Quantum mechanics emerging from complex Brownian motions

A Preprint

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January 31, 2022

Abstract

The connection between the Schrodinger equation and Einstein diffusion theory on basis of Brownian motion of independent particles is well known. However, in contrast to diffusion theory, quantum mechanics theory has suffered controversial interpretations due to the counterintuitive concept of wavefunction. Here, while we confirm there is no difference in the mathematical form of these two equations, we derive the complex version of displacement. Using diffusion theory of particles in a medium, as simple as it is, we describe that quantum mechanics is just an elegant and subtle equation to describe the probability of all the trajectories that a particle can take to propagate in time by a predictive wavefunction. Therefore, information on the position of particles through time in quantum theory is embedded in the wavefunction which predicts the evolution of an ensemble of individual Brownian particles.

Keywords Quantum Mechanics · Wavefunction · Browning Motion · Diffusion

1 Introduction

In Rayleigh-Jean's mathematical model of the spectral radiance per frequency of the black body radiation using equipartition theorem, a drawback in the classical theory was realized for higher frequencies. Based on equipartition theory in a microcanonical ensemble where all the microstates are equally accessible, there would be an equal probability for each radiation frequency of the entire continuous spectrum. This theory allocates the same radiation probability for an infinitesimally small state that can vibrate with corresponding infinite frequency, and therefore infinite energy. Not only does this contradicts the experimental data, but also it is not sensible as any matter under this condition would evaporate instantaneously. This could have been one of the reasons that Ludwig Boltzmann was conceptualizing the existence of a quanta particle and energy a few decades earlier. To understand this ultraviolet catastrophe and come up with better models to fit the experimental data Max Planck improved the Wien's distribution for lower frequencies. However, he did not understand the physics behind his model mechanistically. Planck eventually adopted the Boltzmann conjecture on the existence of energy packets and particles as an act of despair. Planck was against this conjecture because the presence of quanta particles can violate the second law of thermodynamics in measurements on entropy for infinitesimal system and time — the entropy of a system with a small number of particles under stochastic Brownian motion can decrease for an epsilon time. Using Boltzmann probability distribution, he imagined there are oscillators with different strengths that absorb or emit a certain discrete amount of radiation frequencies — Kirchhoff had earlier described that a perfect black body emits the same frequency radiations that it absorbs at a given temperature in thermal equilibrium. The oscillator that vibrates with the energy $h\nu$ equal to kT has a higher probability of oscillations at a given temperature, and the extreme cases have no oscillations. Later the Photoelectric effect of Einstein confirmed the presence of quanta energy after Lenard's realization that the velocity of emitted electrons is independent of the intensity of ultraviolet light but dependent on its frequency. [1]

After the discovery of the electron by J. J. Thomson, he proposed the plum pudding model in which the plums represent the electrons inside the pudding of positively charged particles. This was a continuation of William Thomson's (Kelvin) description of the inner structure of an atom in the vortex theory of an atom in which atom as vortex was inside an aether medium. Later on, Rutherford discovered that the most mass of the atom is concentrated in the center of it. Bohr put Planck's description of discrete energy packets and Rutherford's model of an atom together to build a model of an atom with discrete energy orbits through which electrons can rotate around the nucleus, and they can absorb and emit radiation by moving from one orbit to another. Bohr described that there is a fundamental ground state level of energy for electrons in which they are stable and do not lose energy. But this picture does not reconcile with Newtonian classical mechanics since the electrons should lose energy and eventually get absorbed by electromagnetic forces of the nucleus at the center of the atom. Therefore, it was thought that electrons behave like waves spread around the nucleus with a concentration in a certain region that is called orbital. [1] Using the concept of the wavefunction, Erwin Schrodinger invented a formula to calculate the energy and probabilistic position of electrons in a system [2]:

$$\hat{H}\Psi = i\hbar\frac{\partial\Psi}{\partial t} \tag{1}$$

Hamiltonian, the total energy of the system, is composed of the kinetic and potential energy of the constituents of the system:

$$\hat{H} = \hat{T} + \hat{V} \tag{2}$$

where

$$\hat{T} = \hat{p}^2/2M \tag{3}$$

and

$$\hat{p} = -i\hbar\frac{\partial\psi}{\partial x} \tag{4}$$

Therefore;

$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + \hat{V}\frac{\partial^2\Psi}{\partial x^2} = i\hbar\frac{\partial\Psi}{\partial t} \tag{5}$$

Regardless of the beauty and precise prediction power of this equation, it cannot explain the particle behavior of electrons as it is observed in radioactive experiments. To justify this contradiction, and also to understand the physical reality behind the Schrodinger equation, there have been several interpretations of quantum mechanics equations explained briefly in the next paragraph.

