Gauss' Law for Magnetism

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Abstract

Gauss' law for magnetism states that the magnetic flux across any closed surface is zero. In this paper a rigorous proof of the Gauss' law for magnetism from the first principle has been presented.

Keywords : Magnetic flux, Closed surface.

1 DERIVATION

Let the origin of a cartesian coordinate system be situated at the center of a sphere S_1 of radius ρ. Consider a charge q moving with velocity **v** along the x axis and situated at the center of the sphere at the current instant. Magnetic field **B** at a distance **r** due to the moving charge q is

$$\mathbf{B} = \frac{\mu\, q\, \mathbf{v}\times\mathbf{r}}{4\pi\, r^3} \qquad \text{[Law of Magnetism]}$$

$$= \left(\frac{\mu q}{4\pi}\right)\mathbf{F} = C\,\mathbf{F}$$

where

$$C = \left(\frac{\mu q}{4\pi}\right)$$

and

$$\mathbf{F} = \frac{\mathbf{v}\times\mathbf{r}}{r^3} = \frac{v\,\mathbf{i}\times\left(x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k}\right)}{r^3}$$

$$= \frac{v\left(y\,\mathbf{k} - z\,\mathbf{j}\right)}{r^3}$$

let **n** be the unit vector normal to the sphere S_1, then

$$\mathbf{F.n} = \frac{v\left(y\,\mathbf{k} - z\,\mathbf{j}\right).\left(x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k}\right)}{\rho^4} \qquad [\, r = \rho \,]$$

$$= \frac{v\left(-zy + zy\right)}{\rho^4} = 0$$

$$\Rightarrow \oint_{S_1} \mathbf{B.dA} = \oint_{S_1} C\,\mathbf{F.n}\; dA = 0 \qquad (\,\text{i}\,)$$

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let

$$\mathbf{F} = f_x\,\mathbf{i} + f_y\,\mathbf{j} + f_z\,\mathbf{k}$$

$$= -\frac{vz}{r^3}\,\mathbf{j} + \frac{vy}{r^3}\,\mathbf{k}$$

now

$$\frac{\partial r}{\partial x} = \frac{x}{r}; \quad \frac{\partial r}{\partial y} = \frac{y}{r}; \quad \frac{\partial r}{\partial z} = \frac{z}{r}$$

so

$$\frac{\partial f_x}{\partial x} = 0$$

$$\frac{\partial f_y}{\partial y} = -v\left(\frac{1}{r^3}\frac{\partial z}{\partial y} - \frac{3\,yz}{r^5}\right) = v\,\frac{3\,yz}{r^5}$$

$$\frac{\partial f_z}{\partial z} = v\left(\frac{1}{r^3}\frac{\partial y}{\partial z} - \frac{3\,yz}{r^5}\right) = -v\,\frac{3\,yz}{r^5}$$

now

$$\nabla .\mathbf{F} = \frac{\partial f_x}{\partial x} + \frac{\partial f_y}{\partial y} + \frac{\partial f_z}{\partial z}$$

$$= \frac{3\,yz\,(v - v)}{r^5} = 0$$

$$\Rightarrow \nabla .\mathbf{B} = C\,\nabla .\mathbf{F} = 0$$

now consider an arbitrary shaped closed surface S_2 that lies inside the sphere S_1 and encloses the origin, then

$$\int_V (\nabla .\mathbf{B})\; dV = 0$$

where V is the volume between the sphere S_1 and closed surface S_2

$$\Rightarrow \oint_{S_1} \mathbf{B.dA} + \oint_{S_2} \mathbf{B.dA} = 0$$ [using Divergence theorem]

$$\Rightarrow 0 + \oint_{S_2} \mathbf{B.dA} = 0$$ [using (i)]

$$\Rightarrow \oint_{S_2} \mathbf{B.dA} = 0$$

2 CONCLUSION

Magnetic flux for arbitrary closed surface S_2 due to a moving charge is zero. Since all magnetic sources consist of moving charges therefore by Superposition principle, Gauss' law holds for all magnetic sources.

References

1. Hugh D. Young, Roger A. Freedman, Albert Lewis Ford, *“Sears’ and Zemansky’s University Physics with Modern Physics 13th edition.”*