Analyzing some integrals linked with the MRB Constant. New possible mathematical connections with various parameters of Ramanujan's mathematics and several equations concerning some sectors of String Theory

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Abstract

In this paper, we analyze some integrals linked with the MRB Constant (Marvin Ray Burns Constant). We describe the new possible mathematical connections with various parameters of Ramanujan's mathematics and several equations concerning some sectors of String Theory

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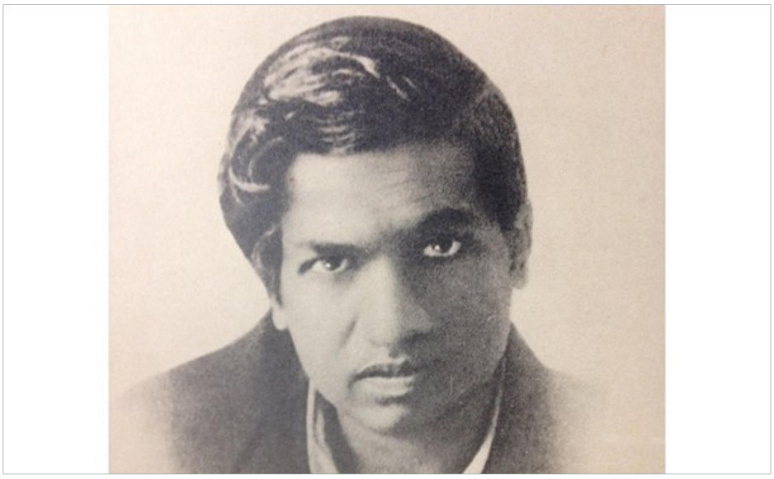
Srinivasa Ramanujan (1887-1920)



https://www.moduscc.it/ramanujan-il-grande-matematico-indiano-13453-131115/

Vesuvius Landscape with gorses – Naples (Italy)

https://www.pinterest.it/pin/95068242114589901/

From:**The MRB constant: ALL ABOARD!** POSTED BY: ***Marvin Ray Burns***

We now let's take a cue from the following integral

$$\text{CMRB} = -2\,i \int_1^{i\,\infty} \text{Im}\left[\frac{e^{\frac{\text{Log}[t]}{t}}}{-e^{(-i\,\pi\,t)} + e^{(i\,\pi\,t)}}\right] dt,\ x \in \mathbb{C}.$$

We perform the following calculations:

[-2*I*Integrate[Im[E^(x + Log[t]/t)/ (-E^((-I)*Pi*t + x) + E^(I*Pi*t + x))]]]

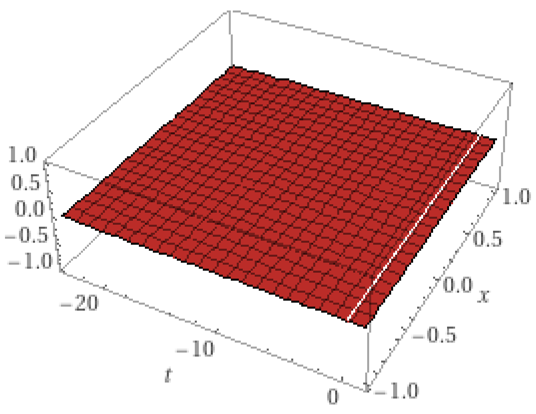
Indefinite integral

$$-2\,i \int \text{Im}\left(\frac{e^{x+\log(t)/t}}{-e^{-i\,\pi\,t+x} + e^{i\,\pi\,t+x}}\right) dx = -2\,i\,x\,\text{Im}\left(\frac{e^{i\,\pi\,t}\,\sqrt[t]{t}}{-1+e^{2\,i\,\pi\,t}}\right) + \text{constant}$$

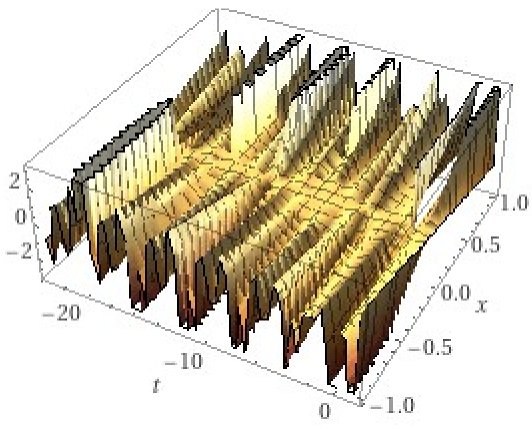
$\log(x)$ is the natural logarithm
$\text{Im}(z)$ is the imaginary part of z
i is the imaginary unit

3D plots

Real part

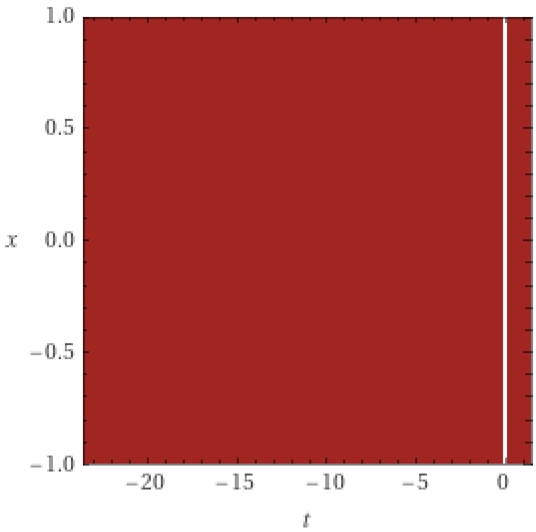


Imaginary part

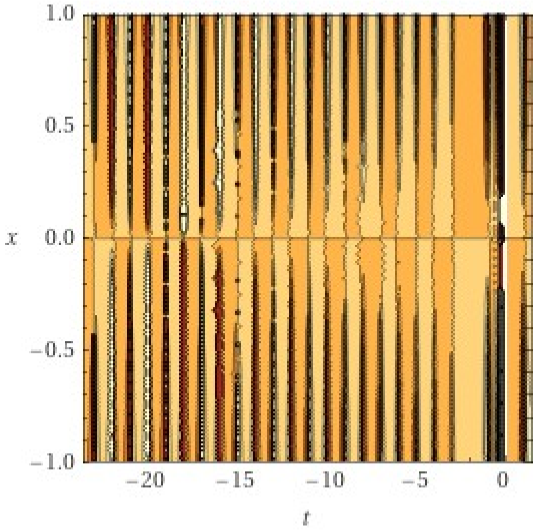


Contour plots

Real part



Imaginary part



Alternate forms

$i\,x\,\mathrm{Re}\left(\sqrt[t]{t}\,\csc(\pi\,t)\right)$

$-2\,i\,x\,\mathrm{Im}\left(\frac{e^{i\,\pi\,t}\,\sqrt[t]{t}}{\left(-1+e^{i\,\pi\,t}\right)\left(1+e^{i\,\pi\,t}\right)}\right)$

csc(x) is the cosecant function
Re(z) is the real part of z

Alternate forms assuming t and x are positive

$-2\,i\,\sqrt[t]{t}\;x\,\mathrm{Im}\left(\frac{e^{i\,\pi\,t}}{-1+e^{2\,i\,\pi\,t}}\right)$

$i\,\sqrt[t]{t}\;x\,\mathrm{Re}(\csc(\pi\,t))$

Alternate form assuming t and x are real

$i\,\sqrt[2\,t]{t^2}\;x\,\csc(\pi\,t)\cos\left(\frac{\arg(t)}{t}\right)$

arg(z) is the complex argument

Dividing the MRB Constant by the following alternate form:

$$-2\,i\,x\,\mathrm{Im}\left(\frac{e^{i\pi t}\sqrt[t]{t}}{\left(-1+e^{i\pi t}\right)\left(1+e^{i\pi t}\right)}\right)$$

we obtain:

(MRB constant)/(-2 i x Im((e^(i π t) t^(1/t))/((-1 + e^(i π t)) (1 + e^(i π t)))))

Input

$$-\frac{C_{\mathrm{MRB}}}{2\,i\,x\,\mathrm{Im}\left(\frac{e^{i\pi t}\sqrt[t]{t}}{\left(-1+e^{i\pi t}\right)\left(1+e^{i\pi t}\right)}\right)}$$

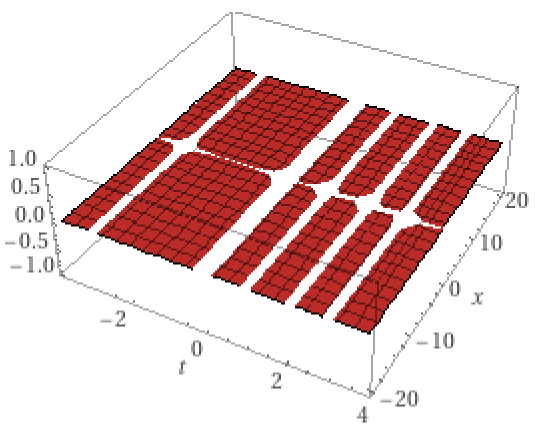
$\mathrm{Im}(z)$ is the imaginary part of z
C_{MRB} is the MRB constant
i is the imaginary unit

Exact result

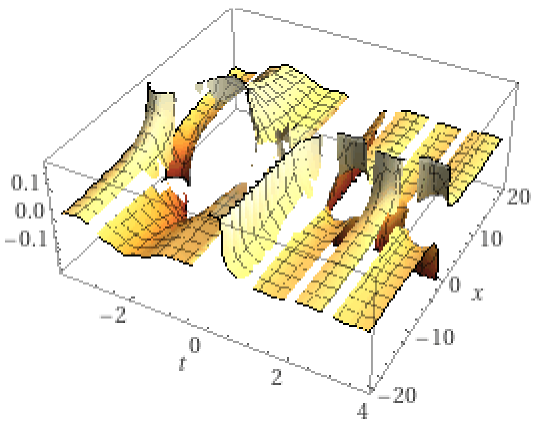
$$\frac{i\,C_{\mathrm{MRB}}}{2\,x\,\mathrm{Im}\left(\frac{e^{i\pi t}\sqrt[t]{t}}{\left(-1+e^{i\pi t}\right)\left(1+e^{i\pi t}\right)}\right)}$$

3D plots

Real part

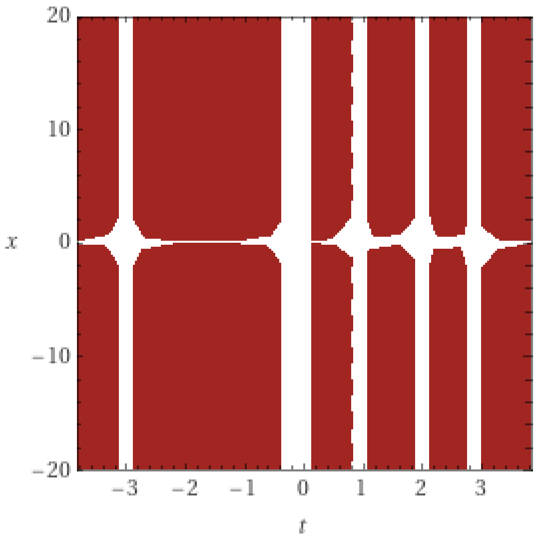


Imaginary part

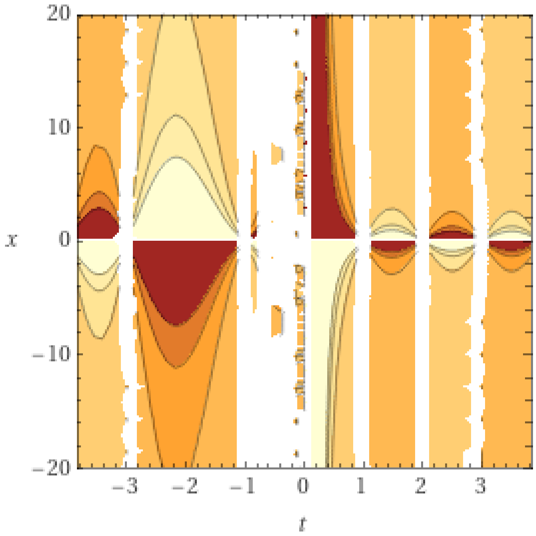


Contour plots

Real part



Imaginary part



Alternate forms

$$-\frac{i\, C_{\mathrm{MRB}}}{x\, \mathrm{Re}\left(\sqrt[t]{t}\, \csc(\pi t)\right)}$$

$$\frac{i\, C_{\mathrm{MRB}}}{2\, x\, \mathrm{Im}\left(\frac{e^{i \pi t}\, \sqrt[t]{t}}{-1+e^{2 i \pi t}}\right)}$$

$\csc(x)$ is the cosecant function
$\mathrm{Re}(z)$ is the real part of z

Alternate form assuming t and x are positive

$$-\frac{i\, t^{-1/t}\, C_{\mathrm{MRB}}}{x\, \mathrm{Re}(\csc(\pi t))}$$

Alternate form assuming t and x are real

$$(i\,C_{\text{MRB}}) \Bigg/ \left(2\,x\left(\sqrt[2t]{t^2}\,\cos\!\left(\frac{\arg(t)}{t}\right)\left(-\frac{\sin(\pi\,t)\left(\frac{\sin^2(\pi\,t)}{\sin^2(\pi\,t)+(\cos(\pi\,t)-1)^2}+\frac{(\cos(\pi\,t)-1)\cos(\pi\,t)}{\sin^2(\pi\,t)+(\cos(\pi\,t)-1)^2}\right)}{\sin^2(\pi\,t)+(\cos(\pi\,t)+1)^2} - \frac{\sin(\pi\,t)\,(\cos(\pi\,t)+1)}{\left(\sin^2(\pi\,t)+(\cos(\pi\,t)-1)^2\right)\left(\sin^2(\pi\,t)+(\cos(\pi\,t)+1)^2\right)}\right) + \sqrt[2t]{t^2}\,\sin\!\left(\frac{\arg(t)}{t}\right)\left(\frac{(\cos(\pi\,t)+1)\left(\frac{\sin^2(\pi\,t)}{\sin^2(\pi\,t)+(\cos(\pi\,t)-1)^2}+\frac{(\cos(\pi\,t)-1)\cos(\pi\,t)}{\sin^2(\pi\,t)+(\cos(\pi\,t)-1)^2}\right)}{\sin^2(\pi\,t)+(\cos(\pi\,t)+1)^2} - \frac{\sin^2(\pi\,t)}{\left(\sin^2(\pi\,t)+(\cos(\pi\,t)-1)^2\right)\left(\sin^2(\pi\,t)+(\cos(\pi\,t)+1)^2\right)}\right)\right)\right)$$

$\arg(z)$ is the complex argument

Roots

(no roots exist)

Derivative

$$\frac{\partial}{\partial x}\left(-\frac{C_{\text{MRB}}}{2\,i\,x\,\text{Im}\!\left(\frac{e^{i\,\pi\,t}\,\sqrt[t]{t}}{(-1+e^{i\,\pi\,t})(1+e^{i\,\pi\,t})}\right)}\right) = -\frac{i\,C_{\text{MRB}}}{2\,x^2\,\text{Im}\!\left(\frac{e^{i\,\pi\,t}\,\sqrt[t]{t}}{-1+e^{2\,i\,\pi\,t}}\right)}$$

Indefinite integral

$$\int \frac{i\,C_{\text{MRB}}}{2\,x\,\text{Im}\!\left(\frac{e^{i\,\pi\,t}\,\sqrt[t]{t}}{(-1+e^{i\,\pi\,t})(1+e^{i\,\pi\,t})}\right)}\,dx = \frac{i\log(x)\,C_{\text{MRB}}}{2\,\text{Im}\!\left(\frac{e^{i\,\pi\,t}\,\sqrt[t]{t}}{-1+e^{2\,i\,\pi\,t}}\right)} + \text{constant}$$

(assuming a complex-valued logarithm)

$\log(x)$ is the natural logarithm

From the exact result

$$\frac{i\,C_{\text{MRB}}}{2\,x\,\text{Im}\left(\frac{e^{i\,\pi\,t}\,\sqrt[t]{t}}{\left(-1+e^{i\,\pi\,t}\right)\left(1+e^{i\,\pi\,t}\right)}\right)}$$

for t = -5 , we obtain:

(i MRB const)/(2 x Im((e^(i π -5) t^(1/-5))/((-1 + e^(i π -5)) (1 + e^(i π -5)))))

Input

$$\frac{i\,C_{\text{MRB}}}{2\,x\,\text{Im}\left(\frac{e^{i\,\pi-5}\,t^{-1/5}}{\left(-1+e^{i\,\pi-5}\right)\left(1+e^{i\,\pi-5}\right)}\right)}$$

