GEOMETRIC APPROACH TO PLANCK SCALE FIELD

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Abstract. In this paper i present a try to explain how gravity works on Planck scale and beyond it.

1. Clock and rules as general approach to field

In physics base ideas need to be measured quantities, simplest measuring tools for given space-time is ruler and clock or in general sense a transformation that says how clocks and rules behave. To explain fully a physical field i need both set of rules and clocks (coordinates) and rule how they transform from one frame to another or more precise how they transform from one point of field to another point of field. In general sense field should be independent of how we chose to measure distance by what ruler or clock, so it should be not frame dependent but free of any dependence of chosen coordinate system. Most general approach to field should be field without dependence on how we measure geometry of that field in addition it's only free chosen parameter should be it's energy- so field should be only energy dependent , where energy represents matter in that field. But let's go step back, how do i measure distance in a field? I chose a coordinate system and measure how much distance it takes in that coordinate system from one point of field to another. Field itself doesn't have a coordinate system assign to it, but it's easier to use it to map how field changes. Most general transformation of field is a transformation that gives for each dimension of field a vector that represents that field. But that vector can transform in two ways: contravariant and covariant, if I want to make thinking as general as possible , vector transformation should be same covariant and contravariant way, so field is independent of both how coordinate system is chosen and how chosen objects transform.

2. Coordinate system

Let's say i have a coordinate system F and rules of all possible covariant and contravariant transformations of any vector in that system F^{α}_{β}, i want those transformations to obey few rules, first one is that transformation is symmetric so it has to obey equality:

$$F^{\alpha}_{\beta} = F^{\beta}_{\alpha} \tag{2.1}$$

Second rule is that transformation does work for any number of vectors i want to transform for given space-time, so in this case i took two vectors one in covariant form and one in contravariant form i want to make generalize that transformation for any number of vectors so i get in general sense a transformation: $F^{\alpha_1...\alpha_n}_{\beta_1...\beta_n}$ but for n dimension space-time i need only n vector pairs to fully explain that space-time so for general coordinate system of n dimensions i will get transformation that has n pair of covariant and contravariant vectors. So it's second rule for given coordinate system of n dimensions i have n covariant and contravariant pairs that gives a transformation:

$$F(\mathbf{x}) : F^{\alpha_1...\alpha_n}_{\beta_1...\beta_n} \tag{2.2}$$

Now i don't have any information about transformation itself. If i want transformation to be only rules of how long are rules and how fast clocks tick i can use relation with metric tensor. Metric tensor says how fast coordinates change with respect to flat coordinates, i can use change of that is equal to change of my transformation. How it works? If transformation changes fast metric gives slowly changing clocks and short rules that change by short amount, i can write that third rule as equation:

$$\partial_{\alpha_1}...\partial_{\alpha_n} g^{\beta_1\gamma_1}...g^{\beta_n\gamma_n}\partial_{\gamma_1}...\partial_{\gamma_n} F^{\alpha_1...\alpha_n}_{\beta_1...\beta_n} = \partial_{\alpha_1}\partial_{\beta_1}...\partial_{\alpha_n}\partial_{\beta_n} g^{\alpha_1\beta_1}...g^{\alpha_n\beta_n} \tag{2.3}$$

Now i both have a scalar field (coordinate system) and transformation of field in general sense that has to obey three rules i explain before. Still i miss energy dependence of field but i will arrive at it later. Field is now in most general sense a field where only how i measure clocks and rules changes.

