On the solution of the strong gravitational field, the solution of the Singularity problem, the origin of Dark energy and Dark matter

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Abstract

In order to apply general relativity to a strong gravitational field, the gravitational self-energy of the object itself must be considered. By considering the gravitational self-energy, it is possible to solve the singularity problem, which is the biggest problem with general relativity. When an object acts on gravity, the gravitational action of gravitational potential energy is also included. Therefore, even in the case of the universe, the gravitational action of gravitational potential energy must be considered. Gravitational potential energy generates a repulsive force because it has a negative equivalent mass. For the observable universe, I calculated the negative gravitational self-energy, which is approximately three times greater than the positive mass energy, which can explain the accelerated expansion of the universe. The source of dark energy is presumed to be due to gravitational self-energy and an increase in mass due to the expansion of the particle horizon. It is calculated that the effect of dark energy is occurring because matter and galaxies entering the particle horizon contribute to the total gravitational potential energy. While mass energy is proportional to M, gravitational self-energy increases faster because gravitational self-energy is proportional to $-\frac{M^2}{R}$. Accordingly, an effect of increasing dark energy occurs. I present Friedmann's equations and cosmological constant function obtained through gravitational self-energy model. This model predicts an inflection point where dark energy becomes larger and more important than the energy of matter and radiation.

I. Field equation in strong gravitational field and their solution

1. Problem with Einstein's Field Equation

Einstein's field equations are incomplete, despite great achievements. Since the equation is incomplete, it has a singularity problem as a solution. Einstein's field equations do not conform to the equivalence principle of general relativity, which states that all energy is a gravitational source. The gravitational field must also act as a gravitational source, but this is missing from Einstein's field equation.

> *In writing the field equation (48) we have assumed that the quantity $T^{\mu\nu}$ is the energy-momentum tensor of matter. In order to obtain a linear field equation we have left out the effect of the gravitational field upon itself. Because of this omission, our linear field equation has several (related) defects: (1) According to (48) matter acts on the gravitational field (changes the fields), but there is no mutual action of gravitational fields on matter; that is, the gravitational field can acquire energy-momentum from matter, but nevertheless the energy-momentum of matter is conserved ($\partial_\nu T^{\mu\nu} = 0$). This is an inconsistency. (2)Gravitational energy does not act as source of gravitation, in contradiction to the principle of equivalence. Thus, although Eq. (48) may be a fair approximation in the equivalence. Thus, although Eq. (48) may be a fair approximation in the case of weak gravitational fields, it cannot be an exact equation.* [1]

> *The obvious way to correct for our sin of omission is to include the energy-momentum tensor of the gravitational field in $T^{\mu\nu}$. This means that we take for the quantity $T^{\mu\nu}$ the total energy-momentum tensor of matter plus gravitation:*

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$$T^{\mu\nu} = T^{\mu\nu}_{(m)} + t^{\mu\nu} \tag{1}$$

Here $T^{\mu\nu}_{(m)}$ and $t^{\mu\nu}$ are, respectively, the energy-momentum tensor of matter and gravitation. We assume that the interaction energy of matter and gravitation is always included in $T^{\mu\nu}_{(m)}$; this is a reasonable convention since the interaction energy-density will only differ from zero at those places where there is matter.

Our field equation now becomes

$$\partial_\lambda \partial^\lambda \phi^{\mu\nu} = -\kappa(T^{\mu\nu}_{(m)} + t^{\mu\nu}) \tag{2}$$

As such, since the field equation does not include the interaction between the gravitational field and matter, I know that many people are trying to find a solution by making a new field equation that considers the gravitational field as a source of gravity. However, it does not seem to be able to present an efficient solution comparable to the existing Schwarzschild equation.

2. Hint and solution of new field equation taking into account the energy of the gravitational field

2-1. Take a look at the existing solutions!

Let's take a look at the most famous solution, the Schwarzschild solution.

$$ds^2 = -(1 - \frac{2GM}{c^2 r})c^2 dt^2 + \frac{1}{(1 - \frac{2GM}{c^2 r})} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \tag{3}$$

The only important variables are the mass M and the distance r. So, only the mass M is important. Then, to find the solution of the strong gravitational field, we only need to find the equivalent mass of the gravitational field.

2-2. Equivalent mass of gravitational potential energy

Now, all we need to do is list all the gravitational potential energies and find their equivalent mass.

$$U_T = \sum_i U_i = \sum_i -M_{gp,i} c^2 \tag{4}$$

$-M_{gp,i}$ is the equivalent mass of gravitational potential energy. $M_{gp,i}$ is a positive value.

Since gravitational potential energy is negative energy, $-M_{gp}$ is used to clearly indicate it. Most of the situations we need to analyze are two-body problems. The principle of the multi-body problem is similar.

The gravitational problem we face is to establish and solve the equation of motion when mass M and mass m are separated by a distance r.

The solution is well established. In this solution we just put the energy momentum of the gravitational potential energy. It is only necessary to find the equivalent mass of the gravitational potential energy.

$$U_T = \sum_i U_i = U_{gs-M} + U_{gs-m} + U_{gp-Mm} \tag{5}$$

U_{gs-M}: Total gravitational potential energy of large mass M = gravitational self-energy of large mass M
U_{gs-m}: Total gravitational potential energy of small mass m = gravitational self-energy of small mass m
U_{gp-Mm}: The gravitational potential energy between the large mass M and the small mass m

Although the system contains countless particles, it can be summarized in three terms.

What is necessary is just to find the equivalent mass corresponding to this gravitational potential energy. This value only differs depending on the specific situation. In individual situations, if the internal structure of the mass M and the mass m is presented, it is possible to obtain an accurate value.

For example, suppose that the mass is uniformly distributed in a spherical shape as shown in the figure above.

$$U_{gs-M} = -\frac{3}{5}\frac{GM^2}{R_M} \tag{6}$$

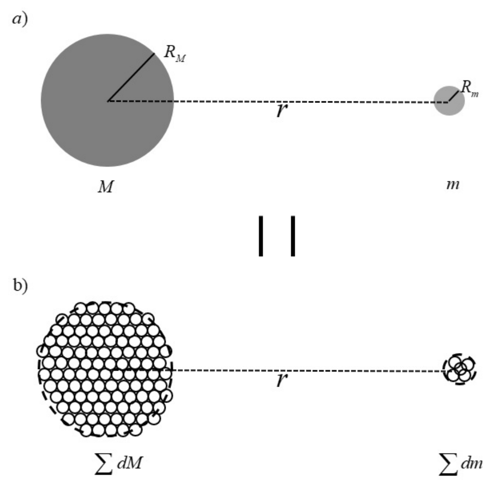

Figure 1: There is a mass M and mass m. Mass M and m are made up of several particles. If you find the gravitational potential energy of all particles, it looks like there are many terms, but if you organize it, you can organize it into three terms. The gravitational self-energy of the mass M + the gravitational self-energy of the mass m + the gravitational potential energy of the mass M and mass m.

$$U_{gs-m} = -\frac{3}{5}\frac{Gm^2}{R_m} \tag{7}$$

$$U_{gp-Mm} = -\frac{GMm}{r} \tag{8}$$

Now, let's compare how much each size has compared to the mass energy Mc^2. Knowing the size of each, we can decide whether we should consider or ignore the corresponding physical quantity.

2-2-1. The gravitational self-energy of the gravitational source M

The magnitude of gravitational self-energy at the Schwarzschild radius

$$U_{gs-M} = -\frac{3}{5}\frac{GM^2}{R_M} = -\frac{3}{5}\frac{GM^2}{R_S} = -\frac{3}{5}\frac{GM^2}{\frac{2GM}{c^2}} = -0.3Mc^2 \tag{9}$$

$$-M_{gs-M} = -0.3M \tag{10}$$

It can be seen that the gravitational self-energy effect is quite large. At the Schwarzschild radius of an object, it can be seen that the magnitude of negative gravitational self-energy is 30% of the free state.

Since the gravitational self-energy is negative, the mass M does not fully work in a black hole, but acts as much as the gravitational potential energy is subtracted, and as a result, it becomes the same as the state with a mass of $0.7M$.

As experienced in elementary particle physics, you can think of mass defect due to binding energy. The bound state is a state in which the total mass is reduced by the difference in binding energy compared to the free state. The good thing about gravitational self-energy is that the principle that assumes that all mass is gathered at the center of mass can be used for the equivalent mass formed by gravitational self-energy.

2-2-2. The gravitational self-energy of a small mass m **or test mass** m

1) The magnitude of gravitational self-energy at the Schwarzschild radius

$$U_{gs-m} = -\frac{3}{5}\frac{Gm^2}{R_m} = -\frac{3}{5}\frac{Gm^2}{R_S} = -\frac{3}{5}\frac{Gm^2}{\frac{2Gm}{c^2}} = -0.3mc^2 \tag{11}$$

$$-M_{gs-m} = -0.3m \tag{12}$$

2) Situations in which the gravitational self-energy of a small mass m is negligible

- In the normal case (unless the object has a strong gravitational field), gravitational self-energy can be neglected because it is very small compared to mass energy. In the case of Earth, the gravitational self-energy is 10^{10} times less than the mass energy.

- If the mass m is taken as the test mass, the gravitational self-energy can be neglected.

- If the total mass of mass m is regarded as a mass including gravitational self-energy, the gravitational self-energy of mass m can be neglected.

2-2-3. Gravitational potential energy between mass M and mass m

The magnitude of gravitational potential energy at the Schwarzschild radius

$$U_{gp-Mm} = -\frac{GMm}{r} = -\frac{GMm}{R_S} = -\frac{GMm}{\frac{2GM}{c^2}} = -(\frac{m}{2M})Mc^2 \tag{13}$$

$$-M_{gp-Mm} = -\frac{m}{2} \tag{14}$$

In the case of $M >> m$, the U_{gp-Mm} term is negligible.

When considering the motion of particles near a black hole, it can be seen that this value is very small compared to the mass energy Mc^2 of the gravitational source and can be ignored. However, in cases such as collisions of black holes of similar mass, this term should be taken into account. This term has a maximum value when $R = R_S$, and since r is in the denominator, if it is considerably farther than the distance of R_S, it is also small compared to the gravitational self-energy of the gravitational source M.