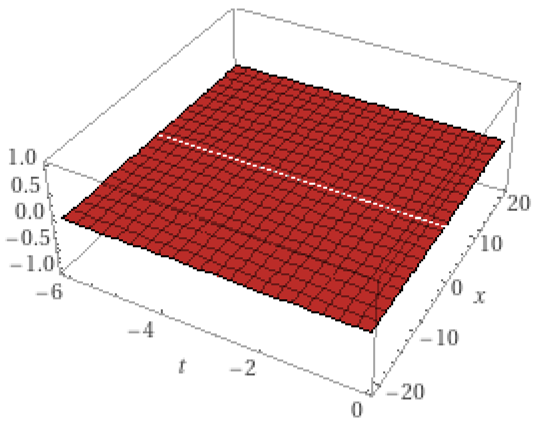
Im(z) is the imaginary part of z
i is the imaginary unit
C_{MRB} is the MRB constant

Exact result

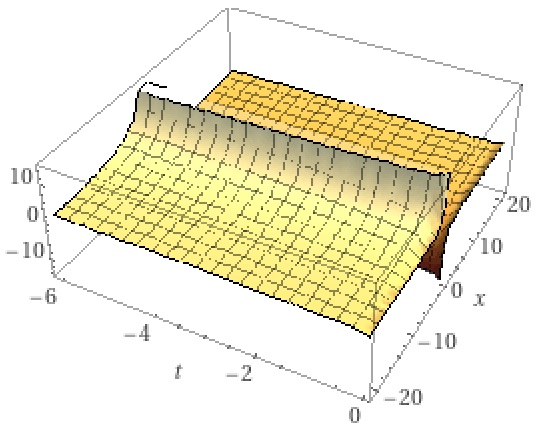
$$-\frac{i\left(-1-\frac{1}{e^5}\right)\left(1-\frac{1}{e^5}\right)e^5\,C_{\text{MRB}}}{2\,x\,\text{Im}\left(\frac{1}{\sqrt[5]{t}}\right)}$$

3D plots

Real part



Imaginary part



Alternate forms

$$\frac{i \sinh(5) C_{\text{MRB}}}{x \operatorname{Im}\left(\frac{1}{\sqrt[5]{t}}\right)}$$

$$\frac{i \left(e^5 - 1\right) \left(1 + e^5\right) C_{\text{MRB}}}{2 e^5 x \operatorname{Im}\left(\frac{1}{\sqrt[5]{t}}\right)}$$

$$\frac{i (e - 1) (1 + e) \left(1 - e + e^2 - e^3 + e^4\right) \left(1 + e + e^2 + e^3 + e^4\right) C_{\text{MRB}}}{2 e^5 x \operatorname{Im}\left(\frac{1}{\sqrt[5]{t}}\right)}$$

$\sinh(x)$ is the hyperbolic sine function

Alternate form assuming t and x are positive

$\tilde{\infty}$

$\tilde{\infty}$ is complex infinity

Expanded form

$$\frac{i\, e^5\, C_{\mathrm{MRB}}}{2\, x\, \mathrm{Im}\left(\frac{1}{\sqrt[5]{t}}\right)} - \frac{i\, C_{\mathrm{MRB}}}{2\, e^5\, x\, \mathrm{Im}\left(\frac{1}{\sqrt[5]{t}}\right)}$$

Alternate form assuming t and x are real

$$-\frac{i\left(e^{10}-1\right)\sqrt[10]{t^2}\, \csc\left(\frac{\arg(t)}{5}\right) C_{\mathrm{MRB}}}{2\, e^5\, x}$$

$\arg(z)$ is the complex argument
$\csc(x)$ is the cosecant function

Roots

(no roots exist)

Derivative

$$\frac{\partial}{\partial x}\left(\frac{i\, C_{\mathrm{MRB}}}{2\, x\, \mathrm{Im}\left(\frac{e^{i\pi-5}\, t^{-1/5}}{\left(-1+e^{i\pi-5}\right)\left(1+e^{i\pi-5}\right)}\right)}\right) = -\frac{i\left(e^{10}-1\right) C_{\mathrm{MRB}}}{2\, e^5\, x^2\, \mathrm{Im}\left(\frac{1}{\sqrt[5]{t}}\right)}$$

Indefinite integral

$$\int -\frac{i\left(-1-\frac{1}{e^5}\right)\left(1-\frac{1}{e^5}\right) e^5\, C_{\mathrm{MRB}}}{2\, x\, \mathrm{Im}\left(\frac{1}{\sqrt[5]{t}}\right)}\, dx = \frac{i\left(e^5-1\right)\left(1+e^5\right)\log(x)\, C_{\mathrm{MRB}}}{2\, e^5\, \mathrm{Im}\left(\frac{1}{\sqrt[5]{t}}\right)} + \text{constant}$$

(assuming a complex-valued logarithm)

$\log(x)$ is the natural logarithm

From:

$$\frac{i\, C_{\mathrm{MRB}}}{2\, x\, \mathrm{Im}\left(\frac{e^{i\pi t}\sqrt[t]{t}}{(-1+e^{i\pi t})(1+e^{i\pi t})}\right)}$$

and

$$-\frac{i\left(-1-\frac{1}{e^5}\right)\left(1-\frac{1}{e^5}\right)e^5\, C_{\mathrm{MRB}}}{2\, x\, \mathrm{Im}\left(\frac{1}{\sqrt[5]{t}}\right)}$$

we observe that:

(i MRB const)/(2 x Im((e^(i π -5) t^(1/-5))/((-1 + e^(i π -5)) (1 + e^(i π -5))))) = -(i (-1 - 1/e^5) (1 - 1/e^5) e^5 MRB const)/(2 x Im(1/t^(1/5)))

Input

$$\frac{i\, C_{\mathrm{MRB}}}{2\, x\, \mathrm{Im}\left(\frac{e^{i\pi-5}\, t^{-1/5}}{(-1+e^{i\pi-5})(1+e^{i\pi-5})}\right)} = -\frac{i\left(-1-\frac{1}{e^5}\right)\left(1-\frac{1}{e^5}\right)e^5\, C_{\mathrm{MRB}}}{2\, x\, \mathrm{Im}\left(\frac{1}{\sqrt[5]{t}}\right)}$$

$\mathrm{Im}(z)$ is the imaginary part of z
i is the imaginary unit
C_{MRB} is the MRB constant

Result
True

Multiplying both the sides by

(2 x Im((e^(i π -5) t^(1/-5))/((-1 + e^(i π -5)) (1 + e^(i π -5)))))

we obtain:

-(i (-1 - 1/e^5) (1 - 1/e^5) e^5 MRB const)/(2 x Im(1/t^(1/5)))*(((2 x Im((e^(i π -5) t^(1/-5))/((-1 + e^(i π -5)) (1 + e^(i π -5)))))))

Input

$$-\frac{i\left(-1-\frac{1}{e^5}\right)\left(1-\frac{1}{e^5}\right)e^5\,C_{\mathrm{MRB}}}{2\,x\,\mathrm{Im}\left(\frac{1}{\sqrt[5]{t}}\right)}\left(2\,x\,\mathrm{Im}\left(\frac{e^{i\,\pi-5}\,t^{-1/5}}{\left(-1+e^{i\,\pi-5}\right)\left(1+e^{i\,\pi-5}\right)}\right)\right)$$

$\mathrm{Im}(z)$ is the imaginary part of z
i is the imaginary unit
C_{MRB} is the MRB constant

Exact result

$i\,C_{\mathrm{MRB}}$

Decimal approximation

0.1878596424620671202485179340542732300559030949001387861720046840...
i

(using the principal branch of the logarithm for complex exponentiation)

Alternate complex forms

$$C_{\mathrm{MRB}}\left(\cos\left(\frac{\pi}{2}\right)+i\sin\left(\frac{\pi}{2}\right)\right)$$

Approximate form

$e^{(i\,\pi)/2}\,C_{\mathrm{MRB}}$

Polar coordinates

$r = C_{\mathrm{MRB}}$ (radius), $\theta = \frac{\pi}{2}$ (angle)

C_{MRB} = 0.187859642462….

0.187859642462… ≈ MRB Constant

Now, we verify the following integral:

$$C_{MRB}=-2i\int_{1}^{i\infty}\Im\left(\frac{e^{\frac{\log(t)}{t}}e^{i\pi t}}{-e^{0}+e^{2i\pi t}}\right)dt$$

indeed, we obtain:

-2 I*NIntegrate[Im[(E^(I*Pi*t + Log[t]/t))/(-1 + E^((2*I)*Pi*t))], {t, 1, 10^7 I}, WorkingPrecision -> 20]

Input

$$-2\,i\,\text{NIntegrate}\left[\text{Im}\left(\frac{e^{i\pi t+\log(t)/t}}{-1+e^{(2i)\pi t}}\right),\{t,1,10^7\,i\},\text{WorkingPrecision}\to 20\right]$$

$\log(x)$ is the natural logarithm
$\text{Im}(z)$ is the imaginary part of z
i is the imaginary unit

Computation result

$$-2\,i\,\text{NIntegrate}\left[\text{Im}\left(\frac{e^{i\pi t+\log(t)/t}}{-1+e^{(2i)\pi t}}\right),\{t,1,10^7\,i\},\text{WorkingPrecision}\to 20\right]=$$

SlowLarge t

Decimal approximation

$0.18785602000739089 6939 +$
$1.8785602000739\times 10^{-8}\,i$

Alternate complex forms

$0.187856020007390026219\left(\cos\left(1.00000000000000\times 10^{-7}\right)+i\sin\left(1.00000000000000\times 10^{-7}\right)\right)$

$0.187856020007390026219\,e^{1.00000000000000\times 10^{-7}\,i}$

Polar coordinates

$r = 0.187856020007390026219$ (radius), $\theta = 1.00000000000000\times 10^{-7}$ (angle)

0.1878596020007390026219 ≈ MRB Constant

From which, after some calculations:

4+MRB const + (((64*5+4)*1/((-2 I*NIntegrate[Im[(E^(I*Pi*t + Log[t]/t))/(-1 + E^((2*I)*Pi*t))], {t, 1, 10^7 I}, WorkingPrecision -> 20]))))

Input

$$4 + C_{\text{MRB}} + (64\times 5+4)\left(-\frac{1}{2\,i\,\text{NIntegrate}\left[\text{Im}\left(\frac{e^{i\pi t+\log(t)/t}}{-1+e^{(2i)\pi t}}\right),\{t,1,10^7\,i\},\text{WorkingPrecision}\to 20\right]}\right)$$

$\log(x)$ is the natural logarithm
$\text{Im}(z)$ is the imaginary part of z
C_{MRB} is the MRB constant
i is the imaginary unit

Computation result

$$4 + C_{\text{MRB}} + -\frac{64 \times 5 + 4}{2\,i\,\text{NIntegrate}\left[\text{Im}\left(\frac{e^{i\pi t+\log(t)/t}}{-1+e^{(2i)\pi t}}\right), \{t, 1, 10^7\, i\}, \text{WorkingPrecision} \to 20\right]} =$$

SlowLarge t

Decimal approximation

1728.91299747542912354 −
0.0001724725137833 i

Alternate complex forms

$$1728.91299747543772628\left(\cos\left(-9.975777499223\times 10^{-8}\right) + i\sin\left(-9.975777499223\times 10^{-8}\right)\right)$$

$$1728.91299747543772628\, e^{-9.975777499223\times 10^{-8}\, i}$$

Polar coordinates

$r = 1728.91299747543772628$ (radius), $\theta = -9.975777499223\times 10^{-8}$ (angle)

1728.91299747543772628

This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. ($1728 = 8^2 * 3^3$) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number) and again:

((4+MRB const + (((64*5+4)*1/((-2 I*NIntegrate[Im[(E^(I*Pi*t + Log[t]/t))/(-1 + E^((2*I)*Pi*t))], {t, 1, 10^7 I}, WorkingPrecision -> 20]))))))^1/15

Input

$$\left(4 + C_{\mathrm{MRB}} + (64 \times 5 + 4)\left(-\frac{1}{2\,i\,\mathrm{NIntegrate}\left[\mathrm{Im}\left(\frac{e^{i\,\pi\,t + \log(t)/t}}{-1+e^{(2\,i)\,\pi\,t}}\right), \{t, 1, 10^7\,i\}, \mathrm{WorkingPrecision} \to 20\right]}\right)\right)^{(1/15)}$$

$\log(x)$ is the natural logarithm
$\mathrm{Im}(z)$ is the imaginary part of z
C_{MRB} is the MRB constant
i is the imaginary unit

Computation result

$$\left(4 + C_{\mathrm{MRB}} + -\frac{64 \times 5 + 4}{2\,i\,\mathrm{NIntegrate}\left[\mathrm{Im}\left(\frac{e^{i\,\pi\,t + \log(t)/t}}{-1+e^{(2\,i)\,\pi\,t}}\right), \{t, 1, 10^7\,i\}, \mathrm{WorkingPrecision} \to 20\right]}\right)^{(1/15)} = \mathrm{SlowLarge}\,t$$

Decimal approximation

$1.643809714214959302521 - 1.0932186640047 \times 10^{-8}\,i$

Alternate complex forms

$1.6438097142149593388873 \left(\cos\left(-6.6505183328156\times 10^{-9}\right) + i \sin\left(-6.6505183328156\times 10^{-9}\right)\right)$

$1.6438097142149593388873\, e^{-6.6505183328156\times 10^{-9}\, i}$

Polar coordinates

$r = 1.6438097142149593388873$ (radius), $\theta = -6.6505183328156\times 10^{-9}$ (angle)

1.6438097142149593388873 $\zeta(2) = \frac{\pi^2}{6} = 1.644934\ldots$ (trace of the instanton shape)

1/27(3+MRB const + (((64*5+4)*1/((-2 I*NIntegrate[Im[(E^(I*Pi*t + Log[t]/t))/(-1 + E^((2*I)*Pi*t))], {t, 1, 10^7 I}, WorkingPrecision -> 20])))))

Input

$$\frac{1}{27}\left(3 + C_{\text{MRB}} + (64\times 5+4)\left(-\frac{1}{2\, i\, \text{NIntegrate}\left[\text{Im}\left(\frac{e^{i\pi t+\log(t)/t}}{-1+e^{(2i)\pi t}}\right), \{t, 1, 10^7 i\}, \text{WorkingPrecision} \to 20\right]}\right)\right)$$

$\log(x)$ is the natural logarithm
$\text{Im}(z)$ is the imaginary part of z
C_{MRB} is the MRB constant
i is the imaginary unit

Computation result

$$\frac{1}{27}\left(3 + C_{\text{MRB}} + -\frac{64 \times 5 + 4}{2\,i\ \text{NIntegrate}\left[\text{Im}\left(\frac{e^{i\pi t+\log(t)/t}}{-1+e^{(2i)\pi t}}\right), \{t, 1, 10^7\, i\}, \text{WorkingPrecision} \to 20\right]}\right) =$$

SlowLarge
t

Decimal approximation

63.996777684275152724 −
$6.3878708808628 \times 10^{-6}\, i$

From the decimal approximation, we obtain:

(63.996777684275152724 -6.3878708808628 × 10^-6 i)^2+1/2

Input interpretation

$$(63.996777684275152724 - 6.3878708808628 \times 10^{-6}\, i)^2 + \frac{1}{2}$$

i is the imaginary unit

Result

4096.0875539704973744… −
0.00081760630527686300356… i

Alternate complex forms

$$4096.0875539705789742\left(\cos(-1.9960664768611 \times 10^{-7}) + i \sin(-1.9960664768611 \times 10^{-7})\right)$$

$4096.0875539705789742\, e^{-1.9960664768611\times 10^{-7}\, i}$

Polar coordinates

$r = 4096.0875539705789742$ (radius), $\theta = -1.9960664768611\times 10^{-7}$ (angle)

$4096.0875539705789742 \approx 4096 = 64^2$

We now let's take a cue from the following integral

$$\mathbf{CMRB} = \mathrm{Re}\left(\int_0^{i\infty} \frac{g(-t)}{\sin(\pi t)\cos(\pi t)\, i + \sin^2(\pi t)}\, dt\right),$$

we perform the following calculations:

NIntegrate[g[-t]/(Sin[Pi*t]*Cos[Pi*t]*I + Sin[Pi*t]^2), {t, 0, I*Infinity}

Input

$$\int_0^{i\infty} \frac{g(-t)}{\sin(\pi t)\cos(\pi t)\, i + \sin^2(\pi t)}\, dt$$

$$\int_0^{i\infty} \frac{g(-t)}{\sin(\pi t)\cos(\pi t)\, i + \sin^2(\pi t)}\, dt$$

integral (((-1)^x (1-(x+1)^(1/(x+1)))/(sin(π t) cos(π t) i + sin^2(π t)))) dt

Indefinite integral

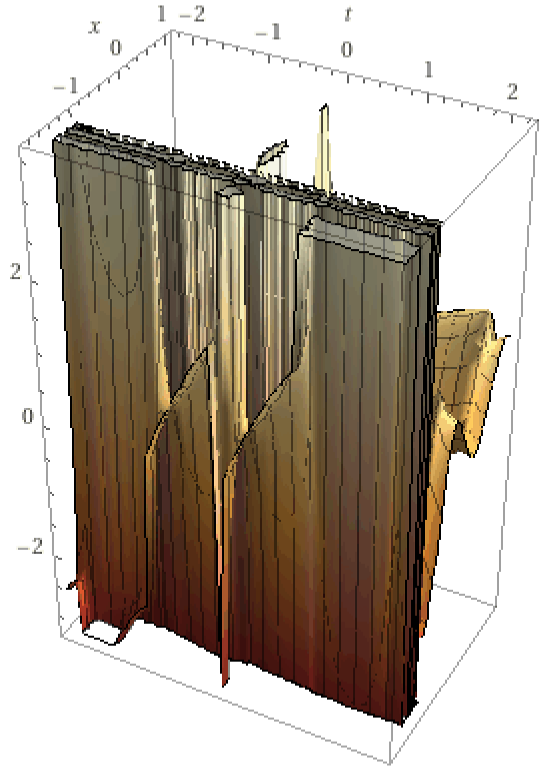
$$\int \frac{(-1)^x\left(1-\sqrt[x+1]{x+1}\right)}{\sin(\pi t)\cos(\pi t)\, i + \sin^2(\pi t)}\, dt = i(-1)^x\left(\sqrt[x+1]{x+1}-1\right)\left(\frac{\log(\sin(\pi t))}{\pi} + i\, t\right) + \text{constant}$$

$\log(x)$ is the natural logarithm

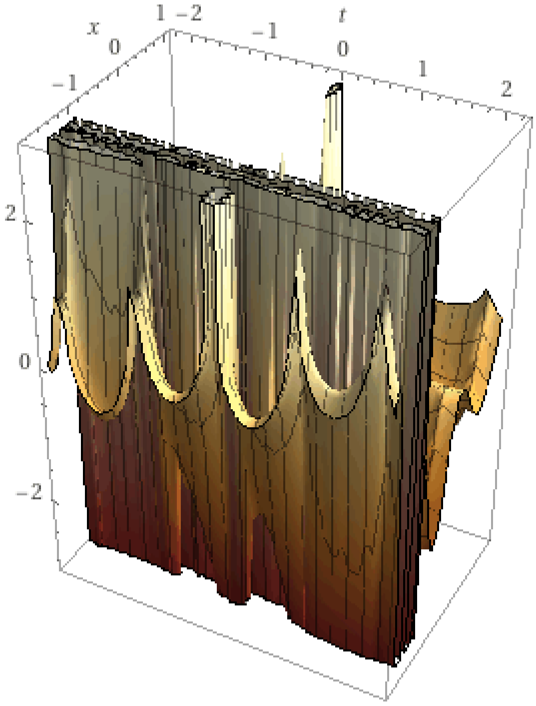
i is the imaginary unit

3D plots

Real part



Imaginary part



Alternate forms of the integral

$$\frac{(-1)^{x+1}\left(\sqrt[x+1]{x+1}-1\right)(\pi t - i \log(\sin(\pi t)))}{\pi} + \text{constant}$$

$$\frac{i(-1)^{x+1/2}\left(\sqrt[x+1]{x+1}-1\right)(\pi t - i \log(\sin(\pi t)))}{\pi} + \text{constant}$$

$$i(-1)^{x}\left(\sqrt[x+1]{x+1}-1\right)\left(i t + \frac{\log\left(\frac{1}{2} i \left(e^{-i \pi t} - e^{i \pi t}\right)\right)}{\pi}\right) + \text{constant}$$

Expanded form of the integral

$$t\left(-(-1)^{x}\right)\sqrt[x+1]{x+1} + t(-1)^{x} + \frac{i(-1)^{x}\sqrt[x+1]{x+1}\log(\sin(\pi t))}{\pi} - \frac{i(-1)^{x}\log(\sin(\pi t))}{\pi} + \text{constant}$$

Alternate forms assuming t and x are positive

$$-\frac{e^{i \pi x} \left(\sqrt[x+1]{x+1} - 1\right) (\pi t - i \log(\sin(\pi t)))}{\pi} + \text{constant}$$

$$t \left(-e^{i \pi x}\right) \sqrt[x+1]{x+1} + t e^{i \pi x} + \frac{i e^{i \pi x} \sqrt[x+1]{x+1} \log(\sin(\pi t))}{\pi} - \frac{i e^{i \pi x} \log(\sin(\pi t))}{\pi} + \text{constant}$$

Series expansion of the integral at x=0

$$x \left(-t + \frac{i \log(\sin(\pi t))}{\pi}\right) + O\left(x^2\right)$$

(Taylor series)

Series expansion of the integral at x=∞

$$e^{i \pi x} \left(\frac{\log(x) \left(\frac{i \log(\sin(\pi t))}{\pi} - t\right)}{x} + O\left(\left(\frac{1}{x}\right)^2\right)\right)$$

From:

$$-\frac{e^{i \pi x} \left(\sqrt[x+1]{x+1} - 1\right) (\pi t - i \log(\sin(\pi t)))}{\pi} + \text{constant}$$

we obtain:

-(e^(i π x) ((x + 1)^(1/(x + 1)) - 1) (π t - i log(sin(π t))))/π

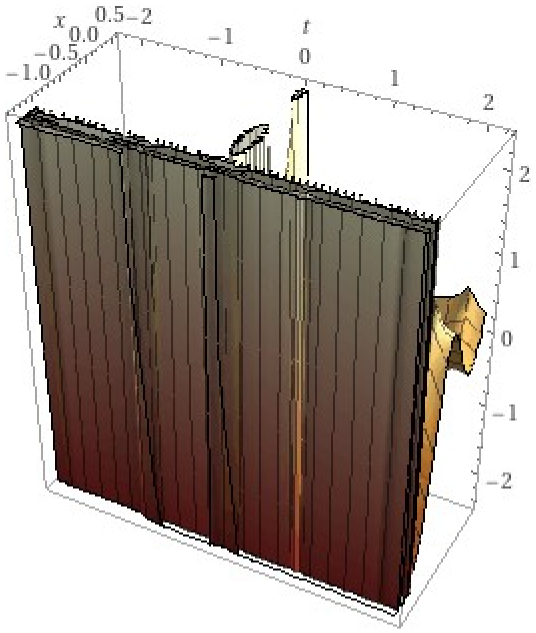
Input

$$-\frac{e^{i\pi x}\left(\sqrt[x+1]{x+1}-1\right)(\pi t - i\log(\sin(\pi t)))}{\pi}$$

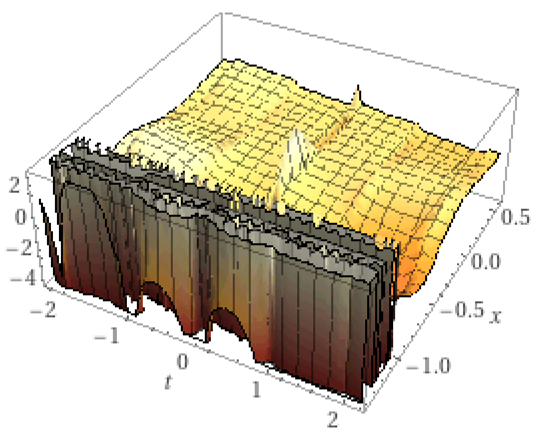
log(x) is the natural logarithm
i is the imaginary unit

3D plots

Real part



Imaginary part



Alternate forms

$$-\frac{e^{i\pi x}\left(\sqrt[x+1]{x+1}-1\right)\left(\pi t-i\log\left(\frac{1}{2}\,i\left(e^{-i\pi t}-e^{i\pi t}\right)\right)\right)}{\pi}$$

$$\frac{e^{i\pi x}(\pi t-i\log(\sin(\pi t)))}{\pi}-\frac{e^{i\pi x}\sqrt[x+1]{x+1}\,(\pi t-i\log(\sin(\pi t)))}{\pi}$$

Expanded form

$$t\left(-e^{i\pi x}\right)\sqrt[x+1]{x+1}+t\,e^{i\pi x}+\frac{i\,e^{i\pi x}\sqrt[x+1]{x+1}\,\log(\sin(\pi t))}{\pi}-\frac{i\,e^{i\pi x}\log(\sin(\pi t))}{\pi}$$

Root for the variable x

$x=0$

Series expansion at x=0

$$x\left(-t+\frac{i\log(\sin(\pi t))}{\pi}\right)+O\left(x^2\right)$$

(Taylor series)

Series expansion at x=∞

$$e^{i\pi x}\left(\frac{\log(x)\left(\frac{i\log(\sin(\pi t))}{\pi}-t\right)}{x}+O\left(\left(\frac{1}{x}\right)^2\right)\right)$$

Derivative

$$\frac{\partial}{\partial x}\left(-\frac{e^{i\pi x}\left(\sqrt[x+1]{x+1}-1\right)(\pi t - i\log(\sin(\pi t)))}{\pi}\right) = \frac{1}{\pi (x+1)^2} e^{i\pi x}\left(-\sqrt[x+1]{x+1} - i\pi\left(\sqrt[x+1]{x+1}-1\right)(x+1)^2 + \sqrt[x+1]{x+1}\log(x+1)\right) (\pi t - i\log(\sin(\pi t)))$$

Alternative representations

$$-\frac{e^{i\pi x}\left(\sqrt[x+1]{x+1}-1\right)(\pi t - i\log(\sin(\pi t)))}{\pi} = -\frac{\left(\pi t - i\log_e(\sin(\pi t))\right) e^{i\pi x}\left(-1+\sqrt[1+x]{1+x}\right)}{\pi}$$

$$-\frac{e^{i\pi x}\left(\sqrt[x+1]{x+1}-1\right)(\pi t - i\log(\sin(\pi t)))}{\pi} = -\frac{(\pi t - i(\log(a)\log_a(\sin(\pi t)))) e^{i\pi x}\left(-1+\sqrt[1+x]{1+x}\right)}{\pi}$$

$$-\frac{e^{i\pi x}\left(\sqrt[x+1]{x+1}-1\right)(\pi t - i\log(\sin(\pi t)))}{\pi} = -\frac{z^{i\pi x}\left(\sqrt[x+1]{x+1}-1\right)\left(\pi t - i\log_e(\sin(\pi t))\right)}{\pi} \quad \text{for } z = e$$

$\log_b(x)$ is the base- b logarithm

Series representations

$$-\frac{e^{i\pi x}\left(\sqrt[x+1]{x+1}-1\right)(\pi t - i\log(\sin(\pi t)))}{\pi} = -\frac{\left(-1+\sqrt[1+x]{1+x}\right)\left(\sum_{k=0}^{\infty}\frac{\pi^k (i x)^k}{k!}\right)\left(\pi t + i\sum_{k=1}^{\infty}\frac{(-1)^k(-1+\sin(\pi t))^k}{k}\right)}{\pi}$$

for $|-1+\sin(\pi t)| < 1$

$$-\frac{e^{i\pi x}\left(\sqrt[x+1]{x+1}-1\right)(\pi t-i\log(\sin(\pi t)))}{\pi}=$$
$$-\frac{e^{z_0}\left(-1+\sqrt[1+x]{1+x}\right)\left(\pi t+i\sum_{k=1}^{\infty}\frac{(-1)^k(-1+\sin(\pi t))^k}{k}\right)\sum_{k=0}^{\infty}\frac{(i\pi x-z_0)^k}{k!}}{\pi}$$
for $|-1+\sin(\pi t)|<1$

$$-\frac{e^{i\pi x}\left(\sqrt[x+1]{x+1}-1\right)(\pi t-i\log(\sin(\pi t)))}{\pi}=$$
$$-\frac{1}{\pi}\left(-1+\sqrt[1+x]{1+x}\right)\left(\sum_{k=0}^{\infty}\frac{\pi^k(i x)^k}{k!}\right)\left(\pi t-i\log(-1+\sin(\pi t))+\right.$$
$$\left.i\sum_{k=1}^{\infty}\frac{(-1)^k(-1+\sin(\pi t))^{-k}}{k}\right)$$
for $|-1+\sin(\pi t)|>1$

$n!$ is the factorial function
$|z|$ is the absolute value of z

Integral representations

$$-\frac{e^{i\pi x}\left(\sqrt[x+1]{x+1}-1\right)(\pi t-i\log(\sin(\pi t)))}{\pi}=$$
$$-\frac{e^{i\pi x}\left(-1+\sqrt[1+x]{1+x}\right)\left(\pi t-i\int_1^{\sin(\pi t)}\frac{1}{\tau}\,d\tau\right)}{\pi}$$

$$-\frac{e^{i\pi x}\left(\sqrt[x+1]{x+1}-1\right)(\pi t-i\log(\sin(\pi t)))}{\pi}=$$
$$-\frac{e^{i\pi x}\left(-1+\sqrt[1+x]{1+x}\right)\left(\pi t-i\log\left(\pi t\int_0^1\cos(\pi t\tau)\,d\tau\right)\right)}{\pi}$$

$$-\frac{e^{i\pi x}\left(\sqrt[x+1]{x+1}-1\right)(\pi t-i\log(\sin(\pi t)))}{\pi}=$$
$$-\frac{e^{i\pi x}\left(-1+\sqrt[1+x]{1+x}\right)\left(\pi t-i\log\left(-\frac{1}{4}i\sqrt{\pi}\,t\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{s-(\pi^2t^2)/(4s)}}{s^{3/2}}\,ds\right)\right)}{\pi}$$
for $\gamma>0$

Multiple-argument formulas

$$-\frac{e^{i\pi x}\left(\sqrt[x+1]{x+1}-1\right)(\pi t-i\log(\sin(\pi t)))}{\pi}=\frac{(-1)^{1+x}\left(-1+\sqrt[1+x]{1+x}\right)(\pi t-i\log(\sin(\pi t)))}{\pi}$$

$$-\frac{e^{i\pi x}\left(\sqrt[x+1]{x+1}-1\right)(\pi t-i\log(\sin(\pi t)))}{\pi}=-\frac{e^{i\pi x}\left(-1+\sqrt[1+x]{1+x}\right)\left(\pi t-i\log\left(3\sin\left(\frac{\pi t}{3}\right)-4\sin^3\left(\frac{\pi t}{3}\right)\right)\right)}{\pi}$$

$$-\frac{e^{i\pi x}\left(\sqrt[x+1]{x+1}-1\right)(\pi t-i\log(\sin(\pi t)))}{\pi}=-\frac{e^{i\pi x}\left(-1+\sqrt[1+x]{1+x}\right)\left(\pi t-i\log\left(2\cos\left(\frac{\pi t}{2}\right)\sin\left(\frac{\pi t}{2}\right)\right)\right)}{\pi}$$

from:

$$\frac{e^{i\pi x}(\pi t-i\log(\sin(\pi t)))}{\pi}-\frac{e^{i\pi x}\sqrt[x+1]{x+1}\,(\pi t-i\log(\sin(\pi t)))}{\pi}$$

for x = -0.5 and t = 1.5

(e^(i π*-0.5) (π *1.5 - i log(sin(π*1.5))))/π - (e^(i π *-0.5) (-0.5 + 1)^(1/(-0.5 + 1)) (π*1.5 - i log(sin(π *1.5))))/π

Input

$$\frac{e^{i\pi\times(-0.5)}(\pi\times 1.5-i\log(\sin(\pi\times 1.5)))}{\pi}-\frac{e^{i\pi\times(-0.5)}\sqrt[-0.5+1]{-0.5+1}\,(\pi\times 1.5-i\log(\sin(\pi\times 1.5)))}{\pi}$$

$\log(x)$ is the natural logarithm
i is the imaginary unit

Result

$-1.875\,i$

(using the principal branch of the logarithm for complex exponentiation)

-1.875 *i*

Alternate complex forms

$1.14807\times10^{-16} - 1.875\,i$

Radians

$1.875\,(\cos(-1.5708) + i\sin(-1.5708))$

$1.875\,e^{-1.5708\,i}$

Polar coordinates

$r = 1.875$ (radius), $\theta = -1.5708$ (angle)

