Are gravimeters sensitive enough to measure gravitational waves?

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Calculations show that the sensitivity of common gravimeters is sufficient to measure GW in wide frequency range around 0.1 Hz. Initial evaluations have confirmed that it is possible to extract the coordinates and drift of known binary star systems with good accuracy from multi-year data records of gravimeters distributed around the world. This opens the possibility of an Earth-based search for continuous GW several years before LISA.

1 Introduction

The detection of continuous gravitational waves opens another window for astronomy to observe certain celestial objects. GW cause extraordinarily small changes in the length of scales, which is why one needs very extended detectors. Since interferometers such as LIGO, with arm lengths of only 4000 m, are apparently too short, plans are underway for the much larger LISA space telescope, which will not provide initial observations for at least 20 years. Earthquakes do not disturb the operation far from Earth - but repairs or readjustments become impossible.

As a way out, one can use the Earth itself as the largest possible test body. Gravimeters have been recording the smallest changes in gravity for decades and provide an excellent database for searching for gravitational waves. A hint for their existence was discovered by geophysicists decades ago: The Earth gives off a relentless hum of countless notes, but the origin of this sound remains a mystery [1]. The researchers found that the hum's amplitude does not vary over time and doesn't depend on the seasons. Do continuous GWs cause the never ending hum of the Earth? We will not discuss that here.

Here, the only question is: Is the intrinsic noise of gravimeters weak enough to detect the periodic changes of the Earth's diameter caused by the GW of distant celestial bodies?

2 The properties of gravimeters

Gravimeters are sensors near the surface of the Earth to determine the value of the acceleration due to gravity (about 9.81 $\frac{m}{s^2}$). Some types measure the absolute value, others only the relative change. Both types can provide years of uninterrupted data every second. To judge whether a GW of constant frequency may be detected in the noise floor of a gravimeter, one has to know the inherent noise of the gravimeter and compare it with the stress value h_0 predicted by the theory of relativity.

The usual benchmark *Power Spectral Density* (PSD) [2] has the unit $\frac{m^2}{s^4 Hz}$ and provides the first part of the sought answer, if one knows the bandwidth BW of the signal processing. Detailed investigations yield the following values:

- The PSD of superconducting gravimeters in the range around 6 mHz is about 10^{-18} $\frac{m^2}{s^4 Hz}$ [3] and [4], Figure 5.

- The PSD of the widely used gravimeter STS1 is even lower at $10^{-19}\ \frac{m^2}{s^4 Hz}$ [5]. It should be possible to improve this value significantly if the gravimeter is not mounted directly on the ground, as has been the case up to now.

- In the successful LISA Pathfinder model experiment, small-scale PSD values around $10^{-29}\ \frac{m^2}{s^4 Hz}$ [6] were achieved, which are also hoped for in the future GW observatory LISA. There is one hurdle to overcome: In the model experiment, the spacing of the test bodies was 0.4 m. For the LISA observatory, a distance of 2.5×10^9 m is planned.

So far gravimeters are instruments of geophysics and therefore mounted directly on the ground. Future *astronomical* gravimeters will be mounted avoiding sound transmission and therefore have little response to earthquakes. This change will lower the PSD of gravimeters to values below $10^{-22}\ \frac{m^2}{s^4 Hz}$.

3 Signal bandwidth and the recording period

Not only does the Earth affect gravimeters, but so do the Moon, the Sun, the planets, and certainly distant binary star systems. The result is a signal mixture of oscillations of different frequencies, which can be selectively investigated with the known methods of communications engineering.

Gravimeters receive a very wide frequency range between 10^{-8} Hz and about 100 Hz, which is dominated by very strong tidal signals ($f \approx 11\ \mu$Hz). The amplitudes of the suspected GW in the range around 10 mHz are at least a factor of 10^7 weaker than the tides and can only be detected if the recordings of the gravimeters are processed with the lowest possible bandwidth. The reason is given by communications engineering: Each kind of signal transmission requires a certain frequency range, called channel bandwidth. Too narrow a bandwidth distorts the signal and generates errors. Too much bandwidth allows unwanted frequencies and unnecessary noise to pass through, degrading the signal-to-noise ratio (SNR).

The choice of bandwidth BW determines the average amplitude A_{noise} of the interfering noise after the filter:

$$A_{noise} = \sqrt{PSD \cdot BW} \tag{1}$$

One cannot narrow the bandwidth of the signal processing arbitrarily in order to eliminate the disturbing noise. Because then the necessary time span T, which the filter needs to settle down, increases. This relationship was first formulated by Küpfmüller and is reminiscent of the Heisenberg uncertainty principle.

$$T \cdot BW \geq 0.5 \tag{2}$$

If one looks for weak GW signals in the recordings of gravimeters, this means: If one wants to reduce the amplitude of the interfering noise to $10^{-14}\ \frac{m}{s^2}$, the bandwidth of the filters must not exceed 1 nHz (formula (1)). Because of (2), gravimeters must be operated for at least 15 years and the frequency of the GW must not vary by more than 0.5 nHz during the entire period to keep the signal within the filter range. Some gravimeters have been recording data for more than 20 years [7].

