Toward a proof of the Riemann Hypothesis

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Abstract: The Möbius function $\mu(j)= -1,0,1$ depending on whether j has an odd number of factors, a square factor, or an even number of factors. The Mertens function $m(n)$ is $j = 1$ *to* n, $\sum \mu(j)$. For all n, $|m(n)| < 2n^{1/2}$. $|m(n)|=O(n^{1/2})$, and therefore the Riemann Hypothesis is true.

Set $f_n = (1/2)(2/3)(3/4)\ldots([n^{1/2}]/([n^{1/2}]+1))(n)=n/([n^{1/2}]+1) < n^{1/2}$. See Appendix 1.

Set $S_n = (4/3)(9/8)(16/15)(25/24)\ \ldots(s^2/(s^2-1))$, $s^2 \leq [n^{1/2}]+1$. $S_n= 2n/(n+1)$ Proof by induction.

$n=2$ $S_n = (4/3)$ assume $S_n= 2n/n+1$ $S_{n+1} = (2n/n+1)(n+1)^2 / ((n+1)^2–1) = 2(n+1)/((n+1)+1)$

$\boldsymbol{|m(n)| < (S_n)(f_n) < (2)(n^{1/2})}$.

A negative cycle is an interval in which $m(s) \leq 0$ for all values of s and a positive cycle is an interval in which $m(s) \geq 0$ for all values of s.

For every $s \geq 1$, $m(s)$ is in a positive or negative cycle or possibly both if $m(s)=0$.

For $|m(s)|$ at maximum value within a positive/negative cycle $m(s+1)$ is in the same cycle.

$\boldsymbol{|m(n)| < (2)(n^{1/2})}$ can be inferred from $\boldsymbol{m(1)=1}$ alone.

For the cycle containing $\boldsymbol{m(n)}$ one or both of the following two conditions hold.
(1) $\boldsymbol{|m(n)|}$ is at maximum value within a cycle and $\boldsymbol{|m(n)| \leq [(2)(n^{1/2})]}$
(2) $\boldsymbol{|m(n)| \leq [(2)(n^{1/2})] – 1}$.

1) $\boldsymbol{|m(n+1)| \leq |m(n)| \leq [(2)(n^{1/2})] \leq [(2)(n+1)^{1/2})]}$.

2) $\boldsymbol{|m(n+1)| \leq |m(n)|+1 \leq [(2)(n^{1/2})] \leq [(2)(n+1)^{1/2})]}$.

$\boldsymbol{|m(n)|=O(n^{1/2})}$**,** and therefore Riemann Hypothesis is true.

Appendix 1. The Mertens function $m(n)$ is applied to the first n positive integers as a set. The reciprocal of each of the ***s*** non-square integers up to $[n^{1/2}]+1$ is a Mertens proportionality factor. The MPF are applied repeatedly to the fractional part of n. $(1 – 1/f_1)(n)=n_1$, $(1 – 1/f_2)(n_1)=n_2$, … $(1 – 1/f_s)(n_{s-1})=n_s < 2n^{1/2}$ $2 = f_1$ thru $f_s \leq [n^{1/2}]+1$. Collectively, the MPF are a measure of the proportion of elements in the Mertens function set whose Möbius function always has a combined value of zero. $m(n)$ has a maximum/minimum possible value depending on $m(n)$ being in a positive/negative cycle.