In summary, in the situation where $M >> m$ or $r >> R_S$, only the first term, the gravitational self-energy of the gravitational source M, needs to be considered, and M and m form a high-density object such as a black hole, $r \approx R_S$. In the case of, all three terms should be considered.

2-2-4. Equivalent mass of gravitational potential energy in the Earth, the Sun, and a Black hole

1) Earth

Earth's mass : $M_E = 5.972 \times 10^{24} kg$, Earth's average radius : $R_E = 6.371 \times 10^6 m$

$$U_{gs-Earth} = -\frac{3}{5}\frac{GM_E{}^2}{R_E} = -2.241 \times 10^{32}[kgm^2s^{-2}] = -M_{gs-Earth}c^2 \tag{15}$$

$$-M_{gs-Earth} = -2.493 \times 10^{15}[kg] \tag{16}$$

Comparison of Earth's mass energy and gravitational self-energy

$$\frac{|U_{gs-Earth}|}{M_E c^2} = \frac{|-M_{gs-Earth}c^2|}{M_E c^2} = 4.174 \times 10^{-10} \tag{17}$$

It is negligible under normal circumstances.

2) Sun

Sun's mass : $M_{sun} = 1.988 \times 10^{30} kg$, Sun's average radius : $R_{sun} = 6.955 \times 10^8 m$

$$U_{gs-sun} = -\frac{3}{5}\frac{GM_{sun}{}^2}{R_{sun}} = -2.275 \times 10^{43}[kgm^2s^{-2}] = -M_{gs-sun}c^2 \tag{18}$$

$$-M_{gs-sun} = -2.531 \times 10^{26}[kg] \tag{19}$$

Comparison of Sun's mass energy and gravitational self-energy

$$\frac{|U_{gs-sun}|}{M_{sun}c^2} = \frac{|-M_{gs-sun}c^2|}{M_{sun}c^2} = 1.273 \times 10^{-4} \tag{20}$$

3) Black hole

$$U_{gs-M} = -\frac{3}{5}\frac{GM^2}{R_M} = -\frac{3}{5}\frac{GM^2}{R_S} = -\frac{3}{5}\frac{GM^2}{\frac{2GM}{c^2}} = -0.3Mc^2 = -M_{gs-M}c^2 \tag{21}$$

Comparison of Black hole's mass energy and gravitational self-energy

$$\frac{|U_{gs-M}|}{Mc^2} = \frac{|-M_{gs-M}c^2|}{Mc^2} = 0.3 \tag{22}$$

These are good results. In the case of the Earth, the magnitude of its gravitational energy was very small (10^{-10}), so it was not necessary to consider it, whereas in the case of a neutron star or a black hole, this value appears to be at a meaningful level.

2-2-5. When the test particle is near a source of gravity with a strong gravitational field

From the previous analysis, since the second and third terms can be ignored, the equivalent mass of the gravitational potential energy is simplified.

$$U_T = \sum_i U_i = U_{gs-M} + U_{gs-m} + U_{gp-Mm} \simeq U_{gs-M} \tag{23}$$

In the case of a black hole, the magnitude of the gravitational potential energy is $U_T = \sum_i U_i \approx -0.3Mc^2$ and only the gravitational self-energy of the gravitational source M needs to be considered. In particular, the fact that the center of the equivalent mass of the gravitational potential energy coincides with the center of mass of the gravitational source makes the situation convenient.

2-3. Under the general theory of relativity, a solution in a strong gravitational field [2] [3]

All energy is a gravitational source, and gravitational potential energy is also a gravitational source. Thus, the gravitational field is self-interacting. This makes the problem very complicated. However, gravitational self-energy is the first set of gravitational potential energies produced by a mass. Thus, the gravitational self-energy will be in the first term when developing the solution of gravitational interaction. That is, it will be the largest and most important term.

It would be nice if we could make a complete field equation that includes the gravitational action of the gravitational field and solve it, but we have not yet created such a complete field equation that is accepted by everyone. Also, simple solution do not exist.

Since gravitational self-energy will be the largest and most important first-order term, if this is used, problems such as the singularity of the Einstein field equation can be solved.

In all existing solutions, the mass term M must be replaced by $(M - M_{gs})$.

$M \to (M) + (-M_{gs})$, $-M_{gs}$ is the equivalent mass of gravitational self-energy. In all existing solutions (Schwarzschild, Kerr, Reissner-Nordström, ...), the mass term M must be replaced by $(M - M_{gs})$.

For example, Schwarzschild solution is,

$$ds^2 = -(1 - \frac{2GM}{c^2 r})c^2dt^2 + \frac{1}{(1 - \frac{2GM}{c^2 r})}dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2 \tag{24}$$

Schwarzschild-Choi solution is

$$ds^2 = -(1 - \frac{2G(M - M_{gs})}{c^2 r})c^2dt^2 + \frac{1}{(1 - \frac{2G(M - M_{gs})}{c^2 r})}dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2 \tag{25}$$

This solution shows a significant difference between the vicinity of a black hole and the interior of a black hole.

For the sphere with uniform density,

$$-M_{gs} = -\frac{3}{5}\frac{GM^2}{Rc^2} \tag{26}$$

1) If $M \gg |-M_{gs}|$, in other words if $r \gg R_S$, we get the Schwarzschild solution.

2) If $M = |-M_{gs}|$ (At $r = R_{gs} = 0.3R_S$)

Looking for the size in which gravitational self-energy becomes equal to rest mass energy by comparing both,

$$U_{gs} = |-\frac{3}{5}\frac{GM^2}{R_{gs}}| = Mc^2 \tag{27}$$

$$R_{gs} = \frac{3}{5}\frac{GM}{c^2} \tag{28}$$

Comparing R_{gs} with R_S, the radius of Schwarzschild black hole,

$$R_{gs} = \frac{3}{5}\frac{GM}{c^2} < R_S = \frac{2GM}{c^2} \tag{29}$$

$$R_{gs} = 0.3R_S \tag{30}$$

At $r = R_{gs} = 0.3R_S$, $M = |-M_{gs}|$. So,

$$ds^2 = -c^2dt^2 + dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2 \tag{31}$$

The singularity disappears, and a flat space-time is obtained.

2-4. Create a new field equation containing the energy of the gravitational field

It is thought that the method applied above can be used to find new field equation and solution.

1) Establish a universal field equation in which the energy of the gravitational field is entered.

$$T^{\mu\nu} = T^{\mu\nu}_{(m)} + t^{\mu\nu} \tag{32}$$

$T^{\mu\nu}$ is the total energy-momentum tensor of matter and gravity. Here $T^{\mu\nu}_{(m)}$ and $t^{\mu\nu}$ are, respectively, the energy-momentum tensor of matter and gravitation.

Our field equation now becomes

$$\partial_\lambda\partial^\lambda\phi^{\mu\nu} = -\kappa(T^{\mu\nu}_{(m)} + t^{\mu\nu}) \tag{33}$$

2) It develops through the existing general relativity principle, and expresses energy terms of the gravitational field in an appropriate part.

$$U_T = \sum_i U_i = U_{gs-M} + U_{gs-m} + U_{gp-Mm} \tag{34}$$

Find the equivalent mass of the gravitational potential energy and approximate it.

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$$\frac{U_T}{c^2} = \frac{\sum_i U_i}{c^2} = \frac{U_{gs-M}}{c^2} + \frac{U_{gs-m}}{c^2} + \frac{U_{gp-Mm}}{c^2} \simeq \frac{U_{gs-M}}{c^2} \tag{35}$$

3) Put it in the Schwarzschild-Choi equation.

Schwarzschild-Choi solution is

$$ds^2 = -(1 - \frac{2G(M - M_{gs})}{c^2r})c^2dt^2 + \frac{1}{(1 - \frac{2G(M-M_{gs})}{c^2r})}dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2 \tag{36}$$

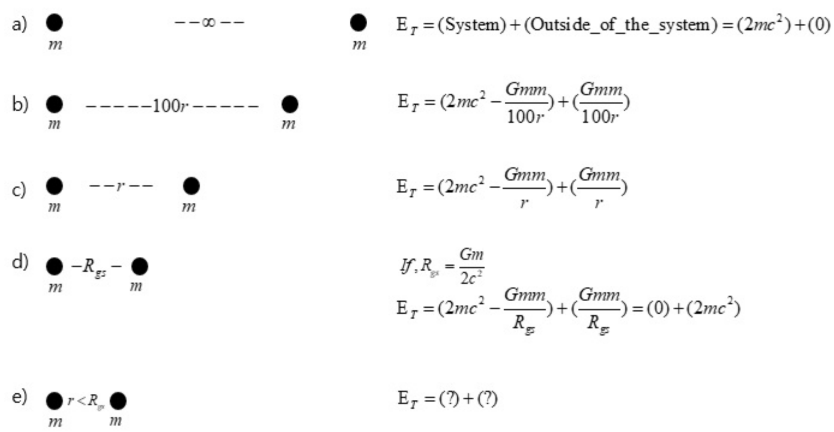

Figure 2: Description from the mass defect. What if we compress more than R_{gs} ?

II. The solution to the black hole singularity problem [2] [3]

1. Binding energy in the mass defect problem

Consider situations a) and c).

In a), the total mass of the two particle system is 2m, and the total energy is $E_T = 2mc^2$

In c), is the total energy of the two-particle system $E_T = 2mc^2$?

In c), when the two particle system act gravitationally on an external particle, will they act gravitationally with a gravitational mass of 2m?

Are a) and c) the same mass or same state?

If you do not apply negative binding energy, the conclusions you get will not match the actual results.