4 The reception of an idealized GW

Let us assume that a binary star system generates a GW of constant frequency and the distance to the Earth remains constant. When the GW passes the Earth, the diameter L oscillates in the same rhythm with the maximum amplitude ΔL. The strain h is calculated with the approach

$$h = \Delta L/L = h_0 \cdot sin(\omega t) \tag{3}$$

How large is h_0? Previous estimates give values between 10^{-25} for fast rotating pulsars and 10^{-19} for binary systems in our Galaxy. Very close binary star systems, which have not been noticed by electromagnetic waves so far, could cause even higher strain.

For the following calculations we assume that a close double star system generates a GW of constant frequency $\omega = 0.5\ s^{-1}$ and the strain h_0 here on Earth has the value 10^{-20}. Thus the change of the local gravity $\ddot{L}$ near the Earth surface is

$$\ddot{L} = L \cdot \omega^2 \cdot h_0 = 12.7 \times 10^6\ m \cdot (0.5\ \frac{1}{s})^2 \cdot 10^{-20} \approx 3 \times 10^{-14}\ \frac{m}{s^2} \tag{4}$$

This value corresponds approximately to the limit $10^{-14}\ \frac{m}{s^2}$ reached by (from astronomical point of view unfavorably mounted) gravimeters. This GW can be detected with gravimeters and appears in the spectrum as a narrow spectral line exceeding the level of background noise (SNR $\approx$ 9). Summing the signal with a selective integrator, the cumulative amplitude increases in proportion to the measurement duration because the amplitude of the GW is constant for years [8].

5 The reception of a real GW

No GW can satisfy these idealized assumptions because, first, the source radiates gravitational energy and therefore increases the orbital frequency. The value of the frequency drift is not known a priori. Secondly, the received frequency oscillates in the annual rhythm (Fig 1) because the Earth moves in the wave field of the GW (Doppler effect).

At $f_{GW} = 0.1$ Hz and low ecliptic latitude of the source, the frequency deviation caused by this phase modulation can reach the maximum value of

$$\Delta f = f_{GW} \cdot \left(\sqrt{\frac{c + v_{Earth}}{c - v_{Earth}}} - 1 \right) \approx 10\ \mu Hz \tag{5}$$

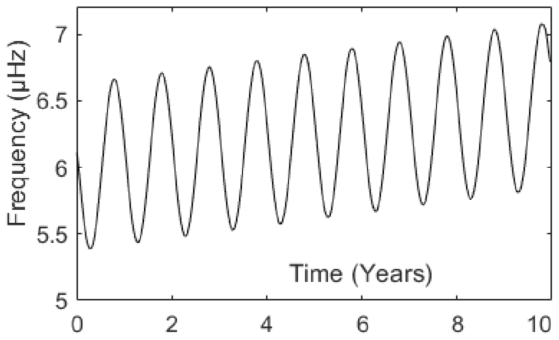

Figure 1): *Example HM Cancri: The image shows the measured frequency wobble and drift of the GW as a function of time. The average value of the GW frequency is 6220 μHz and was reduced to 6 μHz before processing. (Principle of the superhet).*

To process an FM signal with this frequency deviation without distortion, the filter bandwidth must be at least twice as wide (Carson rule). This value exceeds the desired bandwidth of 1 nHz (see Section 3) by orders of magnitude and makes it unlikely to detect a GW. There are several reasons for this:

- When the bandwidth increases by a factor of 20,000, the noise level increases by a factor of 141 (formula (1)) and the SNR drops dramatically.
- Approximately 20 μHz / 31.69 nHz $\approx$ 630 closely spaced spectral lines with mutual separation f_{year} = 31.69 nHz fill the entire bandwidth (see Fig 2). The defined phase relationships should not change.
- The GW transports a certain amount of energy, which is distributed approximately evenly over 630 spectral lines. This reduces the amplitudes of these lines, many of them will disappear in the noise.
- Probably the Milky Way hosts many thousands of GW sources of similar frequency and all of them fill similarly wide Carson bandwidths with their bundle of spectral lines. It is not easy to separate these overlapping spectra. FFT-based methods are hardly suitable for this purpose.

The same problems must be solved for the future LISA observatory; they cannot be eliminated by any modification of the sensors.

Probably the only way out: The signal processing must eliminate both the phase modulation and the frequency drift in order to be able to reduce the receiver bandwidth to $\sim 10^{-9}$ Hz. This measure increases the signal level of the central spectral line and lowers the noise level enough to identify weak GWs. In addition, one can focus on a single spectral line instead of a bundle of many hundreds of lines.

The *modified superhet method* (MSH) described in [9] removes all modulations of the GW signal (drift and phase modulation) and calculates properties of the GW source from it.

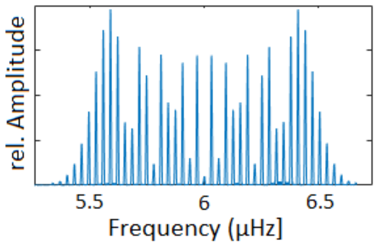

Figure 2): *The spectrum of a phase modulated oscillation of sufficiently long duration covers a large bandwidth. The modulation frequency determines the spacing of the lines.*

6 Summary

The test body *Earth* is sufficiently large and gravimeters are sensitive enough to measure GW in the frequency range 10^{-4} Hz to 1 Hz despite unfavorable mounting. The MSH method is a suitable algorithm to remove all interfering modulations of a GW and to reduce the bandwidth to such an extent that the GWs of many sources exceed the critical threshold value SNR > 1. The MSH method also allows to determine the direction in which to look for the GW source.

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