)^2} +$$

$$\frac{4x a^3}{(2a+1)\left(b^2+3b+2\right)^2} + \frac{b^4\tan^{-1}\left(\frac{x}{a}\right) a^3}{(2a+1)\left(b^2+3b+2\right)^2} + \frac{6b^3\tan^{-1}\left(\frac{x}{a}\right) a^3}{(2a+1)\left(b^2+3b+2\right)^2} +$$

$$\frac{11b^2\tan^{-1}\left(\frac{x}{a}\right) a^3}{(2a+1)\left(b^2+3b+2\right)^2} + \frac{6b\tan^{-1}\left(\frac{x}{a}\right) a^3}{(2a+1)\left(b^2+3b+2\right)^2} - \frac{\tan^{-1}\left(\frac{x}{a}\right) a^3}{(2a+1)\left(b^2+3b+2\right)^2} -$$

$$\frac{b^4\tan^{-1}\left(\frac{x}{a+1}\right) a^3}{(2a+1)\left(b^2+3b+2\right)^2} - \frac{6b^3\tan^{-1}\left(\frac{x}{a+1}\right) a^3}{(2a+1)\left(b^2+3b+2\right)^2} - \frac{7b^2\tan^{-1}\left(\frac{x}{a+1}\right) a^3}{(2a+1)\left(b^2+3b+2\right)^2} +$$

$$\frac{6b\tan^{-1}\left(\frac{x}{a+1}\right) a^3}{(2a+1)\left(b^2+3b+2\right)^2} + \frac{6\tan^{-1}\left(\frac{x}{a+1}\right) a^3}{(2a+1)\left(b^2+3b+2\right)^2} + \frac{x a^2}{(2a+1)\left(b^2+3b+2\right)^2} +$$

$$\frac{2b^4\tan^{-1}\left(\frac{x}{a}\right) a^2}{(2a+1)\left(b^2+3b+2\right)^2} + \frac{12b^3\tan^{-1}\left(\frac{x}{a}\right) a^2}{(2a+1)\left(b^2+3b+2\right)^2} + \frac{26b^2\tan^{-1}\left(\frac{x}{a}\right) a^2}{(2a+1)\left(b^2+3b+2\right)^2} +$$

$$\frac{24b\tan^{-1}\left(\frac{x}{a}\right) a^2}{(2a+1)\left(b^2+3b+2\right)^2} + \frac{8\tan^{-1}\left(\frac{x}{a}\right) a^2}{(2a+1)\left(b^2+3b+2\right)^2} - \frac{b^4\tan^{-1}\left(\frac{x}{a+1}\right) a^2}{(2a+1)\left(b^2+3b+2\right)^2} -$$

$$\frac{6b^3\tan^{-1}\left(\frac{x}{a+1}\right) a^2}{(2a+1)\left(b^2+3b+2\right)^2} - \frac{11b^2\tan^{-1}\left(\frac{x}{a+1}\right) a^2}{(2a+1)\left(b^2+3b+2\right)^2} - \frac{6b\tan^{-1}\left(\frac{x}{a+1}\right) a^2}{(2a+1)\left(b^2+3b+2\right)^2} +$$

$$\frac{b^4\tan^{-1}\left(\frac{x}{a}\right) a}{(2a+1)\left(b^2+3b+2\right)^2} + \frac{6b^3\tan^{-1}\left(\frac{x}{a}\right) a}{(2a+1)\left(b^2+3b+2\right)^2} + \frac{13b^2\tan^{-1}\left(\frac{x}{a}\right) a}{(2a+1)\left(b^2+3b+2\right)^2} +$$

$$\frac{12b\tan^{-1}\left(\frac{x}{a}\right) a}{(2a+1)\left(b^2+3b+2\right)^2} + \frac{4\tan^{-1}\left(\frac{x}{a}\right) a}{(2a+1)\left(b^2+3b+2\right)^2} + \text{constant}$$

Series expansion of the integral at x=0

$$x - \left(x^3 \left(a^4 \left(-\left(2 b^2 + 6 b + 5\right)\right) - 2 a^3 \left(2 b^2 + 6 b + 5\right) + a^2 \left(2 b^4 + 12 b^3 + 24 b^2 + 18 b + 3\right) + 2 a \left(b^2 + 3 b + 2\right)^2 + \left(b^2 + 3 b + 2\right)^2\right)\right) / \left(3 \left(a^2 (a + 1)^2 \left(b^2 + 3 b + 2\right)^2\right)\right) + \left(a (a + 1) x^5 \left(\frac{(a + 1) \left(a^4 - a^2 \left(2 b^2 + 6 b + 5\right) + \left(b^2 + 3 b + 2\right)^2\right)}{5 a^5} - \frac{1}{5 (a + 1)^5} a \left(a^4 + 4 a^3 + a^2 \left(-2 b^2 - 6 b + 1\right) - 2 a \left(2 b^2 + 6 b + 3\right) + b \left(b^3 + 6 b^2 + 11 b + 6\right)\right)\right)\right) / \left((2 a + 1) \left(b^2 + 3 b + 2\right)^2\right) + O\left(x^6\right)$$

(Taylor series)

Series expansion of the integral at x=∞

$$\frac{a^2 (a + 1)^2 x}{\left(b^2 + 3 b + 2\right)^2} + \left(\pi a^2 (a + 1)^2 \left(\sqrt{\frac{1}{a^2}} \left(a^4 - a^2 \left(2 b^2 + 6 b + 5\right) + \left(b^2 + 3 b + 2\right)^2\right) - \sqrt{\frac{1}{(a + 1)^2}} \left(a^4 + 4 a^3 + a^2 \left(-2 b^2 - 6 b + 1\right) - 2 a \left(2 b^2 + 6 b + 3\right) + b \left(b^3 + 6 b^2 + 11 b + 6\right)\right)\right)\right) / \left(2 (2 a + 1) \left(b^2 + 3 b + 2\right)^2\right) + \frac{2 a^2 (a + 1)^2 \left(a^2 + a - b^2 - 3 b - 2\right)}{\left(b^2 + 3 b + 2\right)^2 x} + O\left(\left(\frac{1}{x}\right)^3\right)$$

(Laurent series)

Now, we calculate the expression containing the gamma functions in the right-hand side:

1/2*Pi^0.5 (((gamma(a+1/2) gamma(b+1) gamma(b-a+1/2)))) / (((gamma(a) gamma(b+1/2) gamma(b-a+1))))

Input

$$\frac{1}{2}\sqrt{\pi} \times \frac{\Gamma\left(a+\frac{1}{2}\right)\Gamma(b+1)\Gamma\left(b-a+\frac{1}{2}\right)}{\Gamma(a)\Gamma\left(b+\frac{1}{2}\right)\Gamma(b-a+1)}$$

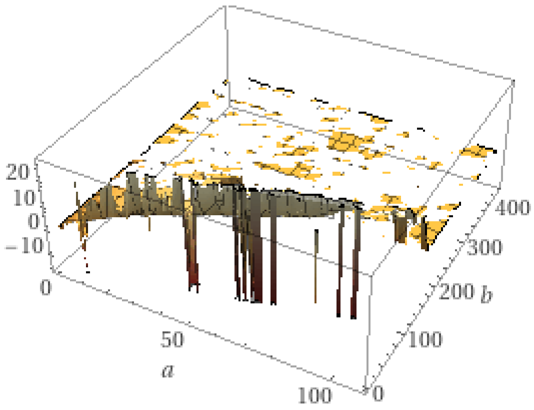
$\Gamma(x)$ is the gamma function

Exact result

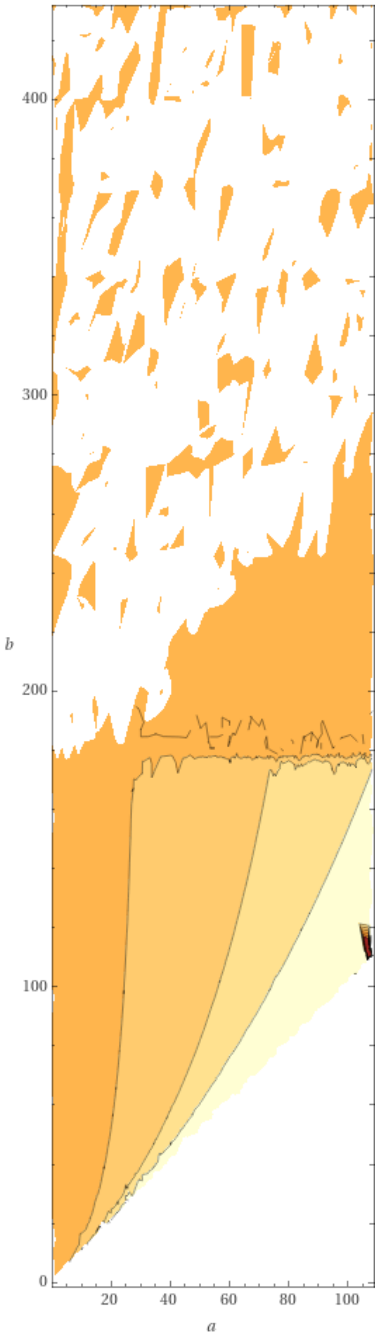
$$\frac{\sqrt{\pi}\,\Gamma\left(a+\frac{1}{2}\right)\Gamma(b+1)\Gamma\left(-a+b+\frac{1}{2}\right)}{2\,\Gamma(a)\Gamma\left(b+\frac{1}{2}\right)\Gamma(-a+b+1)}$$

The study of this function provides the following representations:

3D plot



Contour plot



Roots

(no roots exist)

Series expansion at a = 0

$$
\begin{aligned}
&\frac{\pi a}{2} + \frac{1}{2}\pi a^2 \Bigl(-\psi^{(0)}\bigl(b+\tfrac{1}{2}\bigr) + \psi^{(0)}(b+1) + \gamma + \psi^{(0)}\bigl(\tfrac{1}{2}\bigr)\Bigr) + \\
&\quad \frac{1}{12}\pi a^3 \Bigl(3\psi^{(0)}\bigl(b+\tfrac{1}{2}\bigr)^2 - 6\Bigl(\psi^{(0)}(b+1)+\gamma+\psi^{(0)}\bigl(\tfrac{1}{2}\bigr)\Bigr)\psi^{(0)}\bigl(b+\tfrac{1}{2}\bigr) + \\
&\qquad 3\psi^{(0)}(b+1)^2 + 6\Bigl(\gamma+\psi^{(0)}\bigl(\tfrac{1}{2}\bigr)\Bigr)\psi^{(0)}(b+1) + 3\psi^{(1)}\bigl(b+\tfrac{1}{2}\bigr) - \\
&\qquad 3\psi^{(1)}(b+1) + \pi^2 + 3\gamma^2 + 3\psi^{(0)}\bigl(\tfrac{1}{2}\bigr)^2 + 6\gamma\psi^{(0)}\bigl(\tfrac{1}{2}\bigr)\Bigr) + \\
&\quad \frac{1}{12}\pi a^4 \Bigl(-\psi^{(0)}\bigl(b+\tfrac{1}{2}\bigr)^3 + 3\Bigl(\psi^{(0)}(b+1)+\gamma+\psi^{(0)}\bigl(\tfrac{1}{2}\bigr)\Bigr)\psi^{(0)}\bigl(b+\tfrac{1}{2}\bigr)^2 - \\
&\qquad \Bigl(3\psi^{(0)}(b+1)^2 + 6\Bigl(\gamma+\psi^{(0)}\bigl(\tfrac{1}{2}\bigr)\Bigr)\psi^{(0)}(b+1) + 3\psi^{(1)}\bigl(b+\tfrac{1}{2}\bigr) - \\
&\qquad\quad 3\psi^{(1)}(b+1) + \pi^2 + 3\gamma^2 + 3\psi^{(0)}\bigl(\tfrac{1}{2}\bigr)^2 + 6\gamma\psi^{(0)}\bigl(\tfrac{1}{2}\bigr)\Bigr)\psi^{(0)}\bigl(b+\tfrac{1}{2}\bigr) + \\
&\qquad \psi^{(0)}(b+1)^3 + 3\Bigl(\gamma+\psi^{(0)}\bigl(\tfrac{1}{2}\bigr)\Bigr)\psi^{(0)}(b+1)^2 + 3\psi^{(0)}\bigl(\tfrac{1}{2}\bigr)\psi^{(1)}\bigl(b+\tfrac{1}{2}\bigr) + \\
&\qquad 3\gamma\psi^{(1)}\bigl(b+\tfrac{1}{2}\bigr) + \psi^{(0)}(b+1) \\
&\qquad\ \Bigl(3\psi^{(1)}\bigl(b+\tfrac{1}{2}\bigr) - 3\psi^{(1)}(b+1) + \pi^2 + 3\gamma^2 + 3\psi^{(0)}\bigl(\tfrac{1}{2}\bigr)^2 + 6\gamma\psi^{(0)}\bigl(\tfrac{1}{2}\bigr)\Bigr) - \\
&\qquad 3\psi^{(0)}\bigl(\tfrac{1}{2}\bigr)\psi^{(1)}(b+1) - 3\gamma\psi^{(1)}(b+1) - \psi^{(2)}\bigl(b+\tfrac{1}{2}\bigr) + \\
&\qquad \psi^{(2)}(b+1) + \gamma\pi^2 + \gamma^3 - \psi^{(2)}(1) + \psi^{(2)}\bigl(\tfrac{1}{2}\bigr) + \\
&\qquad \psi^{(0)}\bigl(\tfrac{1}{2}\bigr)^3 + 3\gamma\psi^{(0)}\bigl(\tfrac{1}{2}\bigr)^2 + \pi^2\psi^{(0)}\bigl(\tfrac{1}{2}\bigr) + 3\gamma^2\psi^{(0)}\bigl(\tfrac{1}{2}\bigr)\Bigr) + \\
&\quad \frac{1}{720}\pi \Bigl(15\psi^{(0)}\bigl(b+\tfrac{1}{2}\bigr)^4 - 60\Bigl(\psi^{(0)}(b+1)+\psi^{(0)}\bigl(\tfrac{1}{2}\bigr)+\gamma\Bigr)\psi^{(0)}\bigl(b+\tfrac{1}{2}\bigr)^3 + \\
&\qquad 30\Bigl(3\psi^{(0)}(b+1)^2 + 6\Bigl(\gamma+\psi^{(0)}\bigl(\tfrac{1}{2}\bigr)\Bigr)\psi^{(0)}(b+1) + 3\psi^{(1)}\bigl(b+\tfrac{1}{2}\bigr) - \\
&\qquad\quad 3\psi^{(1)}(b+1) + 3\psi^{(0)}\bigl(\tfrac{1}{2}\bigr)^2 + 6\gamma\psi^{(0)}\bigl(\tfrac{1}{2}\bigr) + \pi^2 + 3\gamma^2\Bigr)\psi^{(0)}\bigl(b+\tfrac{1}{2}\bigr)^2 - \\
&\qquad 60\Bigl(\psi^{(0)}(b+1)^3 + 3\Bigl(\gamma+\psi^{(0)}\bigl(\tfrac{1}{2}\bigr)\Bigr)\psi^{(0)}(b+1)^2 + \\
&\qquad\quad \Bigl(3\psi^{(1)}\bigl(b+\tfrac{1}{2}\bigr) - 3\psi^{(1)}(b+1) + 3\psi^{(0)}\bigl(\tfrac{1}{2}\bigr)^2 + 6\gamma\psi^{(0)}\bigl(\tfrac{1}{2}\bigr) + \pi^2 + 3\gamma^2\Bigr) \\
&\qquad\quad\ \psi^{(0)}(b+1) + 3\Bigl(\gamma+\psi^{(0)}\bigl(\tfrac{1}{2}\bigr)\Bigr)\psi^{(1)}\bigl(b+\tfrac{1}{2}\bigr) - 3\psi^{(0)}\bigl(\tfrac{1}{2}\bigr)\psi^{(1)}(b+1) - \\
&\qquad\quad 3\gamma\psi^{(1)}(b+1) - \psi^{(2)}\bigl(b+\tfrac{1}{2}\bigr) + \psi^{(2)}(b+1) - \psi^{(2)}(1) + \psi^{(2)}\bigl(\tfrac{1}{2}\bigr) + \\
&\qquad\quad \psi^{(0)}\bigl(\tfrac{1}{2}\bigr)^3 + 3\gamma\psi^{(0)}\bigl(\tfrac{1}{2}\bigr)^2 + \pi^2\psi^{(0)}\bigl(\tfrac{1}{2}\bigr) + 3\gamma^2\psi^{(0)}\bigl(\tfrac{1}{2}\bigr) + \gamma\pi^2 + \gamma^3\Bigr) \\
&\qquad\ \psi^{(0)}\bigl(b+\tfrac{1}{2}\bigr) + 15\psi^{(0)}(b+1)^4 + 60\Bigl(\gamma+\psi^{(0)}\bigl(\tfrac{1}{2}\bigr)\Bigr)\psi^{(0)}(b+1)^3 + \\
&\qquad 45\psi^{(1)}\bigl(b+\tfrac{1}{2}\bigr)^2 + 45\psi^{(1)}(b+1)^2 + 90\psi^{(0)}\bigl(\tfrac{1}{2}\bigr)^2\psi^{(1)}\bigl(b+\tfrac{1}{2}\bigr) + \\
&\qquad 180\gamma\psi^{(0)}\bigl(\tfrac{1}{2}\bigr)\psi^{(1)}\bigl(b+\tfrac{1}{2}\bigr) + 30\pi^2\psi^{(1)}\bigl(b+\tfrac{1}{2}\bigr) + \\
&\qquad 90\gamma^2\psi^{(1)}\bigl(b+\tfrac{1}{2}\bigr) + 30\psi^{(0)}(b+1)^2 \\
&\qquad\ \Bigl(3\psi^{(1)}\bigl(b+\tfrac{1}{2}\bigr) - 3\psi^{(1)}(b+1) + 3\psi^{(0)}\bigl(\tfrac{1}{2}\bigr)^2 + 6\gamma\psi^{(0)}\bigl(\tfrac{1}{2}\bigr) + \pi^2 + 3\gamma^2\Bigr) - \\
&\qquad 90\psi^{(1)}\bigl(b+\tfrac{1}{2}\bigr)\psi^{(1)}(b+1) - 90\psi^{(0)}\bigl(\tfrac{1}{2}\bigr)^2\psi^{(1)}(b+1) - \\
&\qquad 180\gamma\psi^{(0)}\bigl(\tfrac{1}{2}\bigr)\psi^{(1)}(b+1) - 30\pi^2\psi^{(1)}(b+1) - \\
&\qquad 90\gamma^2\psi^{(1)}(b+1) - 60\psi^{(0)}\bigl(\tfrac{1}{2}\bigr)\psi^{(2)}\bigl(b+\tfrac{1}{2}\bigr) - \\
&\qquad 60\gamma\psi^{(2)}\bigl(b+\tfrac{1}{2}\bigr) + 60\psi^{(0)}\bigl(\tfrac{1}{2}\bigr)\psi^{(2)}(b+1) + 60\gamma\psi^{(2)}(b+1) + \\
&\qquad 60\psi^{(0)}(b+1)\Bigl(3\Bigl(\gamma+\psi^{(0)}\bigl(\tfrac{1}{2}\bigr)\Bigr)\psi^{(1)}\bigl(b+\tfrac{1}{2}\bigr) - 3\Bigl(\gamma+\psi^{(0)}\bigl(\tfrac{1}{2}\bigr)\Bigr)\psi^{(1)}(b+1) - \\
&\qquad\quad \psi^{(2)}\bigl(b+\tfrac{1}{2}\bigr) + \psi^{(2)}(b+1) - \psi^{(2)}(1) + \psi^{(2)}\bigl(\tfrac{1}{2}\bigr) + \psi^{(0)}\bigl(\tfrac{1}{2}\bigr)^3 + \\
&\qquad\quad 3\gamma\psi^{(0)}\bigl(\tfrac{1}{2}\bigr)^2 + \pi^2\psi^{(0)}\bigl(\tfrac{1}{2}\bigr) + 3\gamma^2\psi^{(0)}\bigl(\tfrac{1}{2}\bigr) + \gamma\pi^2 + \gamma^3\Bigr) + \\
&\qquad 15\psi^{(3)}\bigl(b+\tfrac{1}{2}\bigr) - 15\psi^{(3)}(b+1) - 60\psi^{(0)}\bigl(\tfrac{1}{2}\bigr)\psi^{(2)}(1) - 60\gamma\psi^{(2)}(1) + \\
&\qquad 60\psi^{(0)}\bigl(\tfrac{1}{2}\bigr)\psi^{(2)}\bigl(\tfrac{1}{2}\bigr) + 60\gamma\psi^{(2)}\bigl(\tfrac{1}{2}\bigr) + 15\psi^{(0)}\bigl(\tfrac{1}{2}\bigr)^4 + 60\gamma\psi^{(0)}\bigl(\tfrac{1}{2}\bigr)^3 + \\
&\qquad 30\pi^2\psi^{(0)}\bigl(\tfrac{1}{2}\bigr)^2 + 90\gamma^2\psi^{(0)}\bigl(\tfrac{1}{2}\bigr)^2 + 60\gamma\pi^2\psi^{(0)}\bigl(\tfrac{1}{2}\bigr) + \\
&\qquad 60\gamma^3\psi^{(0)}\bigl(\tfrac{1}{2}\bigr) + 19\pi^4 + 30\gamma^2\pi^2 + 15\gamma^4\Bigr)a^5 + O\bigl(a^6\bigr)
\end{aligned}
$$

