Electron Mass and Proton Mass:

The Derivation

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Abstract:

By using the concept of matter waves and the resulting limit velocities for particle movements the author comes to a calculative mass ${m_{gx}}^2 = \frac{h^2}{GRm_x}$. This mass and three other calculative masses formed by physical constants, among them the Planck mass (${m_x}^2 = m_e * m_p$, ${m_{pl}}^2 = \frac{ch}{G}$, ${m_{eq}}^2 = \frac{e^2}{4\pi\epsilon_o G}$), help the author to form the following highly symmetrical proportion:

$$\frac{{m_{gx}}^2}{{m_x}^2} = \frac{{m_{pl}}^2}{{m_{eq}}^2}$$

Inserting into this relation the underlying physical constants, forming the calculative masses, we get the following equation:

$$\frac{{m_{gx}}^2}{{m_x}^2} = \frac{h^2}{GR{m_x}^3} = \frac{2\pi}{\alpha} = \frac{4\pi\epsilon_o ch}{e^2} = \frac{{m_{pl}}^2}{{m_{eq}}^2}$$

And in fact it was the author's intention to derive this equation ${m_e}^3 * {m_p}^3 = [\frac{e^2 h}{4\pi\epsilon_o cGR}]^2$, which he found in 2012 through systematic numerical investigations.

$${m_{gx}}^2 = h^2/GRm_x \qquad {m_{pl}}^2 = ch/G$$

$${m_{gx}}^2/\,{m_x}^2 = {m_{pl}}^2/\,{m_{eq}}^2$$

$${m_x}^2 = m_e * m_p \qquad {m_{eq}}^2 = e^2/4\pi\epsilon_o G$$

$${m_{gx}}^2/\,{m_x}^2 = h^2/GR\,{m_x}^3 = 2\pi/\alpha$$

$$2\pi/\alpha = 4\pi\epsilon_o ch/e^2 = {m_{pl}}^2/\,{m_{eq}}^2$$

$${m_x}^3 = \alpha/2\pi * (h^2/GR) =$$

$${m_x}^3 = e^2 h/4\pi\epsilon_o cGR$$

Introduction:

As already investigated systematically in the author's previous work "The Code of Nature", the values of the electron mass and the proton mass (m_e and m_p) can be represented in a convincing manner by five physical constants plus a time-varying parameter. The five constants are the elementary electric charge e, the vacuum electric permittivity ϵ_o, the Planck constant h, the speed of light c and the gravitational constant G (see [1]). As a time-varying parameter, either the Hubble radius R or the Hubble constant H can be used. The straightforwardness and simplicity of the relation as found by the author in 2012 speak for themselves:

$$m_e^3 * m_p^3 = [\frac{e^2h}{4\pi\epsilon_o cGR}]^2 = [\frac{e^2Hh}{4\pi\epsilon_o c^2G}]^2 \qquad (1)$$

In the past few years it was the author's intention to derive and interpret this relation systematically. In 2014 the author made an important step with his cosmological interpretation of this equation (see [2]). Now a matter wave based derivation of the equation above shall be elaborated.

Abbreviations:

Speed of light $c = 2.9979 * 10^8 \frac{m}{s}$

Gravitational constant $G = 6.6743 * 10^{-11} \frac{m^3}{kgs^2}$

Planck constant $h = 6.6261 * 10^{-34} \frac{kgm^2}{s}$

Electron mass $m_e = 9.1094 * 10^{-31}$ kg

Proton mass $m_p = 1.6726 * 10^{-27}$ kg

Mass $m_x = [m_e * m_p]^{\frac{1}{2}} = 3.9034 * 10^{-29}$ kg

Elementary electric charge $e = 1.6022 * 10^{-19}$ As

Coulomb constant $k_c = \frac{1}{4\pi\epsilon_0} = 8.9876 * 10^9 \frac{kgm^3}{A^2s^4}$

Hubble constant $H = 2.33 * 10^{-18} \frac{1}{s}$

Hubble radius $R = \frac{c}{H} = 1.285 * 10^{26}$ m

Fine-structure constant $\alpha = \frac{e^2}{2\epsilon_0 ch} = \frac{1}{137.036}$

$\frac{2\pi}{\alpha} = \frac{4\pi\epsilon_0 ch}{e^2} = \frac{2\pi*137,036}{1} = 861.023$

$\frac{m_p}{m_e} = 1836.153$

Investigation:

Louis de Broglie's famous idea of matter waves should be at the beginning of our investigation. As Louis de Broglie impressively showed, every particle, such as an electron, can be correlated with a wave whose wavelength λ depends on the momentum p of the particle (see [3]):

$$\lambda = \frac{h}{p} \qquad (2)$$

If the particle moves with a speed v, the absolute value of which is very small compared to the speed of light c, then the non-relativistic momentum p = mv (m being the rest mass of the particle) can be inserted in equation (2) and the wavelength of the matter wave results in:

$$\lambda = \frac{h}{mv} \qquad (3)$$

The distinction between group velocity and phase velocity in wave packets should serve as a further basis for our investigation. Wave packets consist of waves of different frequencies and wavelengths in a frequency band, whereby the speed at which individual wave components or frequencies propagate can differ significantly from the speed of the envelope of the wave packet. The velocity of a single frequency component is called the phase velocity and the velocity of the enveloping wave packet is called the group velocity. Signals and information are always transmitted at the group velocity, since a single harmonic wave is not suitable for information transmission (see [4]).

