A simple mathematical solution of the cosmological constant problem.

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Abstract / Introduction

Assuming a constant cosmological vacuum density in quantum mechanics, we provide a simple mathematical solution to the cosmological constant problem, i.e. the disagreement of the order of a factor 10^{122} between the theoretical and the measured value of the vacuum energy . We give an interpretation with Casimir effect and a non-exclusive route for our solution to make physical sense.

Keywords : Cosmological constant problem, Vacuum catastrophe, cosmological constant, zero point energy, Casimir effect, Hildebrand solubility parameter.

With :

m_p Planck mass,

l_p Planck length,

$\hbar$ reduced Planck constant,

c speed of light in vacuum,

Λ cosmological constant,

A energy density of the zero point energy in Quantum fied theory.

B energy density of the vacuum assumed for the cosmological constant in quantum mechanics.

C cosmological constant's energy density of the ΛCDM model,

Finally ***A/C*** which is the usual value of the vacuum catastrophe (="cosmological constant problem"), egal about 10^{122}.We will show that ***A/C=C/B*** so ***C²=A*B***.

– let's consider ***A***, the energy density of the zero point exprimed in J/m³ :

$$A = m_p c^2 / l_p^3 = \hbar (l_p^{-4}) . c\ ^*$$

$$= \hbar (l_p^{-2})^2 . c$$

$$B = \frac{1}{(8\pi)^2} . \hbar (\Lambda_{m^{-2}})^2 . c$$

-* $m_p . l_p = \frac{\hbar}{c}$ so $m_p = \frac{\hbar}{l_p c}$

– The energy density of the cosmological constant ***C***, with J/m³, is the geometric mean of A and B :

$$A/C = C/B$$

$$C = \sqrt{A.B} = \sqrt{\hbar(l_p^{-2})^2 . c . \hbar(\Lambda_{m^{-2}})^2 . c/(8\pi)^2}$$

$$= \sqrt{\hbar^2 (l_p^{-2})^2 . c^2 (\Lambda_{m^{-2}})^2/(8\pi)^2}$$

$$= \frac{\hbar c . \Lambda_{m^{-2}}}{l_p^2 8\pi} \;{}^{**}$$

$$= \frac{F_p . \Lambda_{m^{-2}}}{8\pi}$$

where

$F_p = \frac{c^4}{G}$ is the Planck force,

$$\ldots = \frac{c^4 . \Lambda_{m^{-2}}}{8\pi G} = \rho_\Lambda c^2$$

in other words, the classical formula of the energy density of the cosmological constant in the ΛCDM model.

with this addition to simplify the verification :

-** $l_p = \sqrt{\frac{\hbar G}{c^3}}$

$$l_p^2 = \frac{\hbar G}{c^3}$$

so

$$\frac{\hbar . c}{l_p^2} = \frac{\hbar . c . c^3}{\hbar G} = \frac{c^4}{G} = F_p$$

Method used:

- We write density energy exprimed in J/m^3 of the zero point energy in the quantum field theory ***A***, with the reduced Planck constant to make appear a unit of dimension [L^{-2}]
- We assume a vacuum volume density of the cosmological constant in quantum mechanics, ***B***, always with the reduced Planck constant, on the same dimensional model as ***A*** . *the cosmological constant of dimension [L^{-2}] is of the same dimension as lp^{-2}*
- It is shown that the vacuum volume density of the cosmological constant of general relativity, ***C***, is the geometric mean of ***A*** and ***B***.

Interpretation of this mathematical solution with the Casimir effect :

We have,

- volume density of energy of the vacuum in quantum mechanics ***A***:

$$A = m_p c^2 / l_p^3 = \hbar (l_p^{-4}) . c$$

- the hypothetical vacuum energy density of the cosmological constant in quantum mechanics, ***B*** :

$$B = \frac{1}{(8\pi)^2} . \hbar (\Lambda_{m^{-2}})^2 . c$$

- energy density of the cosmological constant ***C*** is the geometric mean of ***A*** and ***B*** :

$$\mathrm{C} = \sqrt{A\ B} = \Lambda_{m^{-2}}\ l_{Pl}^{-2}\ \frac{\hbar\ c}{8\ \pi}$$

The last formula could then be interpreted as a Casimir effect mixing the zero point energy of the vacuum and the hypothetical energy density of the vacuum of the cosmological constant in quantum mechanics because this equality is exactly the equality (1) of this document [1]. In this case it would validate the hypothetical energy density of the vacuum of the cosmological constant in quantum mechanics.

As a reminder in Casimir formula we assumed:

$$\frac{1}{L^4} = \Lambda_{m^{-2}}\ l_{Pl}^{-2}$$

However, a question arises : what is the physical meaning of the square root of an energy density (for ***A*** or ***B***)?

There is no reference on this subject for cosmology. But there is one for the "cohesive energy density" in relation with an ideal gas. cf. the English Wikipedia : Hildebrand solubility parameter.

Of course to speak of vacuum solubility of the QFT in the "quantum" vacuum of the cosmological constant is a physical nonsense. On the other hand, by noting that the value of the energy density of the QFT vacuum is exactly that of the energy density of the Planck mass ($\mathbf{m_{Pl}\ c^2/l_{Pl}^3}$), the solubility of the latter in the "quantum" vacuum of the cosmological constant makes physical sense.

References:

[1] S. Wojnow, Invention of a link between the Casimir effect, the quantum vacuum energy and the cosmological constant. https://vixra.org/abs/2112.0121

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