(Taylor series)

$\psi^{(n)}(x)$ is the n^{th} derivative of the digamma function

γ is the Euler-Mascheroni constant

Derivative

$$\frac{\partial}{\partial a}\left(\frac{\sqrt{\pi}\,\left(\Gamma\left(a+\frac{1}{2}\right)\Gamma(b+1)\,\Gamma\left(b-a+\frac{1}{2}\right)\right)}{2\left(\Gamma(a)\,\Gamma\left(b+\frac{1}{2}\right)\Gamma(b-a+1)\right)}\right)=$$
$$\left(\sqrt{\pi}\,\Gamma\left(a+\frac{1}{2}\right)\Gamma(b+1)\,\Gamma\left(-a+b+\frac{1}{2}\right)\left(-\psi^{(0)}\left(-a+b+\frac{1}{2}\right)+\psi^{(0)}(-a+b+1)-\psi^{(0)}(a)+\psi^{(0)}\left(a+\frac{1}{2}\right)\right)\right)\Big/\left(2\,\Gamma(a)\,\Gamma\left(b+\frac{1}{2}\right)\Gamma(-a+b+1)\right)$$

For a = 2 and b = 3, we obtain from the initial expression :

integrate((1+(x/(3+1))^2) / (1+(x/2)^2) * (1+(x/(3+2))^2) / (1+(x/(2+1))^2))dx

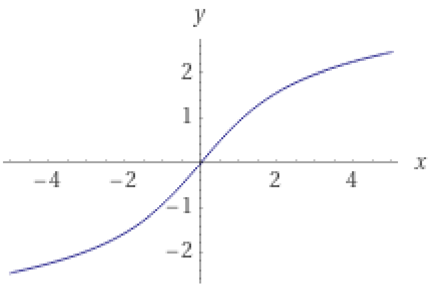
Indefinite integral

$$\int\frac{\left(1+\left(\frac{x}{3+1}\right)^2\right)\left(1+\left(\frac{x}{3+2}\right)^2\right)}{\left(1+\left(\frac{x}{2}\right)^2\right)\left(1+\left(\frac{x}{2+1}\right)^2\right)}\,dx=$$
$$\frac{3}{500}\left(15\,x-112\tan^{-1}\left(\frac{x}{3}\right)+378\tan^{-1}\left(\frac{x}{2}\right)\right)+\text{constant}$$

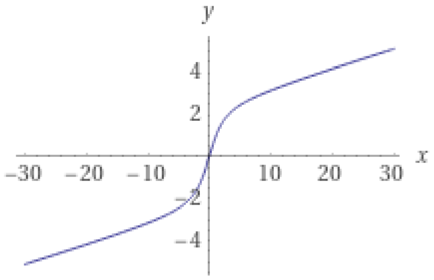
$\tan^{-1}(x)$ is the inverse tangent function

The study of this function provides the following representations:

Plots of the integral **(figures that can be related to the open strings)**



(x from −5 to 5)



(x from −30 to 30)

Alternate forms of the integral

$$\frac{3}{500}\left(15x - 14\left(8\tan^{-1}\left(\frac{x}{3}\right) - 27\tan^{-1}\left(\frac{x}{2}\right)\right)\right) + \text{constant}$$

$$\frac{9x}{100} - \frac{42}{125} i\log\left(1 - \frac{ix}{3}\right) + \frac{42}{125} i\log\left(1 + \frac{ix}{3}\right) + \frac{567}{500} i\log\left(1 - \frac{ix}{2}\right) - \frac{567}{500} i\log\left(1 + \frac{ix}{2}\right) + \text{constant}$$

Expanded form of the integral

$$\frac{9x}{100} - \frac{84}{125}\tan^{-1}\left(\frac{x}{3}\right) + \frac{567}{250}\tan^{-1}\left(\frac{x}{2}\right) + \text{constant}$$

Series expansion of the integral at x=0

$$x - \frac{931x^3}{10800} + \frac{8827x^5}{648000} + O\left(x^6\right)$$

(Taylor series)

Series expansion of the integral at x=-2 i

$$\frac{1}{250}\left(\left(\frac{3}{4}\left(378 i\log(x+2i) + 4i\left(56\tanh^{-1}\left(\frac{2}{3}\right) - 3(5 + 63\log(2))\right) + 189\pi\right) - \frac{297}{40}(x+2i) - \frac{78687 i (x+2i)^2}{1600} + \frac{427959(x+2i)^3}{16000} + \frac{27191367 i (x+2i)^4}{1280000} - \frac{536000157(x+2i)^5}{32000000} + O\left((x+2i)^6\right)\right) - 567\pi\left\lfloor\frac{3}{4} - \frac{\arg(x+2i)}{2\pi}\right\rfloor\right)$$

Series expansion of the integral at x=2 i

$$\frac{1}{250}\left(\left(\frac{3}{4}\left(-378\,i\log(x-2\,i)+756\,i\log(2)-224\,i\tanh^{-1}\left(\frac{2}{3}\right)+189\,\pi+60\,i\right)-\frac{297}{40}(x-2\,i)+\frac{78\,687\,i\,(x-2\,i)^2}{1600}+\frac{427\,959\,(x-2\,i)^3}{16\,000}-\frac{27\,191\,367\,i\,(x-2\,i)^4}{1\,280\,000}-\frac{536\,000\,157\,(x-2\,i)^5}{32\,000\,000}+O\left((x-2\,i)^6\right)\right)+567\,\pi\left\lfloor\frac{\pi-2\arg(x-2\,i)}{4\,\pi}\right\rfloor\right)$$

Series expansion of the integral at x=-3 i

$$\frac{1}{250}\left(\left(\left(-84\,i\log(x+3\,i)-\frac{3}{2}\,i\left(45+378\tanh^{-1}\left(\frac{3}{2}\right)-56\log(2)-56\log(3)\right)+525\,\pi\right)-\frac{2183}{10}(x+3\,i)+\frac{20\,587}{150}\,i\,(x+3\,i)^2+\frac{633\,647\,(x+3\,i)^3}{6750}-\frac{19\,110\,007\,i\,(x+3\,i)^4}{270\,000}-\frac{573\,925\,667\,(x+3\,i)^5}{10\,125\,000}+O\left((x+3\,i)^6\right)\right)-399\,\pi\left\lfloor\frac{3}{4}-\frac{\arg(x+3\,i)}{2\,\pi}\right\rfloor-567\,\pi\left\lfloor\frac{\arg(x+3\,i)}{2\,\pi}+\frac{3}{4}\right\rfloor\right)$$

Series expansion of the integral at x=3 i

$$\frac{1}{250}\left(\left(\left(84\,i\log(x-3\,i)+\frac{3}{2}\,i\left(45+378\tanh^{-1}\left(\frac{3}{2}\right)-56\log(2)-56\log(3)\right)-42\,\pi\right)-\frac{2183}{10}(x-3\,i)-\frac{20\,587}{150}\,i\,(x-3\,i)^2+\frac{633\,647\,(x-3\,i)^3}{6750}+\frac{19\,110\,007\,i\,(x-3\,i)^4}{270\,000}-\frac{573\,925\,667\,(x-3\,i)^5}{10\,125\,000}+O\left((x-3\,i)^6\right)\right)+399\,\pi\left\lfloor\frac{\pi-2\arg(x-3\,i)}{4\,\pi}\right\rfloor+567\,\pi\left\lfloor\frac{2\arg(x-3\,i)+\pi}{4\,\pi}\right\rfloor\right)$$

Series expansion of the integral at x=∞

$$\frac{9x}{100}+\frac{399\pi}{500}-\frac{63}{25x}+\frac{2268}{125x^5}+O\left(\left(\frac{1}{x}\right)^6\right)$$

(Laurent series)

Definite integral after subtraction of diverging parts

$$\int_0^\infty\left(\frac{\left(1+\frac{x^2}{25}\right)\left(1+\frac{x^2}{16}\right)}{\left(1+\frac{x^2}{9}\right)\left(1+\frac{x^2}{4}\right)}-\frac{9}{100}\right)dx=\frac{399\pi}{500}\approx 2.50699$$

From the solution of

$$\int\frac{\left(1+\left(\frac{x}{3+1}\right)^2\right)\left(1+\left(\frac{x}{3+2}\right)^2\right)}{\left(1+\left(\frac{x}{2}\right)^2\right)\left(1+\left(\frac{x}{2+1}\right)^2\right)}dx=$$
$$\frac{3}{500}\left(15x-112\tan^{-1}\left(\frac{x}{3}\right)+378\tan^{-1}\left(\frac{x}{2}\right)\right)+\text{constant}$$

we obtain:

3/500 (15 x - 112 tan^(-1)(x/3) + 378 tan^(-1)(x/2))

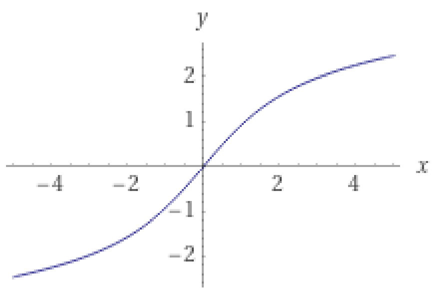
Input

$$\frac{3}{500}\left(15x-112\tan^{-1}\left(\frac{x}{3}\right)+378\tan^{-1}\left(\frac{x}{2}\right)\right)$$

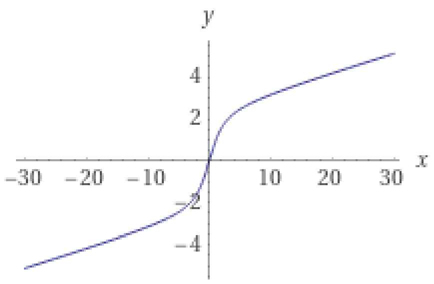
$\tan^{-1}(x)$ is the inverse tangent function

The study of this function provides the following representations:

Plots **(figures that can be related to the open strings)**



(x from −5 to 5)



(x from −30 to 30)

Alternate forms

$$\frac{3}{500}\left(15\,x - 14\left(8\tan^{-1}\left(\frac{x}{3}\right) - 27\tan^{-1}\left(\frac{x}{2}\right)\right)\right)$$

$$\frac{9\,x}{100} - \frac{42}{125}\,i\log\left(1 - \frac{i\,x}{3}\right) + \frac{42}{125}\,i\log\left(1 + \frac{i\,x}{3}\right) + \frac{567}{500}\,i\log\left(1 - \frac{i\,x}{2}\right) - \frac{567}{500}\,i\log\left(1 + \frac{i\,x}{2}\right)$$

$\log(x)$ is the natural logarithm

Expanded form

$$\frac{9\,x}{100} - \frac{84}{125}\tan^{-1}\left(\frac{x}{3}\right) + \frac{567}{250}\tan^{-1}\left(\frac{x}{2}\right)$$

Integer root

$x = 0$

Properties as a real function

Domain

$\mathbb{R}$ (all real numbers)

Range

$\mathbb{R}$ (all real numbers)

Bijectivity

bijective from its domain to $\mathbb{R}$

Parity

odd

Series expansion at x=0

$$x - \frac{931\,x^3}{10\,800} + \frac{8827\,x^5}{648\,000} + O\left(x^6\right)$$

(Taylor series)