3. Field equation

Field is solution to scalar equation that comes out tensors that transform coordinate system. Those coordinates depend only on how coordinate transformation changes thus how field changes. But it leaves out of picture energy let's say i have a general covariant and contravariant tensor that takes two vectors and gives as output their energy, i can write relation between field as scalar equation:

$$\partial_{\alpha_1}...\partial_{\alpha_n} g^{\beta_1\gamma_1}...g^{\beta_n\gamma_n}\partial_{\gamma_1}...\partial_{\gamma_n} F^{\alpha_1...\alpha_n}_{\beta_1...\beta_n} - \partial_{\alpha_1}\partial_{\beta_1}...\partial_{\alpha_n}\partial_{\beta_n} g^{\alpha_1\beta_1}...g^{\alpha_n\beta_n}$$
$$= g_{\gamma_1\alpha_1}...g_{\gamma_n\alpha_n} g^{\gamma_1\beta_1}...g^{\gamma_n\beta_n} T^{\alpha_1...\alpha_n}_{\beta_1...\beta_n} \qquad (3.1)$$

Now i have general transformation rule for field solution to that equation is a scalar field that can be be thought as coordinate system function, it takes n coordinates and transform into number. So let's sum it all up:

1. I take a coordinate system F.
2. I write most general form of transformation of that system that when solved gives back a function of coordinates that generates field.
3. Field itself is a coordinate transformation that takes n coordinates and gives a number.
4. Field itself represents how coordinates transform for a given point.

So scalar field that is key in this equation is a function that transforms n coordinates into number, so how a n coordinates transform for given point of field it's its value. Space-time created out of this function has a metric but not one metric tensor but n ones for n dimensional space-time. Meaning behind it is that scalar function of n dimensions is not a point object so i measure distance not as line but as hyper-volume that function generates. From field equation follows that faster field changes (coordinate system) slower metric tensor does change- so those two are in opposite balance if i use short ruler and slow ticking clock my coordinates need to change by large amount. Or opposite if I use long rulers and fast ticking clocks my coordinates change by small amount. Last part is that energy has to be conserved so change of energy tensor is zero:

$$\partial_{\alpha_1}...\partial_{\alpha_n} g^{\beta_1\gamma_1}...g^{\beta_n\gamma_n}\partial_{\gamma_1}...\partial_{\gamma_n} T^{\alpha_1...\alpha_n}_{\beta_1...\beta_n} = 0 \qquad (3.2)$$

4. Space-time

In whole paper i assume unit used are Planck units of space, time and energy. From field equation comes that field and metric tensor are in opposite to each other, it means that each object moves in space-time by unit of space and time but that unit is not equal for each particle -faster field moves lower units of space and time it uses so space-time shrinks more so unit in that sense takes a bigger part of space-time. Massless particles move with change of field equal to one over Planck units, it means they form an event horizon when moving but from fact that they have lower than Planck energy they do not form a black hole so horizon is only in form of casual structure. If speed is bigger than one over Planck units (that gives shrink of space-time by Planck units in form of metric tensor) field goes faster than massless particles so it goes back in time from point of view of any observer. This particles moving faster than speed of light travel backwards in time and are key to understand big bang in this model. So there are two key components of understanding space-time, first one is speed of field correlation with metric and it's shrinking by Planck units and second one is energy that particle has and can transfer to another particle or in more general sense energy of one part of field can be transfer to another part of field. Now in previous section i told that space-time interval is no longer a line but a hyper-dimensional volume that connects two regions of field, i can write it formally as:

$$ds^{2n} = g_{\alpha_1\beta_1}...g_{\alpha_n\beta_n}dx^{\alpha_1}dx^{\beta_1}...dx^{\alpha_n}dx^{\beta_n} \tag{4.1}$$

It can be understood as shortest volume that connects two volumes of space-time. Each part of field is understood as scalar coordinate transformation function that has n coordinates so it is an n dimensional object, a hyper-surface. I can connect some part of that hyper-surface with another part by using metric tensor but not one but n ones, where each one represents transformation of two vectors in covariant and contravariant form and in coordinate scalar field one coordinate. Proper time is now not a distance in space-time but a shortest hyper-volume connecting two parts of field that are hyper-volumes themself:

$$\tau^n = \frac{1}{c^n}\sqrt{\int_{P(\mathbf{x})\in R^n} g_{\alpha_1\beta_1}...g_{\alpha_n\beta_n}dx^{\alpha_1}dx^{\beta_1}...dx^{\alpha_n}dx^{\beta_n}} \tag{4.2}$$

So history of object is no longer a line but hyper-volume, i assume space-time interval is in space units that's why i divided it by speed of light. Where $P(\mathbf{x}) \in R^n$ is path of field that belongs to some closed n-dimensional region.

