# A model accounting for lepton-quark generations, mass hierarchies and finiteness of Higgs mass: A clue to go beyond Standard Model invoking gravity

K Tennakone

55 Amberville Road, North Andover MA01845, United States

Email: ktenna@yahoo.co.uk

#### Abstract

Despite unprecedented success, Standard Model (SM) fails to determine masses of elementary fermions explaining why they replicate into three generations and resolve the problem of divergence of the Higgs mass and fixing it at the observed value. A simple idea presented here suggests incorporation of gravity into SM provides a clue. In the model elementary fermions are as considerd as one dimensional strings. The string possess self-energy contributions originating from string tension, gauge forces and gravity. The gravitational corrections are determined on basis of dimensional considerations and the requirement of proportionality of all corrections to observed fermion mass itself, to guarantee chiral symmetry in the limit of vanishing mass Extremization of total self-energy lead to three equilibrium states agreeing with observed mass ratios and attributing sizes to fermions close to Planck scale. Second and third generation particle found to be metastable and unstable, could decay the into the lower stable states of same charge via non-standard forces. The same idea of extremization of the Higgs selfenergy for stabilization, fixes its mass at the observed value, when cut-off energy is again close to the Planck scale. The model explain dynamically, the proportionality of fermion masses to vacuum expectation value of the Higgs field. The idea of the model also hints existence of three massive neutrinos of masses of specified masses in the normal order and stable heavy singlet states possibly representing dark matter. Model suggest that there exists an universal ultraviolet cut-off 2.6 x 10<sup>20</sup> GeV and the inverse of gauge coupling constants converge at a value  $\sim 21$ .

Keywords:Beyond Standard Model, Fermion Selfenergy, Gravity and Standard Model, Lepton -Quark Generation problem, Higgs Selfenergy, Fermion Strings

### 1. Introduction

The Standard Model (SM) enables computation high energy scattering amplitudes to an utmost precision in terms of quarks and leptons with arbitarialy assigned physical parameters [1-2]. Spontaneous breaking of SU (2) x U (1) symmetry consistently accommodates massive vector bosons and charged leptons [1-6]. The popular statement that Higgs mechanism (HM) generate the fermion masses is misleading. A more accurately, HM essentially require charged leptons and quarks to be massive, but does not provide an avenue to calculate their masses and answer the question why elementary fermions replicate into three generations. Again divergences of Higgs mass continue to be a problem [7-12]. The massiveness of neutrinos is also not accounted in SM and the mechanism of generating neutrino masses remain obscure [13,14]. Again SM has no candidates to explain dark energy or cold dark matter. A glaring gap in SM is that it has no place for gravity- the other known universal interaction [15]. Supersymmetric extensions of SM of has been considered as the next step to go beyond and even find ways to incorporate gravity.Despite theoretical consistency, such models invoke more arbitrary parameters and provide no hint to determine masses of the elementary fermions. Furthermore, LHC has not hinted existence of supersymmetric particles [16-17].

Perhaps, the only interactions existing in nature are SM forces and gravity - experimentally, no evidence has been found to the contrary. In this situation, the energy desert between electroweak and Planck scales doesn't seem to be unnatural for the following reason. The renormalization

group evolution of SM gauge couplings, extrapolated from low energy experimental data, suggest unification around  $10^{16}$  GeV – a GUT scale close to the Planck scale [18 -19]. The inability of SM to determine the masses of leptons and quarks and previously the electron self-energy in QED, could be the omission of gravity and many ideas have been made to relsove the problem electron self-energy incoperating gravity . In a previous preliminary work , the author has suggested that incorporation of gravity to SM could generate three generations of massive elementary fermions [20].

Here a model is presented, where the self-energies of elementary fermions considered as strings are determined by a unified gauge coupling, string tension and gravity. Model explains their replication into three generations and account for leption – quark mass hiracrachies, accommodating mass ratios agreeing with observation and resolve the problem of divergence of the Higgs mass squared, fixing its mass The assumption that neutrino masses are also generated by the same mechanism ,yield values in agreement with known constraints. The moder permits accommodation of non-SM massive dark matter particles.