While the wavefunction itself is a counterintuitive concept, Max Born found out that the square of a wavefunction would provide the probability of the presence of an electron in a specific location at a given time. [3] The wavefunction provides all the information in regards to the superpositions of an electron in spacetime. However, it collapses through the act of measurement as the electron will always be found in a particular position. This implies that the wave behavior of an electron during the act of measurement instantly converts to particle behavior. This literal interpretation of the Schrodinger equation is called the Copenhagen interpretation which believes in the mathematical strength of quantum mechanics but does not provide a mechanistic meaning to it. [4] As a continuation of this interpretation, soon after the invention of the Schrodinger equation, the indeterministic statistical interpretation was realized by Born and promoted by Heisenberg's uncertainty relation of position and momentum. [5, 6] The wavefunction for an electron is developed in such a way that it encompasses all the possibilities of electron propagation in time which contains information about infinite ensemble measurements on the electron position in time yet before consciousness comes to the place. The efforts to understand the transition from wave to particle paradigm in a realistic context have led to many other interpretations that here we discuss three of them below.

Subjective updating of the wavefunction using the Bayesian principle in QBism interpretation is one of the smooth ways for this transition. [7, 8, 9] Also, the collapse of the wavefunction as an objective property of the wavefunction regardless of external consciousness by adding extra terms to the Schrodinger equation is another way to smooth out the transition from quantum to the classical regime. [10, 11, 12] The other interesting theory is de Broglie's pilot-wave theory of David Bohm which accounts for both particle and wave in the Schrodinger equation. [13, 14] The Bohemian mechanics states that the particles are carried around by wavefunction spread through the entire spacetime continuously. However, even in this nonlocal theory, the particle in the equation only emerges after the conscious measurement is performed in a process called quantum decoherence. [13, 14] All these theories cannot correspond to reality without changing the form of the equation or being assisted by consciousness. Even the superstition many world interpretations [15] that thrives to find the literal realistic meaning of the Schrodinger equation through a split or copied universes would confront with the same issue of consciousness and besides that, it is not clear how such a system would conserve energy.

Here, we explain that the form of the Schrodinger equation in quantum mechanics is a consequence of infinite ensemble measurements of free Browning motions of particles in phase space. We confirm that the Schrodinger equation for a free particle with zero potential is a version of the diffusion equation with a complex diffusion constant [16]:

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} \Longrightarrow \frac{\partial\rho}{\partial t} = D\frac{\partial\rho}{\partial x^2} \tag{6}$$

therefore:

$$\langle\Delta S\rangle = 2\pi ik \tag{7}$$

2 Results and Discussion

We use the Born concept of probability to create the Einstein form of diffusion equation from the Schrodinger equation for a free particle where $V = 0$ as follows:

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} \tag{8}$$

To build the density ρ from Ψ we multiply the Schrodinger equation with the complex conjugate of the wavefunction, Ψ^*, and then integral over the entire spacetime which gives the following equation with a few rearrangements:

$$i\hbar\{\frac{\partial\rho}{\partial t} + [\Psi^*, \frac{\partial}{\partial t}]\Psi\} = -\frac{\hbar^2}{2m}\{\frac{\partial^2\rho}{\partial x^2} + [\Psi^*, \frac{\partial^2}{\partial x^2}]\Psi\} \tag{9}$$

In the above equation if wavefunction Ψ commutes with the first derivative of time t and second derivative of position x, we straightforwardly derive a form of Schrodinger equation similar to one of the diffusion equation. However, wavefunctions in quantum mechanics depend on time and position to propagate in time, and therefore, the uncertainty principle of Heisenberg tells us that these two commutators never commute in a real system. [5] Nonetheless if we consider the wavefunction of a free particle corresponding to the free Brownian particles,

$$\Psi = A\exp(i\frac{(p\cdot x - Et)}{\hbar}) \tag{10}$$

we find out that the uncertainty of these two commutators of Ψ^* with the first derivative of time multiplied by a constant is equivalent to the commutator of the Ψ^* with the second derivative of position x as it is shown below:

$$[\Psi^*, \frac{\partial}{\partial t}] = -i\frac{E}{\hbar}\Psi^* \tag{11}$$

$$[\Psi^*, \frac{\partial^2}{\partial x^2}] = -\frac{p^2}{\hbar^2}\Psi^* \tag{12}$$

and therefore, it is derived that

$$i\hbar[\Psi^*, \frac{\partial}{\partial t}] = -\frac{\hbar^2}{2m}[\Psi^*, \frac{\partial^2}{\partial x^2}] \tag{13}$$

This relation is equivalent to the classical Newtonian laws of motion $F = ma$, and therefore, the Schrodinger equation follows the classical mechanics. [17] From substituting these two uncertainties of commutators into the Schrodinger equation following returns:

$$i\hbar\frac{\partial\rho}{\partial t} + E\rho = -\frac{\hbar^2}{2m}\frac{\partial^2\rho}{\partial x^2} + \frac{p^2}{2m}\rho \tag{14}$$

and if

$$E\rho = \frac{p^2}{2m}\rho \tag{15}$$

a complex version of Fick's second law recovers as follows:

$$\frac{\partial\rho}{\partial t} = i\frac{\hbar}{2m}\frac{\partial^2\rho}{\partial x^2} \tag{16}$$