According to Einstein's theory of relativity, the transfer of information or energy must always take place at speeds that are less than the speed of light, hence the group velocity v_g cannot exceed the speed of light:

$$v_g \leq c \qquad (4)$$

In contrast, the phase velocity v_{ph} can exceed the speed of light because a single harmonic wave is not suitable for information or energy transmission.

As source [5] shows on pages 97 and 98, the phase velocity of matter waves is equal to the square of the speed of light divided by the particle velocity, which corresponds to the group velocity:

$$v_{ph} = \frac{c^2}{v_g} \qquad (5)$$

For our further considerations, the phase velocity and the group velocity of matter waves or wave packets per (5) should be used. Since matter waves can be used to transmit information and energy, the velocity used in formula (3) to calculate the wavelength must be the group velocity of the matter wave. The impulse for determining the classic de Broglie wavelength can therefore be calculated with the group velocity of the matter wave:

$$\lambda = \frac{h}{mv_g} \qquad (6)$$

The phase velocity that we assign to the matter waves should fulfil condition (5). Only with condition (5) will we succeed in deriving the equation $m_x{}^3 = \frac{e^2 h}{4\pi\epsilon_o cGR} = \frac{e^2 Hh}{4\pi\epsilon_o c^2 G}$ with the concept of matter waves ($m_x = [\, m_e * m_p \,]^{1/2}$).

The questions that guide us along the way are:

What is the maximum matter wavelength of a particle and what is the corresponding group velocity? According to (5), how large is the phase velocity corresponding to this group velocity?

According to (6) the wavelength of a particle at rest in the inertial system (v_g = 0) would grow to infinity, which cannot and must not be the case for various reasons. A reasonable theoretical upper limit for the wavelength of matter waves is the so-called Hubble radius, calculated with the Hubble constant and the speed of light:

$$\lambda_{\max} = R = \frac{c}{H} \qquad (7)$$

A particle or a matter wave with a wavelength larger than R or 2R is difficult to imagine, as there would have to be a momentary causal connection beyond this distance. The reason why R is used and not 2R, i.e. the diameter of the Hubble sphere, is explained on page 11.

In contrast to cosmological processes, where light quanta or gravitational waves move between distant objects "only" at the speed of light and can therefore travel billions of years before they are recorded by the observer's measuring instrument, quantum mechanical phenomena must be taken into account with matter waves.

Let's imagine an almost resting electron with an ultra-long wavelength, which is pushed by another very fast object. In this case, the almost stationary electron is instantaneously accelerated to a very high speed and its wavelength is suddenly reduced from ultra-long to ultra-short. This process cannot last billions of years, but the impact reduces the wavelength of the electron instantaneously and with spooky speed. So the impact should show its effect instantaneously over the ultra-long wavelength distance. This is what was meant by the term "momentary causal connection" beyond R.

So while the physical description of a cosmological process depends on the cosmological model used and the temporal development of its parameters, a quantum mechanical description of current processes should also only include currently measurable parameters. In the absence of an alternative, the Hubble constant is used to calculate the theoretical upper limit of the wavelength of a matter wave. And also very pragmatically thought in the sense of achieving the goal: only with the help of the Hubble constant was the author able to derive the numerical relationship

${m_x}^3 = \frac{e^2 h}{4\pi\epsilon_o cGR} = \frac{e^2 Hh}{4\pi\epsilon_o c^2 G}$ found years ago using dimensional analysis.

Using (7) in this sense, let the minimum theoretical group velocity for a matter wave be:

$$v_{g\,\min} = \frac{h}{mR} = \frac{Hh}{cm} \qquad (8)$$

The maximum phase velocity corresponding to this group velocity according to (5) then becomes:

$$v_{ph\,\max} = \frac{c^2}{v_{g\,\min}} = \frac{c^2 mR}{h} = \frac{c^3 m}{Hh} \qquad (9)$$

In the case of the electron, this results in this group velocity and this phase velocity

$$v_{ge} = \frac{Hh}{cm_e} = \frac{2.3338*10^{-18} * 6.6261*10^{-34}}{2.9979*10^{8} * 9.1094*10^{-31}} = 5.6625 * 10^{-30}\ \frac{m}{s} \qquad (10)$$

$$v_{ph,e} = \frac{c^3 m_e}{Hh} = \frac{(2.9979*10^{8})^3 * 9.1094*10^{-31}}{2.3338*10^{-18} * 6.6261*10^{-34}} = 1.5872 * 10^{46}\ \frac{m}{s} \qquad (11)$$

In the case of the proton, this group velocity and this phase velocity result in

$$v_{gp} = \frac{Hh}{cm_p} = \frac{2.3338*10^{-18} * 6.6261*10^{-34}}{2.9979*10^{8} * 1.6726*10^{-27}} = 3.0839 * 10^{-33}\ \frac{m}{s} \qquad (12)$$

$$v_{ph,p} = \frac{c^3 m_p}{Hh} = \frac{(2.9979*10^8)^3 * 1.6726*10^{-27}}{2.3338*10^{-18} * 6.6261*10^{-34}} = 2.9144 * 10^{49} \ \frac{m}{s} \qquad (13)$$

In order to get a feeling for the determined values of the maximum phase velocities or the minimum group velocities, it is helpful to relate them to specific distances or to multiply them by specific transit times. So the question arises: What time t_R does the determined phase of the matter wave need to traverse the Hubble radius?

$$t_R = \frac{R}{v_{ph}} = \frac{h}{c^2 m} \qquad (14)$$

It is interesting that this transit time multiplied by the speed of light results in the Compton wavelength λ_c of the particle in question:

$$c * t_R = \frac{h}{cm} = \lambda_c \qquad (15)$$

So while a phase of the particle traverses the Hubble radius, light covers the distance of one Compton wavelength of the particle in the same time.