1.875

Alternative representations

$$\frac{e^{i\pi(-0.5)}\,(\pi\,1.5 - i\log(\sin(\pi\,1.5)))}{\pi} - \frac{e^{i\pi(-0.5)}\,\sqrt[-0.5+1]{-0.5+1}\,(\pi\,1.5 - i\log(\sin(\pi\,1.5)))}{\pi} = \frac{(1.5\,\pi - i\log(\cos(-\pi)))\,e^{-0.5\,i\,\pi}}{\pi} - \frac{(1.5\,\pi - i\log(\cos(-\pi)))\,\sqrt[0.5]{0.5}\,e^{-0.5\,i\,\pi}}{\pi}$$

$$\frac{e^{i\pi(-0.5)}\,(\pi\,1.5 - i\log(\sin(\pi\,1.5)))}{\pi} - \frac{e^{i\pi(-0.5)}\,\sqrt[-0.5+1]{-0.5+1}\,(\pi\,1.5 - i\log(\sin(\pi\,1.5)))}{\pi} = \frac{(1.5\,\pi - i\log(-\cos(2\,\pi)))\,e^{-0.5\,i\,\pi}}{\pi} - \frac{(1.5\,\pi - i\log(-\cos(2\,\pi)))\,\sqrt[0.5]{0.5}\,e^{-0.5\,i\,\pi}}{\pi}$$

$$\frac{e^{i\pi(-0.5)}(\pi\, 1.5 - i\log(\sin(\pi\, 1.5)))}{\pi} - \frac{e^{i\pi(-0.5)}\sqrt[-0.5+1]{-0.5+1}\,(\pi\, 1.5 - i\log(\sin(\pi\, 1.5)))}{\pi} = \frac{(1.5\,\pi - i(\log(a)\log_a(\sin(1.5\,\pi))))\,e^{-0.5\,i\,\pi}}{\pi} - \frac{(1.5\,\pi - i(\log(a)\log_a(\sin(1.5\,\pi))))\,\sqrt[0.5]{0.5}\,e^{-0.5\,i\,\pi}}{\pi}$$

$\log_b(x)$ is the base- b logarithm

Series representations

$$\frac{e^{i\pi(-0.5)}(\pi\, 1.5 - i\log(\sin(\pi\, 1.5)))}{\pi} - \frac{e^{i\pi(-0.5)}\sqrt[-0.5+1]{-0.5+1}\,(\pi\, 1.5 - i\log(\sin(\pi\, 1.5)))}{\pi} = 1.125\,e^{-0.5\,i\,\pi} - \frac{0.75\,e^{-0.5\,i\,\pi}\,i\log\left(\sum_{k=0}^{\infty}\frac{(-1)^k\pi^{2k}}{(2k)!}\right)}{\pi}$$

$$\frac{e^{i\pi(-0.5)}(\pi\, 1.5 - i\log(\sin(\pi\, 1.5)))}{\pi} - \frac{e^{i\pi(-0.5)}\sqrt[-0.5+1]{-0.5+1}\,(\pi\, 1.5 - i\log(\sin(\pi\, 1.5)))}{\pi} = 1.125\,e^{-0.5\,i\,\pi} - \frac{0.75\,e^{-0.5\,i\,\pi}\,i\log\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,1.5^{1+2k}\,\pi^{1+2k}}{(1+2k)!}\right)}{\pi}$$

$$\frac{e^{i\pi(-0.5)}(\pi\, 1.5 - i\log(\sin(\pi\, 1.5)))}{\pi} - \frac{e^{i\pi(-0.5)}\sqrt[-0.5+1]{-0.5+1}\,(\pi\, 1.5 - i\log(\sin(\pi\, 1.5)))}{\pi} = 1.125\,e^{-0.5\,i\,\pi} - \frac{0.75\,e^{-0.5\,i\,\pi}\,i\log\left(2\sum_{k=0}^{\infty}(-1)^k J_{1+2k}(1.5\,\pi)\right)}{\pi}$$

$n!$ is the factorial function

$J_n(z)$ is the Bessel function of the first kind

Integral representations

$$\frac{e^{i\pi(-0.5)}(\pi\, 1.5 - i\log(\sin(\pi\, 1.5)))}{\pi} - \frac{e^{i\pi(-0.5)}\sqrt[-0.5+1]{-0.5+1}\,(\pi\, 1.5 - i\log(\sin(\pi\, 1.5)))}{\pi} = 1.125\, e^{-0.5\, i\,\pi} - \frac{0.75\, e^{-0.5\, i\,\pi}\, i}{\pi}\int_1^{\sin(1.5\,\pi)}\frac{1}{t}\,dt$$

$$\frac{e^{i\pi(-0.5)}(\pi\, 1.5 - i\log(\sin(\pi\, 1.5)))}{\pi} - \frac{e^{i\pi(-0.5)}\sqrt[-0.5+1]{-0.5+1}\,(\pi\, 1.5 - i\log(\sin(\pi\, 1.5)))}{\pi} = 1.125\, e^{-0.5\, i\,\pi} - \frac{0.75\, e^{-0.5\, i\,\pi}\, i\log\left(1.5\,\pi\int_0^1\cos(1.5\,\pi\, t)\,dt\right)}{\pi}$$

$$\frac{e^{i\pi(-0.5)}(\pi\, 1.5 - i\log(\sin(\pi\, 1.5)))}{\pi} - \frac{e^{i\pi(-0.5)}\sqrt[-0.5+1]{-0.5+1}\,(\pi\, 1.5 - i\log(\sin(\pi\, 1.5)))}{\pi} = 1.125\, e^{-0.5\, i\,\pi} - \frac{0.75\, e^{-0.5\, i\,\pi}\, i\log\left(\frac{0.375\sqrt{\pi}}{\mathcal{B}}\int_{-\mathcal{B}\,\infty+\gamma}^{\mathcal{B}\,\infty+\gamma}\frac{\mathcal{A}^{-(0.5625\,\pi^2)/s+s}}{s^{3/2}}\,ds\right)}{\pi} \quad \text{for } \gamma > 0$$

$$\frac{e^{i\pi(-0.5)}(\pi\, 1.5 - i\log(\sin(\pi\, 1.5)))}{\pi} - \frac{e^{i\pi(-0.5)}\sqrt[-0.5+1]{-0.5+1}\,(\pi\, 1.5 - i\log(\sin(\pi\, 1.5)))}{\pi} = 1.125\, e^{-0.5\, i\,\pi} - \frac{0.75\, e^{-0.5\, i\,\pi}\, i\log\left(\frac{0.375\sqrt{\pi}}{\pi\,\mathcal{A}}\int_{-\mathcal{A}\,\infty+\gamma}^{\mathcal{A}\,\infty+\gamma}\frac{e^{0.575364\, s}\,\pi^{1-2s}\,\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\,ds\right)}{\pi} \quad \text{for } 0 < \gamma < 1$$

$\Gamma(x)$ is the gamma function

Multiple-argument formulas

$$\frac{e^{i\pi(-0.5)}(\pi\, 1.5 - i\log(\sin(\pi\, 1.5)))}{\pi} - \frac{e^{i\pi(-0.5)}\sqrt[-0.5+1]{-0.5+1}\,(\pi\, 1.5 - i\log(\sin(\pi\, 1.5)))}{\pi} = \frac{e^{-0.5\,i\,\pi}\left(1.125\,\pi - 0.75\,i\log\left(3\sin(0.5\,\pi) - 4\sin^3(0.5\,\pi)\right)\right)}{\pi}$$

$$\frac{e^{i\pi(-0.5)}(\pi\, 1.5 - i\log(\sin(\pi\, 1.5)))}{\pi} - \frac{e^{i\pi(-0.5)}\sqrt[-0.5+1]{-0.5+1}\,(\pi\, 1.5 - i\log(\sin(\pi\, 1.5)))}{\pi} = \frac{e^{-0.5\,i\,\pi}(1.125\,\pi - 0.75\,i\log(2\cos(0.75\,\pi)\sin(0.75\,\pi)))}{\pi}$$

$$\frac{e^{i\pi(-0.5)}(\pi\, 1.5 - i\log(\sin(\pi\, 1.5)))}{\pi} - \frac{e^{i\pi(-0.5)}\sqrt[-0.5+1]{-0.5+1}\,(\pi\, 1.5 - i\log(\sin(\pi\, 1.5)))}{\pi} = \frac{e^{-0.5\,i\,\pi}(1.125\,\pi - 0.75\,i\log(U_{0.5}(\cos(\pi))\sin(\pi)))}{\pi}$$

$U_n(x)$ is the Chebyshev polynomial of the second kind

From which:

1/10(((e^(i π*-0.5) (π *1.5 - i log(sin(π*1.5))))/π - (e^(i π *-0.5) (-0.5 + 1)^(1/(-0.5 + 1)) (π*1.5 - i log(sin(π *1.5))))/π))

Input

$$\frac{1}{10}\left(\frac{e^{i\pi\times(-0.5)}(\pi\times 1.5 - i\log(\sin(\pi\times 1.5)))}{\pi} - \frac{e^{i\pi\times(-0.5)}\sqrt[-0.5+1]{-0.5+1}\,(\pi\times 1.5 - i\log(\sin(\pi\times 1.5)))}{\pi}\right)$$

$\log(x)$ is the natural logarithm

i is the imaginary unit

Result

$-0.1875\,i$

(using the principal branch of the logarithm for complex exponentiation)

Alternate complex forms

$1.14807\times 10^{-17} - 0.1875\,i$

$0.1875\,(\cos(-1.5708) + i\sin(-1.5708))$

$0.1875\,e^{-1.5708\,i}$

Polar coordinates

$r = 0.1875$ (radius), $\theta = -1.5708$ (angle)

0.1875 result quite near to the MRB constant value 0.187859642

Alternative representations

$$\frac{1}{10}\left(\frac{e^{i\pi(-0.5)}\,(\pi\,1.5 - i\log(\sin(\pi\,1.5)))}{\pi} - \frac{e^{i\pi(-0.5)}\,\sqrt[-0.5+1]{-0.5+1}\,(\pi\,1.5 - i\log(\sin(\pi\,1.5)))}{\pi}\right) = \frac{1}{10}\left(\frac{(1.5\pi - i\log(\cos(-\pi)))\,e^{-0.5\,i\pi}}{\pi} - \frac{(1.5\pi - i\log(\cos(-\pi)))\,\sqrt[0.5]{0.5}\,e^{-0.5\,i\pi}}{\pi}\right)$$

$$\frac{1}{10}\left(\frac{e^{i\pi(-0.5)}\,(\pi\,1.5 - i\log(\sin(\pi\,1.5)))}{\pi} - \frac{e^{i\pi(-0.5)}\,\sqrt[-0.5+1]{-0.5+1}\,(\pi\,1.5 - i\log(\sin(\pi\,1.5)))}{\pi}\right) = \frac{1}{10}\left(\frac{(1.5\pi - i\log(-\cos(2\pi)))\,e^{-0.5\,i\pi}}{\pi} - \frac{(1.5\pi - i\log(-\cos(2\pi)))\,\sqrt[0.5]{0.5}\,e^{-0.5\,i\pi}}{\pi}\right)$$

$$\frac{1}{10}\left(\frac{e^{i\pi(-0.5)}(\pi\,1.5 - i\log(\sin(\pi\,1.5)))}{\pi} - \frac{e^{i\pi(-0.5)}\sqrt[-0.5+1]{-0.5+1}\,(\pi\,1.5 - i\log(\sin(\pi\,1.5)))}{\pi}\right) = \frac{1}{10}\left(\frac{(1.5\pi - i(\log(a)\log_a(\sin(1.5\pi))))\,e^{-0.5i\pi}}{\pi} - \frac{(1.5\pi - i(\log(a)\log_a(\sin(1.5\pi))))\sqrt[0.5]{0.5}\,e^{-0.5i\pi}}{\pi}\right)$$

$\log_b(x)$ is the base- b logarithm

Series representations

$$\frac{1}{10}\left(\frac{e^{i\pi(-0.5)}(\pi\,1.5 - i\log(\sin(\pi\,1.5)))}{\pi} - \frac{e^{i\pi(-0.5)}\sqrt[-0.5+1]{-0.5+1}\,(\pi\,1.5 - i\log(\sin(\pi\,1.5)))}{\pi}\right) = 0.1125\,e^{-0.5i\pi} - \frac{0.075\,e^{-0.5i\pi}\,i\log\left(\sum_{k=0}^{\infty}\frac{(-1)^k\pi^{2k}}{(2k)!}\right)}{\pi}$$

$$\frac{1}{10}\left(\frac{e^{i\pi(-0.5)}(\pi\,1.5 - i\log(\sin(\pi\,1.5)))}{\pi} - \frac{e^{i\pi(-0.5)}\sqrt[-0.5+1]{-0.5+1}\,(\pi\,1.5 - i\log(\sin(\pi\,1.5)))}{\pi}\right) = 0.1125\,e^{-0.5i\pi} - \frac{0.075\,e^{-0.5i\pi}\,i\log\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,1.5^{1+2k}\,\pi^{1+2k}}{(1+2k)!}\right)}{\pi}$$

$$\frac{1}{10}\left(\frac{e^{i\pi(-0.5)}(\pi\,1.5 - i\log(\sin(\pi\,1.5)))}{\pi} - \frac{e^{i\pi(-0.5)}\sqrt[-0.5+1]{-0.5+1}\,(\pi\,1.5 - i\log(\sin(\pi\,1.5)))}{\pi}\right) = 0.1125\,e^{-0.5i\pi} - \frac{0.075\,e^{-0.5i\pi}\,i\log\left(2\sum_{k=0}^{\infty}(-1)^k J_{1+2k}(1.5\pi)\right)}{\pi}$$

$n!$ is the factorial function

$J_n(z)$ is the Bessel function of the first kind

Integral representations

$$\frac{1}{10}\left(\frac{e^{i\pi(-0.5)}(\pi\,1.5 - i\log(\sin(\pi\,1.5)))}{\pi} - \frac{e^{i\pi(-0.5)}\sqrt[-0.5+1]{-0.5+1}\,(\pi\,1.5 - i\log(\sin(\pi\,1.5)))}{\pi}\right) = 0.1125\,e^{-0.5 i\pi} - \frac{0.075\,e^{-0.5 i\pi}\,i}{\pi}\int_1^{\sin(1.5\pi)}\frac{1}{t}\,dt$$

$$\frac{1}{10}\left(\frac{e^{i\pi(-0.5)}(\pi\,1.5 - i\log(\sin(\pi\,1.5)))}{\pi} - \frac{e^{i\pi(-0.5)}\sqrt[-0.5+1]{-0.5+1}\,(\pi\,1.5 - i\log(\sin(\pi\,1.5)))}{\pi}\right) = 0.1125\,e^{-0.5 i\pi} - \frac{0.075\,e^{-0.5 i\pi}\,i\log\left(1.5\,\pi\int_0^1\cos(1.5\,\pi\,t)\,dt\right)}{\pi}$$

$$\frac{1}{10}\left(\frac{e^{i\pi(-0.5)}(\pi\,1.5 - i\log(\sin(\pi\,1.5)))}{\pi} - \frac{e^{i\pi(-0.5)}\sqrt[-0.5+1]{-0.5+1}\,(\pi\,1.5 - i\log(\sin(\pi\,1.5)))}{\pi}\right) = 0.1125\,e^{-0.5 i\pi} - \frac{0.075\,e^{-0.5 i\pi}\,i\log\left(\frac{0.375\sqrt{\pi}}{\mathcal{B}}\int_{-\mathcal{B}\,\infty+\gamma}^{\mathcal{B}\,\infty+\gamma}\frac{\mathcal{A}^{-(0.5625\,\pi^2)/s+s}}{s^{3/2}}\,ds\right)}{\pi} \quad \text{for } \gamma > 0$$

$$\frac{1}{10}\left(\frac{e^{i\pi(-0.5)}(\pi\,1.5 - i\log(\sin(\pi\,1.5)))}{\pi} - \frac{e^{i\pi(-0.5)}\sqrt[-0.5+1]{-0.5+1}\,(\pi\,1.5 - i\log(\sin(\pi\,1.5)))}{\pi}\right) = 0.1125\,e^{-0.5 i\pi} - \frac{0.075\,e^{-0.5 i\pi}\,i\log\left(\frac{0.375\sqrt{\pi}}{\pi\,\mathcal{A}}\int_{-\mathcal{A}\,\infty+\gamma}^{\mathcal{A}\,\infty+\gamma}\frac{e^{0.575364\,s}\,\pi^{1-2s}\,\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\,ds\right)}{\pi} \quad \text{for } 0 < \gamma < 1$$