In c), the total energy of the two particle system is

$$E_T = 2mc^2 - \frac{Gmm}{r} \tag{37}$$

In the dimensional analysis of energy, E has $kg(m/s)^2$, so all energy can be expressed in the form of $massX(velocity)^2$. So, $E = mc^2$ holds true for all kinds of energy. Here, m does not mean rest mass. If we introduce the equivalent mass $-m_{gp}$ for the gravitational potential energy,

$$-\frac{Gmm}{r} = -m_{gp}c^2 \tag{38}$$

$$E_T = 2mc^2 - \frac{Gmm}{r} = 2mc^2 - m_{gp}c^2 = (2m - m_{gp})c^2 = m^*c^2 \tag{39}$$

The gravitational force acting on a relatively distant third mass m_3 is

$$F = -\frac{Gm^*m_3}{R^2} = -\frac{G(2m - m_{gp})m_3}{R^2} = -\frac{G(2m)m_3}{R^2} - \frac{G(-m_{gp})m_3}{R^2} \tag{40}$$

That is, **when considering the gravitational action of a bound system, not only the mass in its free state but also the binding energy term should be considered.** Alternatively, the gravitational force acting on the bound system can be decomposed into a free-state mass term and an equivalent mass term of binding energy.

While we usually use the mass m^* of the bound system, we forget that m^* is "$m - m_{binding-energy}$". Gravitational potential energy is also a kind of binding energy. Therefore, when dealing with the expansion problem of the universe, the negative gravitational potential energy must also be considered. And, it is very likely that this is the source of dark energy.

In the above equation, what happens if the second term ($F_{gp} = -\frac{G(-m_{gp})m}{R^2}$) is greater than the first term?

Now consider the situations d) and e).

When two particles form a binding state, energy corresponding to the binding energy must be released from the system to the outside of the system. In order to keep the two particles close enough so that $r = R_{gs}$, the total energy of the system must be zero and the initial (in free state) total mass energy of the system must be released to the outside of the system.

Now, in order for these two particles to compress further and achieve a stable state, positive energy must be released from the system to the outside as much as the difference in binding energy. In the case of allowing only positive energy, this compression must be inhibited because there is no more positive energy to withdraw from the system. If a negative energy state is allowed, the inside of the system reaches a negative mass state, and since there is a repulsive gravitational effect between the negative masses, the system expands until it becomes non-negative mass state. In summary, gravitational potential energy, a type of binding energy, has the potential to solve the singularity problem.

2. Gravitational self-energy

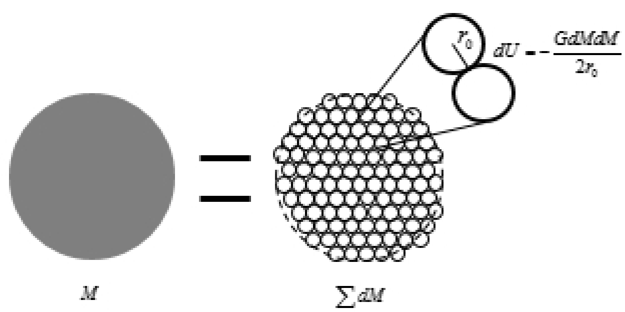

Figure 3: Since all mass M is a set of infinitesimal mass dMs and each dM is gravitational source, too. There exists gravitational potential energy among each of dMs. Generally, mass of an object measured from its outside corresponds to the value of dividing the total of all energy into c^2.

The concept of gravitational self-energy(U_{gs}) is the total of gravitational potential energy possessed by a certain object M itself. Since a certain object M itself is a binding state of infinitesimal mass dMs, it involves the existence of gravitational potential energy among these dMs and is the value of adding up these. $M = \sum dM$.

Gravitational self-energy or Gravitational binding energy($-U_{gs}$) in case of uniform density is given by

$$U_{gs} = -\frac{3}{5}\frac{GM^2}{R} \tag{41}$$

In the generality of cases, the value of gravitational self-energy is small enough to be negligible, compared to mass energy Mc^2. So generally, there was no need to consider gravitational self-energy. However the smaller R becomes, the higher the absolute value of U_{gs}. For this reason, we can see that U_{gs} is likely to offset the mass energy in a certain radius.

Thus, looking for the size in which gravitational self-energy becomes equal to rest mass energy by comparing both,

$$U_{gs} = |-\frac{3}{5}\frac{GM^2}{R_{gs}}| = Mc^2 \tag{42}$$

$$R_{gs} = \frac{3}{5}\frac{GM}{c^2} \tag{43}$$

This equation means that if mass M is uniformly distributed within the radius R_{gs}, gravitational self-energy for such an object equals mass energy in size. So, in case of such an object, mass energy and gravitational self-energy can be completely offset while total energy is zero. Since total energy of such an object is 0, gravity exercised on another object outside is also 0.

Comparing R_{gs} with R_S, the radius of Schwarzschild black hole,

$$R_{gs} = \frac{3}{5}\frac{GM}{c^2} < R_S = \frac{2GM}{c^2} \tag{44}$$

$$R_{gs} = 0.3R_S \tag{45}$$

This means that there exists the point where gravitational self-energy becomes equal to mass energy within the radius of black hole, and that, supposing a uniform distribution, the value exists at the point $0.3R_S$, a 30% level of the black hole radius. Regarding the R_{gs} value, it is necessary to refer to 2-5-1)New calculation of gravitational self-energy in Chapter III.

Even with kinetic energy and virial theorem applied only the radius diminishes as negative energy counterbalances positive energy, but no effects at all on this point: "There is a zone which cannot be compressed anymore due to the negative gravitational potential energy." Although potential energy changes to kinetic energy, in order to achieve a stable bonded state, a part of the kinetic energy must be released to the outside of the system.

Considering the virial theorem (K=-U/2)),

$$R_{gs-vir} = \frac{1}{2}R_{gs} = 0.15R_S \tag{46}$$

Since this value is on a level not negligible against the size of black hole, we should never fail to consider "gravitational self-energy" for case of black hole. In case of the smallest black hole with three times the solar mass, $R_S = 9km$. R_{gs} of this black hole is as far as 3km. In other words, even in a black hole with smallest size that is made by the contraction of a star, the mass distribution can't be reduced to at least radius 3km($R_{gs-vir} = 1.5km$).

3. Black hole does not have a singularity, but it has a Zero Energy Zone

From the equation above, even if some particle comes into the radius of black hole, it is not a fact that it contracts itself infinitely to the point $R = 0$. From the point R_{gs} (or R_{gs-vir}), gravity is 0, and when it enters into the area of R_{gs} (or R_{gs-vir}), total energy within R_{gs} (or R_{gs-vir}) region corresponds to negative values enabling anti-gravity to exist. This R_{gs} (or R_{gs-vir}) region comes to exert repulsive gravity effects on the particles outside of it, therefore it interrupting the formation of singularity at the near the area $R = 0$.

4. In a strong gravitational field, the solution of the general theory of relativity In all existing solutions, the mass term M must be replaced by $(M - M_{gs})$.

We can solve the problem of singularity by separating the term $(-M_{gs})$ of gravitational self-energy from mass and including it in the solutions of field equation.

$M \rightarrow (M) + (-M_{gs})$, $-M_{gs}$ is the equivalent mass of gravitational self-energy. In all existing solutions (Schwarzschild, Kerr, Reissner-Nordström, ...), the mass term M must be replaced by $(M - M_{gs})$.

For example, Schwarzschild solution is,

$$ds^2 = -(1 - \frac{2GM}{c^2 r})c^2dt^2 + \frac{1}{(1 - \frac{2GM}{c^2 r})}dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2 \tag{47}$$

Schwarzschild-Choi solution is

$$ds^2 = -(1 - \frac{2G(M - M_{gs})}{c^2 r})c^2dt^2 + \frac{1}{(1 - \frac{2G(M-M_{gs})}{c^2 r})}dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2 \tag{48}$$

For the sphere with uniform density,

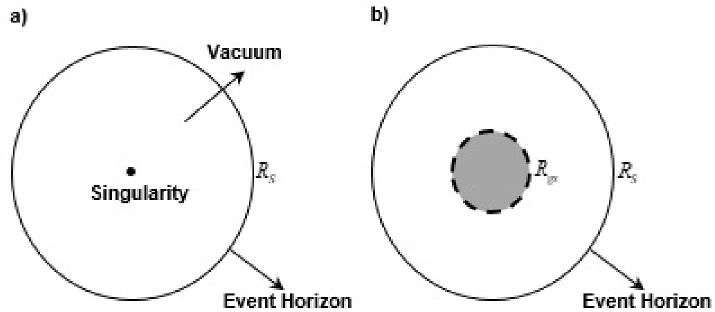

Figure 4: a) Existing Model. b) New Model. The area of within R_{gs} (or R_{gs-vir}) has gravitational self-energy(potential energy) of negative value, which is larger than mass energy of positive value. If r is less than R_{gs}, this area becomes negative energy (mass) state. There is a repulsive gravitational effect between the negative masses, which causes it to expand again. This area (within R_{gs}) exercises anti-gravity on all particles entering this area, and accordingly prevents all masses from gathering to $R = 0$. Therefore the distribution of mass (energy) can't be reduced to at least radius R_{gs} (or R_{gs-vir}).

$$-M_{gs} = -\frac{3}{5}\frac{GM^2}{Rc^2} \tag{49}$$

1) If $M \gg |-M_{gs}|$, in other words if $r \gg R_S$, we get the Schwarzschild solution.
2) If $M = |-M_{gs}|$

$$ds^2 = -c^2dt^2 + dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2 \tag{50}$$

At $r = R_{gs}$, a flat space-time is obtained.
3) If $M \ll |-M_{gs}|$, in other words if $0 \le r \ll R_{gs}$,

$$ds^2 \simeq -(1+\frac{2GM_{gs}}{c^2r})c^2dt^2 + \frac{1}{(1+\frac{2GM_{gs}}{c^2r})}dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2 \tag{51}$$

In the domain of $0 \le r \ll R_{gs}$,

The area of within R_{gs} has gravitational self-energy of negative value, which is larger than mass energy of positive value. Negative mass has gravitational effect which is repulsive to each other. So, we can assume that $-M_{gs}$ is almost evenly distributed. Therefore ρ_{gs} is constant. And we must consider the Shell Theorem.

$$-M_{gs} = -\frac{4\pi r^3}{3}\rho_{gs} \tag{52}$$

$$(1+\frac{2GM_{gs}}{c^2r}) = 1 + \frac{2G(\frac{4\pi}{3}r^3\rho_{gs})}{c^2r} = 1 + \frac{8\pi G\rho_{gs}r^2}{3c^2} \tag{53}$$

$$ds^2 \simeq -(1+\frac{8\pi G\rho_{gs}r^2}{3c^2})c^2dt^2 + \frac{1}{(1+\frac{8\pi G\rho_{gs}r^2}{3c^2})}dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2 \tag{54}$$

If $r \to 0$,

$$ds^2 \simeq -c^2dt^2 + dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2 \tag{55}$$

There is no singularity.
In practice, mass contraction must be stopped at the point where $M_{shell} = M_{shell-gs}$.