5. Matter,anti-matter symmetry and Big Bang

Field equation works for normal matter, that goes forward in time but I can multiply field equation by negative one so i get:

$$\partial_{\alpha_1}\partial_{\beta_1}...\partial_{\alpha_n}\partial_{\beta_n}g^{\alpha_1\beta_1}...g^{\alpha_n\beta_n}-\partial_{\alpha_1}...\partial_{\alpha_n}g^{\beta_1\gamma_1}...g^{\beta_n\gamma_n}\partial_{\gamma_1}...\partial_{\gamma_n}F^{\alpha_1...\alpha_n}_{\beta_1...\beta_n} = -g_{\gamma_1\alpha_1}...g_{\gamma_n\alpha_n}g^{\gamma_1\beta_1}...g^{\gamma_n\beta_n}T^{\alpha_1...\alpha_n}_{\beta_1...\beta_n} \tag{5.1}$$

Now field moves opposite in all directions both of space and time and generates negative energy. But coordinates more natural for this state of matter are opposite coordinates. I will denote them with bar under notation, where i take opposite not only of coordinates but field itself:

$$F(\mathbf{x}) = -\bar{F}(\bar{\mathbf{x}}) \tag{5.2}$$

This bar notation represents matter moving in opposite direction for both space and time. I can re-write field equation using this notation for anti-field:

$$\bar{\partial}_{\alpha_1}...\bar{\partial}_{\alpha_n}\bar{g}^{\beta_1\gamma_1}...\bar{g}^{\beta_n\gamma_n}\bar{\partial}_{\gamma_1}...\bar{\partial}_{\gamma_n}\bar{F}^{\alpha_1...\alpha_n}_{\beta_1...\beta_n}-\bar{\partial}_{\alpha_1}\bar{\partial}_{\beta_1}...\bar{\partial}_{\alpha_n}\bar{\partial}_{\beta_n}\bar{g}^{\alpha_1\beta_1}...\bar{g}^{\alpha_n\beta_n} = \bar{g}_{\gamma_1\alpha_1}...\bar{g}_{\gamma_n\alpha_n}\bar{g}^{\gamma_1\beta_1}...\bar{g}^{\gamma_n\beta_n}\bar{T}^{\alpha_1...\alpha_n}_{\beta_1...\beta_n} \tag{5.3}$$

Conservation of energy for anti-field takes same form but with bar notation:

$$\bar{\partial}_{\alpha_1}...\bar{\partial}_{\alpha_n}\bar{g}^{\beta_1\gamma_1}...\bar{g}^{\beta_n\gamma_n}\bar{\partial}_{\gamma_1}...\bar{\partial}_{\gamma_n}\bar{T}^{\alpha_1...\alpha_n}_{\beta_1...\beta_n} = 0 \tag{5.4}$$

Now I can create simplest model of Big Bang that comes of form combining field and anti-field equation and creating symmetry between matter and anti-matter, i assume that anti-matter moves same as matter but in opposite direction with opposite energy so when i sum both fields i will get zero- same with energy, energy sums to zero:

$$\begin{aligned}&\partial_{\alpha_1}...\partial_{\alpha_n}g^{\beta_1\gamma_1}...g^{\beta_n\gamma_n}\partial_{\gamma_1}...\partial_{\gamma_n}F^{\alpha_1...\alpha_n}_{\beta_1...\beta_n}+\bar{\partial}_{\alpha_1}...\bar{\partial}_{\alpha_n}\bar{g}^{\beta_1\gamma_1}...\bar{g}^{\beta_n\gamma_n}\bar{\partial}_{\gamma_1}...\bar{\partial}_{\gamma_n}\bar{F}^{\alpha_1...\alpha_n}_{\beta_1...\beta_n}\\ &-\partial_{\alpha_1}\partial_{\beta_1}...\partial_{\alpha_n}\partial_{\beta_n}g^{\alpha_1\beta_1}...g^{\alpha_n\beta_n}-\bar{\partial}_{\alpha_1}\bar{\partial}_{\beta_1}...\bar{\partial}_{\alpha_n}\bar{\partial}_{\beta_n}\bar{g}^{\alpha_1\beta_1}...\bar{g}^{\alpha_n\beta_n}\\ &= g_{\gamma_1\alpha_1}...g_{\gamma_n\alpha_n}g^{\gamma_1\beta_1}...g^{\gamma_n\beta_n}T^{\alpha_1...\alpha_n}_{\beta_1...\beta_n}+\bar{g}_{\gamma_1\alpha_1}...\bar{g}_{\gamma_n\alpha_n}\bar{g}^{\gamma_1\beta_1}...\bar{g}^{\gamma_n\beta_n}\bar{T}^{\alpha_1...\alpha_n}_{\beta_1...\beta_n} = 0\end{aligned} \tag{5.5}$$

Now energy conservation is sum of both matter and anti-matter:

$$\partial_{\alpha_1}...\partial_{\alpha_n}g^{\beta_1\gamma_1}...g^{\beta_n\gamma_n}\partial_{\gamma_1}...\partial_{\gamma_n}T^{\alpha_1...\alpha_n}_{\beta_1...\beta_n} +\bar{\partial}_{\alpha_1}...\bar{\partial}_{\alpha_n}\bar{g}^{\beta_1\gamma_1}...\bar{g}^{\beta_n\gamma_n}\bar{\partial}_{\gamma_1}...\bar{\partial}_{\gamma_n}\bar{T}^{\alpha_1...\alpha_n}_{\beta_1...\beta_n} = 0 \tag{5.6}$$

So at moment of big bang there are equal amounts of matter and anti-matter but matter moves forward in time , anti-matter backwards in time so they split. Anti-matter has to be mirror image of matter universe with time reversed from our perspective but from anti-matter perspective it's opposite we are moving backwards in time, reversed are directions in space same as in time.

6. Spin and rotational symmetry

One of most important features of quantum mechanics is spin, in this model spin is connected to rotation of coordinate system- coordinate field is a scalar field so only way a spin can manifest is by rotation of coordinates. First i write rotation angle for spin only that depends on time and spin space-time function that for each point of field gives spin state:

$$\phi_k(\mathbf{x}) = \hbar\sigma_k(\mathbf{x})\, x^n \tag{6.1}$$

But beyond spin there is angular momentum that i will denote J that depends both on position of space-time and assign space vector pairs, each pair i will denote by $n-1$ it comes from convention that time is n coordinate so space coordinate can have only values $n-1$:

$$\varphi_{n-1,k}(\mathbf{x}) = J_{n-1}(\mathbf{x})\, |\sigma_k(\mathbf{x})|\, x^n \tag{6.2}$$

From both angles i can create rotation operator, that is equal to multiplication of both spin part and angular momentum part:

$$\hat{O}_{n-1,k} = \hat{O}_k\left(\phi_k(\mathbf{x})\right)\hat{O}_{n-1,k}\left(\varphi_{n-1,k}(\mathbf{x})\right) \tag{6.3}$$

That operator acts on each of coordinate field vectors, so i can write field as:

$$F\left(\hat{O}_{1,k}\mathbf{x}^1, ..., \hat{O}_{n-1,k}\mathbf{x}^{n-1}, \mathbf{x}^n\right) \tag{6.4}$$