$\Gamma(x)$ is the gamma function

Multiple-argument formulas

$$\frac{1}{10}\left(\frac{e^{i\pi(-0.5)}(\pi\,1.5 - i\log(\sin(\pi\,1.5)))}{\pi} - \frac{e^{i\pi(-0.5)}\sqrt[-0.5+1]{-0.5+1}\,(\pi\,1.5 - i\log(\sin(\pi\,1.5)))}{\pi}\right) = \frac{e^{-0.5 i\pi}\left(0.1125\pi - 0.075\, i\log\left(3\sin(0.5\pi) - 4\sin^3(0.5\pi)\right)\right)}{\pi}$$

$$\frac{1}{10}\left(\frac{e^{i\pi(-0.5)}(\pi\,1.5 - i\log(\sin(\pi\,1.5)))}{\pi} - \frac{e^{i\pi(-0.5)}\sqrt[-0.5+1]{-0.5+1}\,(\pi\,1.5 - i\log(\sin(\pi\,1.5)))}{\pi}\right) = \frac{e^{-0.5 i\pi}(0.1125\pi - 0.075\, i\log(2\cos(0.75\pi)\sin(0.75\pi)))}{\pi}$$

$$\frac{1}{10}\left(\frac{e^{i\pi(-0.5)}(\pi\,1.5 - i\log(\sin(\pi\,1.5)))}{\pi} - \frac{e^{i\pi(-0.5)}\sqrt[-0.5+1]{-0.5+1}\,(\pi\,1.5 - i\log(\sin(\pi\,1.5)))}{\pi}\right) = \frac{e^{-0.5 i\pi}(0.1125\pi - 0.075\, i\log(U_{0.5}(\cos(\pi))\sin(\pi)))}{\pi}$$

$U_n(x)$ is the Chebyshev polynomial of the second kind

From the previous expression, i.e.

$$\frac{e^{i\pi\times(-0.5)}(\pi\times 1.5 - i\log(\sin(\pi\times 1.5)))}{\pi} - \frac{e^{i\pi\times(-0.5)}\sqrt[-0.5+1]{-0.5+1}\,(\pi\times 1.5 - i\log(\sin(\pi\times 1.5)))}{\pi}$$

we obtain also:

12*(((((((e^(i π*-0.5) (π *1.5 - i log(sin(π*1.5))))/π - (e^(i π *-0.5) (-0.5 + 1)^(1/(-0.5 + 1)) (π*1.5 - i log(sin(π *1.5))))/π))))^8-8))-8-MRB const

Input

$$12\left(\left(\frac{e^{i\pi\times(-0.5)}\,(\pi\times 1.5 - i\log(\sin(\pi\times 1.5)))}{\pi} - \frac{e^{i\pi\times(-0.5)}\,\sqrt[-0.5+1]{-0.5+1}\,(\pi\times 1.5 - i\log(\sin(\pi\times 1.5)))}{\pi}\right)^8 - 8\right) - 8 - C_{\text{MRB}}$$

$\log(x)$ is the natural logarithm
i is the imaginary unit
C_{MRB} is the MRB constant

Result

1728.93…

(using the principal branch of the logarithm for complex exponentiation)

1728.93…

This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. (1728 = 8^2 * 3^3) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

((12*((((((((e^(i π*-0.5) (π *1.5 - i log(sin(π*1.5))))/π - (e^(i π *-0.5) (-0.5 + 1)^(1/(-0.5 + 1)) (π*1.5 - i log(sin(π *1.5))))/π))))^8-8))-8-MRB const))^1/15

Input

$$\left(12\left(\left(\frac{e^{i\pi\times(-0.5)}\,(\pi\times 1.5 - i\log(\sin(\pi\times 1.5)))}{\pi} - \frac{e^{i\pi\times(-0.5)}\,\sqrt[-0.5+1]{-0.5+1}\,(\pi\times 1.5 - i\log(\sin(\pi\times 1.5)))}{\pi}\right)^8 - 8\right) - 8 - C_{\text{MRB}}\right) \hat{}\ (1/15)$$

$\log(x)$ is the natural logarithm
i is the imaginary unit
C_{MRB} is the MRB constant

Result

1.6438110685246343894307439569363981814591760928256969607371565665

...

(using the principal branch of the logarithm for complex exponentiation)

1.64381106852463…...≈ ζ(2) = $\frac{\pi^2}{6}$=1.644934… (trace of the instanton shape)

(1/27((12*(((((((e^(i π*-0.5) (π *1.5 - i log(sin(π*1.5))))/π - (e^(i π *-0.5) (-0.5 + 1)^(1/(-0.5 + 1)) (π*1.5 - i log(sin(π *1.5))))/π))))^8-8))-8-MRB const)-1))^2+2*MRB const

Input

$$\left(\frac{1}{27}\left(\left(12\left(\left(\left(\frac{e^{i\pi\times(-0.5)}(\pi\times 1.5-i\log(\sin(\pi\times 1.5)))}{\pi}-\frac{1}{\pi}e^{i\pi\times(-0.5)}\sqrt[-0.5+1]{-0.5+1}\,(\pi\times 1.5-i\log(\sin(\pi\times 1.5)))\right)^8-8\right)-8-C_{MRB}\right)-1\right)\right)^2+2\,C_{MRB}$$

$\log(x)$ is the natural logarithm
i is the imaginary unit
C_{MRB} is the MRB constant

Result

4096.0645618081243267087312640856805262356105861595006835772236556

...

(using the principal branch of the logarithm for complex exponentiation)

4096.064561808……≈ 4096 = 64^2

Now we let's a cue from the following integral:

$$\mathbf{CMRB = 1/2 + \int_1^{i\infty} Im[(t^{1/t} - t^{2n})] (-Csc[\pi t])\, dt\ /.\ n\epsilon Z^*}$$

We perform the following calculations:

MRB constant - (1/2 + Integrate[Im[(t^(1/t) - t^(2 n))] (-Csc[\[Pi] t])])

Input

$$C_{\text{MRB}} - \left(\frac{1}{2} + \int \text{Im}\left(\sqrt[t]{t} - t^{2n}\right)(-\csc(\pi t))\, dt\right)$$

Im(z) is the imaginary part of z
csc(x) is the cosecant function
C_{MRB} is the MRB constant

Exact result

$$\int \csc(\pi t)\, \text{Im}\left(\sqrt[t]{t} - t^{2n}\right) dt + C_{\text{MRB}} - \frac{1}{2}$$

Alternate forms

$$\int \csc(\pi t)\left(\text{Im}\left(\sqrt[t]{t}\right) - \text{Im}\left(t^{2n}\right)\right) dt + C_{\text{MRB}} - \frac{1}{2}$$

$$-\frac{1}{2} + C_{\text{MRB}} + \int \frac{\text{Im}\left(-t^{2n} + \sqrt[t]{t}\right)}{\sin(\pi t)}\, dt$$

$$\frac{1}{2}\left(2\int \csc(\pi t)\, \text{Im}\left(\sqrt[t]{t} - t^{2n}\right) dt + 2\, C_{\text{MRB}} - 1\right)$$

Alternate form assuming n and t are positive

$$C_{\text{MRB}} - \frac{1}{2}$$

Alternate form assuming n and t are real

$$\text{Re}\left(\int \csc(\pi t)\, \text{Im}\left(\sqrt[t]{t} - t^{2n}\right) dt\right) + i\, \text{Im}\left(\int \csc(\pi t)\, \text{Im}\left(\sqrt[t]{t} - t^{2n}\right) dt\right) + C_{\text{MRB}} - \frac{1}{2}$$

Re(z) is the real part of z

Derivative

$$\frac{\partial}{\partial t}\left(C_{\mathrm{MRB}}-\left(\frac{1}{2}+\int \operatorname{Im}\left(\sqrt[t]{t}-t^{2n}\right)(-\csc(\pi t))\,dt\right)\right)=\csc(\pi t)\operatorname{Im}\left(\sqrt[t]{t}-t^{2n}\right)$$

From:

$$C_{\mathrm{MRB}}-\frac{1}{2}$$

we obtain:

-1/2 + MRB const

Input

$$-\frac{1}{2}+C_{\mathrm{MRB}}$$

C_{MRB} is the MRB constant

Decimal approximation

−0.31214035753793287975148206594572676994409690509986121382799531 5
…

-0.3121403575379….

Alternate form

$$\frac{1}{2}(2C_{\mathrm{MRB}}-1)$$

from which, we obtain:

(((8 2^(1/3))*1/π^2))/((-(Artin's constant / MRB constant)((-1/2 + MRB const))))

where

$$\frac{8\sqrt[3]{2}}{\pi^2}\approx 1.0212535$$

Input

$$-\frac{\left(8\sqrt[3]{2}\right)\times\frac{1}{\pi^2}}{\frac{C_A}{C_{\mathrm{MRB}}}\left(-\frac{1}{2}+C_{\mathrm{MRB}}\right)}$$

C_A is Artin's constant
C_{MRB} is the MRB constant

Exact result

$$-\frac{8\sqrt[3]{2}\,C_{\mathrm{MRB}}}{\pi^2 C_A\left(C_{\mathrm{MRB}}-\frac{1}{2}\right)}$$

Decimal approximation

1.6436027232603526144425063948992941618426648441748576328922403731
...

1.64360272326...≈ ζ(2) = $\frac{\pi^2}{6}$=1.644934… (trace of the instanton shape)

Alternate forms

$$\frac{16\sqrt[3]{2}\,C_{\mathrm{MRB}}}{\pi^2\left(C_A-2\,C_A\,C_{\mathrm{MRB}}\right)}$$

$$-\frac{16\sqrt[3]{2}\,C_{\mathrm{MRB}}}{\pi^2 C_A\left(2\,C_{\mathrm{MRB}}-1\right)}$$

$$-\frac{16\sqrt[3]{2}\,C_{\mathrm{MRB}}}{\pi^2 C_A\left(2\,C_{\mathrm{MRB}}-1\right)}$$

(((((8 2^(1/3))*1/π^2))/((-(Artin's constant / MRB constant)((-1/2 + MRB const))))))^15+e+Φ

Input

$$\left(-\frac{\left(8\sqrt[3]{2}\right)\times\frac{1}{\pi^2}}{\frac{C_A}{C_{\mathrm{MRB}}}\left(-\frac{1}{2}+C_{\mathrm{MRB}}\right)}\right)^{15}+e+\Phi$$

C_A is Artin's constant

C_{MRB} is the MRB constant
Φ is the golden ratio conjugate

Exact result

$$-\frac{1125899906842624\, C_{\text{MRB}}^{15}}{\pi^{30}\, C_A^{15} \left(C_{\text{MRB}} - \frac{1}{2}\right)^{15}} + \Phi + e$$

Decimal approximation

1728.9865809032947949492285528550568174167613535285958913489893643
…

1728.9865809…

This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. (1728 = 8^2 * 3^3) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Alternate forms

$$-\frac{36893488147419103232\, C_{\text{MRB}}^{15}}{\pi^{30}\, C_A^{15} (2\, C_{\text{MRB}} - 1)^{15}} + \Phi + e$$

$$-\frac{36893488147419103232\, C_{\text{MRB}}^{15} - \pi^{30} (\Phi + e)\, C_A^{15} (2\, C_{\text{MRB}} - 1)^{15}}{\pi^{30}\, C_A^{15} (2\, C_{\text{MRB}} - 1)^{15}}$$

$$\begin{aligned}\big(&30\,\pi^{30}\,\Phi\, C_A^{15}\, C_{\text{MRB}} - 420\,\pi^{30}\,\Phi\, C_A^{15}\, C_{\text{MRB}}^2 + 3640\,\pi^{30}\,\Phi\, C_A^{15}\, C_{\text{MRB}}^3 - \\ &21840\,\pi^{30}\,\Phi\, C_A^{15}\, C_{\text{MRB}}^4 + 96096\,\pi^{30}\,\Phi\, C_A^{15}\, C_{\text{MRB}}^5 - \\ &320320\,\pi^{30}\,\Phi\, C_A^{15}\, C_{\text{MRB}}^6 + 823680\,\pi^{30}\,\Phi\, C_A^{15}\, C_{\text{MRB}}^7 - \\ &1647360\,\pi^{30}\,\Phi\, C_A^{15}\, C_{\text{MRB}}^8 + 2562560\,\pi^{30}\,\Phi\, C_A^{15}\, C_{\text{MRB}}^9 - \\ &3075072\,\pi^{30}\,\Phi\, C_A^{15}\, C_{\text{MRB}}^{10} + 2795520\,\pi^{30}\,\Phi\, C_A^{15}\, C_{\text{MRB}}^{11} - \\ &1863680\,\pi^{30}\,\Phi\, C_A^{15}\, C_{\text{MRB}}^{12} + 860160\,\pi^{30}\,\Phi\, C_A^{15}\, C_{\text{MRB}}^{13} - \\ &245760\,\pi^{30}\,\Phi\, C_A^{15}\, C_{\text{MRB}}^{14} + 32768\,\pi^{30}\,\Phi\, C_A^{15}\, C_{\text{MRB}}^{15} + \\ &30\, e\,\pi^{30}\, C_A^{15}\, C_{\text{MRB}} - 420\, e\,\pi^{30}\, C_A^{15}\, C_{\text{MRB}}^2 + 3640\, e\,\pi^{30}\, C_A^{15}\, C_{\text{MRB}}^3 - \\ &21840\, e\,\pi^{30}\, C_A^{15}\, C_{\text{MRB}}^4 + 96096\, e\,\pi^{30}\, C_A^{15}\, C_{\text{MRB}}^5 - \\ &320320\, e\,\pi^{30}\, C_A^{15}\, C_{\text{MRB}}^6 + 823680\, e\,\pi^{30}\, C_A^{15}\, C_{\text{MRB}}^7 - \\ &1647360\, e\,\pi^{30}\, C_A^{15}\, C_{\text{MRB}}^8 + 2562560\, e\,\pi^{30}\, C_A^{15}\, C_{\text{MRB}}^9 - \\ &3075072\, e\,\pi^{30}\, C_A^{15}\, C_{\text{MRB}}^{10} + 2795520\, e\,\pi^{30}\, C_A^{15}\, C_{\text{MRB}}^{11} - \\ &1863680\, e\,\pi^{30}\, C_A^{15}\, C_{\text{MRB}}^{12} + 860160\, e\,\pi^{30}\, C_A^{15}\, C_{\text{MRB}}^{13} - \\ &245760\, e\,\pi^{30}\, C_A^{15}\, C_{\text{MRB}}^{14} + 32768\, e\,\pi^{30}\, C_A^{15}\, C_{\text{MRB}}^{15} - e\,\pi^{30}\, C_A^{15} - \\ &\pi^{30}\,\Phi\, C_A^{15} - 36893488147419103232\, C_{\text{MRB}}^{15}\big)\big/\big(\pi^{30}\, C_A^{15} (2\, C_{\text{MRB}} - 1)^{15}\big)\end{aligned}$$

(1/27(((((((8 2^(1/3))*1/π^2))/((-(Artin's constant / MRB constant)((-1/2 + MRB const))))))^15+e+Φ)-1))^2

Input

$$\left(\frac{1}{27}\left(\left(\left(-\frac{\left(8\sqrt[3]{2}\right)\times\frac{1}{\pi^2}}{\frac{C_A}{C_{\mathrm{MRB}}}\left(-\frac{1}{2}+C_{\mathrm{MRB}}\right)}\right)^{15}+e+\Phi\right)-1\right)\right)^2$$

C_A is Artin's constant
C_{MRB} is the MRB constant
Φ is the golden ratio conjugate

Exact result

$$\frac{1}{729}\left(-\frac{1125899906842624\,C_{\mathrm{MRB}}^{15}}{\pi^{30}\,C_A^{15}\left(C_{\mathrm{MRB}}-\frac{1}{2}\right)^{15}}+\Phi-1+e\right)^2$$

