Geometric Qubits
Leptons, Quarks and Gravitons

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January 22, 2022

Abstract

I present an axiomatically constructed model for an underlying description of particles and their interactions, in particular the fermions and gravitation. Using set axioms as a guide, qubits are the fundamental building blocks. It is proposed that the existence of fundamental laws of physics is precluded and only random events occur at the fundamental level. The uncertainty relations and the complex state vectors of quantum theory are a consequence of a Gibbs measure on random variances. This leads to a simple resolution of the measurement problem in quantum mechanics. Quarks and Leptons in 3 generations are Fock states of 4d spaces and their calculated electric charges agree with observations. In addition spin 2 massless gravitons are a 4d Fock state and is the maximum spin state for these 4d Fock states. All particles are geometric, and the dynamics of particles and Space-Time are governed by CAR algebra. One consequence of the model is the cosmological constant being a result of the modification of momentum in curved space.

Contents

Introduction

The Standard Model of particle physics and General Relativity are the current theories matching a wide range of observations. One would like a deeper understanding of these theories and how they are connected. In the Standard Model of particle physics, (the latest reviews and experimentally determined parameters can be viewed on the Particle Data Group website [1]), the particle content, particle properties, and interactions are assumed, guided by experimental results. The classical theory of gravitation, General Relativity [2] is yet to be linked to the Standard Model in a non-trivial way.

The discovery of neutrino oscillations [3] and neutrino mass [4], the accelerated expansion of the universe [5], the flattening of the rotation curves of galaxies [6] and galaxy cluster stability [7] appear to show that the Standard Model and General Relativity are incomplete. This has prompted the formulation of a plethora of models/theories. These include but not limited to GUT models, supersymmetry, dark energy, dark matter, MOND and modifications to General Relativity. However, the particle physics Lagrangians and those of gravitational models/theories do not provide a mechanism of how the system changes. As Richard Feynmann noted in The Messenger Lectures 1964, a particle does not calculate its trajectory in phase space [8].

The measurement problem and the collapse of the wavefunction have also been open problems since the development of Quantum Mechanics [9]. Some interpretations of Quantum Mechanics avoid wavefunction collapse but have there own problems.

In the construction of models/theories of particle physics and gravitation, assumptions about which mathematical structures are to be used are made. For instance, the incorporation of the Standard Model gauge groups into a larger gauge group, extra dimensions and supersymmetry. This freedom to choose the mathematical structure is not accessible by Nature.

Since Axiomatic Set Theory is a foundation of Mathematics, I will use this as a guide for an axiomatically constucted model of nature at the fundamental level.

1 Axioms

It is important to be clear about what physical assumptions are being made when formulating models/theories. More importantly, it is the mathematical assumptions which constrains the physics. Here, I state the axioms and deduce their consequences. Consider a set S of elements ϵ which represents the state of a system.

$$\textbf{A1: No equations of motion for a fundamental system} \tag{1.1}$$
$$\textbf{A2: preclude the creation of same element in sucession} \tag{1.2}$$
$$A1 \Rightarrow \text{no specified relational measure on S} \tag{1.3}$$
$$A1 \Rightarrow \text{no functional relations on } \epsilon \tag{1.4}$$
$$1.4 \Rightarrow \epsilon \text{ are linearly independent} \tag{1.5}$$
$$1.5 \Rightarrow \epsilon \text{ are random} \tag{1.6}$$
$$1.5 \Rightarrow \epsilon \text{ are orthogonal} \tag{1.7}$$
$$A2 \Rightarrow \epsilon \text{ acted on by a CAR operator algebra} \tag{1.8}$$
$$\textbf{A3: minimum 2d CAR,} \tag{1.9}$$
$$1.9 \Rightarrow \epsilon \to \epsilon_a : a = 1, 2 \tag{1.10}$$
$$1.9 \Rightarrow \text{basis vectors } e_a \in \left\{(b_1, 0), (0, b_2) : b_1^2, b_2^2 = \pm 1\right\} \tag{1.11}$$
$$1.11 \Rightarrow \text{ The 2d spaces } V_k \text{ have basis vectors } e_{ka} \tag{1.12}$$
$$1.11 \Rightarrow signature(V_k) \in \{(1, 1), (1, -1), (-1, 1), (-1, -1)\} \tag{1.13}$$
$$1.7 \Rightarrow \text{the 2d vector spaces } V_k \text{ are orthogonal} \tag{1.14}$$
$$\textbf{A4: The random elements are } \epsilon_{ka} = E\left(h_{ka} L^{-1}\right) E\left(x_{ka}\right) \tag{1.15}$$

