A Quantum Tanimoto Coefficient Fidelity Measure for Entanglement Measurement

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Abstract

Fidelity plays an important role in quantum information processing, which provides a basic scale for comparing two quantum states. At present, one of the most commonly used fidelity is Uhlmann-Jozsa (U-J) fidelity. However, U-J fidelity needs to calculate the square root of the matrix, which is not trivial in case of large or infinite density matrices. Moreover, it is found that U-J fidelity has limitations in some cases and can not well reflect the similarity between quantum states. Therefore, a novel quantum fidelity measure called quantum Tanimoto Coefficient (QTC) fidelity is proposed in this paper.
Keywords: Fidelity measure, Quantum Tanimoto Coefficient, Similarity measure.

In recent years, the concept of fidelity has been widely used in quantum communication, quantum cloning, quantum error correction and quantum computing theoretical research. U-J fidelity can reflect the similarity between two quantum states to a certain extent, but it sometimes leads to counter intuitive result. In addition, the U-J fidelity requires one to calculate or

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measure traces of square roots of matrices. This is not trivial in case of large or infinite density matrices. Therefore, we define a novel fidelity called QTC fidelity.

Definition 1. Let ρ and σ be two density matrices of quantum states, the quantum Tanimoto coefficient fidelity measure between them is defined as:

$$F_{TC}(\rho,\sigma) = \frac{tr(\rho\sigma)}{tr(\rho^2+\sigma^2-\rho\sigma)} \tag{1}$$

in which tr denotes the trace operation. Eq. (1) can also be written in the form of:

$$F_{TC}(\rho,\sigma) = \frac{tr(\rho\sigma)}{tr((\rho-\sigma)^2+\rho\sigma)}. \tag{2}$$

As shown in Eq. (2), the commonality and difference between the quantum states are taken into account the fidelity. Since $tr((\rho-\sigma)^2+\rho\sigma) \geqslant tr(\rho\sigma) \geqslant 0$, so $F_{TC}(\rho,\sigma) \geqslant 1$, and $F_{TC}(\rho,\sigma) = 1$ if and only if $\rho = \sigma$. It can be seen that the QTC fidelity measure is self-similarity and does not need to calculate the square root of the matrix.

Eq. (1) applies to any quantum state, including pure state and mixed state. For two pure states $|\phi\rangle$ and $|\varphi\rangle$, the QTC fidelity measure between them also can be calculated by:

$$F_{TC}(|\phi\rangle,|\varphi\rangle) = \frac{|\langle\phi|\varphi\rangle|^2}{|\langle\phi|\phi\rangle|^2+|\langle\varphi|\varphi\rangle|^2-|\langle\phi|\varphi\rangle|^2} = \frac{|\langle\phi|\varphi\rangle|^2}{2-|\langle\phi|\varphi\rangle|^2}. \tag{3}$$

For any pure state $|\phi\rangle$ and mixed state ρ, the QTC fidelity measure between them becomes:

$$F_{TC}(|\phi\rangle,\rho) = \frac{\langle\phi|\rho|\phi\rangle}{|\langle\phi|\phi\rangle|^2+tr(\rho^2)-\langle\phi|\rho|\phi\rangle} = \frac{\langle\phi|\rho|\phi\rangle}{1+tr(\rho^2)-\langle\phi|\rho|\phi\rangle}. \tag{4}$$

The QTC fidelity measure also has the following properties.

(1) $F_{TC}(\rho,\sigma) \in [0,1]$, $F_{TC} = 1$, if and only if $\rho = \sigma$;

(2) $F_{TC}(\rho,\sigma) = F_{TC}(\sigma,\rho)$;

(3) $F_{TC}(U\rho U^{\dagger}, U\sigma U^{\dagger}) = F_{TC}(\rho,\sigma)$ for all unitary U.

Like U-J fidelity, the proposed QTC fidelity measure belongs to 0 to 1. If the two quantum states are more similar, the value of the QTC fidelity measure F_{TC} is closer to 1. When the two quantum states are orthogonal, the QTC fidelity measure between them is 0.

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References