## 2. The Model

As an effective field theory validity of SM extends up to a certain energy scale  $\Lambda$  or equivalently above a length *R*. An attractive hypothesis is  $\Lambda$  resides near Planck energy and invoking gravity resolve the problem of mass and other issues in SM. In this scenario the self-energy of elementary fermions (quarks and leptons) would be a sum of contributions originating from unified SM forces and gravity, all dependent on a cut-off parameter  $\Lambda$  or equvalently a length *R*.

If leptons and quarks are considered as strings, their selfenergies will entail contributions originating from; (1) string tension, (2) unified gauge interaction and (3) gravity. An important

principle to be guided by in derterming the corrections to the fermion selfengry is their proportionality to the mass of the fermion itself. This condition assures preservation of chiral symmetry in the limit of zero fermion mass [21]. Recollect QED expression for electron selfenergy [22]

$$(\delta M)_{\text{QED}} = (3\alpha/2\pi) M [\ln \{\hbar/(McR)\}]$$
(1)

it is proportional to mass M of the electron, where R in (1) is the spatial extension of the electron.

Considering elementary fermion of mass M as an string of length R, the selfenergy correction originating from its tension can be written as,

$$(\delta M)_{\text{STRING TENSION}} = [\kappa(M/M_P) (c^4/G)R]/c^2$$
(2)

where  $(c^4/G)$  Planck tension,  $M_P$  = Planck mass and  $\kappa$  = constant and the expression ensures the correction is proportional to observed mass

Gauge force yield a correction to the selfengery with a lograthmic divergence as in the expression (1) and this correction is written as,

$$(\delta M)_{\text{GAUGE FORCE}} = (3\alpha/2\pi) M [\ln \{\hbar/(McR)\}]$$
(3)

where  $\alpha$  is an unified gauge coupling constant

Now I look for corrections to self-energy originating from gravity proportional to mass M. In absence of an adoptable quantum theory of gravity, simply consider dimensions and arrive at terms inversely proportional to R and  $R^2$ . These terms can be written as ,  $-(GM_pM)/c^2R$  and  $GM\hbar c/R^2$ . The latter term is taken to be positive, because negative sign in this correction leads to an inconsistency. The higher order terms in G, or M are neglected. Thus the correction to the fermion mass arising from gravity can be written as,

$$(\delta M)_{\rm GR} = -(\eta G M M_p)/c^2 R + (\gamma G M \hbar/c^3 R^2)$$
(4)

where  $\eta$  and  $\gamma$  are constants. When  $M \ll M_p$ , the correction  $GM^2/c^2R$  is negligible in comparison to the first term of (4). Again the correction (4) is insignificant for macroscopic objects.

Hereinafter I proceed with units  $\hbar = c = G = 1$  replacing *R* by  $\Lambda^{-1}$  to transfer from length scale *R* to an energy scale  $\Lambda = (R/L_P)^{-1}$  and *M* measured in Planck units. With this simplification, the total selfenergy  $[(\delta M)_{\text{STRING}} + (\delta M)_{\text{GUAG EFORCES}} + (\delta M)_{\text{GR}}]$  takes the form,

$$M = (\kappa M) \Lambda^{-1} + (3\alpha M)/2\pi [\ln (\Lambda M)] - \eta M \Lambda + \gamma M \Lambda^2$$
(5)

Provided  $M \neq 0$ , (5) can also be written as,

$$M = \Lambda \exp \left[ (d/b) \Lambda^{-1} - (a/b) \Lambda + (1/2b) \Lambda^2 - (2\pi/3\alpha) \right]$$
(6)

with 
$$a = \eta/(2\gamma)$$
,  $b = 3\alpha/(4\pi\gamma)$ ,  $d = \kappa/(2\gamma)$  (7)

The condition dM/dA = 0 yield the cubic equation,

$$\Lambda^3 - a\Lambda^2 + b\Lambda - d = 0 \tag{8}$$

Thus real roots  $\Lambda$  of the cubic (8) corresponds to equilibrium values of M and using (6) and (8), the allowed equilibrium masses M can be expressed as,

$$M = \Lambda \exp \left[ (3/2b)\Lambda^2 - (2a/b)\Lambda - 2\pi/3\alpha + 1 \right]$$
(9)

