FOUNDATIONS OF QUANTUM COMPUTING

II. The many faces of quantum uncertainty

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In this Part II we focus on a few key elements of quantum mechanics essential for understanding of quantum technologies and computing.
We begin with a subtle but important similarity between classical and quantum mechanics which is typically overlooked in favor of an apparent differences. Further, it is reminded that classical motion can be obtained via averaging over quantum distributions / wave functions and, conversely, quantum distributions can be recast as a superposition of virtual classical paths. Relatedly, we emphasize the importance of the case intermediate between classical and quantum mechanics – that is, quasi-classical mechanics. The above background facilitates additional insights and heuristics into the mechanisms of widely acclaimed long distance correlations in quantum mechanics and origins of the coherency in quantum ensembles in the context of wave-particle duality.

“…Nothing is more repellent
to normal human beings than
the clinical succession of
of definitions, axioms, and theorems
generated by the labors
of pure mathematicians.”
J.M. Ziman

Introduction

In this Part II we continue (Part I “Foundations of Quantum computinf. I. Demystifying quantum paradoxes”) an honorary attempt to dissolve the haze of mystery around certain facets of Quantum Mechanics randomness and speak about it in a “normal layman” language. There is a caveat though: any classical / heuristic model for a truly quantum event is by necessity bound to some sort of surrogating and should be taken as such. Therefore, a prudent grain of caution is always recommended to avoid improper oversimplifications or even vulgarizations.

Accordingly, in Sec.1 we explore some subtle and often underappreciated similarity between Classical Mechanics (CM) and Quantum Mechanics (QM) in contrast to well acclaimed differences. Sec.2 discusses to what extent CM can be expressed in QM terms and vice versa, i.e. Ehrenfest equations and Feynman path integrals. In Sec.3 we consider classically minded prototypes for long-distance correlations in QM. And, finally, in relation to wave-particle duality, Sec.4 ponders possible mechanisms behind formation of quantum ensembles, in particular, in the context of emerging coherent patterns in experiments with low-intensity beams.

Two final comments are in order: most of formal technical arguments and details in support for the heuristics in the paper are omitted to broaden its accessibility for technically non-savvy readers. For the same reason the list of references is not included: an interested reader should consult any more or less comprehensive text on quantum mechanics.

As in Part I, for compactness, the following intuitive abbreviations are used for most repetitive terms: **CM** – classical mechanics, **QM** – quantum mechanics, **PS** – principle of superposition, **SE** –Shrödinger equation, **PA** – probability amplitude, **WF –** wave function, **EPR** - Einstein- Podolsky-Rosen, **CI** – Copenhagen Interpretation, **WPD** – wave-particle duality.

1. **Quantum vs Classical Mechanics probabilities**

As is well known (and pointed out in part I of "Foundations"), while in CM the motion takes place via paths (called trajectories) fully specified by an initial position and momentum x_1 and p_1, in QM trajectories do not exist simply because the position and momentum cannot be specified simultaneously (Heisenberg uncertainty principle). That is, given initial x_1 in QM, the future particle locations are not known with certainty, but only probabilistically. And here comes a subtle similarity between CM and QM which is widely underappreciated. Indeed, if we specify only x_1, leaving p_1 arbitrary, then even in CM the future paths are undefined. What's more, even if we specify both initial and final x_1 and x_2, keeping initial p_1 or final p_2 arbitrary, then there would still have existed a whole bunch of trajectories connecting x_1 and x_2. A trivial example from elementary physics: projectile motion in an uniform gravitational field, i.e. the motion of a shell fired at some angle to horizon. In other words, in that respect CM and QM are quite similar. However, once we begin to squeeze the range of possible initial p_1 or final p_2 momenta, i.e. when uncertainties Δp_1 or Δp_2 reduce – and this is where the similarity begins to break – so does the spectrum of classically available paths, either emanating from x_1 or connecting x_1 to x_2, so that in the limit of Δp_1 or $\Delta p_2 \rightarrow 0$ we obtain a uniquely defined classical trajectory. Such a refinement is not at all possible in QM even conceptually, because of the uncertainty principle, and thus prohibiting trajectories in QM. We pointed out to this parallelism because it proves helpful for constructing classically inspired heuristics to seemingly mysterious / puzzling quantum phenomena.

Historical aside: to our knowledge, one of the first indications to that subtlety was made as early as in 1933/1934 by prof. Yu.B. Rumer in his “Introduction to Quantum Mechanics”, Moscow, 1935. Once at that, we note in passing that the exposition of QM vs CM in this book is refreshingly clear and concise, yet comprehensive – as opposed to many formidable texts written later - echoing the same of the all time quantum masterpiece “Principles of Quantum Mechanics” by P. Dirac. We wholeheartedly recommend both jewels to all interested readers.

2. **Ehrenfest equations and Feynman paths**.

Given the CM-QM similarity discussed in Sec.1, question arises to what extent it is possible, if at all, to view CM motion as averaged over QM distributions, and vice versa, QM in terms of the CM trajectories.