5. Internal structure of black hole considering gravitational self-energy

When the mass distribution inside the black hole is reduced from R_S to R_{gs} (or R_{gs-vir}), the energy must be released from the inside of the system to the outside of the system in order to reach this binding state.

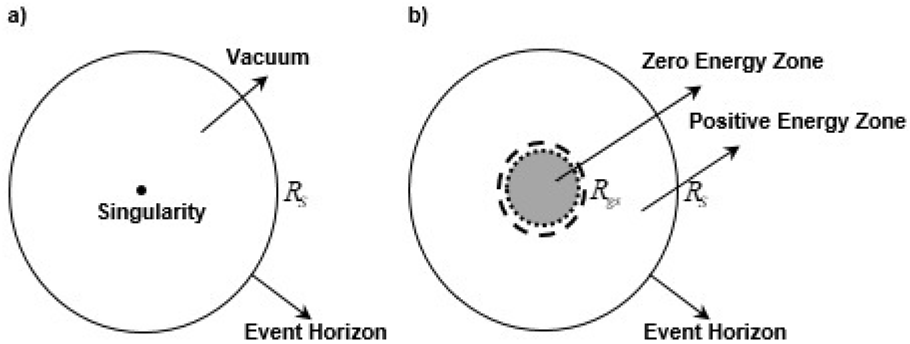

Figure 5: Internal structure of the black hole. a)Existing model b)New model. If, over time, the black hole stabilizes, the black hole does not have a singularity in the center, but it has a Zero (total) Energy Zone.

Here, the system refers to the mass distribution within the radius $0 \leq r \leq R_{gs}$(or R_{gs-vir}). Although potential energy changes to kinetic energy, in order to achieve a stable bonded state, a part of the kinetic energy must be released to the outside of the system. We need to consider the virial theorem.

At this time, the emitted energy does not go out of the black hole. This energy is distributed in the R_{gs} (or R_{gs-vir}) $< r \leq R_S$ region.

If you have only the concept of positive energy, please refer to the following explanation.

From the point of view of mass defect, $r = R_{gs}$ (or R_{gs-vir}) is the point where the total energy of the system is zero. For the system to compress more than this point, there must be an positive energy release from the system. However, since the total energy of the system is zero, there is no positive energy that the system can release. Therefore, the system cannot be more compressed than $r = R_{gs}$ (or R_{gs-vir}). So black hole doesn't have singularity.

By locking horns between gravitational self-energy and mass energy, particles inside black hole or distribution of energy can be stabilized. As a final state, the black hole does not have a singularity in the center, but it has a Zero (total) Energy Zone (ZEZ). R_{gs} is the maximum of ZEZ.

6. Inside the huge black hole, there is enough space for intelligent life to exist [4]

A black hole has no singularity, has a Zero Energy Zone with a total energy of zero, and this region is very large, reaching 15% $\sim$ 30% of the radius of the black hole. It suggests an internal structure of a black hole that is completely different from the existing model. Inside the huge black hole, there is an area where intelligent life can live.

Therefore, by considering gravitational self-energy, it is possible to solve the problem of singularity of black hole, which is the most important problem in general relativity. And, this discovery provides a logic for why we are surviving in universe black hole (formed when only mass energy is considered) without collapsing into singularity.

For example, if the masses are distributed approximately 46.5Gly with the average density of the current universe, the size of the black hole created by this mass distribution will be 491.6Gly, and the size of the Zero Energy Zone will be approximately 73.7Gly $\sim$ 147.5Gly. In other words, there is no strong tidal force and a region with almost flat space-time that can form a stable galaxy structure is much larger than the observable range of 46.5Gly. The entire universe is estimated to be much larger than the observable universe, so it may not be at all unusual for us to observe only the Zero Energy Zone (nearly flat space-time).

Even if humans live inside a black hole called the universe, a sufficiently stable survival area for intelligent life is guaranteed.

III. The sources of dark energy are gravitational self-energy and the expansion of the particle horizon

Size of mass distribution	The size of the universe black hole created by the mass distribution	Size of Zero Energy Zone (15% ~ 30%)
14.3Gly	14.3Gly	2.2Gly ~ 4.3Gly
20.0Gly	39.1Gly	5.9Gly ~ 11.7Gly
46.5Gly	491.6Gly	73.7Gly ~ 147.5Gly
100.0Gly	4883.9Gly	732.6Gly ~ 1465.2gly

Figure 6: Under the observed average density, the size of Universe Black Hole and Zero Energy Zone. If the size of the mass distribution increases R times, the Universe Black Hole and ZEZ created by the new mass distribution become R^3 times larger.

1. Expansion of the universe by gravitational potential energy or gravitational self-energy

Within R_{gs}, negative gravitational self-energy is larger than positive mass energy, and the region within R_{gs} corresponds to a negative mass state, and repulsive force or antigravity exists.

Now, we have repulsion or anti-gravity on a cosmic scale. Therefore, this force will be applicable to various phenomena that require repulsive force on a cosmic scale. For example, Inflation, Dark energy, and the Force that displace the expansion of space and move galaxies.

1-1. Expansion within R_{gs} from the initial mass-energy distribution at the birth of the universe [4]

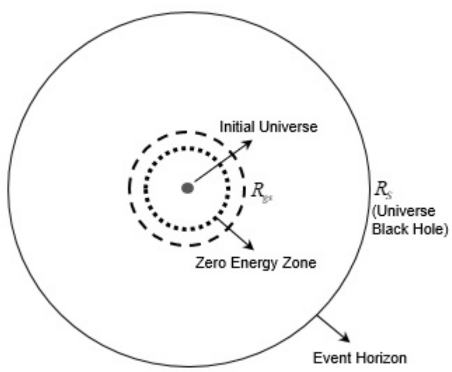

Figure 7: Comparison of the size of the initial universe, the size of the ZEZ, and the size of the Universe Black Hole

Consider the initial state of the universe. The entire universe is larger than the present observable universe, 46.5Gly. Since we do not know the size of the entire universe, after thinking about the state in which all the mass-energy in the present observable universe is concentrated in a very small area, let's apply this logic to the entire universe.

As calculated above, the size of the ZEZ produced by all mass-energy in the observable universe is approximately 73.7Gly $\sim$ 147.5Gly, and the size of the universe black hole is 491.6Gly. Since these materials are concentrated in a very small area, the negative gravitational potential energy of this area exceeds the positive mass energy and corresponds to a negative mass state as a whole. Because there is a repulsive gravitational effect between negative masses, it expands. [3]

This expansion is accelerated up to at least ZEZ (73.7Gly $\sim$ 147.5Gly), and since it is in an accelerated state, expansion continues beyond ZEZ. As time passes, when the distribution of mass is outside the ZEZ, the mass state within the ZEZ is a state in which the positive mass energy is greater than the negative gravitational potential energy, so the total mass (within the ZEZ) is a positive mass, and the attraction is applied to the masses outside the ZEZ. This will have the effect of slowing the expansion.

The universe expansion at the time of the big bang is because all matter started in a region smaller than the ZEZ, and there is a possibility that it corresponds to the accelerated expansion process up to the ZEZ. The size of the ZEZ created by the mass distribution of the observable universe is 73.7Gly $\sim$ 147.5Gly, but the present observable universe is passing 46.5Gly.

1) In the early days of the universe, why didn't the universe become a singularity or black hole?

In mainstream models, this is explained by the expansion of space. In this model, the explanation is different. If the total mass of the universe is collected in a very small area, the negative gravitational potential energy is greater than the positive mass energy, and the whole is placed in a negative mass state. There is a repulsive gravitational effect between the negative masses, so expansion occurs away from each other. At least up to the R_{gs} (Maximum of ZEZ) region, there is an expansion. The singularity problem can be explained as "The singularity itself cannot be formed because of negative gravitational potential energy.".

2) At the beginning of the universe, the problem of escape from a black hole created by the total mass of the universe?

At the beginning of the universe, when considering the expansion in a high-density state, there is a problem that people mistakenly think that this event is the escape of matter from the inside of the black hole created by the total mass of the universe to the outside to form galaxies or stars.

The black hole event horizon created by the total mass of the universe is very large compared to the area where the total mass of the universe is gathered. In other words, in the Black Hole Cosmology model, **matter does not escape the universe black hole, but has not yet reached the event horizon of the universe black hole (formed when only mass energy is considered without considering gravitational potential energy).**

3) About the inflation mechanism

In the above analysis, we hypothesized that all matter in the present observable universe was gathered in a very small area.

By relaxing the conditions, one can assume the sequential birth of mass, or take advantage of the fact that the propagation speed of the field is limited to the speed of light. Also, because of the finite time after birth, the range of interaction is limited. In other words, not everything interacts at the same moment, but it has certain characteristics sequentially according to the birth of the field, the propagation speed of the field, and time (age of the universe). Such circumstances makes it possible to adjust the size of R_{gs} or ZEZ in the early universe to be smaller than the current 73.7 $\sim$ 147.5Gly.

In the standard cosmology, we postulate a rapid accelerated expansion process called inflation before the big bang model. **The accelerated expansion caused by negative gravitational potential energy may be used to explain the inflation mechanism.**

At the birth of the universe, with all matter gathered in a very small area, rapid accelerated expansion occurred due to negative gravitational potential energy, and this accelerated expansion may be terminated due to several factors (end of inflation mechanism). For example, inflation may dissipate after expansion to R_{gs} made by initial materials, or it may end up with problems such as particle horizons.