The following phase running time results for the electron:

$$t_{R,e} = \frac{h}{c^2 m_e} = \frac{6.6261*10^{-34}}{(2.9979*10^8)^2 * 9.1094*10^{-31}} = 8.0933 * 10^{-21} \ s \qquad (16)$$

The phase running time for the proton:

$$t_{R,p} = \frac{h}{c^2 m_p} = \frac{6.6261*10^{-34}}{(2.9979*10^8)^2 * 1.6726*10^{-27}} = 4.4077 * 10^{-24} \ s \qquad (17)$$

While the determined phase and group velocities have extremely large or small amounts, the determined transit times t_R are of the order of magnitude known in particle physics. For example, the average lifetime for a Higgs boson is about 10^{-22} s or for a Z boson $4*10^{-25}$ s.

Compared to the phase of the matter wave, the particle itself would cover the following distance in the time t_R if it were theoretically travelling with the minimum group velocity determined above:

$$l_g = v_g * t_R = \frac{h}{mR} * \frac{h}{c^2 m} = \frac{h^2}{c^2 m^2 R} = \frac{Hh^2}{c^3 m^2} = \frac{{\lambda_c}^2}{R} = \frac{H{\lambda_c}^2}{c} \qquad (18)$$

In the case of the electron, this distance would assume the following value:

$$l_{g,e} = \frac{Hh^2}{c^3 {m_e}^2} = \frac{2.3338*10^{-18}*(6.6261*10^{-34})^2}{(2.9979*10^8)^3 *(9.1094*10^{-31})^2} = 4.5828 * 10^{-50} \ m \qquad (19)$$

In the case of the proton, this distance would assume the following value:

$$l_{g,p} = \frac{Hh^2}{c^3 {m_p}^2} = \frac{2.3338*10^{-18}*(6.6261*10^{-34})^2}{(2.9979*10^8)^3 *(1.6726*10^{-27})^2} = 1.3593 * 10^{-56} \ m \qquad (20)$$

Incidentally, the distance l_g is also obtained if one inserts (9) into (3), i.e. if one forms a wavelength with the phase velocity v_{ph} (which is not permitted for the movement of the particle itself, since $v_{ph} > c$, this λ_{ph} or l_g can so only be seen with reference to the phase of the matter wave):

$$\lambda_{ph} = \frac{h}{v_{ph} m} = \frac{h^2}{c^2 m^2 R} = \frac{Hh^2}{c^3 m^2} = \frac{{\lambda_c}^2}{R} = \frac{H{\lambda_c}^2}{c} = l_g \quad \Rightarrow \quad \mathbf{l_g R = {\lambda_c}^2} \qquad (21)$$

It is interesting that this minimum distance l_g multiplied by the Hubble radius R results in the square of the Compton wavelength λ_c of the particle. Remarkable symmetries can be uncovered here.

What is still missing for the derivation of $m_x{}^3 = \frac{e^2h}{4\pi\epsilon_o cGR} = \frac{e^2Hh}{4\pi\epsilon_o c^2G}$ is a reference to gravitational physics, because the gravitational constant G appears in the desired formula. This reference can be obtained via the so-called orbital speed:

$$v^2 = \frac{GM}{r} \qquad (22)$$

In order for an object to be able to revolve at a distance r on a circular path around the centre of gravity of a celestial body (e.g. the earth) with mass M, it needs a velocity v. Conversely, the radius of the circular path depends on the orbital speed of the object.

This relation can also be applied fictitiously to a particle mass m and the speed of light as the orbital speed:

$$r = \frac{Gm}{c^2} \qquad (23)$$

It can be countered that as soon as the orbital speed approaches the speed of light, the general theory of relativity must be applied and the orbit becomes the event horizon with magnitude $r = \frac{2Gm}{c^2}$. In this regard, however, it must be considered that a particle whose mass is smaller than the so-called Planck mass m_{pl} ($m_{pl}{}^2 = \frac{ch}{G}$) cannot form a black hole with an event horizon. In contrast to black holes, electrons and protons emit light quanta. Moreover considered quite pragmatically, the derivation of m_x primarily depends on the structure of the (orbit) formula and it is the same for Newton´s and Einstein´s formalism except for a factor of 2.

Since the formula $m_x{}^3 = \frac{e^2h}{4\pi\epsilon_o cGR}$ also contains the ratio of the strength of the electromagnetic force to the gravitational force between a proton and an electron after suitable transformation

$\frac{F_e}{F_g} = \frac{e^2}{4\pi\epsilon_o Gm_e m_p} = \frac{m_x cR}{h}$ (because: $m_x = [\, m_e * m_p \,]^{1/2}$), the factor 4π can come entirely from electromagnetism and does not have to be derived in part from gravitational physics.