where $E(h_{ka}L^{-1}), E(x_{ka})$ are the expectation values of the random elements. The inner product of the random elements is

$$\left(\epsilon_{ja}, \epsilon_{kb}\right) = \frac{1}{L^2} \sum_{j,k} E(h_{ja})E(x_{ja})e_{ja}E(h_{kb})E(x_{kb})e_{kb} \tag{1.16}$$
$$\text{where} \quad e_{ja}^2, e_{kb}^2 = \pm 1 \quad \text{Norm of the basis elements} \tag{1.17}$$
$$E(g_{(ja)(kb)}) = E(h_{ja})E(h_{kb}) \quad \text{expectation for the metric} \tag{1.18}$$

The summation in 1.16 is the expectation of the distance measure for the vector space. By the axioms above, $E(h_{ka}L^{-1})$ and $E(x_{ka})$ are independent. Thus ϵ_k are qubits which form the geometry of a space. The space of configurations is acted on by CAR operators by axiom **A3** [10]. I name these qubits ϵ_k as geometric qubits.

2 Gibbs Measure and Quantum Wavefunctions

I will show that there is a Gibbs measure which induces the existence of complex wavefunctions of Quantum Mechanics.

2.1 Uncertainty Relations

Since the random elements are independent, they have the Markov property that the probability of future states depends only upon the present state; that is, given the present, the future does not depend on the past [11][12]. The partition function [13][14] has the property that the random elements are Markovian. By 1.3, I use the statistical measures, the deviations of the components of the 2d vector spaces $\sigma(\epsilon_{ka}) = \sigma(h_{ka}L^{-1})\sigma(x_{ka})$. Let $Z(\beta)$ be the partition function with the potential function H given by

$$H = \sigma^2(h_{ka})\sigma^2(x_{ka})L^{-2} \tag{2.1.1}$$

The partition function for the kth qubit is

$$Z(\beta) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-\beta H}\, \mathrm{d}\sigma(h_{ka})\mathrm{d}\sigma(x_{ka}) \tag{2.1.2}$$

The Gibbs probabilty measure [15] G_k is

$$G_k = P(\sigma(\epsilon_{ka})) = P(\sigma(h_{ka}L^{-1})\sigma(x_{ka})) = \frac{1}{Z(\beta)} e^{-\beta\sigma^2(h_{ka})\sigma^2(x_{ka})L^{-2}} \tag{2.1.3}$$

Since probability P satisfies the strict inequality $0 \leqslant P \leqslant 1$ it follows that

$$\sigma(h_{ka})\sigma(x_{ka}) \geqslant L\sqrt{ln(Z(\beta)} \tag{2.1.4}$$

Thus 2.1.4 are the uncertainity relations for h_{ka}, x_{ka} in curved space.

2.2 Wavefunctions

The uncertainty relations 2.1.4, imply the existence of square integrable complex vectors $\psi(h_{ka})$ and the Fourier transform $\psi(x_{ka})$ [16]

$$\forall e_{ka}, \exists \psi_{ka} \in \mathbb{C} \quad k = 1...4, a = 1, 2 \tag{2.2.1}$$

The kth qubit state can be written as

$$|\psi_k\rangle = \sqrt{G_{k1}}|\psi_{k1}\rangle + \sqrt{G_{k2}}|\psi_{k2}\rangle \tag{2.2.2}$$

$$G_{k1} + G_{k2} = 1 \tag{2.2.3}$$

$$P(|\psi\rangle = \psi_{ka}) = G_{ka} \tag{2.2.4}$$

The probabilities are inherently determined by the Gibbs measures.
Introduce some notation which will be required in section 3.3:

$$\epsilon_{j,0}, \psi_{j,0} \in \{\psi_{ka} : e_{ka}^2 = +1\} \quad j = 1, 2, 3, 4 \tag{2.2.5}$$

$$\epsilon_{0,j}, \psi_{0,j} \in \{\psi_{ka} : e_{ka}^2 = -1\} \quad j = 1, 2, 3, 4 \tag{2.2.6}$$