Since the force from the mass distribution within R_{gs} is anti-gravity, it will ensure the expansion and uniform density of the universe. Anyway, repulsion is now possible on a cosmic scale, so please try using it for various purposes.

1-2. Decelerating expansion after R_{gs} [4]

If we do not assume the birth or influx of new mass-energy, then slowed expansion occurs after R_{gs}. However, if we assume the birth or influx of new mass-energy, such as vacuum energy or a cosmological constant, the situation becomes more complex and dependent on assumptions.

Also, when the particle horizon becomes large, the R_{gs} also becomes large. Accordingly, continuous accelerated expansion may occur.

2. The sources of dark energy are gravitational self-energy and the expansion of the particle horizon

2-1. The need for negative energy (mass) density

1) Negative mass (energy) density in standard cosmology

From the second Friedmann equation or acceleration equation, [5]

$$\frac{1}{R}(\frac{d^2R}{dt^2}) = -(\frac{4\pi G}{3})(\rho + 3P) \tag{56}$$

In standard cosmology, it is explained by introducing an entity that has a positive mass (energy) density but exerts a negative pressure. [6] [7]

$$\rho_\Lambda + 3P = \rho_\Lambda + 3(-\rho_\Lambda) = -2\rho_\Lambda \tag{57}$$

However, if we expand the dark energy term, the final result is a negative mass (energy) density of $-2\rho_\Lambda$.

There are too many people who have an aversion to negative mass (energy). However, in the standard cosmology, accelerated expansion is impossible without negative mass (energy) density. It is just that the negative mass (energy) density term is called negative pressure, so it is not recognized.

The notion (dark energy term) created by the mainstream has an inertial mass density of $+1(\rho)$, equivalent mass density of kinetic energy (or negative pressure) of -3(ρ), with gravitational mass density of -2(ρ). Not only different signs, but different values. It violates the principle of equivalence of inertial mass and gravitational mass, which is the basis of general relativity theory.

2) The energy of a gravitational field is negative

In his lecture, [8] Alan Guth said:

> *The energy of a gravitational field is negative!*
>
> ***The positive energy of the false vacuum was compensated by the negative energy of gravity.***

Stephen Hawking also argued that [9]

> *The matter in the universe is made out of positive energy. However, the matter is all attracting itself by gravity. Two pieces of matter that are close to each other have less energy than the same two pieces a long way apart, because you have to expend energy to separate them against the gravitational force that is pulling them together. Thus, in a sense, the gravitational field has negative energy. In the case of a universe that is approximately uniform in space,* ***one can show that this negative gravitational energy exactly cancels the positive energy represented by the matter.*** *So the total energy of the universe is zero.*
>
> *Now twice zero is also zero. Thus the universe can double the amount of positive matter energy and also double the negative gravitational energy without violation of the conservation of energy.*

Both argued that gravitational potential energy is negative energy and is the true energy that can cancel positive mass energy.

3) Earth's and Moon's Gravitational self-energy

If we want to discover whether gravity gravitates, we must examine the behavior of large masses, of planetary size, with significant and calculable amounts of gravitational self-energy. Treating the Earth as a continuous, classical mass distribution (with no gravitational self-energy in the elementary, subatomic particles), we find that its gravitational self-energy is about 4.6×10^{-10} *times its rest-mass energy. The gravitational self-energy of the Moon is smaller, only about* 0.2×10^{-10} *times its rest-mass energy. - GRAVITATION AND SPACETIME* [1]

4) The gravitational action of gravitational potential energy

When a bound system exerts gravity, the gravitational action of gravitational potential energy, a type of binding energy, must also be considered. Since it is very important in relation to the dark energy problem, let's look again at the equation mentioned earlier.

$$E_T = 2mc^2 - \frac{Gmm}{r} = 2mc^2 - m_{gp}c^2 = (2m - m_{gp})c^2 = m^*c^2 \tag{58}$$

The gravitational force acting on a relatively distant third mass m_3 is

$$F = -\frac{Gm^*m_3}{R^2} = -\frac{G(2m - m_{gp})m_3}{R^2} = -\frac{G(2m)m_3}{R^2} - \frac{G(-m_{gp})m_3}{R^2} \tag{59}$$

That is, when considering the gravitational action of a bound system, not only the mass in its free state but also the binding energy term should be considered. Gravitational potential energy is also a kind of binding energy. Therefore, when dealing with the expansion problem of the universe, the negative gravitational potential energy must also be considered.

In the universe,

$$E_T = \sum m_i c^2 + \sum_{i>j} -\frac{Gm_i m_j}{r_{ij}} = Mc^2 + U_{gs} = Mc^2 + (-M_{gs}c^2) \tag{60}$$

In the above equation, what happens if the second term is greater than the first term?

2-2. Comparison of magnitudes of mass energy and gravitational self-energy in the cosmic event horizon

1) Total mass energy in the cosmic event horizon 16.7Gly

Simply put, the particle horizon is important because it refers to the range of the interaction. The critical density value was used. $\rho_c = 8.50 \times 10^{-27} [kgm^{-3}]$ [10], cosmic event horizon 16.7Gly.

Since the universe is almost flat spacetime, the total mass energy (include radiation energy) in the particle horizon is

$$Mc^2 = \frac{4\pi r^3 \rho c^2}{3} = 1.28 \times 10^{70} [kgm^2 s^{-2}] \tag{61}$$

2) Gravitational self-energy in the cosmic event horizon

$$U = -\frac{3}{5}\frac{GM^2}{R} = -\frac{16\pi^2 G R^5 \rho^2}{15} = -4.99 \times 10^{69} [kgm^2 s^{-2}] \tag{62}$$

3) Comparison of magnitudes of total mass energy and gravitational self-energy in the cosmic event horizon

$$\frac{U}{Mc^2} = \frac{-4.99 \times 10^{69}}{1.28 \times 10^{70}} = -0.39 \tag{63}$$

In the calculation, the current critical density value was used, but when the particle horizon is 16.7Gly, the density is different from now. So, just look at the logic.

In the cosmic event horizon (16.7Gly), the repulsive component is smaller than the attractive component. In this period, the universe is decelerating expansion. That is, when the particle horizon is 16.7Gly, the dark energy component is smaller than that of matter. This period is a period of decelerated expansion.

2-3. The inflection point that changes from decelerated expansion to accelerated expansion

The inflection point is that the particle horizon at the transition from material dominance to dark energy dominance. The particle horizon at the transition from a period of predominant attraction to a period of predominant repulsion.

If the mass energy and the gravitational self-energy are equal, then

$$Mc^2 = |-\frac{3}{5}\frac{GM^2}{R_{gs}}| \tag{64}$$

$$R_{gs} = \sqrt{\frac{5c^2}{4\pi G\rho}} \tag{65}$$

I do not know the magnitude R_{gs} at which the positive mass energy and the negative gravitational potential energy are equal, since I do not have the data of the density. We only need to understand the general flow and possibility, so let's get R_{gs} by putting in the current critical density value.

$$R_{gs} = \sqrt{\frac{5c^2}{4\pi G\rho}} = 26.2Gly \tag{66}$$

Assuming that the average density is approximately 1.25 times the current average density,
we get $R_{gs} = 23.7Gly$.
Assuming that the average density is approximately 1.5 times the current average density,
we get $R_{gs} = 21.4Gly$.
Assuming that the average density is approximately 2 times the current average density,
we get $R_{gs} = 18.7Gly$.

Comparing the data from the existing particle horizon graph, it is estimated that it is approximately 5 to 7 billion years ago from the present. This is similar to the result of standard cosmology. [1] It is necessary to review this model because it includes the transition of the universe to the period of decelerated expansion, the inflection point, and the period of accelerated expansion.

2-4. Comparison of magnitudes of mass energy and gravitational self-energy in the observable universe

1) Total mass energy of the observable universe R=46.5Gly

$$Mc^2 = \frac{4\pi r^3\rho c^2}{3} = 2.75\times 10^{71}[kgm^2s^{-2}] \tag{67}$$

2) Gravitational self-energy of the observable universe

$$U = -\frac{3}{5}\frac{GM^2}{R} = -\frac{16\pi^2GR^5\rho^2}{15} = -8.35\times 10^{71}[kgm^2s^{-2}] \tag{68}$$

3) In the observable universe, the ratio of total mass energy to gravitational self-energy

$$\frac{U}{Mc^2} = \frac{-8.35\times 10^{71}}{2.75\times 10^{71}} = -3.04 \tag{69}$$

The repulsive force component is approximately 3.04 times the attractive force component. So, the universe is accelerating expansion.

4) In the standard cosmology, the ratio of attractive and repulsive components

The following equation is an approximate form of the energy density of the standard cosmology model. [11]

$$\frac{M_{repulsive}}{M_{attractive}} = \frac{-3\rho_\Lambda}{\rho_m+\rho_\Lambda} = \frac{-3(0.683\rho)}{0.317\rho+0.683\rho} = -2.05 \tag{70}$$

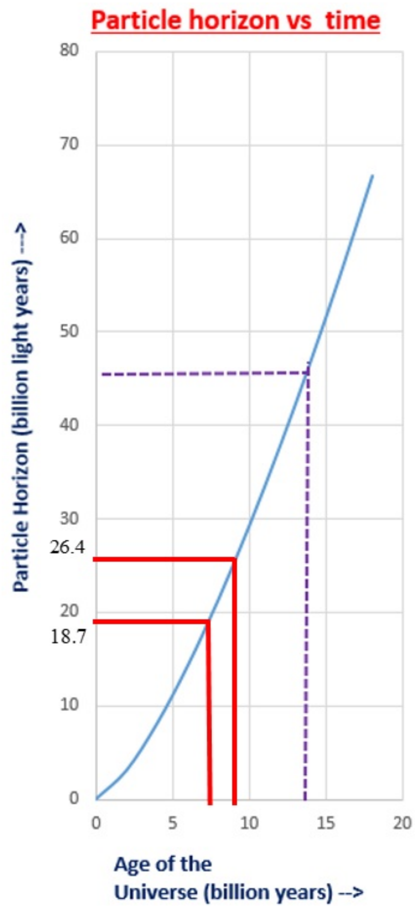

Figure 8: Particle horizon vs time. [11] **About 5-7 billion years ago, it passed the inflection point 18.7 $\sim$ 26.4Gly.** About 5 to 7 billion years ago, it can be said that the universe entered a period of accelerated expansion. The inflection point is affected by the density.