In this sense, according to (23) a fictitious orbital radius r results for a particle, which can be set in relation to λ_{ph} or l_g according to (21):

$$\frac{\lambda_{ph}}{r} = \frac{l_g}{r} = \frac{h^2}{c^2m^2R} \Big/ \frac{Gm}{c^2} = \frac{h^2}{GRm^3} = \frac{Hh^2}{cGm^3} \qquad (24)$$

In the case of the electron, the ratio of l_g/r is:

$$\frac{\lambda_{ph,e}}{r_e} = \frac{l_{g,e}}{r_e} = \frac{h^2}{GRm_e{}^3} = \frac{Hh^2}{cGm_e{}^3} = \frac{2.3338*10^{-18}*(6.6261*10^{-34})^2}{2.9979*10^8*6.6743*10^{-11}*(9.1094*10^{-31})^3} = 6.7745 * 10^7 = \mathbf{a} \qquad (25)$$

In the case of the proton, the ratio of lg/r is:

$$\frac{\lambda_{ph,p}}{r_p} = \frac{l_{g,p}}{r_p} = \frac{h^2}{GRm_p{}^3} = \frac{Hh^2}{cGm_p{}^3} = \frac{2.3338*10^{-18}*(6.6261*10^{-34})^2}{2.9979*10^8*6.6743*10^{-11}*(1.6726*10^{-27})^3} = 1{,}0943 * 10^{-2} = \mathbf{b} \qquad (26)$$

With (25) and (26) we already have all the ingredients for the goal of our derivation. If we multiply **a** from (25) by **b** from (26), the result is the square of the number 861.023.

861.023 in turn corresponds to 2π/α. α is the so-called fine-structure constant and is defined in physics as follows or has the following value:

$$\alpha = \frac{e^2}{2\epsilon_0 ch} = \frac{1}{137.036} \qquad (27)$$

Here is the rest of the derivation in detail:

$$a * b = 6.7745 * 10^7 * 1.0943 * 10^{-2} = 7.4136 * 10^5 = 861.023^2 = [\frac{2\pi}{\alpha}]^2 = [\frac{4\pi\epsilon_o ch}{e^2}]^2 \rightarrow$$

$$a * b = [\frac{4\pi\epsilon_o ch}{e^2}]^2 = \frac{h^2}{GRm_e{}^3} * \frac{h^2}{GRm_p{}^3} \rightarrow m_e{}^3 * m_p{}^3 = [\frac{e^2}{4\pi\epsilon_o ch} * \frac{h^2}{GR}]^2 = [\frac{e^2 h}{4\pi\epsilon_o cGR}]^2 = [\frac{e^2 Hh}{4\pi\epsilon_o c^2 G}]^2$$

with $\mathbf{m_e * m_p = m_x{}^2}$ → $$\mathbf{m_x{}^3 = \frac{e^2 h}{4\pi\epsilon_o cGR} = \frac{e^2 Hh}{4\pi\epsilon_o c^2 G}} \qquad \textbf{(28)}$$

With the experience gained so far, the formula (28) can be derived directly in a few steps from m_x instead of from m_e and m_p:

1. $m_x = (m_e * m_p)^{\frac{1}{2}} = (9.1094 * 10^{-31} * 1.6726 * 10^{-27})^{\frac{1}{2}} = 3.9034 * 10^{-29}$ kg
2. $v_{g,x} = \frac{h}{Rm_x} = \frac{Hh}{cm_x}$
3. $v_{ph,x} = \frac{c^2}{v_{g,x}} = \frac{c^2 m_x R}{h} = \frac{c^3 m_x}{Hh}$
4. $\lambda_{ph,x} = \frac{h}{v_{ph,x} m_x} = \frac{h^2}{c^2 m_x{}^2 R} = \frac{Hh^2}{c^3 m_x{}^2}$
5. $r_x = \frac{Gm_x}{c^2}$
6. $\frac{\lambda_{ph,x}}{r_x} = \frac{Hh^2}{cGm_x{}^3} = \frac{h^2}{GRm_x{}^3} = \frac{2.3338*10^{-18}*(6.6261*10^{-34})^2}{2.9979*10^8*6.6743*10^{-11}*(3.9034*10^{-29})^3} = 861.023 = \frac{4\pi\epsilon_o ch}{e^2}$ →
7. $\frac{h^2}{GRm_x{}^3} = \frac{4\pi\epsilon_o ch}{e^2}$ → $\mathbf{m_x{}^3 = \frac{e^2 h}{4\pi\epsilon_o cGR} = \frac{e^2 Hh}{4\pi\epsilon_o c^2 G}}$ **(29)**

Discussion of the results:

On page 14 of the work "Units and Reality" [6] the author found the following approximation for the proton mass by transforming the unit system:

$$m_p{}^3 \approx \left(\frac{2\pi}{\alpha}\right)^{2/3} * \frac{h^2}{GR} \qquad (30)$$

Considering that $\left(\frac{2\pi}{\alpha}\right)^{2/3} = 90.5058$, (30) can be put into a form analogous to (26) and compared with the b from (26):

$$\frac{h^2}{GRm_p{}^3} = \frac{1}{90.5058} = 1.1049 * 10^{-2} \approx \mathbf{b} \text{ from } (26) = 1.0943 * 10^{-2}$$

The approximation from "Units and Reality" with $\left(\frac{2\pi}{\alpha}\right)^{2/3}$ for 1/b is accurate to 0.96% (the approximation is slightly larger) and impressively confirms the assumption expressed in this work that physical relationships can be revealed by transforming the unit system.

