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A Class of Super Perfect Magic Squares of Order Six

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Absrtact: in the construction of magic square, single even magic square is the most difficult. In this paper, we find a kind of super perfect magic square of order 6, which has a variety of dynamic change patterns and can be called a ten thousand square graph.

Key words: 6th order magic square, super perfect magic square, transformation, ten thousand square graph

Magic square, also known as vertical and horizontal chart, is recognized as the "Luoshu" originated in China. It has a history of more than 4300 years in China, and was first recorded in Dadaili, a historical literature in the Spring and Autumn Period [1]. Magic square is a natural number matrix with the same number of rows and columns, filled with the natural numbers starting from 1 to the square of row or column. The sum of the numbers on each row, column and two diagonals is equal. The value of the number of rows or columns is called the order of magic square. For example, "Luoshu" ("nine palace vertical and horizontal diagram") is a third-order magic square, which is filled with nine numbers from 1 to 9 in the 3x3 grid, so that the sum of the numbers on the row, column and diagonal is 15. People have made a lot of discussions on magic squares [2] [3], and obtained many precious, diverse, ingenious and interesting magic squares [4]. For example, the central part of the seventh order magic square constructed by Yang Hui 1275 in the Southern Song Dynasty is a fifth order magic square, which again contains a third-order magic square and so on[5].

Magic squares can be divided into odd, single even and double even orders according to order number. In the construction of magic square, the magic square of single even order is the most difficult. For thousands of years, with the spread of magic square all over the world, it has aroused widespread international interest. The concept of magic square has been expanded and given more characteristics. Such as magic square composed of prime numbers. In the study of magic squares, determining the number of magic squares of each order is an unsolved problem [5] [6]. The number of magic squares of order 5 and below is certain, but the number of magic squares of order 6 is an astronomical number, which cannot be solved even with powerful computers, and can only be estimated at present. On the other hand, the construction of magic squares with special properties or extensions has attracted the attention of researchers. However, most of them focus on how to complete the static structure of magic square, and lack of further change exploration.

By observing the structure of Rubik's cube and its solution [7], we found a class of super perfect magic squares of order 6, that is, magic squares with multiple changes and mutual conversion. Since natural number 6 is the smallest single even number, the construction of magic square of order 6 has typical significance. The super perfect magic square of order 6 found in this paper is highly changeable and can be called a ten thousand square diagram.

1. Structural characteristics of super perfect 6th order magic square

1.1 As the basic pattern, the magic square of order 6 of Figure 1 can be regarded as composed of 3x3 areas, each area is a sub pattern of 2x2, and the area can be divided into three parts: corner, edge and center;

1.2 Each area consists of four connected numbers, i.e. n; n+1; n+2; N + 3, n is the natural number 1 ~ 33;

1.3 On the diagonal, the sum of the corresponding numbers of central symmetry is equal. As shown in Figure 1, 23 + 14, 24 + 13, 17 + 20, 5 + 32, 6 + 31 and 19 + 18 are all equal to 37.

The following discussion, unless otherwise specified, is based on the basic pattern of Figure 1.

23	21	4	1	30	32
22	24	2	3	31	29
27	28	17	18	12	9
26	25	19	20	10	11
8	6	35	33	13	16
5	7	34	36	15	14

Figure 1. The basic pattern.

2.Dynamic change of super perfect magic square of order 6

For the convenience of narration, the corresponding numbers before and after the transformation are placed in two brackets respectively, which are corresponding and interchanged one by one according to the position. For example:

（1，2，3）-> （6，7，8）

Indicates that 1 and 6 are interchangeable, 2 and 7 are interchangeable, 3 and 8 are interchangeable, etc.

2.1 Transformations that keep the numbers on the diagonal unchanged

(1) Row and column exchanges

Another magic square of order 6 can be derived by swapping lines 1, 2 and 3 with lines 6, 5 and 4, that is:

（23，21，4，1，30，32）-> （5，7，34，36，15，14）and

（22，24，2，3，31，29）-> （8，6，35，33，13，16）and

（27，28，17，18，12，9）-> （26，25，19，20，10，11）

The same for exchanging columns 1, 2 and 3 with columns 6, 5 and 4, that is:

（23，22，27，26，8，5）-> （32，29，9，11，16，14）and

（21，24，28，25，6，7）-> （30，31，12，10，13，15）and

（4，2，17，19，35，34）-> （1，3，18，20，33，36）

(2) Exchanges within a single edge area

The following exchanges deduce a new magic square, namely:

（4，1）-> （2，3）, or

（9，12）-> （11，10）, or

（28，25）-> （27，26）, or

（34，35）-> （36，33）, or

(3) Exchanges of two edge areas simultaneously

Namely:

（4，2，34）-> （1，3，36）

(4) Exchanges in both the central area and the edge area simultaneously

Namely:

（17，19，35）-> （18，20，33）, or

（4，2，17，19）-> （1，3，18，20）, or

（28，27，17，18）-> （25，26，19，20）

(5) Exchanges in both the corner areas and the edge areas

simultaneously

Namely:

（22，30，9，16，36，34，7，26）-> （21，29，11，15，33，35，8，28）

The total number of magic squares generated from the above transformations in Figure 1 is 65536.