At 16.7 Gly, the attraction component is larger than the repulsive component, whereas at 46.5 Gly, the repulsive component is 3.04 times larger than the attractive component. In the middle, there is an inflection point that changes from decelerated expansion to accelerated expansion. About 5-7 billion years ago, similar to the predictions of standard cosmology.

Although there is a slight difference between the two values, it is not an astronomical error, so it is likely to be resolved later. It is possible that kinetic energy, general relativity, cosmological effects, assumption of uniformly distribution, or differences in cosmological models may explain the difference between the two values.

2-5. Regarding the cause of the error

1) New formula for calculating gravitational self-energy

According to the principle of general relativity, gravitational potential energy is also a source of gravity. Therefore, there is a problem in the conventional expression of gravitational self-energy.

Looking at the formula we use to find gravitational self-energy,

$$M' = \frac{4\pi}{3} r^3 \rho \tag{71}$$

$$dm = \rho r^2 dr \sin\theta d\theta d\varphi \tag{72}$$

$$dU_{gs} = -G\frac{M'm}{r} \tag{73}$$

In the existing gravitational self-energy equation, the gravitational action due to negative gravitational self-energy is omitted from the internal mass M' term. The equivalent mass of gravitational self-energy must also be reflected in the M' term.

$$M^* = M' - {M_{gs}}' = M' - \frac{3}{5}\frac{GM'^2}{rc^2} = \frac{4\pi}{3}r^3\rho - \frac{16\pi^2 G}{15c^2}r^5\rho^2 \tag{74}$$

$$dU_{gs} = -G\frac{M^* dm}{r} = -G\frac{(M' - {M_{gs}}')(\rho r^2 dr\sin\theta d\theta d\varphi)}{r} = -(\frac{4\pi G\rho^2}{3}r^4 - \frac{16\pi^2 G^2\rho^3}{15c^2}r^6)dr\sin\theta d\theta d\varphi \tag{75}$$

$$U_{gs} = -\frac{4\pi G\rho^2}{3}\int_0^R r^4 dr\int_0^\pi \sin\theta d\theta\int_0^{2\pi} d\varphi + \frac{16\pi^2 G^2\rho^3}{15c^2}\int_0^R r^6 dr\int_0^\pi \sin\theta d\theta\int_0^{2\pi} d\varphi \tag{76}$$

$$U_{gs} = -\frac{(4\pi\rho)^2 G}{3}\int_0^R r^4 dr + \frac{(4\pi\rho)^3 G^2}{15c^2}\int_0^R r^6 dr = -\frac{3}{5}\frac{GM^2}{R} + \frac{(4\pi\rho)^3 G^2}{15c^2}\frac{1}{7}R^7 \tag{77}$$

$$U_{gs} = -\frac{3}{5}\frac{GM^2}{R}(1 - \frac{3}{7}\frac{GM}{Rc^2}) \tag{78}$$

$$U_{gs-T} = U_{gs-M} + U_{gs-gs} = (-\frac{3}{5}\frac{GM^2}{R}) + (\frac{3}{7}\frac{GM}{Rc^2}\frac{3}{5}\frac{GM^2}{R}) \tag{79}$$

The first term is the gravitational self-energy term produced by the positive mass M, and the second term is the gravitational self-energy term created by the equivalent mass of the gravitational self-energy of the internal mass M'. The second term is very small in the case of ordinary matter and can be neglected, but it cannot be neglected when there are sufficiently many substances such as the universe.

Comparing the first and second terms, At R=46.5 Gly,

$$U_{gs} = -\frac{3}{5}\frac{GM^2}{R}(1 - \frac{3}{7}\frac{GM}{Rc^2}) = (1.2)(\frac{3}{5}\frac{GM^2}{R}) \tag{80}$$

If we look for the case where $(1 - \frac{3}{7}\frac{GM}{Rc^2}) = 0$,

$$R = \frac{3}{7}\frac{GM}{c^2} = \frac{3}{14}R_S = \frac{5}{7}R_{gs} \tag{81}$$

2) Universe structure correction variable or Gravitational self-energy correction variable $\beta(t)$

Another possibility related to the cause of the error is the mass problem of strongly bound objects, such as stars, black holes and galaxies. The mass of a star or galaxy observed from the outside will already be the total mass reflecting the value of its own gravitational energy. On the other hand, the gravitational self-energy equation assumes a uniform distribution and calculates all gravitational potential terms. In other words, there is a possibility that the gravitational self-energy term is over-calculated.

By introducing the universe structure (correction) variable $\beta(t)$, if we correct the gravitational self-energy value,

$$\beta(t) = \frac{2.05}{3.04} = 0.674 \tag{82}$$

$${U_{gs}}' = \beta(t)U_{gs} = (0.674)(-\frac{3}{5}\frac{GM^2}{R}) \tag{83}$$

Although the universe structure correction variable $\beta(t)$ is a variable in all time, it is thought that it can be treated almost like a constant since the change will be small after the galactic structure is formed. Therefore, β **can be called a universe structure constant**, a name corresponding to the fine structure constant of elementary particle physics, and can also be called a huge structure constant.

3) Because of the propagation velocity of the field, it is possible that some matter within the particle horizon is not participating in gravitational interactions

That is, it is possible that the gravitational field has not yet been transmitted from one end to the other. There is a possibility of overcalculation of gravitational self-energy.

In addition to 1) - 3) mentioned here, there may be factors that need to be corrected. In summary, I will use the universe structure correction variable $\beta(t)$.

2-6. New Friedmann equations and cosmological constant function obtained by the gravitational self-energy model

The Friedmann equation can be obtained from the field equation. The basic form can also be obtained through Newtonian mechanics.

If the object to be analyzed has spherical symmetry, from the second Newton's law, $R(t) = a(t)R$

$$m\frac{d^2R(t)}{dt^2} = -\frac{GM(t)m}{R(t)^2} \tag{84}$$

$$m\frac{d^2a(t)R}{dt^2} = -Gm\frac{\frac{4\pi}{3}a^3(t)R^3\rho(t)}{a^2(t)R^2} = -Gm\frac{4\pi}{3}a(t)R\rho(t) \tag{85}$$

$$\frac{\ddot{a}(t)}{a(t)} = -\frac{4\pi G}{3}\rho(t) \tag{86}$$

By adding pressure, we can create an acceleration equation.

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + \frac{3P}{c^2}) \tag{87}$$

Returning to the starting point, we put in the internal mass $M(t)$. However, according to the gravitational self-energy model, this mass M only needs to be replaced by $(M) + (-M_{gs})$. The gravitational self-energy of the particles making up the mass $M(t)$ and the mass $M(t)$ have the same center of mass. That is, when the internal mass M is entered, the gravitational self-energy also has the same center of mass, so it can be applied immediately.

Even when using the FLRW metric, there is a relative distance $a(t)R$ or radius defined by the scale factor $a(t)$, and there is a mass density. If we multiply the mass (energy) density by the coefficient, it will be restored to the mass in $a(t)R$. As long as internal masses are applied, the gravitational self-energy between their internal mass terms always seems applicable.

The current cosmic constant model is

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu} \tag{88}$$

$$(\frac{\dot{a}}{a})^2 = \frac{8\pi G\rho}{3} - \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3} \tag{89}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + \frac{3P}{c^2}) + \frac{\Lambda c^2}{3} \tag{90}$$

1) A model that corresponds to the gravitational self-energy to the cosmological constant

$$-M_{gs} = -\frac{16\pi^2 GR^5(t)\rho^2}{15c^2}\beta(t) = (-\frac{4\pi G}{5c^2}R^2(t)\rho^2)\beta(t)(\frac{4\pi R^3(t)}{3}) = (-\rho_{gs})V \tag{91}$$

$$-\rho_{gs} = (-\frac{4\pi G}{5c^2}a^2R^2\rho^2)\beta(t) \tag{92}$$

$-\rho_{gs}$ is the gravitational self-energy density, and $\beta(t)$ is the universe structure correction variable. Instead of energy density ρ, we have $\rho + (-\rho_{gs})$.

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}((\rho - \rho_{gs}) + \frac{3P}{c^2}) = -\frac{4\pi G}{3}(\rho + \frac{3P}{c^2}) + \frac{4\pi G}{3}\rho_{gs} \tag{93}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + \frac{3P}{c^2}) + \frac{\beta(t)}{15}(\frac{4\pi GaR\rho}{c})^2 \tag{94}$$

Cosmological constant function

$$\frac{\Lambda c^2}{3} = \frac{\beta(t)}{15}(\frac{4\pi GaR\rho}{c})^2 \tag{95}$$

$$\Lambda(t) = \beta(t)(\frac{4\pi Ga(t)R\rho(t)}{\sqrt{5}c})^2 = \beta(\frac{4\pi GaR\rho}{\sqrt{5}c})^2 \tag{96}$$

First Friedmann-Choi equation

$$(\frac{\dot{a}}{a})^2 = \frac{8\pi G\rho}{3} - \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3} = \frac{8\pi G\rho}{3} - \frac{kc^2}{a^2} + \frac{\beta(t)}{15}(\frac{4\pi GaR\rho}{c})^2 \tag{97}$$

Second Friedmann-Choi equation, acceleration equation

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + \frac{3P}{c^2}) + \frac{\Lambda c^2}{3} = -\frac{4\pi G}{3}(\rho + \frac{3P}{c^2}) + \frac{\beta(t)}{15}(\frac{4\pi GaR\rho}{c})^2 \tag{98}$$

To see if the derived equation is probable, let's find the value of the cosmological constant.