Writing  $\lambda = \Lambda/a$ , equations (8) and (9) simplifies to read

$$\lambda^3 - \lambda^2 + \beta \lambda - \delta = 0 \tag{10}$$

$$M = a\lambda \exp\left[\left(\frac{3}{2\beta}\right)\lambda^2 - \left(\frac{2}{\beta}\right)\lambda - \frac{2\pi}{3\alpha} + 1\right]$$
(11)

where 
$$\beta = b/a^2$$
 and  $\delta = d/a^3$ 

For values of  $\beta \le 0.25$ , the cubic equation (10) has three positive real roots, provided  $\delta < \beta^2/4$ and when this condition is not satisfied only one real positive root. Again, when all the three roots are real and positive ; value of each root is less than unity. The parameter *a* has dimensions of energy measured in units of Planck energy. Mass ratios are independent of *a* interpreted as the energy scale (cut-off energy) of the model (measured in Planck units). Essentially  $\lambda = \Lambda/a \le 1$ . If higher order corrections  $\Lambda^n$  (n > 2) are introduced to the selfenergy expression (5); the polynomial equation in  $\lambda$  ( =  $\Lambda/a$ ) resulting from the condition  $dM/d\Lambda = 0$ , gain additional (n-3) roots. Numerical analysis reveal all these additional roots are either complex, negative or positive and greater than unity. As the limiting cut-off energy *a* should not be exceeded, the expression (5) for fermion selfenery is physically meaningful only if its extermal values corresponds values of  $\lambda$  less than unity and this condition is satisfied only if terms higher than  $\Lambda^2$  in the expression (5) for fermion selfenergy are excluded. Similarly as the objects are one dimensional, the inverse powers of  $\Lambda$  greater than unity are not incoporated in (5). These considerations further justify writing the fermion selfenergy as in (5).

In the case where (10) has three distinct positive roots ,(11) corresponds three masses. Suggesting masses of fermions of charges Q = 1, 2/3,1/3 are represented by this equation. Numerical solution of (10) and (11) reveal, that each value of Q there exists unique values of the parameters  $\beta$ ,  $\delta$  yielding mass ratios amazingly close to charged leptons and quark masses ( roots of the cubic equations calculated using CASIO polynomial equation solver [23]). From (11) it follows that the mass ratios are explicitly independent of the value of the parameter a and gauge coupling constant  $\alpha$ .

Case I When  $\beta = 4.06265 \times 10^{-2}$ ,  $\delta = 1.37887 \times 10^{-5}$  roots of (10) are  $\lambda_1 = 0.957589225604$ ,  $\lambda_2 = 0.04206848996$ ,  $\lambda_3 = 3.422844 \times 10^{-4}$  and the mass ratio calculated from (11) is  $M_1$ :  $M_3 : M_2 = 1: 206.72: 3477.20$  compared to the observed ratio  $M_e : M_{mu} : M_{tau} = 1$ : 206.768:3477.228 for charged leptons.

*Case II.* When  $\beta = 3.37850 \ge 10^{-2}$ ,  $\delta = 1.2000 \ge 10^{-6}$  roots of (10) are  $\lambda_1 = 0.964990584351$ ,  $\lambda_2 = 0.03497385950848$ ,  $\lambda_3 = 3.5556 \ge 10^{-5}$  and the mass ratio calculated from (11) is  $M_1$ :  $M_3 : M_2 = 1: 262.5: 34465$  compared to  $M_{up} : M_c : M_{top} = 1: 260: 34400$ .for Q = 2/3 quarks mass eigen values

*Case III.* When  $\beta = 4.95 \times 10^{-2}$ ,  $\delta = 1.4595 \times 10^{-5}$  roots (10) are  $\lambda_1 = 0.9477894583278$ ,  $\lambda_2 = 0.051913916207$ ,  $\lambda_3 = 2.96625 \times 10^{-4}$  and the mass ratio calculated from (11) is  $M_1$ :  $M_3 : M_2 = 1: 20:: 469$  compared to  $M_d : M_s : M_b = 1: 20: : 465$  for Q = 1/3 quark mass eigen values.. In *Case I*, where masses of leptons are accurately known, the chosen values of  $\beta$  and  $\delta$ , fits mass ratios almost exactly. Whereas in the *Cases II* and *III*, there are uncertainties in light quark mass eigen values (calculated using the CKM matrix ).

It is important to note that the above analysis is not mere fitting of data but a demonstration that solutions of (10) and (11), yielding observed mass ratios exist. It is not possible to determine values of  $\beta$  and  $\delta$  to satisfy any arbitrary three mass ratios. The model demonstrates the mass hierarchy of elementary fermions.