2.3 Fock space

Since each qubit must be in a non-empty state, it follows that the spaces are all 4d and are one of $\mathbb{R}^{1,3}, \mathbb{R}^{3,1}, \mathbb{R}^{2,2}$ spaces. For intance, the 4d spaces $\mathbb{R}^{1,3}, \mathbb{R}^{3,1}$ will have states as show below, where e_{ik} are unit basis vectors.

$$\{(\epsilon_{11}e_{11}, 0), (0, \epsilon_{22}e_{22}), (\epsilon_{31}e_{31}, 0), (\epsilon_{41}e_{41}, 0)\} \in \mathbb{R}^{1,3} \tag{2.3.1}$$

$$\{(\epsilon_{11}e_{11}, 0), (\epsilon_{21}e_{21}, 0), (\epsilon_{31}e_{31}, 0), (0, \epsilon_{42}e_{42})\} \in \mathbb{R}^{3,1} \tag{2.3.2}$$

Construct the set of 4d real vector spaces $\{(V_\alpha, \sigma(\epsilon_k))\}$ The Gibbs measures on this set of deviations induces the complex vector spaces and the corresponding states $F_{\alpha,ab}$.

$$V_{1,ab} = \mathbb{R}^{1,3} \quad |F_{1,ab}\rangle = |\sqrt{G_{1a}G_{22}G_{31}G_{4b}}\,\psi_{1a}\psi_{22}\psi_{31}\psi_{4b}\rangle \tag{2.3.3}$$

$$V_{2,ab} = \mathbb{R}^{3,1} \quad |F_{2,ab}\rangle = |\sqrt{G_{1a}G_{21}G_{32}G_{4b}}\,\psi_{1a}\psi_{21}\psi_{32}\psi_{4b}\rangle \tag{2.3.4}$$

$$V_{3,ab} = \mathbb{R}^{2,2} \quad |F_{3,ab}\rangle = |\sqrt{G_{1a}G_{21}G_{31}G_{4b}}\,\psi_{1a}\psi_{21}\psi_{31}\psi_{4b}\rangle \tag{2.3.5}$$

$$V_{4,ab} = \mathbb{R}^{2,2} \quad |F_{4,ab}\rangle = |\sqrt{G_{1a}G_{32}G_{22}G_{4b}}\,\psi_{1a}\psi_{32}\psi_{22}\psi_{4b}\rangle \tag{2.3.6}$$

The Fock space at point $X \in \mathbb{R}^4$, $|F(X)\rangle$ [17] consists of the states $F_{\alpha,ab}$, and is

$$|F(X)\rangle = \bigoplus_{\alpha=1...4,a,b=1,2} |F_{\alpha,ab}\rangle \tag{2.3.7}$$

The sum of the probabilities of the Fock states at X must be equal to 1, results in the following condition on the Gibbs measures:

$$G_{1a}^2 G_{4b}^2 \left[(G_{22}G_{31})^2 + (G_{21}G_{32})^2 + (G_{21}G_{31})^2 + (G_{32}G_{22})^2\right] = 1 \tag{2.3.8}$$

The Fock space over all points $X_i \in \mathbb{R}^4$ is the tensor product

$$|F\rangle = \bigotimes_{i=1}^{\infty} |F(X_i)\rangle \tag{2.3.9}$$

Construct the 2 component Fock states:

$$|F_1\rangle = |F_{1,ab}F_{2,ab}\rangle, \quad |F_2\rangle = |F_{1,ab}F_{3,ab}\rangle, \quad |F_3\rangle = |F_{2,ab}F_{4,ab}\rangle \tag{2.3.10}$$

These are for the construction of fermions, section 3.

2.4 Measurement Problem

In the original formulation of Quantum mechanics, the wavefunction collapse is the discontinuous change in the wavefunction (as a superpostion of eigenfunctions) to an eigenfunction whose eigenvalue is the observable. This contradicts the unitary evolution of the wavefunction [9][18]. This is a postulate of the original quantum mechanics and no universally accepted mechanism has been found to explain how the wavefunction changes to the eigenfunction following a measurement.