First of all, an averaging of the Shrödinger equation (SE) over a spatial coordinates leads to the equivalent Ehrenfest equation, which reads as the modified Newton second law ma = F + **Quantum corrections** (ψ), where “**Quantum corrections**” is a cumulative notation for additional terms, arising from quantum effects, and ψ is the wave function of a system / particle. In other words, this equation can be alternatively viewed as averaging over quantum states spaced around some “mean” trajectory. Further, under normal conditions **Quantum Corrections** term is comparable with F in the Right Hand Side (RHS) and, as expected, the standard Newton equation does not apply. However, when $\hbar$ reduces the **Quantum corrections** term reduces commensurately and totally vanish in the limit of $\hbar \rightarrow 0$, recovering thereby the pure classical Newton equation ma = F, or $m\, d^2x(t)/dt^2 = \partial U/\partial x$. In other words, in the quasi-classical limit $\hbar \rightarrow 0$, a classical motion is contributed by few quantum states tightly packed around the particle center of mass.

The construction of the opposite view, QM in terms of CM trajectories, follows from the R.Feynman milestone result: namely, Feynman showed that the quantum motion can be rendered, in a sense, as an interference of classical trajectories. Specifically, the probability amplitude of getting, say, from x_1 to x_2, which normally stems from SE, can be alternatively, **but equivalently**, obtained by summing amplitudes along all classical paths from x_1 to x_2. More precisely, if for each and every imaginable trajectory connecting x_1 and x_2 – and trajectories need not necessarily be real "physical" trajectories – calculate an ordinary classical action S_k, then the sum $\sum \exp[(i/\hbar)S_k(x_1, x_2)]$ over all trajectories (k is the summation index) gives a quantum amplitude $K(x_1,x_2,t)$, which otherwise would come as a solution to SE. Without delving into this any further, we point out only three key points. First, Feynman path sum (or integral) became a standard technical tool in the modern Quantum Field Theory (QFT). Second, similar to Ehrenfest equations, as $\hbar \to 0$, i.e. in a quasi-classical situation, all exponents in the sum wildly oscillate and effectively cancel each other, except for those corresponding to paths in the neighborhood of classic paths (where $S'_k \approx 0$ – stationary points of S_k). That is, in the classical limit, quantum amplitudes are dominated by classical paths and their vicinity, as expected. In other words, QM is possible to construct from CM trajectories, and, the other way around, CM motion naturally arises in the $\hbar \to 0$ limit of QM. Last, but not the least, a quantum motion can be perceived, at least heuristically, as happening over the web of classical "virtual" trajectories. Clearly, a transparency and heuristic appeal of Feynman path integral is rather irresistible.

To bottomline, the transition CM $\longleftrightarrow$ QM looks as follows. As we move from CM to QM, a classical trajectory splits into a tight bundle of paths which continue to diverge as $\hbar$ grows. Conversely, when $\hbar$ reduces, quantum / Feynman paths coalesce around classical trajectory, and eventually fully collapse on it in the limit $\hbar = 0$. In the intermediate region – traditionally known as quasi-classical – where the system is already not classical, but not yet fully quantum, quantum amplitudes (and probabilities) follow directly from classical actions obtained along classical trajectories (see, for instance, R. Feynman, A. Hibbs, "Quantum mechanics and path integrals", McGrawhill, 1965).

3. **Some general heuristics on long – distance correlations.**

In Part I, we touched base on long-distance correlations - resulting from conservation laws - as a true wave phenomenon via a well reputable concept of wave-particle duality. We then emphasized, that even though SE is a statistical equation, the conservation laws hold in quantum mechanics not statistically, but – surprisingly in some sense – in every individual outcome: we can dub this as a "detailed" conservation, rather than statistical one. Here, we'll offer additional qualitative arguments that this detailed conservation is not surprising, but is, in fact, what to be naturally expected from the two way CM $\longleftrightarrow$ QM heuristics (Sec. 2).

Intending for a sort of classically minded prototype for quantum long-distance correlations, consider first a shell at rest exploding into two equal pieces. At any time - and distance! - after the explosion the total momentum remains 0, i.e. the momenta (angular momenta, spins, etc.) of each piece are equal and opposite, as long as there are no external actions. We can even imagine a sequence of random explosions, producing every time a directionally random distribution of fragments, but as long as they are pair-wise balanced, the conservation still holds for all random realizations in every possible direction. Therefore, if we view quantum amplitudes as a virtual superposition of classical events (in a sense of classical imitation of quantum ensemble) - loosely speaking, a la Feynman paths superposition and not necessarily in a coordinate space, but in some suitable representation - we can expect a detailed translation of classical conservation to the quantum world. Obviously, this classical heuristics is only a surrogate imitation of a true quantum reality, but it helps understand that consistency and a smooth transition between classical and quantum cases obviates the need for an artificial quantum non-locality.