Series expansion at x=-2 i

$$\frac{1}{250}\left(\left(\frac{3}{4}\left(378\,i\log(x+2\,i) + 4\,i\left(56\tanh^{-1}\left(\frac{2}{3}\right) - 3\,(5 + 63\log(2))\right) + 189\,\pi\right) - \frac{297}{40}(x+2\,i) - \frac{78\,687\,i\,(x+2\,i)^2}{1600} + \frac{427\,959\,(x+2\,i)^3}{16\,000} + \frac{27\,191\,367\,i\,(x+2\,i)^4}{1\,280\,000} - \frac{536\,000\,157\,(x+2\,i)^5}{32\,000\,000} + O\left((x+2\,i)^6\right)\right) - 567\,\pi\left\lfloor\frac{3}{4} - \frac{\arg(x+2\,i)}{2\,\pi}\right\rfloor\right)$$

Series expansion at x=2 i

$$\frac{1}{250}\left(\left(\frac{3}{4}\left(-378\,i\log(x-2\,i)+756\,i\log(2)-224\,i\tanh^{-1}\left(\frac{2}{3}\right)+189\,\pi+60\,i\right)-\frac{297}{40}(x-2\,i)+\frac{78\,687\,i\,(x-2\,i)^2}{1600}+\frac{427\,959\,(x-2\,i)^3}{16\,000}-\frac{27\,191\,367\,i\,(x-2\,i)^4}{1\,280\,000}-\frac{536\,000\,157\,(x-2\,i)^5}{32\,000\,000}+O\left((x-2\,i)^6\right)\right)+567\,\pi\left\lfloor\frac{\pi-2\arg(x-2\,i)}{4\,\pi}\right\rfloor\right)$$

Series expansion at x=-3 i

$$\frac{1}{250}\left(\left(\left(-84\,i\log(x+3\,i)-\frac{3}{2}\,i\left(45+378\tanh^{-1}\left(\frac{3}{2}\right)-56\log(2)-56\log(3)\right)+525\,\pi\right)-\frac{2183}{10}(x+3\,i)+\frac{20\,587}{150}\,i\,(x+3\,i)^2+\frac{633\,647\,(x+3\,i)^3}{6750}-\frac{19\,110\,007\,i\,(x+3\,i)^4}{270\,000}-\frac{573\,925\,667\,(x+3\,i)^5}{10\,125\,000}+O\left((x+3\,i)^6\right)\right)-399\,\pi\left\lfloor\frac{3}{4}-\frac{\arg(x+3\,i)}{2\,\pi}\right\rfloor-567\,\pi\left\lfloor\frac{\arg(x+3\,i)}{2\,\pi}+\frac{3}{4}\right\rfloor\right)$$

Series expansion at x=3 i

$$\frac{1}{250}\left(\left(\left(84\,i\log(x-3\,i)+\frac{3}{2}\,i\left(45+378\tanh^{-1}\left(\frac{3}{2}\right)-56\log(2)-56\log(3)\right)-42\,\pi\right)-\frac{2183}{10}(x-3\,i)-\frac{20\,587}{150}\,i\,(x-3\,i)^2+\frac{633\,647\,(x-3\,i)^3}{6750}+\frac{19\,110\,007\,i\,(x-3\,i)^4}{270\,000}-\frac{573\,925\,667\,(x-3\,i)^5}{10\,125\,000}+O\left((x-3\,i)^6\right)\right)+399\,\pi\left\lfloor\frac{\pi-2\arg(x-3\,i)}{4\,\pi}\right\rfloor+567\,\pi\left\lfloor\frac{2\arg(x-3\,i)+\pi}{4\,\pi}\right\rfloor\right)$$

Series expansion at x=∞

$$\frac{9x}{100} + \frac{399\pi}{500} - \frac{63}{25x} + \frac{2268}{125x^5} + O\left(\left(\frac{1}{x}\right)^6\right)$$

(Laurent series)

Derivative

$$\frac{d}{dx}\left(\frac{3}{500}\left(15x - 112\tan^{-1}\left(\frac{x}{3}\right) + 378\tan^{-1}\left(\frac{x}{2}\right)\right)\right) = \frac{9(x^4 + 41x^2 + 400)}{100(x^2+4)(x^2+9)}$$

Indefinite integral

$$\int \frac{3}{500}\left(15x - 112\tan^{-1}\left(\frac{x}{3}\right) + 378\tan^{-1}\left(\frac{x}{2}\right)\right)dx = \frac{3\left(15x^2 - 756\log(x^2+4) + 336\log(x^2+9) - 224x\tan^{-1}\left(\frac{x}{3}\right) + 756x\tan^{-1}\left(\frac{x}{2}\right)\right)}{1000} + \text{constant}$$

From

$$\frac{3}{500}\left(15x - 112\tan^{-1}\left(\frac{x}{3}\right) + 378\tan^{-1}\left(\frac{x}{2}\right)\right)$$

For $x = 1.6579679871623^2$, we obtain:

3/500 (15*(1.6579679871623^2)- 112 tan^(-1)((1.6579679871623^2)/3) + 378 tan^(-1)((1.6579679871623^2)/2))

Input interpretation

$$\frac{3}{500}\left(15 \times 1.6579679871623^2 - 112\tan^{-1}\left(\frac{1.6579679871623^2}{3}\right) + 378\tan^{-1}\left(\frac{1.6579679871623^2}{2}\right)\right)$$

$\tan^{-1}(x)$ is the inverse tangent function

Result

1.8849555921538…

(result in radians)

1.8849555921538….

The study of this function provides the following representations:

Alternative representations

$$\frac{1}{500}\left(15\times 1.65796798716230000^2-112\tan^{-1}\left(\frac{1.65796798716230000^2}{3}\right)+378\tan^{-1}\left(\frac{1.65796798716230000^2}{2}\right)\right)3=\frac{3}{500}\left(-112\,\mathrm{sc}^{-1}\left(\frac{1.65796798716230000^2}{3}\,\middle|\,0\right)+378\,\mathrm{sc}^{-1}\left(\frac{1.65796798716230000^2}{2}\,\middle|\,0\right)+15\times 1.65796798716230000^2\right)$$

$$\frac{1}{500}\left(15\times 1.65796798716230000^2-112\tan^{-1}\left(\frac{1.65796798716230000^2}{3}\right)+378\tan^{-1}\left(\frac{1.65796798716230000^2}{2}\right)\right)3=\frac{3}{500}\left(-112\tan^{-1}\left(1,\frac{1.65796798716230000^2}{3}\right)+378\tan^{-1}\left(1,\frac{1.65796798716230000^2}{2}\right)+15\times 1.65796798716230000^2\right)$$

$$\frac{1}{500}\left(15\times 1.65796798716230000^2 - 112\tan^{-1}\left(\frac{1.65796798716230000^2}{3}\right)+ 378\tan^{-1}\left(\frac{1.65796798716230000^2}{2}\right)\right)3 = \frac{3}{500}\left(112\,i\tanh^{-1}\left(\frac{i\,1.65796798716230000^2}{3}\right) - 378\,i\tanh^{-1}\left(\frac{i\,1.65796798716230000^2}{2}\right) + 15\times 1.65796798716230000^2\right)$$

$sc^{-1}(x\mid m)$ is the inverse of the Jacobi elliptic function sc

$\tan^{-1}(x,\ y)$ is the inverse tangent function

$\tanh^{-1}(x)$ is the inverse hyperbolic tangent function

i is the imaginary unit

Series representations

$$\frac{1}{500}\left(15\times 1.65796798716230000^2 - 112\tan^{-1}\left(\frac{1.65796798716230000^2}{3}\right)+ 378\tan^{-1}\left(\frac{1.65796798716230000^2}{2}\right)\right)3 = 0.247397206180950772 + 0.79800000000000000\,i\log(2) - 1.13400000000000000\,i\log(-1.00000000000000000\,(-1.37442892322750429 + i)\,i) + 0.33600000000000000\,i\log(-1.00000000000000000\,(-0.91628594881833619 + i)\,i) + \sum_{k=1}^{\infty} -\frac{1}{k}1.134000000000000\times 0.500000000000000000^k\,i\left(1.000000000000000\,(-1.00000000000000000\,(-1.37442892322750429 + i)\,i)^k - 0.296296296296296\,(-1.00000000000000000\,(-0.91628594881833619 + i)\,i)^k\right)$$

$$\frac{1}{500}\left(15\times 1.65796798716230000^2 - 112\tan^{-1}\left(\frac{1.65796798716230000^2}{3}\right)+ 378\tan^{-1}\left(\frac{1.65796798716230000^2}{2}\right)\right)3 = 0.247397206180950772 - 0.79800000000000000\,i\log(2) - 0.33600000000000000\,i \log(-1.00000000000000000\,i\,(0.91628594881833619+i)) + 1.13400000000000000\,i \log(-1.00000000000000000\,i\,(1.37442892322750429+i)) + \sum_{k=1}^{\infty} -\frac{1}{k}0.336000000000000\times 0.500000000000000000^k\, i \left(1.000000000000000\,(-1.00000000000000000\,i\,(0.91628594881833619+i))^k - 3.37500000000000\,(-1.00000000000000000\,i\,(1.37442892322750429+i))^k\right)$$

$$\frac{1}{500}\left(15\times 1.65796798716230000^2 - 112\tan^{-1}\left(\frac{1.65796798716230000^2}{3}\right)+ 378\tan^{-1}\left(\frac{1.65796798716230000^2}{2}\right)\right)3 = 0.247397206180950772 + \sum_{k=0}^{\infty}\frac{1}{1+2k}\left(-\frac{1}{5}\right)^k F_{1+2k}\left(-1.23148831521184384\, e^{1.21144077747140964\,k}\left(\frac{1}{1+\sqrt{1.67166395200153489}}\right)^{1+2k} + 6.2344095957599595\, e^{2.02237099368773840\,k}\left(\frac{1}{1+\sqrt{2.51124389200345350}}\right)^{1+2k}\right)$$

$\log(x)$ is the natural logarithm

F_n is the n^{th} Fibonacci number

Integral representations

$$\frac{1}{500}\left(15\times 1.65796798716230000^2 - 112\tan^{-1}\left(\frac{1.65796798716230000^2}{3}\right)+ 378\tan^{-1}\left(\frac{1.65796798716230000^2}{2}\right)\right)3 = 0.2473972061809508 + \int_0^1 \frac{1.5772010179756 + 0.9167443864912\,t^2}{0.630512026686442 + 1.72043706099165\,t^2 + 1.00000000000000\,t^4}\,dt$$

$$\frac{1}{500}\left(15\times 1.65796798716230000^2-112\tan^{-1}\left(\frac{1.65796798716230000^2}{3}\right)+378\tan^{-1}\left(\frac{1.65796798716230000^2}{2}\right)\right)3 = 0.2473972061809508+\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}-\frac{1}{\pi^{3/2}}\,0.779301199469995\,e^{-1.67046666438636529\,s}\left(1.000000000000000\,e^{0.60953725208075350\,s}-0.1975308641975309\,e^{1.06092941230561179\,s}\right)i\,\Gamma\left(\frac{1}{2}-s\right)\Gamma(1-s)\,\Gamma(s)^2\,ds \quad \text{for } 0<\gamma<\frac{1}{2}$$

$$\frac{1}{500}\left(15\times 1.65796798716230000^2-112\tan^{-1}\left(\frac{1.65796798716230000^2}{3}\right)+378\tan^{-1}\left(\frac{1.65796798716230000^2}{2}\right)\right)3 = 0.247397206180951+\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}-\frac{1}{i\,\pi\,\Gamma\left(\frac{3}{2}-s\right)}\,0.1539360394014805\,e^{-0.63607663256784778\,s}\left(-5.06250000000000+1.000000000000000\,e^{0.81093021621632876\,s}\right)\Gamma\left(\frac{1}{2}-s\right)\Gamma(1-s)\,\Gamma(s)\,ds \quad \text{for } 0<\gamma<\frac{1}{2}$$

Continued fraction representations

$$\frac{1}{500}\left(15\times 1.65796798716230000^2 - 112\tan^{-1}\left(\frac{1.65796798716230000^2}{3}\right) + 378\tan^{-1}\left(\frac{1.65796798716230000^2}{2}\right)\right)3 =$$

$$0.247397206180950772 - \frac{0.61574415760592192}{1 + \underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{0.83957994000191861\,k^2}{1+2k}} + \frac{3.11720479787997973}{1 + \underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{1.88905486500431688\,k^2}{1+2k}} =$$

$$0.247397206180950772 - \frac{0.61574415760592192}{1 + \frac{0.83957994000191861}{3+\frac{3.3583197600076745}{5+\frac{7.5562194600172675}{7+\frac{13.4332790400306978}{9+\dots}}}}} + \frac{3.11720479787997973}{1 + \frac{1.88905486500431688}{3+\frac{7.5562194600172675}{5+\frac{17.0014937850388519}{7+\frac{30.224877840069070}{9+\dots}}}}}$$

$$\frac{1}{500}\left(15\times 1.65796798716230000^2-112\tan^{-1}\left(\frac{1.65796798716230000^2}{3}\right)+378\tan^{-1}\left(\frac{1.65796798716230000^2}{2}\right)\right)3=0.247397206180950772-\frac{0.61574415760592192}{1.00000000000000000+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{0.83957994000191861\,(1-2k)^2}{1.839579940001919+0.3208401199961628\,k}}+\frac{3.11720479787997973}{1.00000000000000000+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{1.88905486500431688\,(1-2k)^2}{2.889054865004317-1.778109730008634\,k}}=$$

$$0.247397206180950772-0.61574415760592192\Bigg/\left(1.00000000000000000+0.83957994000191861\Bigg/\left(2.160420059998081+7.5562194600172675\Bigg/\left(2.481260179994244+\frac{20.9894985000479653}{2.802100299990407+\frac{41.139417060094012}{3.122940419986570+\ldots}}\right)\right)\right)+$$

$$3.11720479787997973\Bigg/\left(1.00000000000000000+1.88905486500431688\Bigg/\left(1.110945134995683+17.0014937850388519\Bigg/\left(-0.667164595012951+\frac{47.226371625107922}{-2.445274325021584+\frac{92.563688385211527}{-4.223384055030218+\ldots}}\right)\right)\right)$$

$$\frac{1}{500}\left(15\times 1.65796798716230000^2 - 112\tan^{-1}\left(\frac{1.65796798716230000^2}{3}\right)+ 378\tan^{-1}\left(\frac{1.65796798716230000^2}{2}\right)\right)3 =$$

$$2.74885784645500858 + \frac{0.51696644289931184}{3+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{0.83957994000191861\left(1+(-1)^{1+k}+k\right)^2}{3+2k}} - \frac{5.8885708886499740}{3+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{1.88905486500431688\left(1+(-1)^{1+k}+k\right)^2}{3+2k}} =$$

$$2.74885784645500858 + \frac{0.51696644289931184}{3+\frac{7.5562194600172675}{5+\frac{3.3583197600076745}{7+\frac{20.9894985000479653}{9+\frac{13.4332790400306978}{11+\dots}}}}} - \frac{5.8885708886499740}{3+\frac{17.0014937850388519}{5+\frac{7.5562194600172675}{7+\frac{47.226371625107922}{9+\frac{30.224877840069070}{11+\dots}}}}}$$

$$\frac{1}{500}\left(15\times 1.65796798716230000^2 - 112\tan^{-1}\left(\frac{1.65796798716230000^2}{3}\right) + 378\tan^{-1}\left(\frac{1.65796798716230000^2}{2}\right)\right)3 = 0.247397206180950772 - \frac{0.61574415760592192}{1.83957994000191861 + \underset{k=1}{\overset{\infty}{\mathrm{K}}} \frac{1.67915988000383723\left(1-2\left\lfloor\frac{1+k}{2}\right\rfloor\right)\left\lfloor\frac{1+k}{2}\right\rfloor}{\left(1.41978997000095931+0.41978997000095931(-1)^k\right)(1+2k)}} + \frac{3.11720479787997973}{2.88905486500431688 + \underset{k=1}{\overset{\infty}{\mathrm{K}}} \frac{3.7781097300086338\left(1-2\left\lfloor\frac{1+k}{2}\right\rfloor\right)\left\lfloor\frac{1+k}{2}\right\rfloor}{\left(1.94452743250215844+0.94452743250215844(-1)^k\right)(1+2k)}} =$$

$$0.247397206180950772 - 0.61574415760592192 \Big/ \left(1.83957994000191861 + -\left(1.67915988000383723 \Big/ \left(3.0000000000000000 - 1.67915988000383723 \Big/ \left(9.1978997000095931 - \frac{10.0749592800230234}{7.0000000000000000 - \frac{10.0749592800230234}{16.5562194600172675+\ldots}}\right)\right)\right)\right) + 3.11720479787997973 \Big/ \left(2.88905486500431688 + -\left(3.7781097300086338 \Big/ \left(3.0000000000000000 - 3.7781097300086338 \Big/ \left(14.4452743250215844 - \frac{22.6686583800518025}{7.0000000000000000 - \frac{22.6686583800518025}{26.0014937850388519+\ldots}}\right)\right)\right)\right)$$

$\underset{k=k_1}{\overset{k_2}{\mathrm{K}}} a_k / b_k$ is a continued fraction

We note that the value 1.6579679871623 is very near to the 14th root of the following Ramanujan's class invariant $Q = \left(G_{505}/G_{101/5}\right)^3 = 1164.2696$ i.e. 1.65578...