In section 2.1, the Gibbs probabilty measure $P(\sigma(x_{ka})\sigma(h_{ka}L^{-1}))$ was given, which induces complex wavefunctions ψ_{ka}, normalised to 1 induces the Born rule [19]. The expectation values for the random variables h_{ka} and x_{ka} are calculated using the complex vectors ψ_{ka}. Let the initial configuration be S_i with induced wavefunction ψ_i and final configuration S_f with induced wavefunction ψ_f. Since the configuration space changes randomly, this can be represented as follows

$$S_i \to S_f \Rightarrow \exists(\psi_i \to \psi_f) \tag{2.4.1}$$

A change in the configuration space resuts in the initial wavefunction being replaced by the induced wavefunction of the final configuration. Thus no wavefunction collapse is involved.

2.5 Momentum

Applying the de-Broglie relation for momentum [20]

$$\psi(h_k x_k/L) = \psi(P_k x_k/\hbar) \tag{2.5.1}$$

$$P_k = \frac{\hbar h_k}{L} \tag{2.5.2}$$

Consider a 'local' unitary transformation of the wavefunctions, so that the transformation is applied to the same x_k, (not 'local' if transformation is applied at $-x_k$)

$$\begin{aligned}\psi_k(h_k x_k/L) &\to e^{ik_k x_k}\psi_k(h_k x_k/L) = \psi_k(h_k x_k/L + k_k x_k)\\ &\to \psi_k((h_k/L + k_k)x_k)\end{aligned} \tag{2.5.3}$$

The momentum is transformed to

$$P_k \to P_k + \hbar k_k \tag{2.5.4}$$

Following the 'local' unitary transformation 2.5.3, the uncertainty relations 2.1.4 become

$$\sigma(h_{ka}/L + k_{ka})\sigma(x_{ka}) \geqslant \sqrt{ln(Z(\beta)} \tag{2.5.5}$$

Using the property of the variance $Var(X + c) = Var(X)$ where X is a random variable and c is constant, then in the limit that h_{ka} is constant, the uncertainty relation simplifies to

$$\sigma(k_{ka})\sigma(x_{ka}) \geqslant \sqrt{ln(Z(\beta)} \tag{2.5.6}$$

The uncertainity relation [21][22] is the Heisenberg uncertainity relation for the momentum $\hbar k$ and position. Since all particle momenta are changed by a positive amount, I identify the constant $L^{-2} = \Lambda$, the cosmological constant.

3 Fermions

3.1 Chirality

Adapting the defintion of chirality [23]
Chirality is the eigenvalues of the σ_5 matrix, where σ_5 is given by

$$\sigma_5 = -\sigma_0\sigma_1\sigma_2\sigma_3, \sigma_i^2 = -I_2 \tag{3.1.1}$$

$$\text{On } \mathbb{R}^{1,3}, \sigma_0^2 = I_2, \sigma_5 = -I_2 \tag{3.1.2}$$

$$\text{On } \mathbb{R}^{3,1}, \sigma_0^2 = -I_2, \sigma_5 = -I_2 \tag{3.1.3}$$

Thus, fermions are chiral -1.

3.2 Spin 1/2

The action of the CAR operators on the qubits include transformations isomorphic to SU(2). The spin s=1/2 is generated by SU(2) acting on the a and/or b components of the Fock states $F_{\alpha,ab}$

$$SU(2,\mathbb{C}) \times (F_{\alpha,a(b)}, F_{\alpha,(a)b}) \to s = \left(\pm\frac{1}{2}, \pm\frac{1}{2}\right) \tag{3.2.1}$$