$$\Lambda = \frac{\beta(t)}{5}(\frac{4(3.14)(6.67 \times 10^{-11}m^3kg^{-1}s^{-2})(46.5 \times 9.46 \times 10^{24}m)(8.50 \times 10^{-27}kgm^{-3})}{(2.99 \times 10^8 ms^{-1})^2})^2 \tag{99}$$

$$\Lambda = \frac{\beta(t)}{5}(\frac{4\pi GaR\rho}{c^2})^2 = (2.455 \times 10^{-52}m^{-2})\beta(t) \tag{100}$$

The value obtained from the Planck satellite is,

$$\Lambda = 3(\frac{H_0}{c})^2\Omega_\Lambda = 1.1056 \times 10^{-52}m^{-2} \tag{101}$$

Now, we get the universe structure variable (or constant)

$$\beta(t_{now}) = \frac{1.1056 \times 10^{-52}m^{-2}}{2.455 \times 10^{-52}m^{-2}} = 0.450 \tag{102}$$

First Friedmann equation

$$(\frac{\dot{a}}{a})^2 = \frac{8\pi G\rho}{3} - \frac{kc^2}{a^2} + 3(\frac{2\pi GaR\rho}{5c})^2 \tag{103}$$

Second Friedmann equation, acceleration equation

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + \frac{3P}{c^2}) + 3(\frac{2\pi GaR\rho}{5c})^2 \tag{104}$$

Cosmological constant function

$$\Lambda(t) = (\frac{6\pi GaR\rho}{5c^2})^2 \tag{105}$$

$a(t)R$ is the particle horizon at the point in time to be analyzed. In the gravitational self-energy model, if $\rho(t) \propto \frac{1}{a(t)R}$, it is possible that the density of dark energy seems to be constant. However, I think that the density of dark energy will change.

Note that the gravitational self-energy model and the cosmological constant model have different elements. For example, the gravitational self-energy model has a negative energy density and does not assume negative pressure. In addition, the energy density is a constant in the cosmological constant model, but the energy density is a variable in the gravitational self-energy model. Since $a(t)$ and $\rho(t)$ are functions of time, **the dark energy (or cosmological constant term) is a function of time**.

2) Negative mass model

First Friedmann equation ($\rho > |-\rho_{gs}|$)

$$(\frac{\dot{a}}{a})^2 = \frac{8\pi G}{3}(\rho - \rho_{gs}) - \frac{kc^2}{a^2} \tag{106}$$

$$(\frac{\dot{a}}{a})^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} + \frac{8\pi G}{3}(-\rho_{gs}) \tag{107}$$

$$(\frac{\dot{a}}{a})^2 = \frac{8\pi G\rho}{3} - \frac{kc^2}{a^2} - \frac{2\beta(t)}{15}(\frac{4\pi GaR\rho}{c})^2 \tag{108}$$

Second Friedmann equation, acceleration equation

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}((\rho - \rho_{gs}) + \frac{3P}{c^2}) \tag{109}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + \frac{3P}{c^2}) + \frac{\beta(t)}{15}(\frac{4\pi GaR\rho}{c})^2 \tag{110}$$

In the existing cosmological constant model, $+\frac{\Lambda c^2}{3}$ entered the two equations equally. However, in the model in which the cosmological constant $\Lambda = 0$ and negative mass density is introduced, the two values are different.

Also, in the first Friedmann equation, when the negative mass density is greater than the positive mass density, it is assumed that there is a sign inversion. It seems that the sign of the Ricci scalar term needs to be changed in the process of deriving the equation.

$$-(\frac{\dot{a}}{a})^2 - \frac{kc^2}{a} = \frac{8\pi G}{3}(\rho - \rho_{gs}) \tag{111}$$

First Friedmann equation ($\rho < |-\rho_{gs}|$)

$$(\frac{\dot{a}}{a})^2 = \frac{8\pi G}{3}(\rho_{gs} - \rho) - \frac{kc^2}{a} \tag{112}$$

$$(\frac{\dot{a}}{a})^2 = \frac{2\beta(t)}{15}(\frac{4\pi GaR\rho}{c})^2 - \frac{kc^2}{a^2} - \frac{8\pi G\rho}{3} \tag{113}$$

Second Friedmann equation, acceleration equation

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}((\rho - \rho_{gs}) + \frac{3P}{c^2}) = -\frac{4\pi G}{3}(\rho + \frac{3P}{c^2}) + \frac{4\pi G}{3}\rho_{gs} \tag{114}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + \frac{3P}{c^2}) + \frac{\beta(t)}{15}(\frac{4\pi GaR\rho}{c})^2 \tag{115}$$

The first Friedmann equation is different from the existing form, so a new approach is needed.

This value $(\frac{\dot{a}}{a})^2$ is the square of the Hubble parameter, and there are observed values. If we find the value through standard cosmology,

$$(\frac{\dot{a}}{a})^2 = \frac{8\pi G\rho}{3} - \frac{kc^2}{a^2} + (\frac{\Lambda c^2}{3}) = (4.75 \times 10^{-36}s^{-2}) + (3.29 \times 10^{-36}s^{-2}) - \frac{kc^2}{a^2} \tag{116}$$

$$(\frac{\dot{a}}{a})^2 = (8.04 \times 10^{-36}s^{-2}) - \frac{kc^2}{a^2} \tag{117}$$

In the new equation,

$$\frac{2\beta(t)}{15}(\frac{4\pi G R\rho}{c})^2 = (14.63 \times 10^{-36} s^{-2})\beta(t) \tag{118}$$

$$(\frac{\dot{a}}{a})^2 = ((14.63)\beta(t) - 4.75) \times 10^{-36} s^{-2} - \frac{kc^2}{a^2} \tag{119}$$

Since $(\frac{\dot{a}}{a})^2$ is the square of the Hubble parameter, assuming that this equation is the same as the equation obtained from the existing standard cosmology,

$$((14.63)\beta(t) - 4.75) \times 10^{-36} s^{-2} = (8.04 \times 10^{-36} s^{-2}) \tag{120}$$

$$\beta(t_{now}) = \frac{12.79}{14.63} = 0.874 \tag{121}$$

For similarity with the standard model, they are presented in the form of Friedmann's equations. However, the Friedmann equation needs more research. It is necessary to construct a field equation when positive and negative mass exist together and to find a solution.

2-7. One way to explain why dark energy density appears to be a constant

According to the observation results so far, the dark energy density seems to be a constant. One way to explain these results is to hypothesize the introduction or creation of new substances. Since the cosmological constant model explains the phenomenon by assuming a uniform energy density, let's try to explain it through dynamics here.

The dark energy term suggested by gravitational potential energy model is as follows.

$$\Lambda(t) = \frac{\beta(t)}{5}(\frac{4\pi G a(t) R\rho(t)}{c^2})^2 = (\frac{6\pi G a(t) R\rho(t)}{5c^2})^2 \tag{122}$$

Then, in order for this dark energy term to look like a constant, the following relationship needs to be established.

$$\rho(t) \propto \frac{1}{a(t)R} \tag{123}$$

When a material within a radius R expands to $2R$, its density drops to $1/2^3$. However, the above equation suggests that when R is changed to $2R$, the density drops only by $1/2$. Therefore, other kinetic property must exist.

Let's assume the following situation. Matter and galaxies are moving according to the Hubble-Lemaitre law. Assume that the gravitational field is moving at a higher speed than these. The propagation speed of the gravitational field will have "speed of space expansion" + "speed of light".

Then, over time, the amount of matter and galaxies that enter into gravitational interactions will increase. This increase in matter and galaxies has the effect of increasing gravitational self-energy. When R_0 is changed to $2R_0$, if the mass is increased by 8, the density remains the same. When R_0 is changed to $2R_0$, if the mass is 4 times, the density is halved.

Therefore, the point we need to find is R_x, which has a mass 4 times greater than its mass at R_0.

$$M_x = \frac{4\pi {R_x}^3}{3}\rho_0 = 4(\frac{4\pi {R_0}^3}{3}\rho_0) \tag{124}$$

$$R_x = (4)^{\frac{1}{3}} R_0 \simeq 1.587 R_0 \tag{125}$$

When R_0 expands by k times, the value of R_x whose mass is k times: $R_x = (\frac{k^3}{2})R_0$

Comparing the average speed of the Field with the average speed of the Matter in R_x,

$$V_{R_0-Field} = \frac{\Delta R}{\Delta t} = \frac{2R_0 - R_0}{\Delta t} = \frac{R_0}{\Delta t} \tag{126}$$

$$V_{R_x-Matter} = \frac{\Delta R}{\Delta t} = \frac{2R_0 - R_x}{\Delta t} = (0.413)\frac{R_0}{\Delta t} = (0.413)V_{R_0-Field} \tag{127}$$

$$V_{R_0-Field} = \frac{V_{R_x-Matter}}{(0.413)} = 2.42V_{R_x-Matter} \tag{128}$$

When $a(t)R$ is about half of the current universe, if the speed of the field is about 2.42 times faster than the speed of matter, the phenomenon in which dark energy appears to be a constant can be explained.

2-8. Increase in dark energy

2-8-1. The ratio of increase in gravitational self-energy to increase in mass energy

$$\frac{d(Mc^2)}{dR} = 4\pi R^2 \rho c^2 \tag{129}$$

$$\frac{d(U_{gs})}{dR} = -\frac{16\pi^2 G}{3}R^4\rho^2 = (-\frac{4\pi R^3 \rho G}{3Rc^2})(4\pi R^2 \rho c^2) = -\frac{GM}{Rc^2}(\frac{d(Mc^2)}{dR}) \tag{130}$$

$$\frac{d(U_{gs})}{dR} = -\frac{R_S}{2R}\frac{d(Mc^2)}{dR} \tag{131}$$

R_S is the Schwarzschild radius of the black hole formed by Particle Horizon.
The size of the event horizon formed by the mass distribution of 46.5 Gly is 477.8 Gly.

$$\frac{d(U_{gs})}{dR} = -\frac{R_S}{2R}(\frac{d(Mc^2)}{dR}) = -\frac{477.8Gly}{2(46.5Gly)}(\frac{d(Mc^2)}{dR}) = -(5.14)(\frac{d(Mc^2)}{dR}) \tag{132}$$

If the particle horizon increases and a positive mass is produced by M, the equivalent mass of negative gravitational potential energy is produced by $-5.14M$. This value is not a fixed value, it depends on the density and the size of the particle horizon.