Indeed, from:

$$G_{505} = P^{-1/4}Q^{1/6} = (\sqrt{5}+2)^{1/2}\left(\frac{\sqrt{5}+1}{2}\right)^{1/4}(\sqrt{101}+10)^{1/4}$$
$$\times\left((130\sqrt{5}+29\sqrt{101})+\sqrt{169440+7540\sqrt{505}}\right)^{1/6}.$$

Thus, it remains to show that

$$(130\sqrt{5}+29\sqrt{101})+\sqrt{169440+7540\sqrt{505}}=\left(\sqrt{\frac{113+5\sqrt{505}}{8}}+\sqrt{\frac{105+5\sqrt{505}}{8}}\right)^3,$$

which is straightforward. □

$$\sqrt[14]{\left(\sqrt{\frac{113+5\sqrt{505}}{8}}+\sqrt{\frac{105+5\sqrt{505}}{8}}\right)^3} = 1{,}65578\ldots$$

Now, for a = 2 and b = 3 , from the previous expression containing the gamma functions, we obtain:

(sqrt(π) Γ(2 + 1/2) Γ(3 + 1) Γ(-2 + 3 + 1/2))/(2 Γ(2) Γ(3 + 1/2) Γ(-2 + 3 + 1))

Input

$$\frac{\sqrt{\pi}\,\Gamma\left(2+\frac{1}{2}\right)\Gamma(3+1)\,\Gamma\left(-2+3+\frac{1}{2}\right)}{2\,\Gamma(2)\,\Gamma\left(3+\frac{1}{2}\right)\Gamma(-2+3+1)}$$

$\Gamma(x)$ is the gamma function

Exact result

$$\frac{3\pi}{5}$$

Decimal approximation

1.884955592153875943077586029967701730518301639625063492584966755 3

...

1.8849555921….

Property

$\frac{3\pi}{5}$ is a transcendental number

The study of this function provides the following representations:

Alternative representations

$$\frac{\sqrt{\pi}\left(\Gamma\left(2+\frac{1}{2}\right)\Gamma(3+1)\Gamma\left(-2+3+\frac{1}{2}\right)\right)}{2\,\Gamma(2)\,\Gamma\left(3+\frac{1}{2}\right)\Gamma(-2+3+1)}=\frac{\frac{1}{2}!\times\frac{3}{2}!\times 3!\sqrt{\pi}}{2\,(1!)^2\,\frac{5}{2}!}$$

$$\frac{\sqrt{\pi}\left(\Gamma\left(2+\frac{1}{2}\right)\Gamma(3+1)\Gamma\left(-2+3+\frac{1}{2}\right)\right)}{2\,\Gamma(2)\,\Gamma\left(3+\frac{1}{2}\right)\Gamma(-2+3+1)}=\frac{e^{-\log(2)+\log(12)}\,e^{-\log G(3/2)+\log G(5/2)}\,e^{-\log G(5/2)+\log G(7/2)}\sqrt{\pi}}{2\left(e^0\right)^2 e^{-\log G(7/2)+\log G(9/2)}}$$

$$\frac{\sqrt{\pi}\left(\Gamma\left(2+\frac{1}{2}\right)\Gamma(3+1)\Gamma\left(-2+3+\frac{1}{2}\right)\right)}{2\,\Gamma(2)\,\Gamma\left(3+\frac{1}{2}\right)\Gamma(-2+3+1)}=\frac{\Gamma\left(\frac{3}{2},0\right)\Gamma\left(\frac{5}{2},0\right)\Gamma(4,0)\sqrt{\pi}}{2\,\Gamma(2,0)^2\,\Gamma\left(\frac{7}{2},0\right)}$$

Series representations

$$\frac{\sqrt{\pi}\,\left(\Gamma\left(2+\frac{1}{2}\right)\Gamma(3+1)\,\Gamma\left(-2+3+\frac{1}{2}\right)\right)}{2\,\Gamma(2)\,\Gamma\left(3+\frac{1}{2}\right)\Gamma(-2+3+1)} = \frac{12}{5}\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}$$

$$\frac{\sqrt{\pi}\,\left(\Gamma\left(2+\frac{1}{2}\right)\Gamma(3+1)\,\Gamma\left(-2+3+\frac{1}{2}\right)\right)}{2\,\Gamma(2)\,\Gamma\left(3+\frac{1}{2}\right)\Gamma(-2+3+1)} = \sum_{k=0}^{\infty}\frac{12\,(-1)^k\left(956\times 5^{-2k}-5\times 239^{-2k}\right)}{5975\,(1+2k)}$$

$$\frac{\sqrt{\pi}\,\left(\Gamma\left(2+\frac{1}{2}\right)\Gamma(3+1)\,\Gamma\left(-2+3+\frac{1}{2}\right)\right)}{2\,\Gamma(2)\,\Gamma\left(3+\frac{1}{2}\right)\Gamma(-2+3+1)} = \frac{3}{5}\sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k}+\frac{2}{1+4k}+\frac{1}{3+4k}\right)$$

Integral representations

$$\frac{\sqrt{\pi}\,\left(\Gamma\left(2+\frac{1}{2}\right)\Gamma(3+1)\,\Gamma\left(-2+3+\frac{1}{2}\right)\right)}{2\,\Gamma(2)\,\Gamma\left(3+\frac{1}{2}\right)\Gamma(-2+3+1)} = \frac{12}{5}\int_0^1\sqrt{1-t^2}\,dt$$

$$\frac{\sqrt{\pi}\,\left(\Gamma\left(2+\frac{1}{2}\right)\Gamma(3+1)\,\Gamma\left(-2+3+\frac{1}{2}\right)\right)}{2\,\Gamma(2)\,\Gamma\left(3+\frac{1}{2}\right)\Gamma(-2+3+1)} = \frac{6}{5}\int_0^1\frac{1}{\sqrt{1-t^2}}\,dt$$

$$\frac{\sqrt{\pi}\,\left(\Gamma\left(2+\frac{1}{2}\right)\Gamma(3+1)\,\Gamma\left(-2+3+\frac{1}{2}\right)\right)}{2\,\Gamma(2)\,\Gamma\left(3+\frac{1}{2}\right)\Gamma(-2+3+1)} = \frac{6}{5}\int_0^{\infty}\frac{1}{1+t^2}\,dt$$

Now, we calculate the whole equation:

1/2*Pi^0.5 (((gamma(a+1/2) gamma(b+1) gamma(b-a+1/2)))) / (((gamma(a) gamma(b+1/2) gamma(b-a+1)))) = integrate((1+(x/(b+1))^2) / (1+(x/a)^2) * (1+(x/(b+2))^2) / (1+(x/(a+1))^2))dx

Input

$$\frac{1}{2}\sqrt{\pi}\times\frac{\Gamma\left(a+\frac{1}{2}\right)\Gamma(b+1)\,\Gamma\left(b-a+\frac{1}{2}\right)}{\Gamma(a)\,\Gamma\left(b+\frac{1}{2}\right)\Gamma(b-a+1)}=\int\frac{1+\left(\frac{x}{b+1}\right)^2}{1+\left(\frac{x}{a}\right)^2}\times\frac{1+\left(\frac{x}{b+2}\right)^2}{1+\left(\frac{x}{a+1}\right)^2}\,dx$$

$\Gamma(x)$ is the gamma function

Result

$$\frac{\sqrt{\pi}\,\Gamma\left(a+\frac{1}{2}\right)\Gamma(b+1)\,\Gamma\left(-a+b+\frac{1}{2}\right)}{2\,\Gamma(a)\,\Gamma\left(b+\frac{1}{2}\right)\Gamma(-a+b+1)}=$$

$$\left(a\,(a+1)\left((a+1)\left(a^4-a^2\left(2b^2+6b+5\right)+\left(b^2+3b+2\right)^2\right)\tan^{-1}\left(\frac{x}{a}\right)+a\left(\left(2a^2+3a+1\right)x-\left(a^4+4a^3+a^2\left(-2b^2-6b+1\right)-2a\left(2b^2+6b+3\right)+b\left(b^3+6b^2+11b+6\right)\right)\tan^{-1}\left(\frac{x}{a+1}\right)\right)\right)\right)\Big/\left((2a+1)\left(b^2+3b+2\right)^2\right)$$

From:

(a (a + 1) ((a + 1) (a^4 - a^2 (2 b^2 + 6 b + 5) + (b^2 + 3 b + 2)^2) tan^(-1)(x/a) + a ((2 a^2 + 3 a + 1) x - (a^4 + 4 a^3 + a^2 (-2 b^2 - 6 b + 1) - 2 a (2 b^2 + 6 b + 3) + b (b^3 + 6 b^2 + 11 b + 6)) tan^(-1)(x/(a + 1)))))/((2 a + 1) (b^2 + 3 b + 2)^2)

For a = 2 and b = 3 , simplifying, we obtain:

(2 (3) ((3) (16 - 4 (2*9 + 6*3 + 5) + (9 + 9 + 2)^2) tan^(-1)(x/2) + 2 ((8 + 3*2 + 1) x - (16 + 32 + 4 (-2*9 – 6*3 + 1) – 2*2 (2*9 + 6*3 + 3) + 3 (3^3 + 6*9 + 11*3 + 6)) tan^(-1)(x/(2 + 1)))))/((4 + 1) (9 + 9 + 2)^2)

Input

$$\frac{1}{(4+1)(9+9+2)^2} 2\times 3\left(3\left(16-4(2\times 9+6\times 3+5)+(9+9+2)^2\right)\tan^{-1}\left(\frac{x}{2}\right)+2\left((8+3\times 2+1)x-\left(16+32+4(-2\times 9-6\times 3+1)-2\times 2(2\times 9+6\times 3+3)+3\left(3^3+6\times 9+11\times 3+6\right)\right)\tan^{-1}\left(\frac{x}{2+1}\right)\right)\right)$$

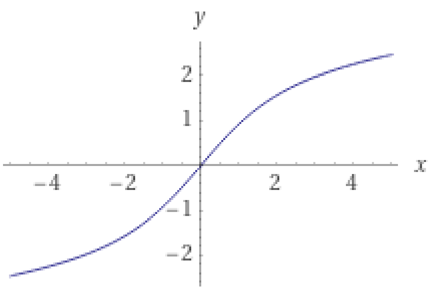
$\tan^{-1}(x)$ is the inverse tangent function

Result

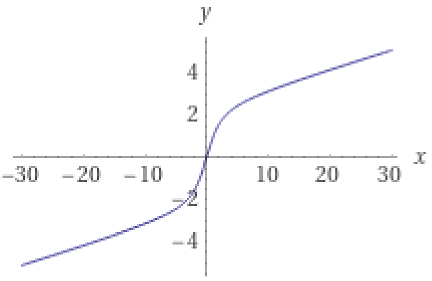
$$\frac{3\left(2\left(15x-112\tan^{-1}\left(\frac{x}{3}\right)\right)+756\tan^{-1}\left(\frac{x}{2}\right)\right)}{1000}$$

The study of this function provides the following representations:

Plots **(figures that can be related to the open strings)**



(x from -5 to 5)



(x from -30 to 30)

Alternate forms

$$\frac{3}{500}\left(15\,x - 112\tan^{-1}\left(\frac{x}{3}\right) + 378\tan^{-1}\left(\frac{x}{2}\right)\right)$$

$$\frac{3}{500}\left(15\,x - 14\left(8\tan^{-1}\left(\frac{x}{3}\right) - 27\tan^{-1}\left(\frac{x}{2}\right)\right)\right)$$

$$\frac{9\,x}{100} - \frac{42}{125}\,i\log\left(1 - \frac{i\,x}{3}\right) + \frac{42}{125}\,i\log\left(1 + \frac{i\,x}{3}\right) + \frac{567}{500}\,i\log\left(1 - \frac{i\,x}{2}\right) - \frac{567}{500}\,i\log\left(1 + \frac{i\,x}{2}\right)$$

$\log(x)$ is the natural logarithm

Expanded form

$$\frac{9\,x}{100} - \frac{84}{125}\tan^{-1}\left(\frac{x}{3}\right) + \frac{567}{250}\tan^{-1}\left(\frac{x}{2}\right)$$

Integer root

$x = 0$

Properties as a real function

Domain

$\mathbb{R}$ (all real numbers)

Range

$\mathbb{R}$ (all real numbers)

Bijectivity

bijective from its domain to $\mathbb{R}$

Parity

odd

$\mathbb{R}$ is the set of real numbers

Series expansion at x=0

$$x - \frac{931 x^3}{10800} + \frac{8827 x^5}{648000} + O(x^6)$$

(Taylor series)

Series expansion at x=-2 i

$$\frac{1}{250}\left(\left(\frac{3}{4}\left(378 i \log(x+2i) + 4 i \left(56 \tanh^{-1}\left(\frac{2}{3}\right) - 3(5 + 63\log(2))\right) + 189\pi\right) - \frac{297}{40}(x+2i) - \frac{78687 i (x+2i)^2}{1600} + \frac{427959 (x+2i)^3}{16000} + \frac{27191367 i (x+2i)^4}{1280000} - \frac{536000157 (x+2i)^5}{32000000} + O\left((x+2i)^6\right)\right) - 567\pi \left\lfloor \frac{3}{4} - \frac{\arg(x+2i)}{2\pi} \right\rfloor\right)$$

Series expansion at x=2 i

$$\frac{1}{250}\left(\left(\frac{3}{4}\left(-378 i \log(x-2i) + 756 i \log(2) - 224 i \tanh^{-1}\left(\frac{2}{3}\right) + 189\pi + 60 i\right) - \frac{297}{40}(x-2i) + \frac{78687 i (x-2i)^2}{1600} + \frac{427959 (x-2i)^3}{16000} - \frac{27191367 i (x-2i)^4}{1280000} - \frac{536000157 (x-2i)^5}{32000000} + O\left((x-2i)^6\right)\right) + 567\pi \left\lfloor \frac{\pi - 2\arg(x-2i)}{4\pi} \right\rfloor\right)$$

Series expansion at x=-3 i

$$\frac{1}{250}\left(\left(\left(-84\,i\log(x+3\,i)-\frac{3}{2}\,i\left(45+378\tanh^{-1}\left(\frac{3}{2}\right)-56\log(2)-56\log(3)\right)+525\,\pi\right)-\frac{2183}{10}(x+3\,i)+\frac{20587}{150}\,i\,(x+3\,i)^2+\frac{633647\,(x+3\,i)^3}{6750}-\frac{19110007\,i\,(x+3\,i)^4}{270000}-\frac{573925667\,(x+3\,i)^5}{10125000}+O\left((x+3\,i)^6\right)\right)-399\,\pi\left\lfloor\frac{3}{4}-\frac{\arg(x+3\,i)}{2\,\pi}\right\rfloor-567\,\pi\left\lfloor\frac{\arg(x+3\,i)}{2\,\pi}+\frac{3}{4}\right\rfloor\right)$$

Series expansion at x=3 i

$$\frac{1}{250}\left(\left(\left(84\,i\log(x-3\,i)+\frac{3}{2}\,i\left(45+378\tanh^{-1}\left(\frac{3}{2}\right)-56\log(2)-56\log(3)\right)-42\,\pi\right)-\frac{2183}{10}(x-3\,i)-\frac{20587}{150}\,i\,(x-3\,i)^2+\frac{633647\,(x-3\,i)^3}{6750}+\frac{19110007\,i\,(x-3\,i)^4}{270000}-\frac{573925667\,(x-3\,i)^5}{10125000}+O\left((x-3\,i)^6\right)\right)+399\,\pi\left\lfloor\frac{\pi-2\arg(x-3\,i)}{4\,\pi}\right\rfloor+567\,\pi\left\lfloor\frac{2\arg(x-3\,i)+\pi}{4\,\pi}\right\rfloor\right)$$