The approximation for the electron mass corresponding to the proton mass approximation is:

$$m_e{}^3 \approx \left(\frac{2\pi}{\alpha}\right)^{-8/3} * \frac{h^2}{GR} \qquad (31)$$

$$\frac{h^2}{GR m_e{}^3} = \frac{1}{\left(\frac{2\pi}{\alpha}\right)^{-8/3}} = \frac{1}{1.4904*10^{-8}} = 6.7097 * 10^7 \approx \mathbf{a} \text{ from } (25) = 6.7745 * 10^7$$

The corresponding approximation with $\left(\frac{2\pi}{\alpha}\right)^{-8/3}$ for 1/a is also accurate to 0.96%, but here the approximation is 0.96% smaller. Therefore, if one multiplies the approximations (30) and (31) together, one arrives exactly at the formula (29) to be derived:

$$m_e{}^3 * m_p{}^3 = \left(\frac{2\pi}{\alpha}\right)^{-8/3} * \left(\frac{2\pi}{\alpha}\right)^{2/3} * [\frac{h^2}{GR}]^2 = \left(\frac{\alpha}{2\pi}\right)^2 * [\frac{h^2}{GR}]^2 = m_x{}^6 \Rightarrow$$

$$m_x{}^3 = \frac{\alpha}{2\pi} * \frac{h^2}{GR} = \frac{e^2}{4\pi\epsilon_o ch} * \frac{h^2}{GR} = \frac{e^2 h}{4\pi\epsilon_o cGR}$$

Considering the core of the derivation of (29), which is given by the equation in point 6 of this derivation above

$$\frac{\lambda_{ph,x}}{r_x} = \frac{h^2}{c^2 m_x{}^2 R} / \frac{Gm_x}{c^2} = \frac{h^2}{GR m_x{}^3} = \frac{Hh^2}{cGm_x{}^3} = \frac{2.3338*10^{-18}*(6.6261*10^{-34})^2}{2.9979*10^8*6.6743*10^{-11}*(3.9034*10^{-29})^3} = 861.023 = \frac{4\pi\epsilon_o ch}{e^2}$$

it can be seen that the ratio of two characteristic lengths $\frac{\lambda_{ph,x}}{r_x}$ results in a dimensionless number $\frac{4\pi\epsilon_o ch}{e^2}$, which stands for electromagnetism.

$\frac{4\pi\epsilon_o ch}{e^2}$ corresponds to $\frac{2\pi}{\alpha}$, i.e. contains the so-called fine-structure constant $\alpha = \frac{e^2}{2\epsilon_0 ch} = \frac{1}{137.036}$. $\lambda_{ph,x} = \frac{h^2}{c^2 m_x{}^2 R} = \frac{\lambda_{c,x}{}^2}{R}$ represents the wavelength of the phase of a matter wave and is equal to the square of the Compton wavelength of mass m_x divided by the Hubble radius. So $\lambda_{ph,x}$ combines a quantum-physical parameter with a cosmological parameter and thus describes a wave of cosmological effect.

$r_x = \frac{Gm_x}{c^2}$ represents the radius of the orbit around the mass m_x associated with the speed of light. r_x thus represents a gravitational parameter for a mass, being the geometric mean of electron and proton mass, and therefore stands for gravitation on a very small scale.

The quotient $\frac{\lambda_{ph,x}}{r_x} = \frac{h^2}{GR m_x{}^3}$ formed from the quantum physical constant h, the gravitational physical constant G, the cosmological constant H (R = c/H) and the particle masses ($m_x = (m_e * m_p)^{\frac{1}{2}}$) results in that dimensionless number $\frac{2\pi}{\alpha} = \frac{4\pi\epsilon_o ch}{e^2}$ that can be formed from the constants that describes the electromagnetism. These are the elementary charge e, the speed of light c, the Planck constant h, and the Coulomb constant $k_c = \frac{1}{4\pi\epsilon_0}$. It is interesting that the Planck constant h appears as the only constant on both sides of $\frac{2\pi}{\alpha}$ in the equation, on the left with the mass m_x (or m_e and m_p) and on the right with the charge e:

$$\frac{h^2}{GRm_x{}^3} = \frac{2\pi}{\alpha} = \frac{4\pi\epsilon_o ch}{e^2} \qquad (32)$$

However, since quantum mechanics affects both particle physics and electromagnetism, it is plausible that the Planck constant h appears on both sides.

Looking at both sides of the equation in terms of power numbers, a certain asymmetry is noticeable. The mass m_x occurs in the 3rd power, the elementary charge e only in the 2nd power. In addition, there is the square of h on the left and just h on the right side. However, this apparent asymmetry can be made to disappear by considering that the left term of the equation contains the group velocity: $v_{g,x} = \frac{h}{Rm_x}$.

left: $$\frac{h^2}{GRm_x{}^3} = \frac{v_{g,x}h}{Gm_x{}^2} = \frac{v_{g,x}h}{Gm_p m_e} = \frac{v_{g,x}h}{1} * \frac{1}{Gm_p m_e} = \frac{2\pi}{\alpha} \qquad (33)$$

right: $$\frac{4\pi\epsilon_o ch}{e^2} = \frac{ch}{1} * \frac{4\pi\epsilon_o}{e^2} = \frac{2\pi}{\alpha} \qquad (34)$$

In this form, the left side of (33) is - as the terms marked in yellow show - again symmetrical to the right side of (34): The product of the group velocity and h (left) or the product of the speed of light and h (right) are in the respective numerator. The gravitation between proton and electron appears in the denominator on the left and the electromagnetic force between two elementary charges appears in the denominator on the right.