2.2 Transformations that keep the secondary vertex number of the diagonal unchanged

(1) Keep the top left and the bottom right corner numbers unchanged, Figure 2 can be obtained through appropriate derivation from Figure 1, that is:

（7，26，22，4，32，16，35）-> （5，25，22，3，30，15，36）

(2) Similarly, keeping the top left and the bottom right corner numbers unchanged, figure 3 can also be obtained through appropriate derivation from Figure 2, that is:

(7, 3, 30) - > (8, 4, 29), while 36, 33, 35 and 34 rotate 90 degrees clockwise.

(3) Similarly, only four vertex numbers are replaced, and the numbers on other diagonals remain unchanged. After proper derivation, the six magic squares of order 6 in Fig. 4 ~ Fig. 9 can be obtained.

23	22	3	1	32	30
21	24	2	4	31	29
27	28	17	18	9	12
25	26	19	20	10	11
8	6	36	33	13	15
7	5	34	35	16	14

Figure 2. The number 23 in the upper left corner B.

23	22	4	1	32	29
21	24	2	3	31	30
27	28	17	18	9	12
25	26	19	20	10	11
7	6	34	36	13	15
8	5	35	33	16	14

Figure 3. The number 23 in the upper left corner C.

22	23	4	1	32	29
21	24	2	3	31	30
28	27	17	18	9	12
25	26	19	20	10	11
7	6	35	36	13	14
8	5	34	33	16	15

Figure 4. The number 22 in the upper left corner A.

22	21	4	3	29	32
23	24	2	1	31	30
28	27	17	18	12	9
25	26	19	20	10	11
8	6	34	36	13	14
5	7	35	33	16	15

Figure 5. The number 22 in the upper left corner B.

22	23	4	3	29	30
21	24	2	1	31	32
28	27	17	18	12	9
25	26	19	20	10	11
8	6	34	36	13	14
7	5	35	33	16	15

Figure 6. The number 22 in the upper left corner C.

21	23	4	1	32	30
22	24	2	3	31	29
28	27	17	18	9	12
25	26	19	20	11	10
8	6	34	36	13	14
7	5	35	33	15	16

Figure 7. The number 21 in the upper left corner A.

21	22	4	3	32	29
23	24	1	2	31	30
27	28	17	18	9	12
25	26	19	20	11	10
7	6	36	35	13	14
8	5	34	33	15	16

Figure 8. The number 21 in the upper left corner B.

21	22	4	3	29	32
23	24	2	1	31	30
28	27	17	18	12	9
26	25	19	20	11	10
8	6	34	36	13	14
5	7	35	33	15	16

Figure 9. The number 21 in the upper left corner C.

A total of 9 different graphs can be formed, and 202752 magic squares of order 6 can be generated.

2.3 Transformations when the secondary vertex numbers on the diagonal are variable

A group of 9 magic squares can be deduced by simple derivation after the number 23, 22 or 21 is exchanged with the number 24 in Fig. 1; Similarly, another group of 9 magic squares can be deduced after the number 29, 30 or 32 is exchanged with the number 31 in Fig. 1.

For example, Figure 10 is one of the magic squares formed by simple derivation after replacing the secondary vertex numbers 24 and 13 with 23 and 14 in Figure 1, that is:

（23，26，35，30，14）-> （24，25，34，29，13）

24	21	4	1	29	32
22	23	2	3	31	30
27	28	17	18	12	9
25	26	19	20	10	11
8	6	34	33	14	16
5	7	35	36	15	13

Figure 10. The number 23 is in the secondary vertex on the top left corner.

A total of 16 groups of 144 kinds of about 3,244,032 magic squares of order 6 can be obtained.

3.Conclusion and discussion

From the results shown above, we can see that the super perfect magic square of order 6 has the characteristics of variety. Different permutations can produce 16 groups and 144 patterns, and each pattern can produce thousands of sixth order magic squares. The average number of magic squares produced by each pattern is more than 10000, and the cumulative number of sixth order magic squares produced is about 3244032. Therefore, this kind of pattern can also be called ten thousand square graph.

The research of magic square has become a system, but similar super perfect magic square needs further research.

The single even magic square is not easy to construct. The method found in this paper is easy to construct and expand the single even magic square by hand, which is worthy of further exploration.

4.References

[1] Jing Zhang, Xingxiang Liu and Zhao Shi. Definition and construction method of perfect sum magic square. Journal of Hubei University for Nationalities (Natural Science Edition), Vol. 38, No. 4: 420-423, December 2020

[2] Xiangyan Luo, Zean Tian and Quan Xie. A fast magic square arrangement method and its proof. Journal of Mathematics in Practice and Theory, Vol. 50, No. 2: 270-277, January 2020

[3] Xiangdong Xu. Structure of perfect Franklin magic square of order 8N. Journal of Mathematics in Practice and Theory, Vol. 51 No. 10: 308-315, May 2021

[4] Zhiyuan Gao. Magic cube boutique appreciation. Journal of Yan'an Institute of education. supplement (total issue 20) 52-58 pages, 1997

[5] Heling Wu. Magic Square and Others - Classic Problems of Entertainment Mathematics (Second Edition). Beijing, Science Press, 2004

[6] G. Abe. Unsolved Problems on Magic Squares. Disc. Math. 127: 3-13, 1994

[7] Rubik’s Cube: https://www.rubiks.com , Jan 30, 2022