The increasing order of masses  $(M_1:M_3:M_2)$  is not the decreasing order of the roots  $\lambda_1:\lambda_2:\lambda_3$  (increasing order of the particle radii  $R_1$ ,  $R_2$ ,  $R_3$ ), instead  $M_1$  and  $M_3$  are minima, therefore stable equilibria whereas  $M_2$  is a maximum of unstable equilibrium. As seen from the second derivative of M given by,

$$d^{2}M/d\lambda^{2} = M/(\beta\lambda^{2})[3\lambda^{2} - 2\lambda + \beta]$$
(12)

Thus, tau lepton and bottom and top quarks are unstable states; muon, charm and strange quarks metastable and electron, up and down quarks being absolutely stable (Fig.1). One would think stable particles can can oscillate around position of equilibrium generate excited states. When the circular frequency of oscillation  $\omega$  is calculated using (12), I obtain,  $\hbar^-\omega \approx \hbar c/R$ . For first generation fermions whose radii are of the order of Planck length , the excitation energy turns out be  $\sim M_P/\beta$  ( $\beta \ll 1$ ). Thus the electron and other first generation fermions are not only stable but cannot be 'shaken' as well , because the energy required greatly exceed Planck mass. Elementary fermions have no excited states.

The variation of M given by the expression (5) with R (=  $\Lambda^{-1}$ ) is shown in Fig.1. As the particle size  $R \to 0$  or  $\infty$ , self-energy  $\to \infty$ . Presence of two minima necessitates existence of a maximum in between. The forces of SM suppress flavor changing neutral currents, forbidding transitions of  $M_2$  and  $M_3$  to the ground state  $M_1$ . However, the instability and metastability of these two states, indicate transitions could happen via non-SM interactions (possbly gravity) at exceeding small rates. Model admits the possibility of transitions top  $\rightarrow$  charm and bottom  $\rightarrow$  strange. Though uncertain there is some experimental evidence for occurrence of the latter decay and issue is being actively investigated [24].



Fig.1. Sketch of M given in eqn. (6) vs R ( $\Lambda^{-1}$ ) showing minimum  $M_1$  representing masses of electron, up quark or down quark; a maximum  $M_2$  corresponding to masses of tau lepton, top quark or bottom quark and second minimum representing masses of muon, strange quark or charm quark taken that order

The model can be related to the Higgs sector as follows and resolve the problem of the selfenergy of the Higgs boson.

Higgs selfenergy  $M^2_{\rm H}$  is constituted of two components one a quadratic divergence (assumed to be negative – a possibility not ruled out) and the other a lograthmic divergence, so that  $M^2_{\rm H}$  can be written in the form,

$$M^{2}_{\rm H} = -A\Lambda^{2} + (BM^{2}_{\rm T}) \ln (\Lambda/M_{\rm T})$$
(13)

The positive lograthmic component originate mainly from Yukawa interaction with the top quark (mass  $M_T$ ), where *B* is a quantity of the order  $Y^2_T/8\pi^2$  [16], and  $Y_T =$  top quark yukawa coupling constant  $\simeq 1$ 

Expression (13) is extremal (maximum ) when  $\Lambda$  reaches a cut-off value  $\Lambda_c$  given by

$$\Lambda_{\rm c} = \sqrt{\left[1/2(B/A)M_{\rm T}\right]} \tag{14}$$

and extremal value of  $M^{2}_{H}$  is

$$M^{2}_{\rm H} = BM^{2}_{\rm T} \left[ -1/2 + \ln \left( \Lambda_{\rm c} / M_{\rm T} \right) \right]$$
(15)

Higgs selfenergy squared represented by (15) is now finite (unknown bare mass set equal to zero ).

If cut-off energy given by (14) is written as  $\Lambda_c = aM_P$ , expressions (15) and (14) can be written as

$$M_{\rm H}^2 = (M_{\rm T}^2)/8\pi^2 \left[ -\frac{1}{2} + \ln \left[ a \left( M_{\rm P}/M_{\rm T} \right) \right] \right]$$
(16)

$$a = (4\pi\sqrt{A})^{-1} (M_{\rm T}/M_{\rm P})$$
(17)

When a = 1 (16) gives  $M_{\rm H} \sim 120 \,\text{GeV} - a$  value quite close to the observed Higgs mass. However, the value of the coefficient *B* has to be determined more carefully because Higgs mass squared given by (13) needs to include other logarithmic divergences originating from; higgs itself and bosons Z and W, which are negative and possibly similar contributins other sources. Positive contributions of fermions other than top quark though negligible should also be accommodated. So many efforts have been made to evaluate quadratic and logarithmic contributions to Higgs selfenergy, obtaining ambiguous results [8-10,24-31]. Therefore I adopt the following approach to represent logarithmic contribution.