$\frac{2\pi}{\alpha} = 861.023$ is the coupling constant that connects the left-hand side of equation (32) with the right-hand side. Therefore the various sub-areas of physics are numerically linked by it.

In physics, coupling constants are dimensionless parameters that describe the relative strength of the interactions. Since the fine-structure constant α is the coupling constant of the electromagnetic interaction (see [7]), $\frac{2\pi}{\alpha}$ is called the coupling constant of the equation derived in this work.

The question is whether this coupling constant $\frac{2\pi}{\alpha}$ or α is subject to a temporal development, i.e. is α a function of cosmic time or does it really have a constant value? Since there are no reliable facts for a temporally variable α, a temporally constant α can be assumed for the time being.

The fact that α and the other coupling constants of the fundamental interactions are energy-dependent is widely accepted in physics [8]. The author has examined in detail in [2] how the generally recognized time variability of the Hubble radius R or the Hubble constant H on the left-hand side of equation (32) can be reconciled with a time-constant α.

In order to analyse in detail how the value of α affects the particle masses, equations (32) or (33) and (34) should be further transformed:

$$\frac{Gm_{gx}{}^2}{Gm_pm_e} = \left(G.\frac{v_{g,x}h}{G}\right) * \frac{1}{Gm_pm_e} = \frac{v_{g,x}h}{1} * \frac{1}{Gm_pm_e} = \frac{h^2}{GRm_x{}^3} = \frac{2\pi}{\alpha} =$$

$$\frac{4\pi\epsilon_o ch}{e^2} = \frac{ch}{1} * \frac{4\pi\epsilon_o}{e^2} = (G.\frac{ch}{G}) * (\frac{1}{G} * \frac{4\pi\epsilon_o G}{e^2}) = \frac{Gm_{pl}{}^2}{Gm_{eq}{}^2} \Rightarrow$$

$$\frac{Gm_{gx}{}^2}{Gm_pm_e} = \frac{2\pi}{\alpha} = \frac{Gm_{pl}{}^2}{Gm_{eq}{}^2} \qquad (35) \Rightarrow$$

$$\frac{m_{gx}}{m_x} = (\frac{2\pi}{\alpha})^{\frac{1}{2}} = 29{,}343 = \frac{m_{pl}}{m_{eq}} \qquad (36)$$

From (32) we have formed equation (36) with two masses on the left (m_{gx} and m_x) and two masses on the right (m_{pl} and m_{eq}) that need to be explained:

$$m_{pl}{}^2 = \frac{ch}{G} \qquad (37)$$

The mass m_{pl} is the well-known Planck mass and is formed from the speed of light c, the Planck constant h and the gravitational constant G.

$$m_{gx}{}^2 = \frac{v_{g,x}h}{G} = \frac{h^2}{GRm_x} \qquad (38)$$

The mass m_{gx} is formed in a similar way to the Planck mass, but with the group velocity $v_{g,x} = \frac{h}{Rm_x}$ from above instead of the speed of light c. We do not want to attach any real meaning to it in the context of this work (for the time being). Rather, it serves us to show the symmetries of equations (35) and (36) at a glance and thus to reveal the symmetries not recognizable in (1).

$$m_x{}^2 = m_p * m_e \qquad (39)$$

The mass m_x stands for the proton mass m_p and the electron mass m_e and is used, among other things, to present the formulas in this work with the lowest possible power numbers.

$$m_{eq}{}^2 = \frac{e^2}{4\pi\epsilon_o G} \qquad (40)$$

The mass m_{eq} represents the mass equivalent to the elementary charge e, since two masses of size m_{eq} would attract each other with the same force as two elementary charges with different signs (e.g. e- and e+) if they were at the same distance.

Equation (35), i.e. $\frac{Gm_{gx}{}^2}{Gm_pm_e} = \frac{2\pi}{\alpha} = \frac{Gm_{pl}{}^2}{Gm_{eq}{}^2}$, is symmetrical in that the ratio of the strength of the gravitation between two masses m_{gx} to the strength of the gravitation between a proton and an electron behaves in the same way as the ratio of the strength of the gravitation between two Planck masses m_{pl} to the strength of the electromagnetism between a proton and an electron. In both cases the ratio is $\frac{2\pi}{\alpha} = 861.023$.

As for the absolute level of strength of the (gravitational) terms on both sides of equation (35), there is a large imbalance in favour of the electromagnetic side: $\frac{Gm_{eq}{}^2}{Gm_pm_e} = \frac{Gm_{pl}{}^2}{Gm_{gx}{}^2} = 2.269 * 10^{39}$. $2.269 * 10^{39}$ corresponds to the relative strength of electromagnetism to gravity.

As stated above in connection with (7), the theoretical upper limit for the wavelength of matter waves could also be set at twice the Hubble radius R. Then point 6 of the derivation of (29) would result in the following:

$$\frac{\lambda_{ph,x}(2R)}{r_x} = \frac{h^2}{2GRm_x{}^3} = \frac{(6.6261*10^{-34})^2}{2*6.6743*10^{-11}*1.285*10^{26}*(3.9034*10^{-29})^3} = 430.512 = \frac{2\pi\epsilon_o ch}{e^2} = \frac{\pi}{\alpha}$$

Then equation (32) would become $\frac{h^2}{2GRm_x{}^3} = \frac{\pi}{\alpha} = \frac{2\pi\epsilon_o ch}{e^2}$.