To find the ratio $-\frac{R_S}{2R}$ according to R, if $R = k \times Gly$,

$$\frac{\frac{d(U_{gs})}{dR}}{\frac{d(Mc^2)}{dR}} = -\frac{R_S}{2R} \tag{133}$$

$$R_S(k \times Gly) = \frac{2GM}{c^2} = (0.00475 \times Gly)(k^3) \tag{134}$$

$R(Gly)$	$R_S(Gly)$	$-R_S/2R$
10	4.80	-0.238
15	16.0	-0.533
20	38.0	-0.950
25	74.2	-1.48
30	128	-2.13
35	204	-2.91
40	304	-3.80
45	433	-4.81
50	594	-5.94

The density used the current critical density, but the density is a variable. Please see the approximate trend. The rate of increase of gravitational self-energy tends to be greater than the rate of increase of mass energy. Therefore, at some point, a situation arises in which dark energy becomes larger than matter and dark matter.

However, the negative mass ratio of 5.14 produced is very similar to the ratio of matter : dark matter.

In my 2009 paper, assuming that the negative mass is 5.06 (ratio of dark matter/matter) times greater than the positive mass, it was argued that both dark matter and dark energy could be explained. [12] I also argued that the essence of dark matter is negative mass. [2] [12] [13]

2-8-2. Increase in dark energy (gravitational self-energy) due to increase in particle horizon

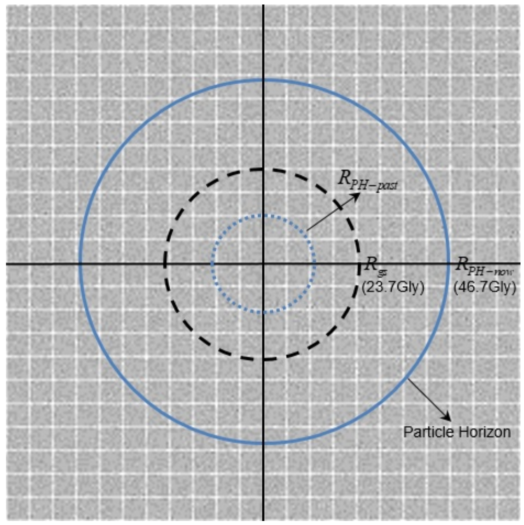

Figure 9: The past particle horizon, the present particle horizon, and R_{gs} 23.7Gly (The inflection point calculated assuming 1.25 times the current average density), the inflection point at which the magnitudes of repulsive and attractive forces are equal. When the particle horizon is smaller than R_{gs}, the attractive component is dominant, and when the particle horizon is larger than R_{gs}, the repulsive component (gravitational self-energy) is dominant.

1) Particles and galaxies spread almost uniformly throughout the universe through the inflation process.

2) Galaxies move according to the Hubble-Lemaitre law.

3) On the other hand, the propagation speed of the field, the range of interaction (particle horizon), has the fastest speed, the speed of light in expanding space.

4) Thus, over time, many new substances (matters and galaxies) enter the particle horizon. In other words, **the newly entering materials undergo gravitational interaction, resulting in an increase in mass and an increase in gravitational potential energy in the region within the particle horizon.**

5) By the way, **while mass energy is proportional to M, total gravitational potential energy (gravitational self-energy) is proportional to $-\frac{M^2}{R}$.** As M increases, the gravitational potential energy increases faster. Accordingly, the repulsive force component increases faster than the attractive force component.

6) The increase in gravitational potential energy due to the newly incorporated matter into the particle horizon is causing the dark energy. The same principle is applicable to the birth of energy within a particle horizon. That is, when the mass energy increases by M, the gravitational self-energy increases by $-\frac{M^2}{R}$. In this model, even if a mass is born while satisfying energy conservation in a local area, as the new gravitational field propagates, the gravitational interaction with other matter will increase. Accordingly, an increase in gravitational self-energy may occur.

7) In the present universe, it is predicted that the dark energy effect (repulsive effect) surpassed the gravitational effect of matter and dark matter about 5 billion years ago. According to this model, it is the point

at which the positive mass energy and the negative gravitational self-energy are equal. Knowing the average density function, we can get the exact value.

8) Gravitational potential energy is a concept that already exists and is negative energy that can create repulsive force. **This model produces similar results to the phenomenon of applying negative pressure while having positive inertial mass.** As the particle horizon expands, the positive mass increases (new influx or birth of matter), but the negative gravitational potential energy created by these positive masses is greater. While having a positive inertial mass, it is creating a negative gravitational mass that is larger than the positive inertial mass. Because of this factor, this model should be reviewed.

2-9. How to validate the dark energy model that gravitational self-energy is the source of dark energy

1) Find the expressions of $\rho(t)$ and $R_{ph}(t)$

$\rho(t)$ is the average density inside the particle horizon. $R_{ph}(t)$ is the particle horizon.

2) At each time, within the particle horizon

Find $E = M(t)c^2$ and $U_{gs} = -\frac{3}{5}\frac{GM(t)^2}{R_{ph}(t)}$

$E = M(t)c^2$ is the attractive energy component and U_{gs} is the repulsive energy component.

3) Compare $\frac{U_{gs}}{E}$ with the observations

Compare $\frac{U_{gs}}{E}$ with the observations. And for the inflection point transitioning from decelerated expansion to accelerated expansion, the theoretical value and the observed value are compared.

4) It is necessary to verify the Friedmann equations and the equation for the cosmological constant

In the gravitational self-energy model, the cosmological constant is a function of time. Therefore, if the dark energy effect is precisely observed, it is possible to determine whether the model is right or wrong.

IV. It is possible that the source of dark matter is also gravitational self-energy

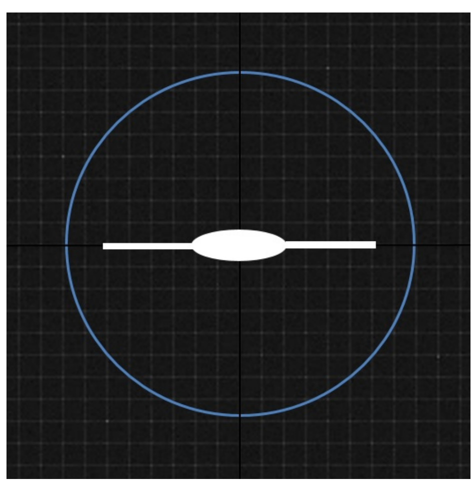

Figure 10: Galaxies are trapped in an ocean full of negative energy (mass). It is possible that negative energies outside the galactic structure are preventing the galaxies from changing their orbits.

The present universe is a universe in which the negative gravitational potential energy is greater than the positive mass energy. And, the magnitude of negative energy (mass) is about 3.04 times the amount of the sum of matter + dark matter, and about 20 times when only matter is taken into account.

Thus, galaxies can be viewed as structures trapped in an ocean full of negative mass. Because the average density of the galaxy itself is higher than the average density of the universe, the galaxy itself is greater than the mass density of negative energy, and thus can retain its structure.

However, the current structure of the galaxy is not a structure without nothing, but a structure with internal and external pressures. Thus, even if the stars in the spiral arms of galaxies have higher velocities than their current orbits, it is likely that they will not be able to transition to orbits with a greater radius than their current orbits. In other words, the dark matter effect may be a phenomenon that occurs because the structure of the galaxy is already in a structure that is under pressure from all sides.

Deep-sea fish living in high water pressure maintain their shape and live in the deep sea, but when they come out of the water with low air pressure, they expand and lose their shape or explode.

If the effect of the gravitational potential energy of the entire universe was the essence of dark matter, then there would be no particle-like substance, and therefore no electromagnetic interaction. The $-5.14M$ obtained in Chapter III is similar to the ratio of matter to dark matter.This may have meaning. [12] [13]

V. Conclusion [2]

In order to apply the general theory of relativity to the strong gravitational field, it is sufficient to consider the gravitational self-energy, which is the binding energy of the object itself.

By considering the gravitational self-energy, it is possible to solve the singularity problem, which is the biggest problem with general relativity, and to rescue general relativity that collapses into a singularity created by oneself.

When the bounded system acts on gravity, the gravitational action of gravitational potential energy is also included. Therefore, even in the case of the universe, we have to calculate the gravitational action of gravitational potential energy. Gravitational potential energy generates a repulsive force because it has a negative equivalent mass. If the gravitational potential energy is greater than the positive mass energy, it can be a candidate for dark energy to explain the current accelerated expansion. And, when we calculated the gravitational potential energy for the observable universe, it is approximately three times larger than the mass energy, so it can explain the accelerated expansion of the universe.

Also, in the field of cosmology, the effect of dark energy occurs because matter and galaxies entering the particle horizon contribute to the total gravitational potential energy. Since the total mass in the particle horizon is proportional to M, whereas the gravitational self-energy is proportional to $-\frac{M^2}{R}$, the repulsive force component increases faster and accelerated expansion occurs.

This model can be verified because it points to the gravitational self-energy and the particle horizon as the causes. This model predicts an inflection point where dark energy becomes larger and more important than the energy of matter and radiation. Through this model, the past, present and future of the universe can be predicted. Therefore, I think experts need to study this model.

[2] Journals have very high barriers to entry, and I understand that. In order to be listed on arXiv, endorsement by other scholars is required. Although endorsed by other scholars, arXiv's administrators are arbitrarily deleting papers. I've experienced it a few times, and it is painful.

Some scientific communities prevent even discussion of content not published in journals. A scientific community that does not allow hypotheses, it was something I could never have imagined when I was a physics student. Who made the rule that only professional soccer players can play soccer? And, is the internet science community a pro league?

No matter how ugly a paper is, you should be very careful about deleting or formatting it.

What we have to keep in mind is that neither Newton's theory nor Einstein's theory. What we must protect are freedom of thought, freedom of research, and freedom of speech and publication. We have the discriminators of the truth of the grand universe and time.

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