The top quark logarithmic contribution to Higgs mass is of the form  $M_T^2 C_T \ln(\Lambda/M_T)$ , where  $C_T$ is a positive constant and other fermions contribute amounts  $C_F M_F^2 \ln(\Lambda/M_F)$  and  $C_F$  again a positive constant. The contributions of Higgs itself and vector bosons are  $-M_H^2 C_H \ln(\Lambda/M_H)$ ,  $-M_Z^2 C_Z \ln(\Lambda/M_Z)$ ,  $-M_W^2 C_W \ln(\Lambda/M_W)$ ; here the  $C_H, C_Z, C_W$  are positive constants. Any other scalar or source S will also contribute similar negative corrections. The sum total all these,  $M_T^2 \Gamma[C_T \ln(\Lambda/M_T) + C_F (M_F/M_T)^2 \ln(\Lambda/M_F) - C_H (M_H/M_T)^2 \ln(\Lambda/M_H) - C_Z (M_Z/M_T)^2 \ln(\Lambda/M_Z) - C_W (M_W/M_T)^2 \ln(\Lambda/M_W) - C_S (M_S/M_T)^2 \ln(\Lambda/M_S)]$ , can be written as  $BM_T^2 \ln[(\Lambda/M_T + D]]$ , where B and D constants or equivalently  $M_T^2 B \ln(\Lambda a/M_T)$ , where a is another constant. Thus Higgs mass squared  $M_H^2$  constituted of quadratic as well as logarithmic corrections is generally of the form,

$$M_{\rm H^2}(\Lambda) = -\Lambda \Lambda^2 + B M_{\rm T}^2 \ln(\Lambda a/M_{\rm T}), \qquad (18)$$

The above quantity is maximum , when  $\Lambda = \Lambda_c$ , where

$$A_{\rm c}^2 = 1/2(B/A)M_{\rm T}^2$$
(19)

Writing  $\Lambda_c = aM_P$ , Higgs mass (maximum value of the expression (21) is,

$$M_{\rm H}^2 = B M_{\rm T}^2 \left[ -1/2 + \ln((a M_P / M_{\rm T})) \right]$$
<sup>(20)</sup>

When cut-off value of  $\Lambda$  is taken as  $aM_P$ , from (19), it follows that quadratic divergence term is  $(B/2)M_T^2(\Lambda/\Lambda_c)^2$  The coefficient of the Higgs quadratic divergence carry the factor  $1/16\pi^2$  originating from the quadratically divergent one loop integral [32, 33, 36], therefore *B* is chosen as equal to  $1/8\pi^2$  to recover the fomula (16) for Higgs mass

$$M_{\rm H}^2 = [(M_{\rm T}^2)/8\pi^2 \cdot] \ [-1/2 + \ln((aM_{\rm P}/M_{\rm T})]$$
(21)

where  $a = (4\pi \sqrt{A})^{-1} (M_T / M_P)$  (22)

Inserting observed values of masses of the Higgs and top quark in (20), we obtain

 $\Lambda_c = aM_P = 2.612 \text{ x } 10^{20} \text{ GeV.}$ , a value about one of magnitude greater than the Planck scale, giving a = 21.41

From (19) it follows that the coefficient A of quadratic divergence in the Higgs selfenergy is finite and exceedingly small, of the order  $(M_T/M_P)^2$ . Making quadratic divergence of  $M^2_H$  small was persistant problem in SM. Model shows that this divergence is indeed minute. However, the question remains, why it is so small.