3.3 Fermion Wavefunctions

From 2.3.10, the 2 component Fock states have the following complex vector space structures

$$\begin{pmatrix}\mathbb{C}^{1,3}\\ \mathbb{C}^{3,1}\end{pmatrix}, \quad \begin{pmatrix}\mathbb{C}^{1,3}\\ \mathbb{C}^{2,2}\end{pmatrix}, \quad \begin{pmatrix}\mathbb{C}^{3,1}\\ \mathbb{C}^{2,2}\end{pmatrix} \tag{3.3.1}$$

$$\mathbb{C}^{2,2} \to \mathbb{C}^{3,1}, \mathbb{C}^{1,3} \tag{3.3.2}$$

The Fock states $|F_2\rangle, |F_3\rangle$ can transform to new states via the transformation 3.3.2 which induces the either of the doublets

$$|F_2\rangle \to |F_2'\rangle \quad \begin{pmatrix}\mathbb{C}^{1,3}\\ \mathbb{C}^{2,2}\end{pmatrix} \to \begin{pmatrix}\mathbb{C}^{1,3}\\ \mathbb{C}^{1,3}\end{pmatrix} \tag{3.3.3}$$

$$|F_3\rangle \to |F_3'\rangle \quad \begin{pmatrix}\mathbb{C}^{3,1}\\ \mathbb{C}^{2,2}\end{pmatrix} \to \begin{pmatrix}\mathbb{C}^{3,1}\\ \mathbb{C}^{3,1}\end{pmatrix} \tag{3.3.4}$$

The special unitary group SU(3) acts on the $\mathbb{C}^3$ is the color space and the unitary group U(1) acts on the $\mathbb{C}$ is the electric charge space.

3.4 Electric Charge Quantum Numbers

Using the relation between electric charge and weak-hypercharge [24]

$$Q = T_3 + \frac{1}{2}Y \tag{3.4.1}$$

where Q is the electric charge operator, $T_3 = \frac{1}{2}\sigma_3$ is the diagonal matrix of $SU(2)$ and Y is the weak-hypercharge operator. Consider a general unitary transformation that leaves the modulus of the phase angle θ unchanged for particle in n states:

$$(nQ - nT_3)\theta = \frac{1}{2}Y\theta \tag{3.4.2}$$

$$Y = \mp I_2 \tag{3.4.3}$$

$$T_3 = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} \tag{3.4.4}$$

For the doublets with singlet states $n = 1$, it follows that the eigenvalues of the charge operator are $(\mp 1, 0)$, the electric charges on the chiral leptons.
For the doublets with triplet states $n = 3$, it follows that the eigenvalues of the charge operator are $(\pm\frac{2}{3}, \mp\frac{1}{3})$, electric charges on the chiral quarks.

The Fock states $|F_1\rangle, F_2'\rangle, F_3'\rangle$ when transformed by 3.4.2 are 3 chiral generations of leptons and quarks.

4 Gravitation and Particles

Consider the states on $\mathbb{C}^{1,3}$ and the transformation to $\mathbb{C}^{2,2}$ as show below.

$$\mathbb{C}^{1,3}: \quad \psi_{22} \to \psi_{21} \quad s = 2 \tag{4.1}$$

$$\mathbb{C}^{1,3}: \quad \psi_{31} \to \psi_{32} \quad s = -2 \tag{4.2}$$

With the spin $\frac{1}{2}$ states aligned, the resulting spin states are $s = \pm 2$ thus forming a graviton spin 2 [25][26]. A different view is that particles such as leptons and quarks are additional symmetries on the spin 2 gravitational field, i.e. pertubations of the gravitational field. Matter is thus entangled to gravitational field.

5 Conclusion

A problem with the Standard Model of particle physics is that the reason for the 3 generations of quarks and leptons and their electric charges are left unanswered. Using qubits, I show that the qubits form spaces of 4d and construct a Fock space of 4d spaces. The signatures of the spaces are $\{(1,3),(3,1),(2,2)\}$. In resolving this question, I have found that a Gibbs meaure on the space of random variances leads to the existence of the quantum wavefunction. The Heisenberg uncertainty relations follow from a 'local' unitary transformation of the wavefunctions. A solution to the wavefunction collapse was presented. The maximum spin of a Fock state is spin 2, hence a graviton state. The cosmological constant is a consequence of the momentum in curved space. Further work is needed on this model, in particular how to derive the Lagrangians of the Standard Model and Gravitation.

6 Acknowledgements

Acknowledgements to the mathematicians and physicists discoveries are given below by providing primary references where possible.

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