Decimal approximation

4095.936383788558254771164510226456225498814728352458443118704410 1…

4095.9363837...≈ 4096 = 64^2

Alternate forms

$$-\frac{73786976294838206464\left(\Phi\,C_{\mathrm{MRB}}^{15}-C_{\mathrm{MRB}}^{15}+e\,C_{\mathrm{MRB}}^{15}\right)}{729\,\pi^{30}\,C_A^{15}\left(2\,C_{\mathrm{MRB}}-1\right)^{15}}+\frac{1361129467683753853853498429727072845824\,C_{\mathrm{MRB}}^{30}}{729\,\pi^{60}\,C_A^{30}\left(2\,C_{\mathrm{MRB}}-1\right)^{30}}+\frac{1}{729}\left(\Phi-1+e\right)^2$$

$$\left(\pi^{30}\left(\Phi-1+e\right)C_A^{15}\left(30\,C_{\mathrm{MRB}}-420\,C_{\mathrm{MRB}}^2+3640\,C_{\mathrm{MRB}}^3-21840\,C_{\mathrm{MRB}}^4+96096\,C_{\mathrm{MRB}}^5-320320\,C_{\mathrm{MRB}}^6+823680\,C_{\mathrm{MRB}}^7-1647360\,C_{\mathrm{MRB}}^8+2562560\,C_{\mathrm{MRB}}^9-3075072\,C_{\mathrm{MRB}}^{10}+2795520\,C_{\mathrm{MRB}}^{11}-1863680\,C_{\mathrm{MRB}}^{12}+860160\,C_{\mathrm{MRB}}^{13}-245760\,C_{\mathrm{MRB}}^{14}+32768\,C_{\mathrm{MRB}}^{15}-1\right)-36893488147419103232\,C_{\mathrm{MRB}}^{15}\right)^2/\left(729\,\pi^{60}\,C_A^{30}\left(2\,C_{\mathrm{MRB}}-1\right)^{30}\right)$$

$$\big(30\,\pi^{30}\,\Phi\,C_A{}^{15}\,C_{\text{MRB}} - 420\,\pi^{30}\,\Phi\,C_A{}^{15}\,C_{\text{MRB}}{}^2 + 3640\,\pi^{30}\,\Phi\,C_A{}^{15}\,C_{\text{MRB}}{}^3 - 21\,840\,\pi^{30}\,\Phi\,C_A{}^{15}\,C_{\text{MRB}}{}^4 + 96\,096\,\pi^{30}\,\Phi\,C_A{}^{15}\,C_{\text{MRB}}{}^5 - 320\,320\,\pi^{30}\,\Phi\,C_A{}^{15}\,C_{\text{MRB}}{}^6 + 823\,680\,\pi^{30}\,\Phi\,C_A{}^{15}\,C_{\text{MRB}}{}^7 - 1\,647\,360\,\pi^{30}\,\Phi\,C_A{}^{15}\,C_{\text{MRB}}{}^8 + 2\,562\,560\,\pi^{30}\,\Phi\,C_A{}^{15}\,C_{\text{MRB}}{}^9 - 3\,075\,072\,\pi^{30}\,\Phi\,C_A{}^{15}\,C_{\text{MRB}}{}^{10} + 2\,795\,520\,\pi^{30}\,\Phi\,C_A{}^{15}\,C_{\text{MRB}}{}^{11} - 1\,863\,680\,\pi^{30}\,\Phi\,C_A{}^{15}\,C_{\text{MRB}}{}^{12} + 860\,160\,\pi^{30}\,\Phi\,C_A{}^{15}\,C_{\text{MRB}}{}^{13} - 245\,760\,\pi^{30}\,\Phi\,C_A{}^{15}\,C_{\text{MRB}}{}^{14} + 32\,768\,\pi^{30}\,\Phi\,C_A{}^{15}\,C_{\text{MRB}}{}^{15} - 30\,\pi^{30}\,C_A{}^{15}\,C_{\text{MRB}} + 30\,e\,\pi^{30}\,C_A{}^{15}\,C_{\text{MRB}} + 420\,\pi^{30}\,C_A{}^{15}\,C_{\text{MRB}}{}^2 - 420\,e\,\pi^{30}\,C_A{}^{15}\,C_{\text{MRB}}{}^2 - 3640\,\pi^{30}\,C_A{}^{15}\,C_{\text{MRB}}{}^3 + 3640\,e\,\pi^{30}\,C_A{}^{15}\,C_{\text{MRB}}{}^3 + 21\,840\,\pi^{30}\,C_A{}^{15}\,C_{\text{MRB}}{}^4 - 21\,840\,e\,\pi^{30}\,C_A{}^{15}\,C_{\text{MRB}}{}^4 - 96\,096\,\pi^{30}\,C_A{}^{15}\,C_{\text{MRB}}{}^5 + 96\,096\,e\,\pi^{30}\,C_A{}^{15}\,C_{\text{MRB}}{}^5 + 320\,320\,\pi^{30}\,C_A{}^{15}\,C_{\text{MRB}}{}^6 - 320\,320\,e\,\pi^{30}\,C_A{}^{15}\,C_{\text{MRB}}{}^6 - 823\,680\,\pi^{30}\,C_A{}^{15}\,C_{\text{MRB}}{}^7 + 823\,680\,e\,\pi^{30}\,C_A{}^{15}\,C_{\text{MRB}}{}^7 + 1\,647\,360\,\pi^{30}\,C_A{}^{15}\,C_{\text{MRB}}{}^8 - 1\,647\,360\,e\,\pi^{30}\,C_A{}^{15}\,C_{\text{MRB}}{}^8 - 2\,562\,560\,\pi^{30}\,C_A{}^{15}\,C_{\text{MRB}}{}^9 + 2\,562\,560\,e\,\pi^{30}\,C_A{}^{15}\,C_{\text{MRB}}{}^9 + 3\,075\,072\,\pi^{30}\,C_A{}^{15}\,C_{\text{MRB}}{}^{10} - 3\,075\,072\,e\,\pi^{30}\,C_A{}^{15}\,C_{\text{MRB}}{}^{10} - 2\,795\,520\,\pi^{30}\,C_A{}^{15}\,C_{\text{MRB}}{}^{11} + 2\,795\,520\,e\,\pi^{30}\,C_A{}^{15}\,C_{\text{MRB}}{}^{11} + 1\,863\,680\,\pi^{30}\,C_A{}^{15}\,C_{\text{MRB}}{}^{12} - 1\,863\,680\,e\,\pi^{30}\,C_A{}^{15}\,C_{\text{MRB}}{}^{12} - 860\,160\,\pi^{30}\,C_A{}^{15}\,C_{\text{MRB}}{}^{13} + 860\,160\,e\,\pi^{30}\,C_A{}^{15}\,C_{\text{MRB}}{}^{13} + 245\,760\,\pi^{30}\,C_A{}^{15}\,C_{\text{MRB}}{}^{14} - 245\,760\,e\,\pi^{30}\,C_A{}^{15}\,C_{\text{MRB}}{}^{14} - 32\,768\,\pi^{30}\,C_A{}^{15}\,C_{\text{MRB}}{}^{15} + 32\,768\,e\,\pi^{30}\,C_A{}^{15}\,C_{\text{MRB}}{}^{15} + \pi^{30}\,C_A{}^{15} - e\,\pi^{30}\,C_A{}^{15} - \pi^{30}\,\Phi\,C_A{}^{15} - 36\,893\,488\,147\,419\,103\,232\,C_{\text{MRB}}{}^{15}\big)^2 \Big/ \left(729\,\pi^{60}\,C_A{}^{30}\,(2\,C_{\text{MRB}} - 1)^{30}\right)$$

Expanded form

$$-\frac{2\,251\,799\,813\,685\,248\,\Phi\,C_{\text{MRB}}{}^{15}}{729\,\pi^{30}\,C_A{}^{15}\left(C_{\text{MRB}} - \frac{1}{2}\right)^{15}} + \frac{1\,267\,650\,600\,228\,229\,401\,496\,703\,205\,376\,C_{\text{MRB}}{}^{30}}{729\,\pi^{60}\,C_A{}^{30}\left(C_{\text{MRB}} - \frac{1}{2}\right)^{30}} + \frac{2\,251\,799\,813\,685\,248\,C_{\text{MRB}}{}^{15}}{729\,\pi^{30}\,C_A{}^{15}\left(C_{\text{MRB}} - \frac{1}{2}\right)^{15}} - \frac{2\,251\,799\,813\,685\,248\,e\,C_{\text{MRB}}{}^{15}}{729\,\pi^{30}\,C_A{}^{15}\left(C_{\text{MRB}} - \frac{1}{2}\right)^{15}} - \frac{2\,\Phi}{729} + \frac{2\,e\,\Phi}{729} + \frac{\Phi^2}{729} + \frac{1}{729} - \frac{2\,e}{729} + \frac{e^2}{729}$$

Observations

We note that, from the number 8, we obtain as follows:

8^2

64

$8^2 \times 2 \times 8$

1024

$8^4 = 8^2 \times 2^6$

True

$8^4 = 4096$

$8^2 \times 2^6 = 4096$

$2^{13} = 2 \times 8^4$

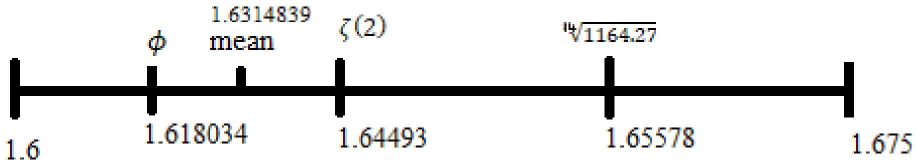
True

$2^{13} = 8192$

$2 \times 8^4 = 8192$

We notice how from the numbers 8 and 2 we get 64, 1024, 4096 and 8192, and that 8 is the fundamental number. In fact $8^2 = 64$, $8^3 = 512$, $8^4 = 4096$. We define it "fundamental number", since 8 is a Fibonacci number, which by rule, divided by the previous one, which is 5, gives 1.6 , a value that tends to the golden ratio, as for all numbers in the Fibonacci sequence

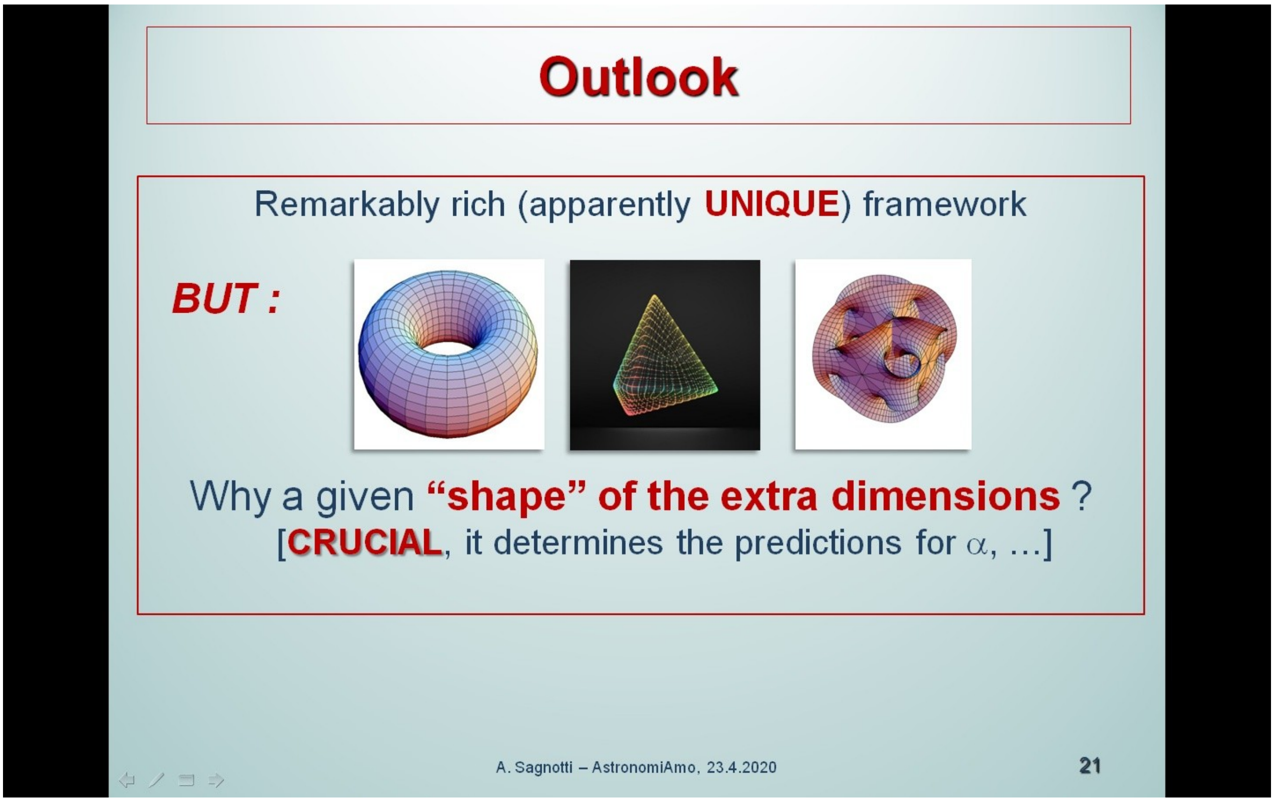
“Golden” Range



Finally we note how $8^2 = 64$, multiplied by 27, to which we add 1, is equal to 1729, the so-called "Hardy-Ramanujan number". Then taking the 15th root of 1729, we obtain a value close to $\zeta(2)$ that 1.6438 ..., which, in turn, is included in the range of what we call "golden numbers"

Furthermore for all the results very near to 1728 or 1729, adding $64 = 8^2$, one obtain values about equal to 1792 or 1793. These are values almost equal to the Planck multipole spectrum frequency 1792.35 and to the hypothetical Gluino mass

Appendix



From: *A. Sagnotti – AstronomiAmo, 23.04.2020*

In the above figure, it is said that: "why a given shape of the extra dimensions? Crucial, it determines the predictions for α".

We propose that whatever shape the compactified dimensions are, their geometry must be based on the values of the golden ratio and ζ(2), (the latter connected to 1728 or 1729, whose fifteenth root provides an excellent approximation to the above mentioned value) which are recurrent as solutions of the equations that we are going to develop. It is important to specify that the initial conditions are **always** values belonging to a fundamental chapter of the work of S. Ramanujan "Modular equations and Appoximations to Pi" (see references). These values are some multiples of 8 (64 and 4096), 276, which added to 4096, is equal to 4372, and finally $e^{\pi\sqrt{22}}$

We have, in certain cases, the following connections:

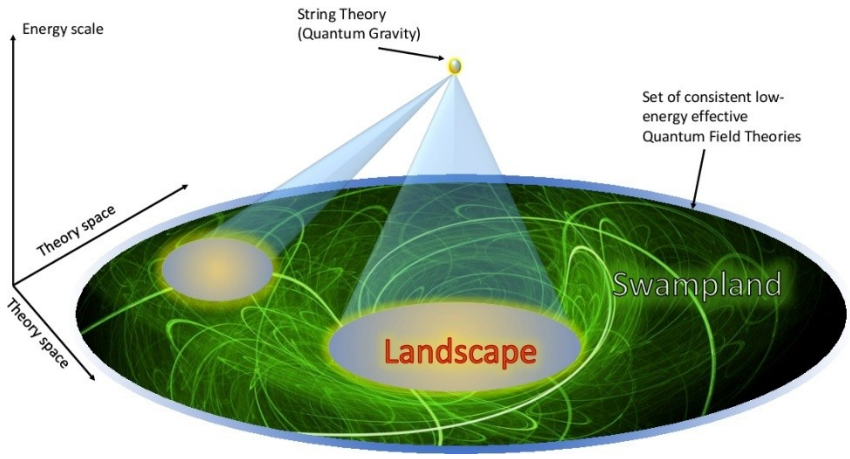


Fig. 1

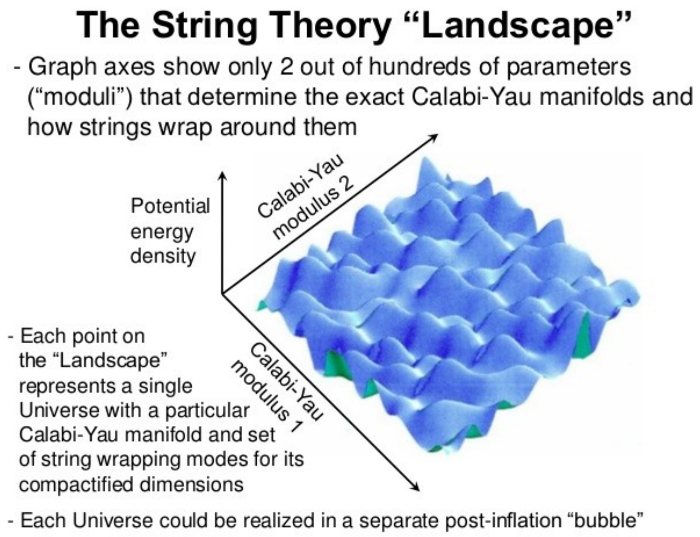


Fig. 2

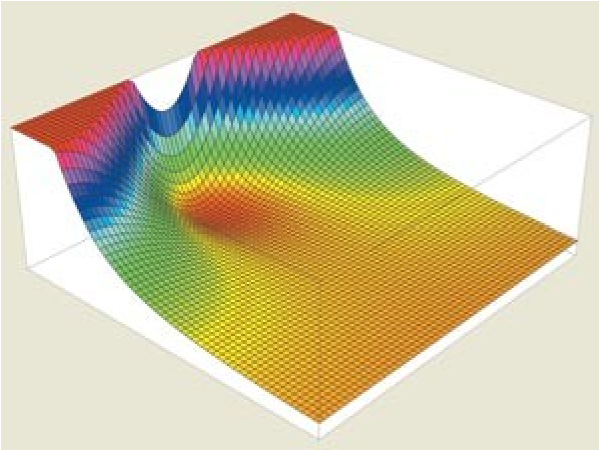


Fig. 3
Stringscape - a small part of the string-theory landscape showing the new de Sitter solution as a local minimum of the energy (vertical axis). The global minimum occurs at the infinite size of the extra dimensions on the extreme right of the figure.