It is reasonable to assume that the above energy scale parameter *a* defining the mass squared of the Higgs boson is same the energy scale for fermions in the expression (11) giving fermion masses That is, the parameter *a* in (11) and (21) have identical numerical values corresponding to a universal cut-off energy =  $aM_P$  = 2.612 x10<sup>20</sup> GeV.Therfore all fermion masses are proportional to the top quark mass (equvalently vacuum expectation value of the Higgs field ) and the mass formula for elementary fermions can also be written as,

$$M = (4\pi\sqrt{A})^{-1} (M_{\rm T}/M_{\rm P}) (\lambda \exp\left[(3/2\beta)\lambda^2 - (2/\beta)\lambda - 2\pi/3\alpha + 1\right]$$
(23)

Thus the model provide an dynamical explanation as to why the fermion and Higgs masses are proportional to the vacuum expectation value of the Higgs field.

Since the value of the parameter *a* is known (calculated from the knowledge of Higgs mass ). All parameters of model can be evaluated using the observed masses of third generation fermions. The values obtained are listed below:

$$1/\alpha = 21.4, \ \eta = 2.6 \ge 10^{-2}, \ \gamma = 6.0 \ge 10^{-4}, \ \kappa = 1.6 \ge 10^{-4} \qquad (Q = 1)$$

$$1/\alpha = 20.3, \ \eta = 3.3 \ge 10^{-2}, \ \gamma = 7.6 \ge 10^{-4}, \ \kappa = 0.22 \ge 10^{-4} \qquad (Q = 2/3)$$

$$1/\alpha = 21.3, \ \eta = 2.1 \ge 10^{-2}, \ \gamma = 4.9 \ge 10^{-4}, \ \kappa = 1.4 \ge 10^{-4} \qquad (Q = 1/3) \qquad (24)$$

The model parameters for all sectors (Q = 1,2/3 and 1/3) are more or less of the same order of magnitude, but nevertheless significant differences generate the observed mass hierarchy of quarks and leptons, because of the exponential dependence in (22).Furthermore, the model connects fermion mass scales to Planck scale via the parameter a determined from the mass of the Higgs boson.

According to the model elementary fermions have sizes of the order  $\Lambda^{-1} = (a\lambda)^{-1}$  in Planck units Electron, up and down quarks being the smallest, having sizes of the order 0.05  $L_P$ . and the second and third generation  $5 \times 10^2 L_P$  and 1.2  $L_P$  respectively. It is interesting to note that the second generation particles have the largest size. Assigning dimensions of this order quarks and leptons would not contradict SM or its predictions.

Using values of parameter given in (13) contribution to self-energy from each term in (5) can computed. It is interesting to note that major contribution (over 90%) to the selfenergy of the electron originate from second term of (5). Nevertheless, all terms in (5) are essential to generate three fermions with observed mass hierarchies and ensure their stability. According to (2), fermion strings have tensions proportional to mass, it turns out thal each generation has nearly same tension, the highest for third generation ~ [ $10^8 \text{ GeV}$ ]<sup>2</sup>. Possibly, flavor is an attribute related string tension and  $10^8 \text{ GeV}$  a threshold for observing charged flavor violations and CP violations (which might be related to flavor violation)

Since three neutrino mass eigenstates exists in nature, it would be interesting if the neutrino masess are also determined by eqns. (10) and (11). Neutrino oscillation experiments, connects their mass<sup>2</sup> differences via the well established relations [13].

$$\Delta m^2 \text{ (atmospheric )} = 2.48 \text{ x } 10^{-3} \text{ eV}^2 \quad (24) \qquad \Delta m^2 \text{ (solar)} = 7.49 \text{ x } 10^{-5} \text{ eV}^2 \quad (25)$$

Thus the knowledge of mass ratios and one of the relations ,either (24) or (25) permits determination of neutrino masses. It is possible to find solutions of (24) and (25) with same ordering masses as in the case of chrged leptons (i.e. root  $\lambda_1$  corresponding to the smallest mass,  $\lambda_3$  to the next and  $\lambda_2$  the largest mass ; which is normal ordering ). When  $\beta = 0.089945$  $\delta = 0.000236$ . The roots of (10) are;  $\lambda_1 = 0.900394800940$ ,  $\lambda_2 = 0.0966931128087$ ,  $\lambda_3 = 0.002693128087$ . Mass ratios obtained combined with (25) and (26), neutrino masses as ; m<sub>1</sub> =  $5.23 \times 10^{-3}$ , m<sub>2</sub> =  $5.\times 10^{-2}$ , m<sub>3</sub> =  $9.8 \times 10^{-3}$  eV. Giving m<sub>2</sub><sup>2</sup> - m<sub>3</sub><sup>2</sup> = 2.47 and m<sub>3</sub><sup>2</sup> - m<sub>1</sub><sup>2</sup> =  $6.9 \times 10^{-5}$  eV<sup>2</sup>. The value of  $1/\alpha$  corresponding to neutrinos turns out be ~ 1/32.