The right side of (35) would look like this: $\frac{\pi}{\alpha} = \frac{Gm_{pl}{}^2}{G*2m_{eq}{}^2} = \frac{Gch}{G} * \frac{4\pi\epsilon_o G}{G*2e^2} = \frac{2\pi\epsilon_o ch}{e^2}$

The equations (32) and (35) would therefore have a less conclusive form, because a multiple of the electromagnetic force would appear on the right-hand side. The same effect would also result, if the event horizon $r_{bl} = \frac{2Gm_x}{c^2}$ would be included in point 6 of the derivation of (29) instead of the orbit radius $r_x = \frac{Gm_x}{c^2}$. In this case point 6 also would be: $\frac{\lambda_{ph,x}(R)}{r_{bl}} = \frac{h^2}{2GRm_x{}^3} = \frac{\pi}{\alpha}$

The approximation formula (30) for the proton mass, then formed on the basis of the terms $\frac{h^2}{2GR}$ and $\frac{\pi}{\alpha}$, would for example assume the form $m_p{}^3 \approx \left(\frac{\pi}{\alpha}\right)^{2/3} * \frac{h^2}{2GR} * 2^{5/3}$. This would be a less elegant form than (30) and would encourage the reduction of the powers of 2 in the fractional terms to get back to the form of (30).

Another interesting question is, how the mass ratios would have to be theoretically so that the coupling constant in the middle of the equation (35), $\frac{Gm_{gx}{}^2}{Gm_p m_e} = \frac{2\pi}{\alpha} = \frac{Gm_{pl}{}^2}{Gm_{eq}{}^2}$ would assume the value 1.

On the right-hand side, things are trivial in that m_{pl} should equal m_{eq}. Equating (37) and (40) gives:

$$m_{pl}{}^2 = m_{eq}{}^2 \Rightarrow \frac{ch}{G} = \frac{e^2}{4\pi\epsilon_o G} \Rightarrow \frac{4\pi\epsilon_o ch}{e^2} = 1 = \frac{2\pi}{\alpha_{th}} \Rightarrow \alpha_{th} = 2\pi$$

In order for m_{pl} to be equal to m_{eq}, the fine-structure constant α_{th} should theoretically have the value 2π.

On the left, matters are a little more subtle. Equating (38) and (39) gives:

1. $m_x{}^2 = m_{gx}{}^2 \Rightarrow m_x{}^2 = \frac{h^2}{GRm_x} \Rightarrow m_{x,th}{}^3 = \frac{h^2}{GR} = \frac{(6.6261*10^{-34})^2}{6.6743*10^{-11}*1.285*10^{26}} = 5.1209 * 10^{-83}$
   $\Rightarrow m_{x,th} = 3.7135 * 10^{-28}\ \mathrm{kg}$
2. $m_x{}^2 = m_{gx}{}^2 \Rightarrow \frac{v_{g,x}h}{G} = m_x{}^2 \Rightarrow v_{g,x,th} = \frac{h}{R_{th}m_x} = \frac{H_{th}h}{cm_x} = \frac{Gm_x{}^2}{h} \Rightarrow H_{th} = \frac{cGm_x{}^3}{h^2} \Rightarrow$
   $H_{th} = \frac{cGm_x{}^3}{h^2} = \frac{2.9979*10^8*6.6743*10^{-11}*(3.9034*10^{-29})^3}{(6.6261*10^{-34})^2} = 2.7105 * 10^{-21}\frac{1}{s}$

In order for the coupling constant to have the value 1 in the middle of equation (35), α would theoretically have to be larger by a factor of $\frac{\alpha_{th}}{\alpha} = \frac{2\pi*137.036}{1} = 861.023$. In addition - if one disregards the theoretical possibility of a c or h or G with a different value - either m_x should theoretically be larger by the factor $\frac{m_{x,th}}{m_x} = \frac{3.7135*10^{-28}}{3.9034*10^{-29}} = 9.5135 = 861.023^{1/3}$ or H should theoretically be smaller by the factor $\frac{H}{H_{th}} = \frac{2.3338*10^{-18}}{2.7105*10^{-21}} = 861.023$. That this is the case can be seen at a glance by rearranging equation (29):

$$m_x{}^3 = \frac{e^2Hh}{4\pi\epsilon_o c^2G} = \frac{\alpha Hh^2}{2\pi cG} = \frac{Hh^2}{861.023*cG} \qquad (41)$$

(41) then becomes either $m_{x,th}{}^3 = \frac{Hh^2}{cG} = \frac{h^2}{RG}$ or $m_x{}^3 = \frac{H_{th}h^2}{cG} = \frac{h^2}{R_{th}G}$. But that would mean that the constants describing electromagnetism would be eliminated from the equation. The properties of a theoretical universe that would fulfil these reduced equations would have to be considered in detail. If one considers that the coupling constant α increases at higher energies, then this would have to be a universe with an energy density many times higher than that of our familiar cosmos. Whether in such a universe the electromagnetic force would already be united with the other basic forces would have to be examined in detail.