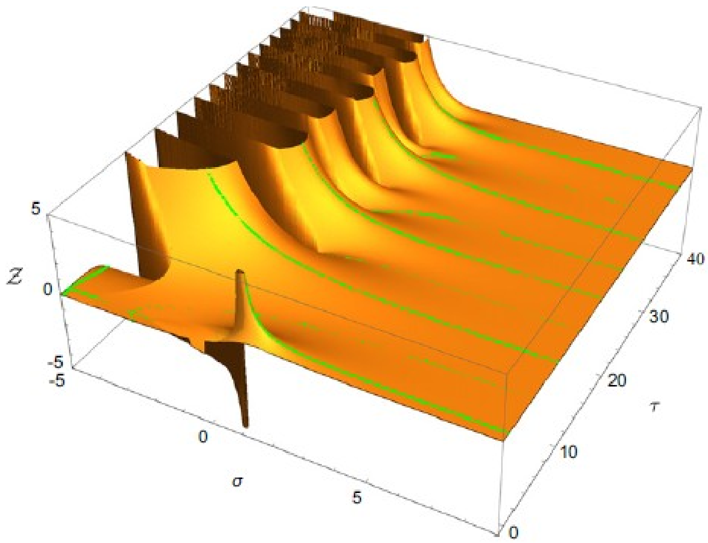


Figure 2. Lines in the complex plane where the Riemann zeta function ζ is real (green) depicted on a relief representing the positive absolute value of ζ for arguments $s \equiv \sigma + \mathrm{i}\tau$ where the real part of ζ is positive, and the negative absolute value of ζ where the real part of ζ is negative. This representation brings out most clearly that the lines of constant phase corresponding to phases of integer multiples of 2π run down the hills on the left-hand side, turn around on the right and terminate in the non-trivial zeros. This pattern repeats itself infinitely many times. The points of arrival and departure on the right-hand side of the picture are equally spaced and given by equation (11).

Fig. 4

From: https://www.mdpi.com/2227-7390/6/12/285/htm

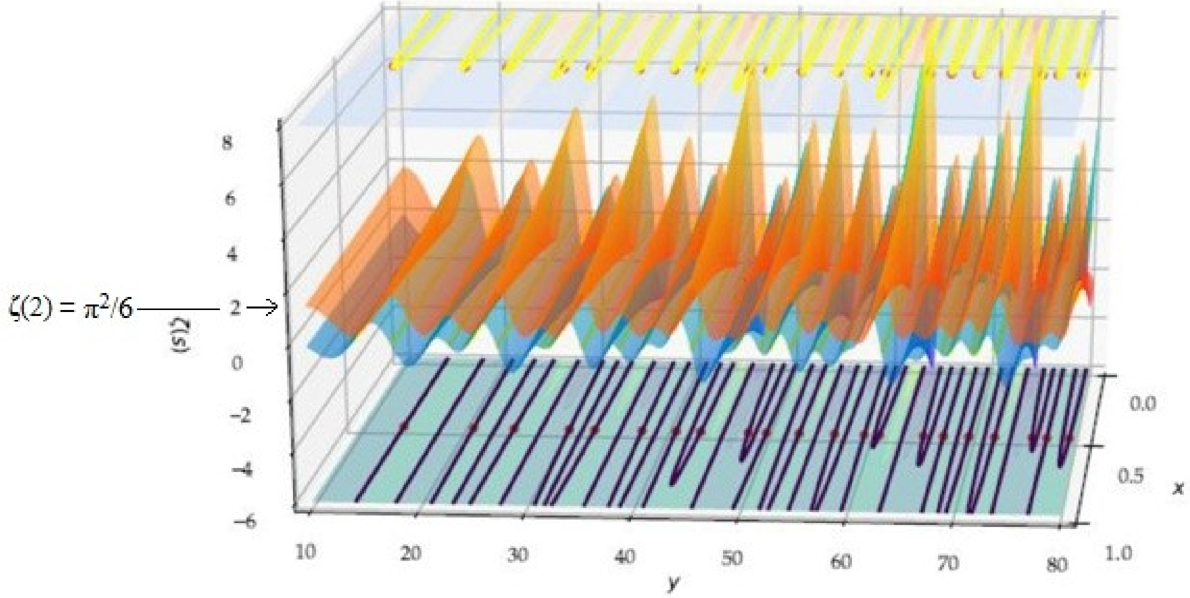


Figure 1. $C(x, y)$ and $S(x, y)$ surfaces of the Riemann $\zeta(x, y) = C - i\,S$ function, in the critical strip $\boldsymbol{s}$: $0 \leqslant x \leqslant 1; 10 \leqslant y \leqslant 80$. On the top and bottom planes, the C and S common zeros are the red points.

Fig. 5

3D plot	$\zeta(2 + i\,t)$

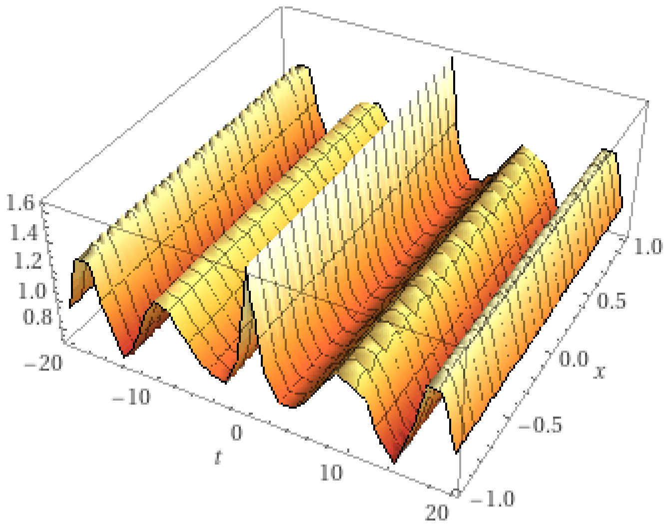


Fig. 6

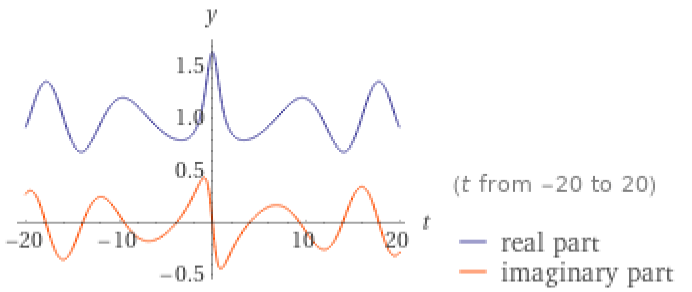
Where ζ(2+it) :

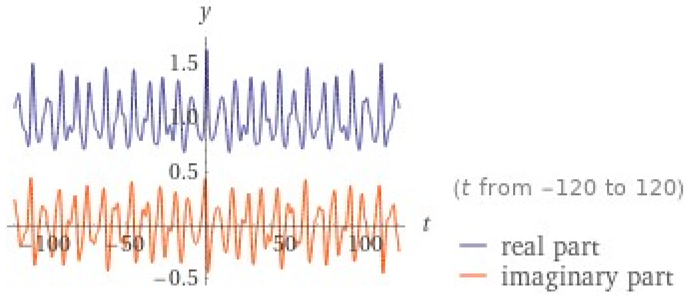
Input

$\zeta(2 + i\,t)$

$\zeta(s)$ is the Riemann zeta function
i is the imaginary unit

Plots





Roots

$t = 2\,i\,(n+1)\,,\quad n \in \mathbb{Z}\,,\quad n \geq 1$

$t = -i\,(\rho_n - 2)\,,\quad n \neq 0\,,\quad n \in \mathbb{Z}$

$\mathbb{Z}$ is the set of integers

ρ_n is the nontrivial n^{th} zero of the Riemann zeta function

Series expansion at t=0

$$\frac{\pi^2}{6} + i\,t\,\zeta'(2) - \frac{1}{2}\,t^2\,\zeta''(2) - \frac{1}{6}\,i\,\zeta^{(3)}(2)\,t^3 + \frac{1}{24}\,\zeta^{(4)}(2)\,t^4 + O\left(t^5\right)$$

(Taylor series)

Alternative representations

$$\zeta(2+i\,t) = \zeta(2+i\,t,\,1)$$

$$\zeta(2+i\,t) = S_{1+i\,t,1}(1)$$

$$\zeta(2+i\,t) = \frac{\zeta\left(2+i\,t,\,\frac{1}{2}\right)}{-1+2^{2+i\,t}}$$

$\zeta(s,\,a)$ is the generalized Riemann zeta function

$S_{n,p}(x)$ is the Nielsen generalized polylogarithm function

Series representations

$$\zeta(2+i\,t) = \sum_{k=1}^{\infty} k^{-2-i\,t} \quad \text{for } \operatorname{Im}(t) < 1$$

$$\zeta(2+i\,t) = \frac{\sum_{k=0}^{\infty} (1+2\,k)^{-2-i\,t}}{1-2^{-2-i\,t}} \quad \text{for } \operatorname{Im}(t) < 1$$

$$\zeta(2+i\,t) = e^{\sum_{k=1}^{\infty} P(k\,(2+i\,t))/k} \quad \text{for } \operatorname{Im}(t) < 1$$

$\operatorname{Im}(z)$ is the imaginary part of z

$P(z)$ gives the prime zeta function

Integral representations

$$\zeta(2+i\,t) = \frac{1}{\Gamma(2+i\,t)} \int_0^{\infty} \frac{\tau^{1+i\,t}}{-1+e^{\tau}}\,d\tau \quad \text{for } \operatorname{Im}(t) < 1$$

$$\zeta(2+i\,t) = \frac{2^{1+i\,t}}{\Gamma(3+i\,t)} \int_0^{\infty} \tau^{2+i\,t} \operatorname{csch}^2(\tau)\,d\tau \quad \text{for } \operatorname{Im}(t) < 1$$

$$\zeta(2+i\,t) = \frac{2^{1+i\,t}}{\Gamma(2+i\,t)} \int_0^{\infty} e^{-\tau}\,\tau^{1+i\,t} \operatorname{csch}(\tau)\,d\tau \quad \text{for } \operatorname{Im}(t) < 1$$

$\Gamma(x)$ is the gamma function

$\operatorname{csch}(x)$ is the hyperbolic cosecant function

Functional equations

$$\zeta(2+i\,t) = -i\,2^{2+i\,t}\,\pi^{1+i\,t}\,\Gamma(-1-i\,t)\sinh\left(\frac{\pi\,t}{2}\right)\zeta(-1-i\,t)$$

$$\zeta(2+i\,t) = \frac{\pi^{3/2+i\,t}\,\Gamma\left(-\frac{1}{2}-\frac{i\,t}{2}\right)\zeta(-1-i\,t)}{\Gamma\left(1+\frac{i\,t}{2}\right)}$$

$$\zeta(2+i\,t) = -\frac{i\sum_{k=0}^{\infty}\frac{\Gamma\left(k-\frac{i\,t}{2}\right)\sum_{j=0}^{k}(-1)^j\,(1+2\,j)\binom{k}{j}\zeta(2+2\,j)}{k!}}{(-i+t)\,\Gamma\left(-\frac{i\,t}{2}\right)}$$

With regard the Fig. 4 the points of arrival and departure on the right-hand side of the picture are equally spaced and given by the following equation:

$$\tau'_k \equiv k \frac{\pi}{\ln 2},$$

with $k = \ldots, -2, -1, 0, 1, 2, \ldots.$

we obtain:
2Pi/(ln(2))

Input:

$$2 \times \frac{\pi}{\log(2)}$$

Exact result:

$$\frac{2\pi}{\log(2)}$$

Decimal approximation:

9.0647202836543876192553658914333336203437229354475911683720330958 …

9.06472028365….

Alternative representations:

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\log_e(2)}$$

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\log(a)\log_a(2)}$$

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{2\coth^{-1}(3)}$$

Series representations:

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{2\, i\, \pi \left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}}{k}} \quad \text{for } x < 0$$

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\log(z_0) + \left\lfloor \frac{\arg(2-z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0)\right) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}}$$

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{2\, i\, \pi \left\lfloor \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right\rfloor + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}}$$

Integral representations:

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\int_1^2 \frac{1}{t}\, dt}$$

$$\frac{2\pi}{\log(2)} = \frac{4\, i\, \pi^2}{\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma} \frac{\Gamma(-s)^2\, \Gamma(1+s)}{\Gamma(1-s)}\, ds} \quad \text{for } -1 < \gamma < 0$$

From which:

(2Pi/(ln(2)))*(1/12 π log(2))

Input:

$$\left(2\times\frac{\pi}{\log(2)}\right)\left(\frac{1}{12}\pi\log(2)\right)$$

$\log(x)$ is the natural logarithm

Exact result:

$$\frac{\pi^2}{6}$$

Decimal approximation:

1.644934066848226436472415166646025189218949901206798437735558229 3

…

1.6449340668…. = ζ(2) = $\frac{\pi^2}{6} = 1.644934\ldots$

From:

Modular equations and approximations to π **-** *Srinivasa Ramanujan*

Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

We have that:

Hence

$$64g_{22}^{24} = e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \cdots,$$
$$64g_{22}^{-24} = 4096e^{-\pi\sqrt{22}} + \cdots,$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\ldots.$$

Again

$$G_{37} = (6+\sqrt{37})^{\frac{1}{4}},$$

$$64G_{37}^{24} = e^{\pi\sqrt{37}} + 24 + 276e^{-\pi\sqrt{37}} + \cdots,$$
$$64G_{37}^{-24} = 4096e^{-\pi\sqrt{37}} - \cdots,$$

so that

$$64(G_{37}^{24} + G_{37}^{-24}) = e^{\pi\sqrt{37}} + 24 + 4372e^{-\pi\sqrt{37}} - \cdots = 64\{(6+\sqrt{37})^6 + (6-\sqrt{37})^6\}.$$

Hence

$$e^{\pi\sqrt{37}} = 199148647.999978\ldots.$$

Similarly, from

$$g_{58} = \sqrt{\left(\frac{5+\sqrt{29}}{2}\right)},$$

we obtain

$$64(g_{58}^{24} + g_{58}^{-24}) = e^{\pi\sqrt{58}} - 24 + 4372e^{-\pi\sqrt{58}} + \cdots = 64\left\{\left(\frac{5+\sqrt{29}}{2}\right)^{12} + \left(\frac{5-\sqrt{29}}{2}\right)^{12}\right\}.$$

Hence

$$e^{\pi\sqrt{58}} = 24591257751.99999982\ldots.$$

We note that, with regard 4372, we can to obtain the following results:
27((4372)^1/2-2-1/2(((√(10-2√5) -2))⁄((√5-1))))+φ

Input

$$27\left(\sqrt{4372}-2-\frac{1}{2}\times\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)+\phi$$

ϕ is the golden ratio

Result

$$\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)$$

Decimal approximation

1729.0526944170905625170637208637148763684189306538457854815447023...

1729.0526944….