Conversely, consider now a classical motion - in light of the Ehrenfest equation – via linearly weighting some tight quantum states. Since the conservation clearly holds for a classical motion, the quantum Ehrenfest averages should do the same. In turn, these averages are made up linearly from quantum states, independent of each other – hence, the conservation should hold individually for each quantum "event" contributing to classical averages.

In the region intermediate between CM and QM – called quasi-classical – both mechanics overlap and coexist so that quantum amplitudes (wave functions) are directly related to classical trajectories. The importance of the quasi-classical mechanics extends way beyond fertile heuristic analogies and technical relationships between CM and QM – it serves for their mutual cross-validation. By way of example: in the above classical model of randomly distributed fragments the long-distance correlation of debris in every possible direction follows immediately, while from the quantum view the randomness in quantum measurements historically contributed to a confusion and even to the so called "quantum non-local" interpretation. However, once we recall the quasi-classical relationships between classical trajectories and quantum amplitudes, the connection of wave functions in any representation to classically balanced outcomes becomes transparent, and so does the conservation in any random realization in QM.

4. Random thoughts on Quantum Vacuum

As mentioned in Part I, the framework of QM and its statistical interpretation rests on the concept of quantum ensembles. In this section we discuss certain heuristics about possible formation mechanisms of those ensembles.

What is invariably observed in all quantum interference experiments with low intensity beams – be it a diffraction on the edge, or on the pinhole or on two narrow slits (in the Young milestone scheme) – is a gradual emergence of a coherent interference pattern on the screen despite the fact, that beam particles are clearly consecutive and independent. And the standard explanation of this striking effect has been traditionally resting on the concept of wave-particle duality (WPD).

Briefly, WPD historically dates back to the Louis de Broglie revolutionary conjecture of 1923, which associated each quantum particle to a corresponding wave – about

60 years later that strange combination of so to speak half-particle and half-wave was somewhat frivolously dubbed as "wavicle" by R.Feynman ("QED: Strange theory of light and matter", 1985). On a more serious note, WPD assumes that all aspects of phenomena in quantum world can be explained either via classical, i.e. particle-like view, or wave-like view, **but not both**. Granted, the wave facet of WPD is clearly conducive for an understanding of a coherent intensity distribution on a screen. However, it is still not enough to explain how this coherence survives across consecutive and independent beam particles. What could be helpful in this regard, is an appreciation of the potential connection of an apparently coherent patterns to an impact of well hidden quantum vacuum.

This footprint of quantum vacuum fluctuations on physical objects can be illustrated via an obvious analogy. Water ripples, i.e. quantum fluctuations, caused by a light summer breeze would not be felt at all by the big ocean cruiser, i.e. heavy classical object. A small sail boat will experience some jittering, but overall its motion would be just slightly affected. And only a small wooden chip, i.e. elementary particle, will be impacted full way. With this parallelism at hand, we reiterate the question: how the consecutive and unequivocally independent (!) particles in beam experiments manage to build-up a coherent interference pattern? Even if we assume - according to WPD - all particles in the beam behaving as waves, they still ought to be independent - analogously to particles they originate from, and that precludes the coherence.

To get this conundrum clarified, let's pretend for a moment that particles appearing as waves are not, in fact, waves per se, but are merely artful cover-ups masquerading the underlying "manipulations" of quantum vacuum. That is, every time free photons, electrons, atoms, or even groups of atoms experience tight interactions with Dirac's "under-sea" of elementary particles hidden in quantum vacuum – and that is exactly what they always do because of their affinity to hidden particles - they exhibit a coherent "wave" behavior, very much like quasi-particles emerging in many-body systems. And further, the quantum vacuum acts as a random, but **stationary and common** "bath" to all members of the beam (quantum ensemble!). This way consecutive and formally independent beam particles become, in fact, mutually correlated. That is, one and the same vacuum hidden fabric superposes with each real beam particle very much like secondary waves in the standard textbook Fresnel diffraction, i.e. it is the engagement of a unique vacuum background that couples all particles / waves to produce an interference picture.

And naturally, that scenario does not apply to heavily macroscopic objects because they are effectively disconnected from vacuum fluctuations.

On the closing note, we reiterate that while the de Broglie hypothesis was not correct literally in associating each particle with a certain wave, it nevertheless paved the way to the whole new paradigm, in which the classical determinism of an individual particle is replaced by the determinism of the wave function in a statistically complete collective of micro-particles, i.e. coherent quantum ensemble.

5. **Some key takeaways.**

1. Along with a well acclaimed difference between QM and CM, there exists yet a subtle and often underappreciated mutual similarity helping better understand the transition between them.
2. In the region intermediate between CM and QM – known as quasi-classical – both mechanics apply and quantum probabilities directly follow from classical mechanics.
3. The long-distance correlations between non-interacting particles in QM is the same manifestation of the conservation laws as in CM and, loosely speaking, can be heuristically pictured as such.
4. It is possible that the formation of quantum ensembles is facilitated via an impact of quantum vacuum.

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