By equalizing the Diffusion equation with the Schrodinger equation the diffusion constant follows in terms of mass and Planck's constant:

$$D = i\frac{\hbar}{2m} \tag{17}$$

the above equation can be rewritten in terms of velocity of light and de Broglie's wavelength using Einstein's $E = mc^2$ and Planck's $E = h\nu$ relations respectively:

$$D = i\frac{c\lambda}{4\pi} \tag{18}$$

This equation tells us that the diffusion of a free particle such as electron increases with its wavelength. Higher internal energy in the particle results in slower diffusion due to the dissipation of the random fluctuating forces emerging from the motions in a medium.

Furthermore, using mean squared displacement shown below:

$$\left\langle(\Delta x)^2\right\rangle = 2Dt \tag{19}$$

and substituting the Eq. 17 with Diffusion coefficient, we receive the following equation:

$$\left\langle(\Delta x)^2\right\rangle = i\frac{\hbar}{m}t \tag{20}$$

For a body that travels with a constant speed of light, a complex version of Hawking-Bekestein displacement equation [18, 19] recovers:

$$\langle\Delta x\rangle = i\frac{\hbar}{mc} \tag{21}$$

This displacement of one degree of freedom results from the force that is applied on the particle by one bit of information that is stored in space at a given temperature.

Furthermore, in a microcanonical ensemble where all the heats convert to work we have:

$$T\Delta S = F\Delta x \tag{22}$$

From Eq. 22, Newton's law, and Unruh's effect we return:

$$\langle\Delta S\rangle = 2\pi i k \tag{23}$$

This is the complex version of the displacement relation of a particle that is one Compton away from the black hole horizon inferred from Bekenstein's thought experiment. He states that the particle now is a part of the black hole and causes an increase in the entropy of the black hole proportional to the increase of its circumference. [18] This hypothesis was supported by the Hawking entropy relation that a complex version of it emerges in Eq. 23 for a spherical object. [19] Erik Verlinde used the displacement and entropy relations of a black hole to derive a general emergent entropic force [20] inspired by holographic principle [21, 22]. The equality between thermodynamic derivation of displacement and the displacement that is derived from equalizing quantum mechanics and classical mechanics —with ignoring the imaginary number— implies that the information for propagating the particle is coming from the underlying degrees of freedom embedded in the space.

Here, Fick's law is also derived and interpreted using different approaches through the implementation of Einstein's theory on Brownian motions of an ensemble of individual particles. Einstein diffusion theory results from ensemble measurements of individual free Brownian particles under random fluctuating forces of a medium. The stochastic fluctuating forces on the particles in random Brownian motions resulted from emergent entropic forces of the degrees of freedom [20] in this presumably elastic quasi-fluid medium. Therefore, using classical mechanics of Brownian motion and diffusion theory, it is conceived that the same principle should apply to a quantum system for ordinary particles of standard model such as electrons. [23]

There is no doubt that the mathematics of the Schrodinger equation manifests an indeterministic and superstitious system. As Copenhagen interpretation also suggests the electron is a wave until one conscious mind performs the measurement and finds a particle in a determined position. Regardless of the mathematics of the Schrodinger equation, according to de Broglie's Pilot-wave nonlocal theory, one can tell the wavefunction does implicitly account for the medium similar to the fluid in Brownian motion that drives the particles by emergent entropic forces through spacetime.

Schrodinger equation is inherently an indeterministic equation from a merely mathematical perspective of seemingly random physical process. However, believing in the presence of a medium from which the statistical forces emerge can be the manifestation of a deterministic mechanical process. [24, 25] This will intuitively be resolved once we have infinite access to all the information of all degrees of freedom of the entire medium. This analogical picture would simplify the understanding of the real physics behind the Schrodinger equation and also would guide experimentalists in finding the particles in space that are dissolving the ordinary matter. There have been many candidates for such particles in the history of science so far such as the existence of an aether [26], Higgs boson particles [27, 28, 29, 30], and dark energy and matter [20, 31, 32].

3 Conclusion

The equality between the Schrodinger equation with the diffusion equation tells us that there should be a medium in spacetime that drives the electrons as an ordinary particle of matter through phase space. The classical system of Brownian motion in a fluid is analogical to the quantum mechanics system in regards to the propagation of electrons in time. Therefore, it is compelling to believe the electrons are solvated in a medium, possibly elastic quasi-fluid.

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4 Statements and Declarations

4.1 Funding

The authors declare that no funds, grants, or other support were received during the preparation of this manuscript.

4.2 Competing Interests

The authors have no relevant financial or non-financial interests to disclose.