Substituting the reduced equation $m_{x,th}{}^3 = \frac{Hh^2}{cG} = \frac{h^2}{RG}$ or $m_{x,th} = \frac{h^{2/3}}{R^{1/3}G^{1/3}}$ into the equation for the minimum group velocity $v_{g,x} = \frac{h}{Rm_x}$ gives $v_{g,x,th} = \frac{h}{Rm_{x,th}} = \frac{G^{1/3}h^{1/3}}{R^{2/3}}$. Inserting this theoretical minimum group velocity into formula (23) for the orbital speed results in a theoretical orbital radius $r_{th} = \frac{Gm_{x,th}}{v_{g,x,th}{}^2} = G * \frac{h^{\frac{2}{3}}}{R^{\frac{1}{3}}G^{\frac{1}{3}}} * \frac{R^{\frac{4}{3}}}{G^{\frac{2}{3}}h^{\frac{2}{3}}} = R$. Such a universe would be symmetric in that the minimum possible group velocity for a particle of mass $m_{x,th}$ would be equal to the orbital speed around that particle at the Hubble radius distance. But obviously this symmetry must be broken as in (29) for the cosmos to be the one in which we live.

Last but not least we make a few considerations regarding the phase velocity of the matter wave. As we discussed above, an almost resting particle with an ultra-long wavelength pushed by another very fast object is instantaneously accelerated to a very high speed and its wavelength is suddenly reduced from ultra-long to ultra-short. This process cannot last billions of years, but the impact reduces the wavelength of the particle instantaneously and with spooky speed. So the impact should show its effect instantaneously over the ultra-long wavelength´s distance.

As we calculated above with the Hubble radius, the maximum possible phase velocity for a matter wave of mass m_x is also spookily fast:

$$v_{ph,x} = \frac{c^2}{v_{g,x}} = \frac{c^2m_xR}{h} = \frac{c^3m_x}{Hh} = \frac{(2.9979*10^8)^3 * 3.9034*10^{-29}}{2.3338*10^{-18} * 6.6261*10^{-34}} = 6.8013 * 10^{47}\ \frac{m}{s} \qquad (42)$$

If one considers that an almost stationary particle should immediately shorten its wavelength during a collision, it would be obvious if this process would take place with the phase velocity. That is why a phase with maximum speed is able to traverse the Hubble radius in about 10^{-22} s:

$$t_{ph,x} = \frac{R}{v_{ph,x}} = \frac{h}{c^2m_x} = \frac{6.6261*10^{-34}}{(2.9979*10^8)^2 * 3.9034*10^{-29}} = 1.8887 * 10^{-22}\ s$$

Because quantum mechanics also allows two particles to be entangled in such a way that they can be described by a common wave equation, a measurement on one of the entangled particles can also immediately influence the state of the second particle. Einstein, who disliked this phenomenon of instantaneity, named it "spooky action at a distance" (see [9]) and therefore criticized quantum mechanics.

If we think further about the previous considerations regarding the phase velocity, then it should be very likely that the phase velocity is behind the phenomenon of "spooky action at a distance". If several entangled particles can "surf" on one wave, then why shouldn't the phases of that wave be able to influence their state almost simultaneously. As we discussed above, matter waves are not local phenomena and can reach cosmic dimensions in extreme cases. Is that the non-locality that causes the "spooky action at a distance"?

If this is the case, the question arises: does the "spooky action at a distance" move with different speeds, or does it always occur with the same speed (at least in a vacuum), in analogy with light, which moves in a vacuum with always the same speed $c = 2.9979 * 10^8 \ \frac{m}{s}$.

Should there be a "spooky action at a distance" with standard speed, then my favourite for that speed would be that given in equation (42):

$v_{ph,x} = \frac{c^2 m_x R}{h} = 6.8013 * 10^{47} \ \frac{m}{s}$, because it is formed with m_x, so it contains both the electron mass and the proton mass just like the formula (29) derived here and would be independent of the actual particle mass and group velocity and would be a physical constant. This would be supported by the fact that the "spooky action at a distance" also comes into play between entangled photons. For photons whose group velocity $v_g = c$, a calculation of the phase velocity via $v_{ph} = \frac{c^2}{v_g}$ would again result in c and would therefore be measurable with current technology and no longer be spookily fast. Even very fast-moving particles with group velocities approaching the speed of light would have a phase velocity that is too low to be called spookily fast.

The ratio of the phase velocity $v_{ph,x}$ to the speed of light c

$\frac{v_{ph,x}}{c} = \frac{c m_x R}{h} = \frac{6.8013*10^{47}}{2.9979*10^8} = 2.269 * 10^{39} = \frac{G {m_{eq}}^2}{G m_p m_e} = \frac{e^2}{4\pi \epsilon_o G m_p m_e} = \frac{F_e}{F_g}$ corresponds exactly to the relative strength of electromagnetism to gravitation and is also implicitly contained in formula (29).

Admittedly, these reflections on "spooky action at a distance" are still speculative. But the aim of physics was always to put an end to any unexplained 'spook' and to replace it with a physical explanation. Someone has to start this, even at the risk that the first approach might turn out to be wrong and better explanations might replace it in the future.

However, the primary goal of the present work was not to explain the "spooky action at a distance", but to derive formula (1) with the help of the concept of matter waves. Readers of this work, who - for whatever reason - oppose the considerations to the "spooky action at a distance" here, may simply ignore them and concentrate on the core goal of this work.

☺

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