Spin has probability connected to it, if i take two vectors $\mathbf{a}^p, \mathbf{b}^p$ that are equal to rotation axis of rotation operator, i can create probability function that is equal to:

$$P_k^{n-1}(\mathbf{x}) = \prod_{p=1}^{n-1}\left(\frac{g\left(\mathbf{a}_k^p, \mathbf{b}_k^p\right)}{\sqrt{g\left(\mathbf{a}_k^p, \mathbf{a}_k^p\right) g\left(\mathbf{b}_k^p, \mathbf{b}_k^p\right)}}\right) \tag{6.5}$$

$$\rho_k^2(\mathbf{x}) = \frac{\cos^2\left(P_k^{n-1}(\mathbf{x})\right)}{\sum_{i\in\sigma}\cos^2\left(P_i^{n-1}(\mathbf{x})\right)} \tag{6.6}$$

If i do measurement and i chose same axis i will have many possible states so function $P_k^{n-1}(\mathbf{x})$ gives one, but probability for example for spin 1/2 particles will take two states positive and negative one, but if i measure spin for any other axis i need to take into account hyper-angle between measured axis and that one im measuring now. So both vectors are equal if i measure only one axis if i do second measurement i need to use one vector of measured axis and second of new axis. Rotation operators can have any $n-1$ axis that are represent by those two vectors. Summation represents i for any possible spin state on same axis, g is metric function of given two vectors.

7. Measurement and summary

Last part and most important one is measurement. There can be many solutions to field equation that represent another possible state of field, when i sum all those possible states and take integral of each state i will get probability function of field being in one possible state:

$$\rho_n^2\left(\mathbf{x}\right)=\frac{\left(\int F_n\left(\hat{O}_{1,k}\mathbf{x}^1,...,\hat{O}_{n-1,k}\mathbf{x}^{n-1},\mathbf{x}^n\right)d^{n-1}\mathbf{x}\right)^2}{\sum_N\left(\int F_N\left(\hat{O}_{1,k}\mathbf{x}^1,...,\hat{O}_{n-1,k}\mathbf{x}^{n-1},\mathbf{x}^n\right)d^{n-1}\mathbf{x}\right)^2} \tag{7.1}$$

Now i need to create a wave function that is essential in quantum physics. Idea is very simple, i take all possible spin states and all possible field states sum them, then when i do measurement i will get only one spin and field state, where there is measurement time $\Delta\mathbf{x}_M^n$ that defines how fast field changes from one state to another:

$$\psi_{n,k}\left(\mathbf{x}\right):=\sum_N\sum_{i\in\sigma}F_N\left(\hat{O}_{1,i}\mathbf{x}^1,...,\hat{O}_{n-1,i}\mathbf{x}^{n-1},\mathbf{x}^n\right)$$
$$\mapsto F_n\left(\hat{O}_{1,k}\mathbf{x}^1,...,\hat{O}_{n-1,k}\mathbf{x}^{n-1},\mathbf{x}^n+\Delta\mathbf{x}_M^n\right):\rho_n^2\left(\mathbf{x}\right)\rho_k^2\left(\mathbf{x}\right) \tag{7.2}$$

Wave function has to be normalized so sum of all possible states has to give one:

$$\sum_n\sum_k\psi_{n,k}\left(\mathbf{x}\right)=1 \tag{7.3}$$

Now i have complete quantum view of field. This model is a try to quantize gravity by using a scalar coordinate field that is solution to field equation. It can explain why there is no anti-matter in our universe as i show in section five, i still did not present solution to field equation - those can be really hard to find but in principle they should exist. This mathematical model does not use normal differential geometry there is a need fo new geometry that has not one metric tensor but n ones that create not a line but a hyper-surface as distance function. It has to be this way in order to make field equation work and create a scalar field as solution. This model is free of singularity to create an infinite density there is need for infinite energy- so black holes don't have property of creating infinite density. Energy tensor is undefined - but it should represent any other forces than gravity- forces that create matter so quantum fields energy. Still definition of energy tensor is that it takes n vector pairs and as output gives their energy- in order for any quantum field to work it has to be described this way.