This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. (1728 = 8^2 * 3^3) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Alternate forms

$$\frac{1}{8}\left(-27\sqrt{5\left(10-2\sqrt{5}\right)}+58\sqrt{5}+432\sqrt{1093}-27\sqrt{2\left(5-\sqrt{5}\right)}-374\right)$$

$$\phi-54+54\sqrt{1093}+\frac{27}{4}\left(1+\sqrt{5}-\sqrt{2\left(5+\sqrt{5}\right)}\right)$$

$$\phi - 54 + 54\sqrt{1093} - \frac{27\left(\sqrt{10 - 2\sqrt{5}} - 2\right)}{2\left(\sqrt{5} - 1\right)}$$

Minimal polynomial

$$256 x^8 + 95\,744 x^7 - 3\,248\,750\,080 x^6 - 914\,210\,725\,504 x^5 + 15\,498\,355\,554\,921\,184 x^4 + 2\,911\,478\,392\,539\,914\,656 x^3 - 32\,941\,144\,911\,224\,677\,091\,680 x^2 - 3\,092\,528\,914\,069\,760\,354\,714\,456 x + 26\,320\,050\,609\,744\,039\,027\,169\,013\,041$$

Expanded forms

$$-\frac{187}{4} + \frac{29\sqrt{5}}{4} + 54\sqrt{1093} - \frac{27}{8}\sqrt{10 - 2\sqrt{5}} - \frac{27}{8}\sqrt{5\left(10 - 2\sqrt{5}\right)}$$

$$-\frac{107}{2} + \frac{\sqrt{5}}{2} + 54\sqrt{1093} + \frac{27}{\sqrt{5} - 1} - \frac{27\sqrt{10 - 2\sqrt{5}}}{2\left(\sqrt{5} - 1\right)}$$

Series representations

$$27\left(\sqrt{4372} - 2 - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{\left(\sqrt{5} - 1\right)2}\right) + \phi = \left(162 - 108\sqrt{1093} - 2\phi - 108\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 2\phi\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} - 27\sqrt{9 - 2\sqrt{5}}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(9 - 2\sqrt{5}\right)^{-k}\right) \Bigg/ \left(2\left(-1 + \sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k}\right)\right)$$

$$27\left(\sqrt{4372}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi=$$
$$\left(162-108\sqrt{1093}-2\phi-108\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+2\phi\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}-27\sqrt{9-2\sqrt{5}}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(9-2\sqrt{5}\right)^{-k}}{k!}\right)\bigg/\left(2\left(-1+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)$$

$$27\left(\sqrt{4372}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi=$$
$$\left(162-108\sqrt{1093}-2\phi-108\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}+108\sqrt{1093}\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}+2\phi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}-27\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(10-2\sqrt{5}-z_0\right)^k z_0^{-k}}{k!}\right)\bigg/\left(2\left(-1+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}\right)\right)$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

Or:

27((4096+276)^1/2-2-1/2(((√(10-2√5) -2))/((√5-1))))+φ

Input

$$27\left(\sqrt{4096+276}-2-\frac{1}{2}\times\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)+\phi$$

ϕ is the golden ratio

Result

$$\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)$$

Decimal approximation

1729.052694417090562517063720863714876368418930653845785481544702 3

...

1729.0526944…. as above

Alternate forms

$$\frac{1}{8}\left(-27\sqrt{5\left(10-2\sqrt{5}\right)}+58\sqrt{5}+432\sqrt{1093}-27\sqrt{2\left(5-\sqrt{5}\right)}-374\right)$$

$$\phi-54+54\sqrt{1093}+\frac{27}{4}\left(1+\sqrt{5}-\sqrt{2\left(5+\sqrt{5}\right)}\right)$$

$$\phi-54+54\sqrt{1093}-\frac{27\left(\sqrt{10-2\sqrt{5}}-2\right)}{2\left(\sqrt{5}-1\right)}$$

Minimal polynomial

$$256x^8+95744x^7-3248750080x^6-914210725504x^5+15498355554921184x^4+2911478392539914656x^3-32941144911224677091680x^2-3092528914069760354714456x+26320050609744039027169013041$$

Expanded forms

$$-\frac{187}{4}+\frac{29\sqrt{5}}{4}+54\sqrt{1093}-\frac{27}{8}\sqrt{10-2\sqrt{5}}-\frac{27}{8}\sqrt{5\left(10-2\sqrt{5}\right)}$$

$$-\frac{107}{2}+\frac{\sqrt{5}}{2}+54\sqrt{1093}+\frac{27}{\sqrt{5}-1}-\frac{27\sqrt{10-2\sqrt{5}}}{2\left(\sqrt{5}-1\right)}$$

Series representations

$$27\left(\sqrt{4096+276}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi=\left(162-108\sqrt{1093}-2\phi-108\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}+108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}+2\phi\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}-27\sqrt{9-2\sqrt{5}}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(9-2\sqrt{5}\right)^{-k}\right)\Bigg/\left(2\left(-1+\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)\right)$$

$$27\left(\sqrt{4096+276}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi=\left(162-108\sqrt{1093}-2\phi-108\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+2\phi\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}-27\sqrt{9-2\sqrt{5}}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(9-2\sqrt{5}\right)^{-k}}{k!}\right)\Bigg/\left(2\left(-1+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)$$

$$27\left(\sqrt{4096+276}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi =$$

$$\left(162-108\sqrt{1093}-2\phi-108\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}+\right.$$
$$108\sqrt{1093}\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}+$$
$$2\phi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}-$$
$$\left.27\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(10-2\sqrt{5}-z_0\right)^k z_0^{-k}}{k!}\right)\Big/$$
$$\left(2\left(-1+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}\right)\right)$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

From which:

(27((4372)^1/2-2-1/2(((√(10-2√5) -2))⁄((√5-1))))+φ)^1/15

Input

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{1}{2}\times\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)+\phi}$$

ϕ is the golden ratio

Exact result

$$\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}$$

Decimal approximation

1.6438185685849862799902301317036810054185756873505184804834183124

...

1.64381856858.... ≈ ζ(2) = $\frac{\pi^2}{6}$=1.6449 34...

Alternate forms

$$\sqrt[15]{\phi - 54 + 54\sqrt{1093} - \frac{27\left(\sqrt{10 - 2\sqrt{5}} - 2\right)}{2\left(\sqrt{5} - 1\right)}}$$

$$\frac{1}{\sqrt[15]{\frac{2\left(\sqrt{5}-1\right)}{166-108\sqrt{5}-108\sqrt{1093}+108\sqrt{5465}-27\sqrt{2\left(5-\sqrt{5}\right)}}}}$$

$$\sqrt[15]{\text{root of } 256x^8 + 95744x^7 - 3248750080x^6 - 914210725504x^5 + 15498355554921184x^4 + 2911478392539914656x^3 - 32941144911224677091680x^2 - 3092528914069760354714456x + 26320050609744039027169013041 \text{ near } x = 1729.05}$$

Minimal polynomial

$$256x^{120} + 95744x^{105} - 3248750080x^{90} - 914210725504x^{75} + 15498355554921184x^{60} + 2911478392539914656x^{45} - 32941144911224677091680x^{30} - 3092528914069760354714456x^{15} + 26320050609744039027169013041$$

Expanded forms

$$\sqrt[15]{\frac{1}{2}\left(1 + \sqrt{5}\right) + 27\left(-2 + 2\sqrt{1093} - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{2\left(\sqrt{5} - 1\right)}\right)}$$

$$\sqrt[15]{-\frac{187}{4}+\frac{29\sqrt{5}}{4}+54\sqrt{1093}-\frac{27}{8}\sqrt{10-2\sqrt{5}}-\frac{27}{8}\sqrt{5\left(10-2\sqrt{5}\right)}}$$

All 15th roots of ϕ + 27 (-2 + 2 sqrt(1093) - (sqrt(10 - 2 sqrt(5)) - 2)/(2 (sqrt(5) - 1)))

$$e^{0}\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}\approx 1.64382 \text{ (real, principal root)}$$

$$e^{(2 i \pi)/15}\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}\approx 1.50170+0.6686\,i$$

$$e^{(4 i \pi)/15}\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}\approx 1.0999+1.2216\,i$$

$$e^{(2 i \pi)/5}\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}\approx 0.5080+1.5634\,i$$

$$e^{(8 i \pi)/15}\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}\approx -0.17183+1.63481\,i$$

Series representations

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi} = \frac{1}{\sqrt[15]{2}}\left(\left(\left(162-108\sqrt{1093}-2\phi-108\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}+108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}+2\phi\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}-27\sqrt{9-2\sqrt{5}}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(9-2\sqrt{5}\right)^{-k}\right)\Big/\left(-1+\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)\right)\text{^}(1/15)\right)$$

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi} = \frac{1}{\sqrt[15]{2}}\left(\left(\left(162-108\sqrt{1093}-2\phi-108\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+2\phi\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}-27\sqrt{9-2\sqrt{5}}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(9-2\sqrt{5}\right)^{-k}}{k!}\right)\Big/\left(-1+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)\text{^}(1/15)\right)$$

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{(\sqrt{5}-1)2}\right)+\phi} =$$

$$\frac{1}{\sqrt[15]{2}}\left(\left(\left(162-108\sqrt{1093}-2\phi-108\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}+\right.\right.\right.$$
$$108\sqrt{1093}\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}+$$
$$2\phi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}-$$
$$\left.27\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(10-2\sqrt{5}-z_0\right)^k z_0^{-k}}{k!}\right)\Big/$$
$$\left.\left.\left(-1+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}\right)\right)\wedge(1/15)\right)$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \le 0$))

Integral representation

$$(1+z)^a = \frac{\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(s)\Gamma(-a-s)}{z^s}\,ds}{(2\pi i)\Gamma(-a)} \quad \text{for } (0<\gamma<-\text{Re}(a) \text{ and } |\arg(z)|<\pi)$$

From:

An Update on Brane Supersymmetry Breaking

J. Mourad and A. Sagnotti - arXiv:1711.11494v1 [hep-th] 30 Nov 2017

From the following vacuum equations:

$$T\,e^{\gamma_E\,\phi} = -\frac{\beta_E^{(p)}\,h^2}{\gamma_E}\,e^{-2(8-p)C+2\beta_E^{(p)}\phi}$$

$$16\,k'\,e^{-2C} = \frac{h^2\left(p+1-\frac{2\beta_E^{(p)}}{\gamma_E}\right)e^{-2(8-p)C+2\beta_E^{(p)}\phi}}{(7-p)}$$

$$(A')^2 = k\,e^{-2A} + \frac{h^2}{16(p+1)}\left(7 - p + \frac{2\,\beta_E^{(p)}}{\gamma_E}\right)\, e^{-2(8-p)C + 2\beta_E^{(p)}\phi}$$

we have obtained, from the results almost equals of the equations, putting

$4096\,e^{-\pi\sqrt{18}}$ instead of

$$e^{-2(8-p)C + 2\beta_E^{(p)}\phi}$$

a new possible mathematical connection between the two exponentials. Thence, also the values concerning *p, C,* β_E and ϕ correspond to the exponents of *e* (i.e. of exp). Thence we obtain for p = 5 and $\beta_E = 1/2$:

$e^{-6C+\phi} = 4096\,e^{-\pi\sqrt{18}}$

Therefore, with respect to the exponentials of the vacuum equations, the Ramanujan's exponential has a coefficient of 4096 which is equal to 64^2, while -6C+ϕ is equal to -$\pi\sqrt{18}$. From this it follows that it is possible to establish mathematically, the dilaton value.

For

exp((-Pi*sqrt(18))   we obtain:

Input:

$\exp\left(-\pi\sqrt{18}\right)$

Exact result:

$e^{-3\sqrt{2}\,\pi}$

Decimal approximation:

$1.6272016226072509292942156739117979541838581136954016\ldots \times 10^{-6}$

1.6272016… * 10^{-6}

Property:

$e^{-3\sqrt{2}\,\pi}$ is a transcendental number

Series representations:

$$e^{-\pi\sqrt{18}} = e^{-\pi\sqrt{17}\sum_{k=0}^{\infty} 17^{-k}\binom{1/2}{k}}$$

$$e^{-\pi\sqrt{18}} = \exp\left(-\pi\sqrt{17}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{17}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$e^{-\pi\sqrt{18}} = \exp\left(-\frac{\pi\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j} 17^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}\right)$$

Now, we have the following calculations:

$$e^{-6C+\phi} = 4096\,e^{-\pi\sqrt{18}}$$

$$e^{-\pi\sqrt{18}} = 1.6272016\ldots * 10\text{^}{-6}$$

from which:

$$\frac{1}{4096}e^{-6C+\phi} = 1.6272016\ldots * 10\text{^}{-6}$$

$$0.000244140625\, e^{-6C+\phi} = e^{-\pi\sqrt{18}} = 1.6272016\ldots * 10\text{^}{-6}$$

Now:

$$\ln\left(e^{-\pi\sqrt{18}}\right) = -13.328648814475 = -\pi\sqrt{18}$$

And:

(1.6272016* 10^-6) *1/ (0.000244140625)

Input interpretation:

$$\frac{1.6272016}{10^6} \times \frac{1}{0.000244140625}$$

Result:

0.0066650177536

0.006665017...

Thence:

$$0.000244140625\, e^{-6C+\phi} = e^{-\pi\sqrt{18}}$$

Dividing both sides by 0.000244140625, we obtain:

$$\frac{0.000244140625}{0.000244140625} e^{-6C+\phi} = \frac{1}{0.000244140625} e^{-\pi\sqrt{18}}$$

$$e^{-6C+\phi} = 0.0066650177536$$

((((exp((-Pi*sqrt(18)))))))*1/0.000244140625

Input interpretation:

$$\exp\left(-\pi\sqrt{18}\right) \times \frac{1}{0.000244140625}$$

Result:

0.00666501785…

0.00666501785…

Series representations:

$$\frac{\exp\left(-\pi\sqrt{18}\right)}{0.000244141} = 4096 \exp\left(-\pi\sqrt{17}\sum_{k=0}^{\infty} 17^{-k}\binom{\frac{1}{2}}{k}\right)$$

$$\frac{\exp(-\pi\sqrt{18})}{0.000244141} = 4096 \exp\left(-\pi\sqrt{17}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{17}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$\frac{\exp(-\pi\sqrt{18})}{0.000244141} = 4096 \exp\left(-\frac{\pi\sum_{j=0}^{\infty}\mathrm{Res}_{s=-\frac{1}{2}+j}\ 17^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}\right)$$

Now:

$$e^{-6C+\phi} = 0.0066650177536$$

$$\exp\left(-\pi\sqrt{18}\right)\times\frac{1}{0.000244140625} =$$

$$e^{-\pi\sqrt{18}}\times\frac{1}{0.000244140625}$$

$$= 0.00666501785\ldots$$

From:
ln(0.00666501784619)

Input interpretation:

log(0.00666501784619)

Result:

–5.010882647757…

-5.010882647757…

Alternative representations:

$$\log(0.006665017846190000) = \log_e(0.006665017846190000)$$

$$\log(0.006665017846190000) = \log(a)\log_a(0.006665017846190000)$$

$$\log(0.006665017846190000) = -\text{Li}_1(0.993334982153810000)$$

Series representations:

$$\log(0.006665017846190000) = -\sum_{k=1}^{\infty} \frac{(-1)^k (-0.993334982153810000)^k}{k}$$

$$\log(0.006665017846190000) = 2 i \pi \left\lfloor \frac{\arg(0.006665017846190000 - x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (0.006665017846190000 - x)^k x^{-k}}{k} \quad \text{for } x < 0$$

$$\log(0.006665017846190000) = \left\lfloor \frac{\arg(0.006665017846190000 - z_0)}{2\pi} \right\rfloor \log\left(\frac{1}{z_0}\right) + \log(z_0) + \left\lfloor \frac{\arg(0.006665017846190000 - z_0)}{2\pi} \right\rfloor \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (0.006665017846190000 - z_0)^k z_0^{-k}}{k}$$

Integral representation:

$$\log(0.006665017846190000) = \int_1^{0.006665017846190000} \frac{1}{t}\, dt$$

In conclusion:

$$-6C + \phi = -5.010882647757\ldots$$

and for C = 1, we obtain:

$$\phi = -5.010882647757 + 6 = 0.989117352243 = \phi$$

Note that the values of n_s (spectral index) 0.965, of the average of the Omega mesons Regge slope 0.987428571 and of the dilaton 0.9891173352243, are also connected to the following two Rogers-Ramanujan continued fractions:

$$\frac{e^{-\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}}-\varphi+1}=1-\cfrac{e^{-\pi}}{1+\cfrac{e^{-2\pi}}{1+\cfrac{e^{-3\pi}}{1+\cfrac{e^{-4\pi}}{1+\dots}}}}\approx 0.9568666373$$

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\cfrac{e^{-\pi\sqrt{5}}}{1+\cfrac{e^{-2\pi\sqrt{5}}}{1+\cfrac{e^{-3\pi\sqrt{5}}}{1+\cfrac{e^{-4\pi\sqrt{5}}}{1+\dots}}}}\approx 0.9991104684$$

(http://www.bitman.name/math/article/102/109/)

Also performing the 512th root of the inverse value of the Pion meson rest mass 139.57, we obtain:
((1/(139.57)))^1/512

Input interpretation:

$$\sqrt[512]{\frac{1}{139.57}}$$

Result:

0.990400732708644027550973755713301415460732796178555551684...

0.99040073.... result very near to the dilaton value 0.989117352243 = ϕ and to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1} = 1-\cfrac{e^{-\pi\sqrt{5}}}{1+\cfrac{e^{-2\pi\sqrt{5}}}{1+\cfrac{e^{-3\pi\sqrt{5}}}{1+\cfrac{e^{-4\pi\sqrt{5}}}{1+\dots}}}} \approx 0.9991104684$$

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References

The MRB constant: ALL ABOARD! POSTED BY: ***Marvin Ray Burns***

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