Series expansion at x=∞

$$\frac{9\,x}{100}+\frac{399\,\pi}{500}-\frac{63}{25\,x}+\frac{2268}{125\,x^5}+O\left(\left(\frac{1}{x}\right)^6\right)$$

(Laurent series)

Derivative

$$\frac{d}{dx}\left(\frac{1}{(4+1)(9+9+2)^2}2\times 3\left(3\left(16-4\,(2\times 9+6\times 3+5)+(9+9+2)^2\right)\tan^{-1}\left(\frac{x}{2}\right)+2\left((8+3\times 2+1)\,x-\left(16+32+4\,(-2\times 9-6\times 3+1)-2\times 2\,(2\times 9+6\times 3+3)+3\left(3^3+6\times 9+11\times 3+6\right)\right)\tan^{-1}\left(\frac{x}{2+1}\right)\right)\right)\right)=\frac{9\left(x^4+41\,x^2+400\right)}{100\left(x^2+4\right)\left(x^2+9\right)}$$

Indefinite integral

$$\int \frac{3\left(2\left(15 x - 112 \tan^{-1}\left(\frac{x}{3}\right)\right) + 756 \tan^{-1}\left(\frac{x}{2}\right)\right)}{1000} \, dx = \frac{3\left(15 x^2 - 756 \log\left(x^2+4\right) + 336 \log\left(x^2+9\right) - 224 x \tan^{-1}\left(\frac{x}{3}\right) + 756 x \tan^{-1}\left(\frac{x}{2}\right)\right)}{1000} + \text{constant}$$

From the above solution

$$\frac{3\left(2\left(15 x - 112 \tan^{-1}\left(\frac{x}{3}\right)\right) + 756 \tan^{-1}\left(\frac{x}{2}\right)\right)}{1000}$$

for $x = 1.6579679871623^2$, we obtain:

(3 (2 (15 (1.6579679871623^2) - 112 tan^(-1)((1.6579679871623^2)/3)) + 756 tan^(-1)((1.6579679871623^2)/2)))/1000

Input interpretation

$$\frac{1}{1000} 3\left(2\left(15 \times 1.6579679871623^2 - 112 \tan^{-1}\left(\frac{1.6579679871623^2}{3}\right)\right) + 756 \tan^{-1}\left(\frac{1.6579679871623^2}{2}\right)\right)$$

$\tan^{-1}(x)$ is the inverse tangent function

Result

1.8849555921538…

(result in radians)

1.8849555921538….

The study of this function provides the following representations:

Alternative representations

$$\frac{1}{1000}3\left(2\left(15\times 1.65796798716230000^2-112\tan^{-1}\left(\frac{1.65796798716230000^2}{3}\right)\right)+756\tan^{-1}\left(\frac{1.65796798716230000^2}{2}\right)\right)=\frac{1}{1000}\times 3\left(756\,\text{sc}^{-1}\left(\frac{1.65796798716230000^2}{2}\,\middle|\,0\right)+2\left(-112\,\text{sc}^{-1}\left(\frac{1.65796798716230000^2}{3}\,\middle|\,0\right)+15\times 1.65796798716230000^2\right)\right)$$

$$\frac{1}{1000}3\left(2\left(15\times 1.65796798716230000^2-112\tan^{-1}\left(\frac{1.65796798716230000^2}{3}\right)\right)+756\tan^{-1}\left(\frac{1.65796798716230000^2}{2}\right)\right)=\frac{1}{1000}3\left(756\tan^{-1}\left(1,\ \frac{1.65796798716230000^2}{2}\right)+2\left(-112\tan^{-1}\left(1,\ \frac{1.65796798716230000^2}{3}\right)+15\times 1.65796798716230000^2\right)\right)$$

$$\frac{1}{1000}3\left(2\left(15\times 1.65796798716230000^2-112\tan^{-1}\left(\frac{1.65796798716230000^2}{3}\right)\right)+756\tan^{-1}\left(\frac{1.65796798716230000^2}{2}\right)\right)=\frac{1}{1000}3\left(-756\,i\tanh^{-1}\left(\frac{i\,1.65796798716230000^2}{2}\right)+2\left(112\,i\tanh^{-1}\left(\frac{i\,1.65796798716230000^2}{3}\right)+15\times 1.65796798716230000^2\right)\right)$$

$\text{sc}^{-1}(x\,|\,m)$ is the inverse of the Jacobi elliptic function sc

Series representations

$$\frac{1}{1000}3\left(2\left(15\times 1.65796798716230000^2-112\tan^{-1}\left(\frac{1.65796798716230000^2}{3}\right)\right)+756\tan^{-1}\left(\frac{1.65796798716230000^2}{2}\right)\right)=0.247397206180950772-0.79800000000000000\,i\log(2)-0.33600000000000000\,i\log(-1.00000000000000000\,i\,(0.91628594881833619+i))+1.13400000000000000\,i\log(-1.00000000000000000\,i\,(1.37442892322750429+i))+\sum_{k=1}^{\infty}-\frac{1}{k}0.336000000000000\times 0.500000000000000000^k\,i\left(1.000000000000000\,(-1.00000000000000000\,i\,(0.91628594881833619+i))^k-3.37500000000000\,(-1.00000000000000000\,i\,(1.37442892322750429+i))^k\right)$$

$$\frac{1}{1000}3\left(2\left(15\times 1.65796798716230000^2-112\tan^{-1}\left(\frac{1.65796798716230000^2}{3}\right)\right)+756\tan^{-1}\left(\frac{1.65796798716230000^2}{2}\right)\right)=0.247397206180950772+\sum_{k=0}^{\infty}\frac{1}{1+2k}\left(-\frac{1}{5}\right)^k F_{1+2k}\left(-1.23148831521184384\,e^{1.21144077747140964\,k}\left(\frac{1}{1+\sqrt{1.67166395200153489}}\right)^{1+2k}+6.2344095957599595\,e^{2.02237099368773840\,k}\left(\frac{1}{1+\sqrt{2.51124389200345350}}\right)^{1+2k}\right)$$

$$\frac{1}{1000} 3 \left(2 \left(15 \times 1.65796798716230000^2 - 112 \tan^{-1}\left(\frac{1.65796798716230000^2}{3}\right)\right) + 756 \tan^{-1}\left(\frac{1.65796798716230000^2}{2}\right)\right) =$$
$$0.247397206180950772 + 1.59600000000000000 \tan^{-1}(x) - 2.26800000000000000 \pi \left\lfloor \frac{\arg(i(-1.37442892322750429 + x))}{2\pi} \right\rfloor + 0.67200000000000000 \pi \left\lfloor \frac{\arg(i(-0.91628594881833619 + x))}{2\pi} \right\rfloor + \sum_{k=1}^{\infty} -\frac{1}{k} 0.336000000000000\, i \left(1.000000000000000 (0.91628594881833619 - x)^k - 3.37500000000000 (1.37442892322750429 - x)^k\right) \left((-i - x)^k - 1.000000000000000 (i - x)^k\right) (-i - x)^{-k} (i - x)^{-k} \quad \text{for } (i\, x \in \mathbb{R} \text{ and } i\, x > 1)$$

$\log(x)$ is the natural logarithm

F_n is the n^{th} Fibonacci number

Integral representations

$$\frac{1}{1000} 3 \left(2 \left(15 \times 1.65796798716230000^2 - 112 \tan^{-1}\left(\frac{1.65796798716230000^2}{3}\right)\right) + 756 \tan^{-1}\left(\frac{1.65796798716230000^2}{2}\right)\right) = 0.2473972061809508 + \int_0^1 \frac{1.5772010179756 + 0.9167443864912\, t^2}{0.630512026686442 + 1.72043706099165\, t^2 + 1.00000000000000\, t^4}\, dt$$

$$\frac{1}{1000} 3 \left(2 \left(15 \times 1.65796798716230000^2 - 112 \tan^{-1}\left(\frac{1.65796798716230000^2}{3}\right)\right) + 756 \tan^{-1}\left(\frac{1.65796798716230000^2}{2}\right)\right) =$$
$$0.247397206180951 + \int_{-i\,\infty+\gamma}^{i\,\infty+\gamma} -\frac{1}{\pi^{3/2}}\, 0.77930119946999\, e^{-1.67046666438636529\, s} \left(1.00000000000000\, e^{0.609537252080753\,50\, s} - 0.197530864197531\, e^{1.06092941230561179\, s}\right) i\, \Gamma\left(\frac{1}{2} - s\right) \Gamma(1 - s)\, \Gamma(s)^2\, ds \quad \text{for } 0 < \gamma < \frac{1}{2}$$

$$\frac{1}{1000}3\left(2\left(15\times 1.65796798716230000^2-112\tan^{-1}\left(\frac{1.65796798716230000^2}{3}\right)\right)+756\tan^{-1}\left(\frac{1.65796798716230000^2}{2}\right)\right)=0.247397206180951+\int_{-i\infty+\gamma}^{i\infty+\gamma}-\frac{1}{i\pi\Gamma\left(\frac{3}{2}-s\right)}0.153936039401480\,e^{-0.636077663256784778\,s}\left(-5.06250000000000+1.00000000000000\,e^{0.810930216216328 76\,s}\right)\Gamma\left(\frac{1}{2}-s\right)\Gamma(1-s)\,\Gamma(s)\,ds\quad\text{for } 0<\gamma<\frac{1}{2}$$

Continued fraction representations

$$\frac{1}{1000}3\left(2\left(15\times 1.65796798716230000^2-112\tan^{-1}\left(\frac{1.65796798716230000^2}{3}\right)\right)+756\tan^{-1}\left(\frac{1.65796798716230000^2}{2}\right)\right)=$$

$$0.247397206180950772-\frac{0.61574415760592192}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{0.83957994000191861\,k^2}{1+2k}}+\frac{3.11720479787997973}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{1.88905486500431688\,k^2}{1+2k}}=$$

$$0.247397206180950772-\frac{0.61574415760592192}{1+\frac{0.83957994000191861}{3+\frac{3.3583197600076745}{5+\frac{7.5562194600172675}{7+\frac{13.4332790400306978}{9+\dots}}}}}+\frac{3.11720479787997973}{1+\frac{1.88905486500431688}{3+\frac{7.5562194600172675}{5+\frac{17.0014937850388519}{7+\frac{30.224877840069070}{9+\dots}}}}}$$

$$\frac{1}{1000} 3 \left(2 \left(15 \times 1.65796798716230000^2 - 112 \tan^{-1}\left(\frac{1.65796798716230000^2}{3}\right)\right) + 756 \tan^{-1}\left(\frac{1.65796798716230000^2}{2}\right)\right) = 0.247397206180950772 - \frac{0.61574415760592192}{1.00000000000000000 + \underset{k=1}{\overset{\infty}{\mathrm{K}}} \frac{0.83957994000191861 (1-2k)^2}{1.839579940001919 + 0.3208401199961628 k}} + \frac{3.11720479787997973}{1.00000000000000000 + \underset{k=1}{\overset{\infty}{\mathrm{K}}} \frac{1.88905486500431688 (1-2k)^2}{2.889054865004317 - 1.778109730008634 k}} =$$

$$0.247397206180950772 - 0.61574415760592192 \Bigg/ \left(1.00000000000000000 + 0.83957994000191861 \Bigg/ \left(2.160420059998081 + 7.5562194600172675 \Bigg/ \left(2.481260179994244 + \frac{20.9894985000479653}{2.802100299990407 + \frac{41.139417060094012}{3.122940419986570 + \ldots}}\right)\right)\right) + 3.11720479787997973 \Bigg/ \left(1.00000000000000000 + 1.88905486500431688 \Bigg/ \left(1.110945134995683 + 17.0014937850388519 \Bigg/ \left(-0.667164595012951 + \frac{47.226371625107922}{-2.445274325021584 + \frac{92.563688385211527}{-4.223384055030218 + \ldots}}\right)\right)\right)$$

$$\frac{1}{1000}3\left(2\left(15\times 1.65796798716230000^2-112\tan^{-1}\left(\frac{1.65796798716230000^2}{3}\right)\right)+756\tan^{-1}\left(\frac{1.65796798716230000^2}{2}\right)\right)=$$

$$2.74885784645500858+\frac{0.51696644289931184}{3+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{0.83957994000191861\left(1+(-1)^{1+k}+k\right)^2}{3+2k}}-\frac{5.8885708886499740}{3+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{1.88905486500431688\left(1+(-1)^{1+k}+k\right)^2}{3+2k}}=$$

$$2.74885784645500858+\cfrac{0.51696644289931184}{3+\cfrac{7.5562194600172675}{5+\cfrac{3.3583197600076745}{7+\cfrac{20.9894985000479653}{9+\cfrac{13.4332790400306978}{11+\ldots}}}}}-\cfrac{5.8885708886499740}{3+\cfrac{17.0014937850388519}{5+\cfrac{7.5562194600172675}{7+\cfrac{47.226371625107922}{9+\cfrac{30.224877840069070}{11+\ldots}}}}}$$

$$\frac{1}{1000}3\left(2\left(15\times 1.65796798716230000^2 - 112\tan^{-1}\left(\frac{1.65796798716230000^2}{3}\right)\right)+ 756\tan^{-1}\left(\frac{1.65796798716230000^2}{2}\right)\right) = 0.247397206180950772 - \frac{0.61574415760592192}{1.83957994000191861 + \underset{k=1}{\overset{\infty}{\mathrm{K}}} \frac{1.67915988000383723\left(1-2\left\lfloor\frac{1+k}{2}\right\rfloor\right)\left\lfloor\frac{1+k}{2}\right\rfloor}{\left(1.41978997000095931+0.41978997000095931(-1)^k\right)(1+2k)}} + \frac{3.11720479787997973}{2.88905486500431688 + \underset{k=1}{\overset{\infty}{\mathrm{K}}} \frac{3.7781097300086338\left(1-2\left\lfloor\frac{1+k}{2}\right\rfloor\right)\left\lfloor\frac{1+k}{2}\right\rfloor}{\left(1.94452743250215844+0.94452743250215844(-1)^k\right)(1+2k)}} =$$

$$0.247397206180950772 - 0.61574415760592192 \Bigg/ \left(1.83957994000191861 + -\left(1.67915988000383723 \Bigg/ \left(3.0000000000000000 - 1.67915988000383723 \Bigg/ \left(9.1978997000095931 - \frac{10.0749592800230234}{7.0000000000000000 - \frac{10.0749592800230234}{16.5562194600172675+\ldots}}\right)\right)\right)\right) + 3.11720479787997973 \Bigg/ \left(2.88905486500431688 + -\left(3.7781097300086338 \Bigg/ \left(3.0000000000000000 - 3.7781097300086338 \Bigg/ \left(14.4452743250215844 - \frac{22.6686583800518025}{7.0000000000000000 - \frac{22.6686583800518025}{26.0014937850388519+\ldots}}\right)\right)\right)\right)$$

$\underset{k=k_1}{\overset{k_2}{\mathrm{K}}} a_k / b_k$ is a continued fraction

From which:

((((3 (2 (15 (1.6579679871623^2) - 112 tan^(-1)((1.6579679871623^2)/3)) + 756 tan^(-1)((1.6579679871623^2)/2)))/1000)))^12-233-55+5

Input interpretation

$$\left(\frac{1}{1000}3\left(2\left(15\times 1.6579679871623^2-112\tan^{-1}\left(\frac{1.6579679871623^2}{3}\right)\right)+756\tan^{-1}\left(\frac{1.6579679871623^2}{2}\right)\right)\right)^{12}-233-55+5$$

$\tan^{-1}(x)$ is the inverse tangent function

Result

1728.932828049…

(result in radians)

1728.932828049…. ≈ 1729

This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. ($1728 = 8^2 * 3^3$) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

The study of this function provides the following representations:

Alternative representations

$$\left(\frac{1}{1000}3\left(2\left(15\times 1.65796798716230000^2-112\tan^{-1}\left(\frac{1.65796798716230000^2}{3}\right)\right)+756\tan^{-1}\left(\frac{1.65796798716230000^2}{2}\right)\right)\right)^{12}-233-55+5=$$
$$-283+\left(\frac{1}{1000}\times 3\left(756\,\mathrm{sc}^{-1}\left(\frac{1.65796798716230000^2}{2}\,\middle|\,0\right)+2\left(-112\,\mathrm{sc}^{-1}\left(\frac{1.65796798716230000^2}{3}\,\middle|\,0\right)+15\times 1.65796798716230000^2\right)\right)\right)^{12}$$

$$\left(\frac{1}{1000}3\left(2\left(15\times 1.65796798716230000^2 - 112\tan^{-1}\left(\frac{1.65796798716230000^2}{3}\right)\right) + 756\tan^{-1}\left(\frac{1.65796798716230000^2}{2}\right)\right)\right)^{12} - 233 - 55 + 5 =$$

$$-283 + \left(\frac{1}{1000}3\left(756\tan^{-1}\left(1, \frac{1.65796798716230000^2}{2}\right) + 2\left(-112\tan^{-1}\left(1, \frac{1.65796798716230000^2}{3}\right) + 15\times 1.65796798716230000^2\right)\right)\right)^{12}$$

$$\left(\frac{1}{1000}3\left(2\left(15\times 1.65796798716230000^2 - 112\tan^{-1}\left(\frac{1.65796798716230000^2}{3}\right)\right) + 756\tan^{-1}\left(\frac{1.65796798716230000^2}{2}\right)\right)\right)^{12} - 233 - 55 + 5 =$$

$$-283 + \left(\frac{1}{1000}3\left(-756\,i\tanh^{-1}\left(\frac{i\,1.65796798716230000^2}{2}\right) + 2\left(112\,i\tanh^{-1}\left(\frac{i\,1.65796798716230000^2}{3}\right) + 15\times 1.65796798716230000^2\right)\right)\right)^{12}$$

$\mathrm{sc}^{-1}(x\,|\,m)$ is the inverse of the Jacobi elliptic function sc

$\tan^{-1}(x, y)$ is the inverse tangent function

$\tanh^{-1}(x)$ is the inverse hyperbolic tangent function

i is the imaginary unit

Series representations

$$\left(\frac{1}{1000}3\left(2\left(15\times 1.65796798716230000^2-112\tan^{-1}\left(\frac{1.65796798716230000^2}{3}\right)\right)+756\tan^{-1}\left(\frac{1.65796798716230000^2}{2}\right)\right)\right)^{12}-233-55+5=-283+\left(531441\left(2\left(41.2328676968251287-56\,i\left(-\log(2)+\log(-i\,(0.91628594881833619+i))+\sum_{k=1}^{\infty}\frac{2^{-k}\,(-i)^k\,(0.91628594881833619+i)^k}{k}\right)\right)+378\,i\left(-\log(2)+\log(-i\,(1.37442892322750429+i))+\sum_{k=1}^{\infty}\frac{2^{-k}\,(-i)^k\,(1.37442892322750429+i)^k}{k}\right)\right)^{12}\right)/1\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000$$

$$\left(\frac{1}{1000}3\left(2\left(15\times 1.65796798716230000^2-112\tan^{-1}\left(\frac{1.65796798716230000^2}{3}\right)\right)+756\tan^{-1}\left(\frac{1.65796798716230000^2}{2}\right)\right)\right)^{12}-233-55+5=-283+\left(531441\left(2\left(41.2328676968251287-112\sum_{k=0}^{\infty}\frac{1}{1+2k}\left(-\frac{1}{5}\right)^k 1.83257189763667239^{1+2k}\,F_{1+2k}\left(\frac{1}{1+\sqrt{1.67166395200153489}}\right)^{1+2k}\right)+756\sum_{k=0}^{\infty}\frac{1}{1+2k}\left(-\frac{1}{5}\right)^k 2.74885784645500858^{1+2k}\,F_{1+2k}\left(\frac{1}{1+\sqrt{2.51124389200345350}}\right)^{1+2k}\right)^{12}\right)/1\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000$$

$$\left(\frac{1}{1000}3\left(2\left(15\times 1.65796798716230000^2-112\tan^{-1}\left(\frac{1.65796798716230000^2}{3}\right)\right)+756\tan^{-1}\left(\frac{1.65796798716230000^2}{2}\right)\right)\right)^{12}-233-55+5=-283+\left(531441\left(2\left(41.2328676968251287-112\left(\tan^{-1}(x)+\pi\left\lfloor\frac{\arg(i(0.91628594881833619-x))}{2\pi}\right\rfloor+\frac{1}{2}i\sum_{k=1}^{\infty}\frac{1}{k}\left(-(-i-x)^{-k}+(i-x)^{-k}\right)(0.91628594881833619-x)^k\right)+756\left(\tan^{-1}(x)+\pi\left\lfloor\frac{\arg(i(1.37442892322750429-x))}{2\pi}\right\rfloor+\frac{1}{2}i\sum_{k=1}^{\infty}\frac{1}{k}\left(-(-i-x)^{-k}+(i-x)^{-k}\right)(1.37442892322750429-x)^k\right)\right)\right)^{12}\right)/1\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000$$

for ($i\,x \in \mathbb{R}$ and $i\,x < -1$)

$\log(x)$ is the natural logarithm

F_n is the n^{th} Fibonacci number

Integral representations

$$\left(\frac{1}{1000}3\left(2\left(15\times 1.65796798716230000^2-112\tan^{-1}\left(\frac{1.65796798716230000^2}{3}\right)\right)+756\tan^{-1}\left(\frac{1.65796798716230000^2}{2}\right)\right)\right)^{12}-233-55+5=-283+0.00297032244892597\left(0.401785714285714+\int_0^1\left((2.56145510841367+1.48883976431969\,t^2)/(0.630512026686442+1.72043706099165\,t^2+1.000000000000000\,t^4)\right)dt\right)^{12}$$

$$\left(\frac{1}{1000} 3 \left(2 \left(15 \times 1.65796798716230000^2 - 112 \tan^{-1}\left(\frac{1.65796798716230000^2}{3}\right)\right) + 756 \tan^{-1}\left(\frac{1.65796798716230000^2}{2}\right)\right)\right)^{12} - 233 - 55 + 5 =$$

$$-283 + \left(531441 \left(2 \left(41.2328676968251287 + \frac{25.6560065669134134\, i}{\pi^{3/2}} \int_{-i\infty+\gamma}^{i\infty+\gamma} 1.83957994000191861^{-s}\, \Gamma\left(\frac{1}{2} - s\right) \Gamma(1-s)\, \Gamma(s)^2\, ds\right) - \frac{259.767066489998311\, i}{\pi^{3/2}} \int_{-i\infty+\gamma}^{i\infty+\gamma} 2.88905486500431688^{-s}\, \Gamma\left(\frac{1}{2} - s\right) \Gamma(1-s)\, \Gamma(s)^2\, ds\right)^{12}\right) \Big/ 1\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000 \quad \text{for } 0 < \gamma < \frac{1}{2}$$

$$\left(\frac{1}{1000} 3 \left(2 \left(15 \times 1.65796798716230000^2 - 112 \tan^{-1}\left(\frac{1.65796798716230000^2}{3}\right)\right) + 756 \tan^{-1}\left(\frac{1.65796798716230000^2}{2}\right)\right)\right)^{12} - 233 - 55 + 5 =$$

$$-283 + \left(531441 \left(2 \left(41.2328676968251287 - \frac{25.6560065669134134}{i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{0.83957994000191861^{-s}\, \Gamma\left(\frac{1}{2} - s\right) \Gamma(1-s)\, \Gamma(s)}{\Gamma\left(\frac{3}{2} - s\right)}\, ds\right) + \frac{259.767066489998311}{i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{1.88905486500431688^{-s}\, \Gamma\left(\frac{1}{2} - s\right) \Gamma(1-s)\, \Gamma(s)}{\Gamma\left(\frac{3}{2} - s\right)}\, ds\right)^{12}\right) \Big/ 1\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000 \quad \text{for } 0 < \gamma < \frac{1}{2}$$

Continued fraction representations

$$\left(\frac{1}{1000}3\left(2\left(15\times 1.65796798716230000^2-112\tan^{-1}\left(\frac{1.65796798716230000^2}{3}\right)\right)+756\tan^{-1}\left(\frac{1.65796798716230000^2}{2}\right)\right)\right)^{12}-233-55+5=$$

$$-283+\left(531441\left(2\left(41.2328676968251287-\frac{102.624026267653654}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{0.83957994000191861\,k^2}{1+2k}}\right)+\frac{1039.06826595999324}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{1.88905486500431688\,k^2}{1+2k}}\right)^{12}\right)\Big/ 1000000000000000000000000000000000000=-283+$$

$$\left(531441\left(2\left(41.2328676968251287-\frac{102.624026267653654}{1+\frac{0.83957994000191861}{3+\frac{3.3583197600076745}{5+\frac{7.5562194600172675}{7+\frac{13.4332790400306978}{9+\dots}}}}}\right)+\frac{1039.06826595999324}{1+\frac{1.88905486500431688}{3+\frac{7.5562194600172675}{5+\frac{17.0014937850388519}{7+\frac{30.224877840069070}{9+\dots}}}}}\right)^{12}\right)\Big/ 1000000000000000000000000000000000000$$

$$\left(\frac{1}{1000}3\left(2\left(15\times 1.65796798716230000^2 - 112\tan^{-1}\left(\frac{1.65796798716230000^2}{3}\right)\right) + 756\tan^{-1}\left(\frac{1.65796798716230000^2}{2}\right)\right)\right)^{12} - 233 - 55 + 5 =$$

$$-283 + \left(531441\left(2\left(41.2328676968251287 - 112\left(0.91628594881833619 - \frac{0.76929530193349977}{3 + \underset{k=1}{\overset{\infty}{\mathrm{K}}} \frac{0.83957994000191861\left(1+(-1)^{1+k}+k\right)^2}{3+2k}}\right)\right) + 756\left(1.37442892322750429 - \frac{2.5963716440255617}{3 + \underset{k=1}{\overset{\infty}{\mathrm{K}}} \frac{1.88905486500431688\left(1+(-1)^{1+k}+k\right)^2}{3+2k}}\right)\right)^{12}\right) / 1000000000000000000000000000000000000 =$$

$$-283 + \left(531441\left(2\left(41.2328676968251287 - 112\left(0.91628594881833619 - \frac{0.76929530193349977}{3 + \frac{7.5562194600172675}{5+\frac{3.3583197600076745}{7+\frac{20.9894985000479653}{9+\frac{13.4332790400306978}{11+\ldots}}}}}\right)\right) + 756\left(1.37442892322750429 - \frac{2.5963716440255617}{3 + \frac{17.0014937850388519}{5+\frac{7.5562194600172675}{7+\frac{47.226371625107922}{9+\frac{30.224877840069070}{11+\ldots}}}}}\right)\right)^{12}\right) / 1000000000000000000000000000000000000$$

$$\left(\frac{1}{1000}3\left(2\left(15\times 1.65796798716230000^2-112\tan^{-1}\left(\frac{1.65796798716230000^2}{3}\right)\right)+756\tan^{-1}\left(\frac{1.65796798716230000^2}{2}\right)\right)\right)^{12}-233-55+5=-283+$$

$$\left(531441\left(2\left(41.2328676968251287-\frac{102.624026267653654}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{0.83957994000191861(-1+2k)^2}{1+2k-0.83957994000191861(-1+2k)}}\right)+\frac{1039.06826595999324}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{1.88905486500431688(-1+2k)^2}{1+2k-1.88905486500431688(-1+2k)}}\right)^{12}\right)\Big/$$

$$1000000000000000000000000000000000000=$$

$$-283+\left(531441\left(2\left(41.2328676968251287-102.624026267653654\Big/\left(1+0.83957994000191861\Big/\left(2.16042005999808139+7.5562194600172675\Big/\left(2.4812601799942442+20.9894985000479653\Big/\left(2.8021002999904069+\frac{41.139417060094012}{3.1229404199865697+\ldots}\right)\right)\right)\right)\right)+1039.06826595999324\Big/\left(1+1.88905486500431688\Big/\left(1.11094513499568312+17.0014937850388519\Big/\left(-0.6671645950129506+47.226371625107922\Big/\left(-2.4452743250215844+\frac{92.563688385211527}{-4.2233840550302181+\ldots}\right)\right)\right)\right)\right)^{12}\right)\Big/$$

$$1000000000000000000000000000000000000$$

$$\left(\frac{1}{1000}3\left(2\left(15\times 1.65796798716230000^2-112\tan^{-1}\left(\frac{1.65796798716230000^2}{3}\right)\right)+756\tan^{-1}\left(\frac{1.65796798716230000^2}{2}\right)\right)\right)^{12}-$$

$$233-55+5=-283+\left(531441\left(2\left(41.2328676968251287-102.624026267653654\middle/\left(1.83957994000191861+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{1.67915988000383723\left(1-2\left\lfloor\frac{1+k}{2}\right\rfloor\right)\left\lfloor\frac{1+k}{2}\right\rfloor}{\left(1+0.419789970000095931\left(1+(-1)^k\right)\right)(1+2k)}\right)\right)+\frac{1039.06826595999324}{2.88905486500431688+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{3.7781097300086338\left(1-2\left\lfloor\frac{1+k}{2}\right\rfloor\right)\left\lfloor\frac{1+k}{2}\right\rfloor}{\left(1+0.94452743250215844\left(1+(-1)^k\right)\right)(1+2k)}}\right)^{12}\right)\middle/ 1000000000000000000000000000000000000=$$

$$-283+\left(531441\left(2\left(41.2328676968251287-102.624026267653654\middle/\left(1.83957994000191861+-\left(1.67915988000383723\middle/\left(3-1.67915988000383723\middle/\left(9.19789970000095931-\frac{10.0749592800230234}{7-\frac{10.0749592800230234}{16.5562194600172675+\ldots}}\right)\right)\right)\right)\right)+1039.06826595999324\middle/\left(2.88905486500431688+-\left(3.7781097300086338\middle/\left(3-3.7781097300086338\middle/\left(14.4452743250215844-\frac{22.6686583800518025}{7-\frac{22.6686583800518025}{26.0014937850388519+\ldots}}\right)\right)\right)\right)\right)^{12}\right)\middle/ 1000000000000000000000000000000000000$$

$\underset{k=k_1}{\overset{k_2}{\mathrm{K}}} a_k / b_k$ is a continued fraction

and we obtain also:

(((((3 (2 (15 (1.6579679871623^2) - 112 tan^(-1)((1.6579679871623^2)/3)) + 756 tan^(-1)((1.6579679871623^2)/2)))/1000)))^12-233-55+5)^1/15

Input interpretation

$$\left(\left(\frac{1}{1000}3\left(2\left(15\times 1.6579679871623^2-112\tan^{-1}\left(\frac{1.6579679871623^2}{3}\right)\right)+756\tan^{-1}\left(\frac{1.6579679871623^2}{2}\right)\right)\right)^{12}-233-55+5\right)\wedge(1/15)$$

$\tan^{-1}(x)$ is the inverse tangent function

Result

1.6438109711710…

(result in radians)

1.6438109711710…. ≈ ζ(2) = $\frac{\pi^2}{6}$ = 1.644934 ... (trace of the instanton shape)

(1/27(((((3 (2 (15 (1.6579679871623^2) - 112 tan^(-1)((1.6579679871623^2)/3)) + 756 tan^(-1)((1.6579679871623^2)/2)))/1000)))^12-233-55+5))^2-5+Φ

Input interpretation

$$\left(\frac{1}{27}\left(\left(\frac{1}{1000}3\left(2\left(15\times 1.6579679871623^2-112\tan^{-1}\left(\frac{1.6579679871623^2}{3}\right)\right)+756\tan^{-1}\left(\frac{1.6579679871623^2}{2}\right)\right)\right)^{12}-233-55+5\right)\right)^2-5+\Phi$$

$\tan^{-1}(x)$ is the inverse tangent function

Φ is the golden ratio conjugate

Result

4096.04152357…

(result in radians)

4096.04152357…. ≈ 4096 = 64^2

And in conclusion:

(((1/27(((((3 (2 (15 (1.6579679871623^2) - 112 tan^(-1)((1.6579679871623^2)/3)) + 756 tan^(-1)((1.6579679871623^2)/2)))/1000)))^12-233-55+5))^2-5+Φ))^34*((-e^(-3 + e + 1/π - 2 π) π^e tan(e π)))

where

$$-e^{-3+e+1/\pi-2\pi}\,\pi^e\tan(e\,\pi)\approx 0.05316943713$$

Input interpretation

$$\left(\left(\frac{1}{27}\left(\left(\frac{1}{1000}3\left(2\left(15\times 1.6579679871623^2-112\tan^{-1}\left(\frac{1.6579679871623^2}{3}\right)\right)+756\tan^{-1}\left(\frac{1.6579679871623^2}{2}\right)\right)\right)^{12}-233-55+5\right)\right)^2-5+\Phi\right)^{34}\left(-e^{-3+e+1/\pi-2\pi}\,\pi^e\tan(e\,\pi)\right)$$

$\tan^{-1}(x)$ is the inverse tangent function

Φ is the golden ratio conjugate

Result

$3.5160090537\ldots\times 10^{121}$

(result in radians)

$0.35160090537\ldots.*10^{122}\approx\Lambda_Q$

The observed value of ρ_Λ or Λ today is precisely the classical dual of its quantum precursor values ρ_Q , Λ_Q in the quantum very early precursor vacuum U_Q as determined by our dual equations

We note that from the above analyzed expression, dividing by 3, multiplying by 10 and in conclusion, dividing by 2 , we obtain:

1/2((1/3((3 (2 (15 (1.6579679871623^2) - 112 tan^(-1)((1.6579679871623^2)/3)) + 756 tan^(-1)((1.6579679871623^2)/2)))/1000))*10)

Input interpretation

$$\frac{1}{2}\left(\left(\frac{1}{3}\times\frac{1}{1000}3\left(2\left(15\times 1.6579679871623^2-112\tan^{-1}\left(\frac{1.6579679871623^2}{3}\right)\right)+756\tan^{-1}\left(\frac{1.6579679871623^2}{2}\right)\right)\right)\times 10\right)$$

$\tan^{-1}(x)$ is the inverse tangent function

Result

3.1415926535897…

(result in radians)

3.1415926535897…. ≈ π

The study of this function provides the following representations:

Series representations

$$\frac{1}{(3\times 1000)\,2}\left(3\left(2\left(15\times 1.65796798716230000^2-112\tan^{-1}\left(\frac{1.65796798716230000^2}{3}\right)\right)+756\tan^{-1}\left(\frac{1.65796798716230000^2}{2}\right)\right)\right)10=$$
$$0.41232867696825129-1.33000000000000000\,i\log(2)-0.560000000000000000\,i\log(-1.00000000000000000\,i\,(0.91628594881833619+i))+1.89000000000000000\,i\log(-1.00000000000000000\,i\,(1.37442892322750429+i))+\sum_{k=1}^{\infty}-\frac{1}{k}0.560000000000000\times 0.500000000000000000^k\,i\left(1.000000000000000\,(-1.00000000000000000\,i\,(0.91628594881833619+i))^k-3.37500000000000\,(-1.00000000000000000\,i\,(1.37442892322750429+i))^k\right)$$

$$\frac{1}{(3\times 1000)\,2}\left(3\left(2\left(15\times 1.65796798716230000^2-112\tan^{-1}\left(\frac{1.65796798716230000^2}{3}\right)\right)+756\tan^{-1}\left(\frac{1.65796798716230000^2}{2}\right)\right)\right)10 = 0.41232867696825129 + \sum_{k=0}^{\infty}\frac{1}{1+2k}\left(-\frac{1}{5}\right)^k F_{1+2k}\left(-2.05248052535307307\,e^{1.21144077747140964\,k}\left(\frac{1}{1+\sqrt{1.67166395200153489}}\right)^{1+2k}+10.3906826595999324\,e^{2.02237099368773840\,k}\left(\frac{1}{1+\sqrt{2.51124389200345350}}\right)^{1+2k}\right)$$

$$\frac{1}{(3\times 1000)\,2}\left(3\left(2\left(15\times 1.65796798716230000^2-112\tan^{-1}\left(\frac{1.65796798716230000^2}{3}\right)\right)+756\tan^{-1}\left(\frac{1.65796798716230000^2}{2}\right)\right)\right)10 =$$
$$0.41232867696825129 + 2.66000000000000000\tan^{-1}(x) - 3.78000000000000000\,\pi\left\lfloor\frac{\arg(i\,(-1.37442892322750429+x))}{2\pi}\right\rfloor + 1.12000000000000000\,\pi\left\lfloor\frac{\arg(i\,(-0.91628594881833619+x))}{2\pi}\right\rfloor + \sum_{k=1}^{\infty}-\frac{1}{k}0.560000000000000\,i\left(1.000000000000000\,(0.91628594881833619-x)^k - 3.37500000000000\,(1.37442892322750429-x)^k\right)\left((-i-x)^k-1.000000000000000\,(i-x)^k\right)(-i-x)^{-k}(i-x)^{-k} \quad \text{for } (i\,x\in\mathbb{R} \text{ and } i\,x>1)$$

$\log(x)$ is the natural logarithm

F_n is the n^{th} Fibonacci number

Continued fraction representations

$$\frac{1}{(3\times 1000)\,2}\left(3\left(2\left(15\times 1.65796798716230000^2-112\tan^{-1}\left(\frac{1.65796798716230000^2}{3}\right)\right)+756\tan^{-1}\left(\frac{1.65796798716230000^2}{2}\right)\right)\right)10 =$$

$$0.412328676968251287-\frac{1.02624026267653654}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{0.83957994000191861\,k^2}{1+2\,k}}+\frac{5.1953413297999662}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{1.88905486500431688\,k^2}{1+2\,k}}=$$

$$0.412328676968251287-\frac{1.02624026267653654}{1+\frac{0.83957994000191861}{3+\frac{3.3583197600076745}{5+\frac{7.5562194600172675}{7+\frac{13.4332790400306978}{9+\dots}}}}}+\frac{5.1953413297999662}{1+\frac{1.88905486500431688}{3+\frac{7.5562194600172675}{5+\frac{17.0014937850388519}{7+\frac{30.224877840069070}{9+\dots}}}}}$$

$$\frac{1}{(3\times 1000)\,2}\left(3\left(2\left(15\times 1.65796798716230000^2 - 112\tan^{-1}\left(\frac{1.65796798716230000^2}{3}\right)\right)+ 756\tan^{-1}\left(\frac{1.65796798716230000^2}{2}\right)\right)\right)10 = 0.412328676968251287 - \frac{1.02624026267653654}{1.00000000000000000 + \underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{0.83957994000191861\,(1-2k)^2}{1.839579940001919+0.3208401199961628\,k}} + \frac{5.1953413297999662}{1.00000000000000000 + \underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{1.88905486500431688\,(1-2k)^2}{2.889054865004317-1.778109730008634\,k}} =$$

$$0.412328676968251287 - 1.02624026267653654 \Big/ \left(1.00000000000000000 + 0.83957994000191861 \Big/ \left(2.160420059998081 + 7.5562194600172675 \Big/ \left(2.481260179994244 + \frac{20.9894985000479653}{2.802100299990407 + \frac{41.139417060094012}{3.122940419986570+\ldots}}\right)\right)\right) +$$

$$5.1953413297999662 \Big/ \left(1.00000000000000000 + 1.88905486500431688 \Big/ \left(1.110945134995683 + 17.0014937850388519 \Big/ \left(-0.667164595012951 + \frac{47.226371625107922}{-2.445274325021584 + \frac{92.563688385211527}{-4.223384055030218+\ldots}}\right)\right)\right)$$

$$\frac{1}{(3\times 1000)\,2}\left(3\left(2\left(15\times 1.65796798716230000^2-112\tan^{-1}\left(\frac{1.65796798716230000^2}{3}\right)\right)+756\tan^{-1}\left(\frac{1.65796798716230000^2}{2}\right)\right)\right)10=$$

$$4.5814297440916810+\frac{0.86161073816551974}{3+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{0.83957994000191861\left(1+(-1)^{1+k}+k\right)^2}{3+2k}}-\frac{9.8142848144166233}{3+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{1.88905486500431688\left(1+(-1)^{1+k}+k\right)^2}{3+2k}}=$$

$$4.5814297440916810+\frac{0.86161073816551974}{3+\frac{7.5562194600172675}{5+\frac{3.3583197600076745}{7+\frac{20.9894985000479653}{9+\frac{13.4332790400306978}{11+\dots}}}}}-\frac{9.8142848144166233}{3+\frac{17.0014937850388519}{5+\frac{7.5562194600172675}{7+\frac{47.226371625107922}{9+\frac{30.224877840069070}{11+\dots}}}}}$$

$$\frac{1}{(3\times 1000)\,2}\left(3\left(2\left(15\times 1.65796798716230000^2 - 112\tan^{-1}\left(\frac{1.65796798716230000^2}{3}\right)\right)+ 756\tan^{-1}\left(\frac{1.65796798716230000^2}{2}\right)\right)\right)10 = 0.412328676968251287 - \frac{1.02624026267653654}{1.83957994000191861 + \underset{k=1}{\overset{\infty}{\mathrm{K}}} \frac{1.67915988000383723\left(1-2\left\lfloor\frac{1+k}{2}\right\rfloor\right)\left\lfloor\frac{1+k}{2}\right\rfloor}{\left(1.41978997000095931+0.41978997000095931(-1)^k\right)(1+2k)}} + \frac{5.1953413297999662}{2.88905486500431688 + \underset{k=1}{\overset{\infty}{\mathrm{K}}} \frac{3.7781097300086338\left(1-2\left\lfloor\frac{1+k}{2}\right\rfloor\right)\left\lfloor\frac{1+k}{2}\right\rfloor}{\left(1.94452743250215844+0.94452743250215844(-1)^k\right)(1+2k)}} =$$

$$0.412328676968251287 - 1.02624026267653654 \Bigg/ \left(1.83957994000191861 + -\left(1.67915988000383723 \Bigg/ \left(3.0000000000000000 - 1.67915988000383723 \Bigg/ \left(9.1978997000095931 - \frac{10.0749592800230234}{7.0000000000000000 - \frac{10.0749592800230234}{16.5562194600172675+\ldots}}\right)\right)\right)\right) + 5.1953413297999662 \Bigg/ \left(2.88905486500431688 + -\left(3.7781097300086338 \Bigg/ \left(3.0000000000000000 - 3.7781097300086338 \Bigg/ \left(14.4452743250215844 - \frac{22.6686583800518025}{7.0000000000000000 - \frac{22.6686583800518025}{26.0014937850388519+\ldots}}\right)\right)\right)\right)$$

$\underset{k=k_1}{\overset{k_2}{\mathrm{K}}} a_k / b_k$ is a continued fraction

From which:

1/6(1/2((1/3((3 (2 (15 (1.6579679871623^2) - 112 tan^(-1)((1.6579679871623^2)/3)) + 756 tan^(-1)((1.6579679871623^2)/2)))/1000))*10))^2

Input interpretation

$$\frac{1}{6}\left(\frac{1}{2}\left(\left(\frac{1}{3}\times\frac{1}{1000}3\left(2\left(15\times 1.6579679871623^2-112\tan^{-1}\left(\frac{1.6579679871623^2}{3}\right)\right)+756\tan^{-1}\left(\frac{1.6579679871623^2}{2}\right)\right)\right)\times 10\right)\right)^2$$

$\tan^{-1}(x)$ is the inverse tangent function

Result

1.644934066848...

(result in radians)

1.644934066848…. = ζ(2) = $\frac{\pi^2}{6}$ (trace of the instanton shape)

The study of this function provides the following representations:

Alternative representations

$$\frac{1}{6}\left(\frac{1}{2(3\times 1000)}\left(3\left(2\left(15\times 1.65796798716230000^2-112\tan^{-1}\left(\frac{1.65796798716230000^2}{3}\right)\right)+756\tan^{-1}\left(\frac{1.65796798716230000^2}{2}\right)\right)\right)10\right)^2=$$

$$\frac{1}{6}\left(\frac{1}{1000}\times 5\left(756\,\mathrm{sc}^{-1}\left(\frac{1.65796798716230000^2}{2}\,\middle|\,0\right)+2\left(-112\,\mathrm{sc}^{-1}\left(\frac{1.65796798716230000^2}{3}\,\middle|\,0\right)+15\times 1.65796798716230000^2\right)\right)\right)^2$$

$$\frac{1}{6}\left(\frac{1}{2(3\times 1000)}\left(3\left(2\left(15\times 1.65796798716230000^2 - 112\tan^{-1}\left(\frac{1.65796798716230000^2}{3}\right)\right) + 756\tan^{-1}\left(\frac{1.65796798716230000^2}{2}\right)\right)\right)10\right)^2 = \frac{1}{6}\left(\frac{1}{1000}5\left(756\tan^{-1}\left(1, \frac{1.65796798716230000^2}{2}\right) + 2\left(-112\tan^{-1}\left(1, \frac{1.65796798716230000^2}{3}\right) + 15\times 1.65796798716230000^2\right)\right)\right)^2$$

$$\frac{1}{6}\left(\frac{1}{2(3\times 1000)}\left(3\left(2\left(15\times 1.65796798716230000^2 - 112\tan^{-1}\left(\frac{1.65796798716230000^2}{3}\right)\right) + 756\tan^{-1}\left(\frac{1.65796798716230000^2}{2}\right)\right)\right)10\right)^2 = \frac{1}{6}\left(\frac{1}{1000}5\left(-756\, i\tanh^{-1}\left(\frac{i\, 1.65796798716230000^2}{2}\right) + 2\left(112\, i\tanh^{-1}\left(\frac{i\, 1.65796798716230000^2}{3}\right) + 15\times 1.65796798716230000^2\right)\right)\right)^2$$

$\text{sc}^{-1}(x \mid m)$ is the inverse of the Jacobi elliptic function sc

$\tanh^{-1}(x)$ is the inverse hyperbolic tangent function

Series representations

$$\frac{1}{6}\left(\frac{1}{2(3\times 1000)}\left(3\left(2\left(15\times 1.65796798716230000^2 - 112\tan^{-1}\left(\frac{1.65796798716230000^2}{3}\right)\right) + 756\tan^{-1}\left(\frac{1.65796798716230000^2}{2}\right)\right)\right)10\right)^2 = 0.209066666666666667\left(-0.368150604435938649 + \sum_{k=0}^{\infty}\frac{1}{1+2k}\left(-\frac{1}{5}\right)^k F_{1+2k}\left(1.83257189763667239^{1+2k}\left(\frac{1}{1+\sqrt{1.67166395200153489}}\right)^{1+2k} - 9.2773952317856539\, e^{2.02237099368773840\,k}\left(\frac{1}{1+\sqrt{2.51124389200345350}}\right)^{1+2k}\right)\right)^2$$

$$\frac{1}{6}\left(\frac{1}{2(3\times 1000)}\left(3\left(2\left(15\times 1.65796798716230000^2 - 112\tan^{-1}\left(\frac{1.65796798716230000^2}{3}\right)\right) + 756\tan^{-1}\left(\frac{1.65796798716230000^2}{2}\right)\right)\right)10\right)^2 = 0.294816666666666667\left(0.310021561630264125 + i\log(2) - 1.42105263157894737\, i \log(-1.00000000000000000\,(-1.37442892322750429 + i)\, i) + 0.421052631578947368\, i \log(-1.00000000000000000\,(-0.91628594881833619 + i)\, i) + \sum_{k=1}^{\infty} -\frac{1}{k} 1.421052631578947 \times 0.500000000000000000^k\, i \left(1.000000000000000\,(-1.00000000000000000\,(-1.37442892322750429 + i)\, i)^k - 0.296296296296296\,(-1.00000000000000000\,(-0.91628594881833619 + i)\, i)^k\right)\right)^2$$

$$\frac{1}{6}\left(\frac{1}{2(3\times 1000)}\left(3\left(2\left(15\times 1.65796798716230000^2 - 112\tan^{-1}\left(\frac{1.65796798716230000^2}{3}\right)\right) + 756\tan^{-1}\left(\frac{1.65796798716230000^2}{2}\right)\right)\right)10\right)^2 =$$

$$0.294816666666666667\left(-0.310021561630264125 + i\log(2) + 0.421052631578947368\, i \log(-1.00000000000000000\, i\,(0.916285948818336 19 + i)) - 1.42105263157894737\, i \log(-1.00000000000000000\, i\,(1.37442892322750429 + i)) + \sum_{k=1}^{\infty} \frac{1}{k} 0.421052631578947 \times 0.500000000000000000^k\, i \left(1.000000000000000\,(-1.00000000000000000\, i\,(0.91628594881833619 + i))^k - 3.37500000000000\,(-1.00000000000000000\, i\,(1.37442892322750429 + i))^k\right)\right)^2$$