There are also values of  $\beta$  and  $\delta$  giving one real root of equation (10) and therefore one mass value from (11). As seen from (12) they are also stable. Although the model does not provide a way of calculating the masses, they could represent neutral singlet fermions- perhaps dark matter particles.

According to the model the gauge coupling constant  $\alpha$  is almost same for charged leptons and quarks ( $1\alpha \sim 1/21$ ), a value close to the expected convergence of coupling constants  $\alpha_1$ ,  $\alpha_2$ ,  $\alpha_3$  in grand unified theories.

Finally, I return to the question why the Higgs quadratic divergence determined by the model is exceedingly small. From (22) the quadratic divergence following from the model is.

$$A\Lambda^2 = - \left[16\pi^2 a^2\right]^{-1} (M_{\rm T}/M_{\rm P})^2 \Lambda^2$$
(26)

The large quadratic divergence of  $M_{\rm H}^2$  seen in many calculations could be an artifact or a cancellation large positive and negative divergences [ 24,26-29 – 36 ]. Model necessitates an exceeding small but finte negative quadratic divergence with coefficient given by (26). The top quark selfenergy given by (5) carries a quadratic divergence  $\gamma M_{\rm T}$  ( $\Lambda/M_{\rm P}$ )<sup>2</sup> [ $\Lambda$  in (5) defined as ( $\Lambda/M_{\rm P}$ )]. The quantum gravitational correction to Higgs mass is expected to include a similar term of the order  $\Delta M_{\rm H} = \gamma M_{\rm H} (\Lambda/M_{\rm P})^2$  with negative sign [ quantum gravitational radiative corrections to bosons and fermions have opposite signs (33,36) ]. Thus the quantum gravitational correction Higgs mass squared is

$$\Delta(M^{2}_{\rm H}) = -2M_{\rm H} (\Delta M_{\rm H}) = -2\gamma (M_{\rm H}/M_{\rm T})^{2} (M_{\rm T}/M_{\rm P})^{2} \Lambda^{2}$$
(27)

From (25)  $\gamma \sim 10^{-4}$ , both (26) and (27) agree in the order of magnitude of Higgs quadratic divergence as less than  $\sim (M_T/M_P)^2 \Lambda^2$ .

#### 3. Conclusion

The model considered lepton and quarks and as strings. The self-energy of the string is constituted of contributions from string tension, unified gauge force and gravity. Gravitational corrections determined on basis of dimensional considerations. A crucial assumption of the model is the proportionality of all corrections to the self-energies of elementary fermions to their observed masses – a requirement guaranteeing chiral symmetry in the limit of vanishing mass. The extremization of total energy leads to three equilibrium states, and which fit into the observed

fermion mass ratios. The first, second and third generation particles are found to be stable, metastable and unstable respectively. Thus the second and third generation fermions could decay into low mass states of same charge via non-standard forces.

According to the model elementary fermions have sizes of the order of Planck length or less. The smallest being first generation particles and largest second generation particles ,while third generation stands intermediate. Strangely, second generation particles are larger than respective entities in the third generation. These sizes does not contradict experimental data or effective field theory approach of SM. A novel approach to Higgs selfenergy presented resolve the problem of finiteness of the Higgs mass and showing fermion selfenergies derived dynamically are indeed proportional the vacuum expectation value of the Higgs field. The assumption that the same dynamical mechanism derive neutrino masses ,combined with neutrino oscillation data provide plausible values for neutrino masses The model allows non-SM heavy fermions that might account for dark matter. The gauge couling constant derived is almost same for all the charged fermions and of the order of magnitude of expected merging of coupling constants in grand unified theories.. The approach used in the model provides a new way dealing with divergencies of selfenergies in QFT. Insead of regarding divergences as pathological, the value of the cut-off parameter leading to stability of the system is regarded as meaningful, irrespective of UV and IR unreachable infinities. Model suggest existence of ultimate ultraviolet cut-off 2.6 x 10<sup>20</sup> GeV.Could pont a way to go beyond standard model. Model needs to be extended to explain flavor properties of fermions other than mass.

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