F_n is the n^{th} Fibonacci number

Continued fraction representations

$$\frac{1}{6}\left(\frac{1}{2\,(3\times 1000)}\left(3\left(2\left(15\times 1.65796798716230000^2-112\tan^{-1}\left(\frac{1.65796798716230000^2}{3}\right)\right)+756\tan^{-1}\left(\frac{1.65796798716230000^2}{2}\right)\right)\right)10\right)^2=$$

$$\frac{\left(82.465735393650257-\frac{205.248052535307307}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{0.83957994000191861\,k^2}{1+2\,k}}+\frac{1039.06826595999324}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{1.88905486500431688\,k^2}{1+2\,k}}\right)^2}{240\,000}$$

$$=\frac{1}{240\,000}\left(82.465735393650257-\frac{205.248052535307307}{1+\frac{0.83957994000191861}{3+\frac{3.3583197600076745}{5+\frac{7.5562194600172675}{7+\frac{13.4332790400306978}{9+\dots}}}}}+\frac{1039.06826595999324}{1+\frac{1.88905486500431688}{3+\frac{7.5562194600172675}{5+\frac{17.0014937850388519}{7+\frac{30.224877840069070}{9+\dots}}}}}\right)^2$$

$$\frac{1}{6}\left(\frac{1}{2\,(3\times 1000)}\left(3\left(2\left(15\times 1.65796798716230000^2 - 112\tan^{-1}\left(\frac{1.65796798716230000^2}{3}\right)\right) + 756\tan^{-1}\left(\frac{1.65796798716230000^2}{2}\right)\right)\right)\right.$$

$$\left.10\right)^2 = \frac{1}{240\,000}\left(82.465735393650257 - \frac{205.248052535307307}{1.00000000000000000 + \underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{0.83957994000191861\,(1-2k)^2}{1.839579940001919+0.3208401199961628\,k}} + \frac{1039.06826595999324}{1.00000000000000000 + \underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{1.88905486500431688\,(1-2k)^2}{2.889054865004317-1.778109730008634\,k}}\right)^2 =$$

$$\frac{1}{240\,000}\left(82.465735393650257 - 205.248052535307307\Big/\left(1.00000000000000000 + 0.83957994000191861\Big/\left(2.160420059998081 + 7.5562194600172675\Big/\left(2.481260179994244 + \frac{20.9894985000479653}{2.802100299990407 + \frac{41.139417060094012}{3.122940419986570+\ldots}}\right)\right)\right) + 1039.06826595999324\Big/\left(1.00000000000000000 + 1.88905486500431688\Big/\left(1.110945134995683 + 17.0014937850388519\Big/\left(-0.667164595012951 + \frac{47.226371625107922}{-2.445274325021584 + \frac{92.563688385211527}{-4.223384055030218+\ldots}}\right)\right)\right)\right)^2$$

$$\frac{1}{6}\left(\frac{1}{2(3\times 1000)}\left(3\left(2\left(15\times 1.65796798716230000^2-112\tan^{-1}\left(\frac{1.65796798716230000^2}{3}\right)\right)+756\tan^{-1}\left(\frac{1.65796798716230000^2}{2}\right)\right)\right)10\right)^2=\frac{1}{240\,000}$$

$$\left(916.28594881833619+\frac{172.32214763310395}{3+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{0.83957994000191861\left(1+(-1)^{1+k}+k\right)^2}{3+2k}}-\frac{1962.8569628833247}{3+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{1.88905486500431688\left(1+(-1)^{1+k}+k\right)^2}{3+2k}}\right)^2=\frac{1}{240\,000}$$

$$\left(916.28594881833619+\frac{172.32214763310395}{3+\frac{7.5562194600172675}{5+\frac{3.3583197600076745}{7+\frac{20.9894985000479653}{9+\frac{13.4332790400306978}{11+\dots}}}}}-\frac{1962.8569628833247}{3+\frac{17.0014937850388519}{5+\frac{7.5562194600172675}{7+\frac{47.226371625107922}{9+\frac{30.224877840069070}{11+\dots}}}}}\right)^2$$

$$\frac{1}{6}\left(\frac{1}{2(3\times 1000)}\left(3\left(2\left(15\times 1.65796798716230000^2 - 112\tan^{-1}\left(\frac{1.65796798716230000^2}{3}\right)\right) + 756\tan^{-1}\left(\frac{1.65796798716230000^2}{2}\right)\right)\right)10\right)^2 =$$

$$\frac{1}{240\,000}\left(82.465735393650257 - 205.248052535307307 \Big/ \left(1.83957994000191861 + \underset{k=1}{\overset{\infty}{\mathrm{K}}}\left(\left(1.67915988000383723\left(1-2\left\lfloor\frac{1+k}{2}\right\rfloor\right)\left\lfloor\frac{1+k}{2}\right\rfloor\right)\Big/\left(\left(1.41978997000095931 + 0.41978997000095931\,(-1)^k\right)(1+2k)\right)\right)\right) + 1039.06826595999324 \Big/ \left(2.88905486500431688 + \underset{k=1}{\overset{\infty}{\mathrm{K}}}\left(\left(3.7781097300086338\left(1-2\left\lfloor\frac{1+k}{2}\right\rfloor\right)\left\lfloor\frac{1+k}{2}\right\rfloor\right)\Big/\left(\left(1.94452743250215844 + 0.94452743250215844\,(-1)^k\right)(1+2k)\right)\right)\right)\right)^2 =$$

$$\frac{1}{240\,000}\left(82.465735393650257 - 205.248052535307307 \Big/ \left(1.83957994000191861 + -\left(1.67915988000383723 \Big/ \left(3.0000000000000000 - 1.67915988000383723 \Big/ \left(9.1978997000095931 - 10.0749592800230234 \Big/ \left(7.0000000000000000 - \frac{10.0749592800230234}{16.5562194600172675 + \ldots}\right)\right)\right)\right)\right) + 1039.06826595999324 \Big/ \left(2.88905486500431688 + -\left(3.7781097300086338 \Big/ \left(3.0000000000000000 - 3.7781097300086338 \Big/ \left(14.4452743250215844 - 22.6686583800518025 \Big/ \left(7.0000000000000000 - \frac{22.6686583800518025}{26.0014937850388519 + \ldots}\right)\right)\right)\right)\right)\right)^2$$

$\underset{k=k_1}{\overset{k_2}{\mathrm{K}}} a_k / b_k$ is a continued fraction

Appendix

From:

Modular equations and approximations to π - *Srinivasa Ramanujan*
Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

We have that:

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \cdots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \cdots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\ldots.$$

Again

$$G_{37} = (6+\sqrt{37})^{\frac{1}{4}},$$

$$\begin{aligned} 64G_{37}^{24} &= e^{\pi\sqrt{37}} + 24 + 276e^{-\pi\sqrt{37}} + \cdots, \\ 64G_{37}^{-24} &= 4096e^{-\pi\sqrt{37}} - \cdots, \end{aligned}$$

so that

$$64(G_{37}^{24} + G_{37}^{-24}) = e^{\pi\sqrt{37}} + 24 + 4372e^{-\pi\sqrt{37}} - \cdots = 64\{(6+\sqrt{37})^{6} + (6-\sqrt{37})^{6}\}.$$

Hence

$$e^{\pi\sqrt{37}} = 199148647.999978\ldots.$$

Similarly, from

$$g_{58} = \sqrt{\left(\frac{5+\sqrt{29}}{2}\right)},$$

we obtain

$$64(g_{58}^{24} + g_{58}^{-24}) = e^{\pi\sqrt{58}} - 24 + 4372e^{-\pi\sqrt{58}} + \cdots = 64\left\{\left(\frac{5+\sqrt{29}}{2}\right)^{12} + \left(\frac{5-\sqrt{29}}{2}\right)^{12}\right\}.$$

Hence

$$e^{\pi\sqrt{58}} = 24591257751.99999982\ldots.$$

From:

An Update on Brane Supersymmetry Breaking
J. Mourad and A. Sagnotti - arXiv:1711.11494v1 [hep-th] 30 Nov 2017

From the following vacuum equations:

$$T\, e^{\gamma_E\, \phi} \;=\; -\,\frac{\beta_E^{(p)}\, h^2}{\gamma_E}\; e^{-\,2\,(8\,-\,p)\,C\,+\,2\,\beta_E^{(p)}\,\phi}$$

$$16\, k'\, e^{-\,2\,C} \;=\; \frac{h^2\left(p\,+\,1\,-\,\frac{2\,\beta_E^{(p)}}{\gamma_E}\right)\, e^{-\,2\,(8\,-\,p)\,C\,+\,2\,\beta_E^{(p)}\,\phi}}{(7\,-\,p)}$$

$$(A')^2 \;=\; k\, e^{-\,2\,A} \;+\; \frac{h^2}{16(p\,+\,1)}\left(7\,-\,p\,+\,\frac{2\,\beta_E^{(p)}}{\gamma_E}\right)\, e^{-\,2\,(8\,-\,p)\,C\,+\,2\,\beta_E^{(p)}\,\phi}$$

we have obtained, from the results almost equals of the equations, putting

$4096\, e^{-\pi\sqrt{18}}$ instead of

$$e^{-\,2\,(8\,-\,p)\,C\,+\,2\,\beta_E^{(p)}\,\phi}$$

a new possible mathematical connection between the two exponentials. Thence, also the values concerning p, C, β_E and ϕ correspond to the exponents of e (i.e. of exp). Thence we obtain for p = 5 and β_E = 1/2:

$$e^{-6C+\phi} = 4096 e^{-\pi\sqrt{18}}$$

Therefore, with respect to the exponentials of the vacuum equations, the Ramanujan's exponential has a coefficient of 4096 which is equal to 64^2, while -6C+ϕ is equal to -$\pi\sqrt{18}$. From this it follows that it is possible to establish mathematically, the dilaton value.

For

exp((-Pi*sqrt(18))   we obtain:

Input:

$\exp\left(-\pi \sqrt{18}\right)$

Exact result:

$e^{-3 \sqrt{2} \pi}$

Decimal approximation:

$1.6272016226072509292942156739117979541838581136954016\ldots \times 10^{-6}$

$1.6272016\ldots * 10^{-6}$

Property:

$e^{-3 \sqrt{2} \pi}$ is a transcendental number

Series representations:

$$e^{-\pi \sqrt{18}} = e^{-\pi \sqrt{17} \sum_{k=0}^{\infty} 17^{-k} \binom{1/2}{k}}$$

$$e^{-\pi \sqrt{18}} = \exp\left(-\pi \sqrt{17} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{17}\right)^k \left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$e^{-\pi \sqrt{18}} = \exp\left(-\frac{\pi \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 17^{-s} \Gamma\left(-\frac{1}{2} - s\right) \Gamma(s)}{2 \sqrt{\pi}}\right)$$

Now, we have the following calculations:

$$e^{-6C+\phi} = 4096e^{-\pi\sqrt{18}}$$

$$e^{-\pi\sqrt{18}} = 1.6272016\ldots * 10\text{^}{-6}$$

from which:

$$\frac{1}{4096}e^{-6C+\phi} = 1.6272016\ldots * 10\text{^}{-6}$$

$$0.000244140625\, e^{-6C+\phi} = e^{-\pi\sqrt{18}} = 1.6272016\ldots * 10\text{^}{-6}$$

Now:

$$\ln\left(e^{-\pi\sqrt{18}}\right) = -13.328648814475 = -\pi\sqrt{18}$$

And:

(1.6272016* 10^-6) *1/ (0.000244140625)

Input interpretation:

$$\frac{1.6272016}{10^6} \times \frac{1}{0.000244140625}$$

Result:

0.0066650177536

0.006665017...

Thence:

$$0.000244140625\, e^{-6C+\phi} = e^{-\pi\sqrt{18}}$$

Dividing both sides by 0.000244140625, we obtain:

$$\frac{0.000244140625}{0.000244140625}\, e^{-6C+\phi} = \frac{1}{0.000244140625}\, e^{-\pi\sqrt{18}}$$

$$e^{-6C+\phi} = 0.0066650177536$$

((((exp((-Pi*sqrt(18)))))))*1/0.000244140625

Input interpretation:

$$\exp\left(-\pi\sqrt{18}\right) \times \frac{1}{0.000244140625}$$

Result:

0.00666501785...

0.00666501785…

Series representations:

$$\frac{\exp\left(-\pi\sqrt{18}\right)}{0.000244141} = 4096 \exp\left(-\pi\sqrt{17} \sum_{k=0}^{\infty} 17^{-k} \binom{\frac{1}{2}}{k}\right)$$

$$\frac{\exp\left(-\pi\sqrt{18}\right)}{0.000244141} = 4096 \exp\left(-\pi\sqrt{17} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{17}\right)^k \left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$\frac{\exp\left(-\pi\sqrt{18}\right)}{0.000244141} = 4096 \exp\left(-\frac{\pi \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 17^{-s}\, \Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}\right)$$

Now:

$e^{-6C+\phi}$ = 0.0066650177536

$$\exp\left(-\pi\sqrt{18}\right)\times\frac{1}{0.000244140625} =$$

$$e^{-\pi\sqrt{18}}\times\frac{1}{0.000244140625}$$

= 0.00666501785…

From:

ln(0.00666501784619)

Input interpretation:

log(0.00666501784619)

Result:

–5.010882647757…

-5.010882647757…

Alternative representations:

$$\log(0.006665017846190000) = \log_e(0.006665017846190000)$$

$$\log(0.006665017846190000) = \log(a)\log_a(0.006665017846190000)$$

$$\log(0.006665017846190000) = -\text{Li}_1(0.993334982153810000)$$

Series representations:

$$\log(0.00666501784619000) = -\sum_{k=1}^{\infty} \frac{(-1)^k (-0.99333498215381000)^k}{k}$$

$$\log(0.00666501784619000) = 2 i \pi \left\lfloor \frac{\arg(0.00666501784619000 - x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (0.00666501784619000 - x)^k x^{-k}}{k} \quad \text{for } x < 0$$

$$\log(0.00666501784619000) = \left\lfloor \frac{\arg(0.00666501784619000 - z_0)}{2\pi} \right\rfloor \log\left(\frac{1}{z_0}\right) + \log(z_0) + \left\lfloor \frac{\arg(0.00666501784619000 - z_0)}{2\pi} \right\rfloor \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (0.00666501784619000 - z_0)^k z_0^{-k}}{k}$$

Integral representation:

$$\log(0.00666501784619000) = \int_1^{0.00666501784619000} \frac{1}{t} \, dt$$

In conclusion:

$$-6C + \phi = -5.010882647757 \ldots$$

and for C = 1, we obtain:

$\phi = -5.010882647757 + 6 =$ **0.989117352243 = ϕ**

Note that the values of n_s (spectral index) 0.965, of the average of the Omega mesons Regge slope 0.987428571 and of the dilaton 0.989117352243, are also connected to the following two Rogers-Ramanujan continued fractions:

$$\frac{e^{-\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}}-\varphi+1}=1-\cfrac{e^{-\pi}}{1+\cfrac{e^{-2\pi}}{1+\cfrac{e^{-3\pi}}{1+\cfrac{e^{-4\pi}}{1+\ldots}}}}\approx 0.9568666373$$

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\cfrac{e^{-\pi\sqrt{5}}}{1+\cfrac{e^{-2\pi\sqrt{5}}}{1+\cfrac{e^{-3\pi\sqrt{5}}}{1+\cfrac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}}\approx 0.9991104684$$

(http://www.bitman.name/math/article/102/109/)

Also performing the 512th root of the inverse value of the Pion meson rest mass 139.57, we obtain:

((1/(139.57)))^1/512

Input interpretation:

$$\sqrt[512]{\frac{1}{139.57}}$$

Result:

0.990400732708644027550973755713301415460732796178555551684...

0.99040073.... result very near to the dilaton value $\mathbf{0.989117352243 = \phi}$ and to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\cfrac{e^{-\pi\sqrt{5}}}{1+\cfrac{e^{-2\pi\sqrt{5}}}{1+\cfrac{e^{-3\pi\sqrt{5}}}{1+\cfrac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}}\approx 0.9991104684$$

Acknowledgments

We would like to thank Professor **Augusto Sagnotti** theoretical physicist at Scuola Normale Superiore (Pisa – Italy) for his very useful explanations and his availability

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