Dark Energy is Gravitational Potential Energy or Energy of the Gravitational Field [1]

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Abstract

When the bound object acts on gravity, the gravitational action of gravitational potential energy is also included. Therefore, even in the case of the universe, the gravitational action of gravitational potential energy must be considered. Gravitational potential energy generates a repulsive force because it has a negative equivalent mass. Mass energy (Mc^2) is an attractive component, and the equivalent mass ($-M_{gs}$) of gravitational potential energy is a repulsive component. Therefore, if $|(-M_{gs})c^2| < Mc^2$, there is a decelerated expansion, and if $|(-M_{gs})c^2| > Mc^2$, accelerated expansion is performed. $|(-M_{gs})c^2| = Mc^2$ is the inflection point from the decelerated expansion to the accelerated expansion. The source of dark energy is presumed to be due to gravitational self-energy and an increase in mass due to the expansion of the particle horizon. The dark energy effect occurs because all positive energy (mass) entering the particle horizon produces negative gravitational potential energy. While mass energy is proportional to M, gravitational self-energy increases faster because gravitational self-energy is proportional to $-\frac{M^2}{R}$. Accordingly, an effect of increasing dark energy occurs. I present Friedmann's equations and dark energy function obtained through gravitational self-energy model. There is no cosmological constant and dark energy is a function of time. This model predicts an inflection point where dark energy becomes larger and more important than the energy of matter and radiation. Since the observable universe is almost flat and the mass density is very low, a correspondence principle between general relativity and Newtonian mechanics will be established. Therefore, gravitational potential energy or gravitational self-energy can be good approximation to the dark energy.

I. Gravitational action of the gravitational field

The basis of modern cosmology is Einstein's 1915 field equation. However, this field equation has a singularity problem inside the black hole. In other words, the field equation created by Einstein is likely to be incomplete.

The fundamental principle of general relativity states that "all energy is a source of gravity". However, the field equation created by Einstein did not fully realize this principle.

> *In writing the field equation (48) we have assumed that the quantity $T^{\mu\nu}$ is the energy-momentum tensor of matter. In order to obtain a linear field equation we have left out the effect of the gravitational field upon itself. Because of this omission, our linear field equation has several (related) defects: (1) According to (48) matter acts on the gravitational field (changes the fields), but there is no mutual action of gravitational fields on matter; that is, the gravitational field can acquire energy-momentum from matter, but nevertheless the energy-momentum of matter is conserved ($\partial_\nu T^{\mu\nu} = 0$). This is an inconsistency. (2)Gravitational energy does not act as source of gravitation, in contradiction to the principle of equivalence. Thus, although Eq. (48) may be a fair approximation in the case of weak gravitational fields, it cannot be an exact equation.*

[1] This paper is an excerpt from the following paper, only items related to dark energy. On the solution of the strong gravitational field, the solution of the Singularity problem, the origin of Dark energy and Dark matter [https://www.researchgate.net/publication/359329109] or [https://vixra.org/abs/2203.0096]

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The obvious way to correct for our sin of omission is to include the energy-momentum tensor of the gravitational field in $T^{\mu\nu}$. This means that we take for the quantity $T^{\mu\nu}$ the total energy-momentum tensor of matter plus gravitation:

$T^{\mu\nu} = T^{\mu\nu}_{(m)} + t^{\mu\nu}$ [1]

The energy of the gravitational field must also function as a gravitational source. However, because it was difficult to define the energy-momentum of the gravitational field, Einstein could not complete the field equation including the gravitational action of the gravitational field. [2] Methods such as the Landau-Lifshitz pseudotensor [3] exist for describing the energy-momentum of a gravitational field, but not everyone agrees. Also, it seems that these pseudotensors are not developed enough to be applied to cosmology. By the way, it may be sufficient to include the equivalent mass density of gravitational self-energy in $T^{\mu\nu}_{(m)}$ without introducing $t^{\mu\nu}$.

Therefore, in this paper, based on the corresponding principle that general relativity is approximated by Newtonian mechanics in a weak field, I approach the gravitational action of gravitational energy as an approximate way (gravitational potential energy). And I will prove that this method suggests sufficiently meaningful results in cosmology.

1. Negative mass (energy) density in standard cosmology

1) The logical structure of standard cosmology

From the second Friedmann equation or acceleration equation, [4]

$$\frac{1}{R}(\frac{d^2R}{dt^2}) = -(\frac{4\pi G}{3})(\rho + 3P) \tag{1}$$

In the standard cosmology, it is explained by introducing an entity that has a positive mass (energy) density but exerts a negative pressure. [4] [5] [6]

$$\rho_\Lambda + 3P = \rho_\Lambda + 3(-\rho_\Lambda) = -2\rho_\Lambda \tag{2}$$

However, if we expand the dark energy term, the final result is a negative mass density of $-2\rho_\Lambda$.

Let's look at the expression expressing $(\rho + 3P)$ as the critical density of the universe.
Matter + Dark Matter : $\rho_m \approx \frac{1}{3}\rho_c$
Dark energy density : $\rho_\Lambda \approx \frac{2}{3}\rho_c$
(Matter + Dark Matter)'s pressure : $P_m \approx 0$
Dark energy's pressure : $P_\Lambda = (-\rho_\Lambda) = (-\frac{2}{3}\rho_c)$

$$\rho + 3P \simeq \rho_m + \rho_\Lambda + 3(P_m + P_\Lambda) \simeq (\frac{1}{3})\rho_c + (\frac{2}{3})\rho_c + 3(-\frac{2}{3})\rho_c = (+1)\rho_c + (-2)\rho_c = (-1)\rho_c \tag{3}$$

$$\rho + 3P \simeq (+1)\rho_c + (-2)\rho_c = (-1)\rho_c \tag{4}$$

Standard cosmology is a form of positive mass density of $+1\rho_c$ and negative mass density of $-2\rho_c$. So, finally, the universe has a negative mass density of "$-\rho_c$", so accelerated expansion is taking place. The current universe is similar to a state where the negative mass density is twice the positive mass density. And the total mass of the observable universe is the negative mass state.

2) The claim that vacuum energy and the cosmological constant have a negative pressure is wrong

In standard cosmology, accelerated expansion is impossible without a negative mass density. Because researchers have an aversion to negative mass density, they are just using negative pressure instead of negative mass density. However, the claim that vacuum energy and the cosmological constant have a negative pressure is wrong.

a) Sign of the negative pressure [7]

Note that the effect of the pressure P is to slow down the expansion (assuming $P > 0$). If this seems counterintuitive, recall that because the pressure is the same everywhere in the universe, both inside and outside the shell, there is no pressure gradient to exert a net force on the expanding sphere. The answer lies in the motion of the particles that creates the fluid's pressure. The equivalent mass of the particle's kinetic energy creates a gravitational attraction that slows down the expansion just as their actual mass does. [4]

The pressure P is related to the momentum or kinetic energy of the particle. Therefore, it seems that in order for the pressure P to have a negative value, it must have negative momentum or negative kinetic energy. So, assuming that the pressure P term has a negative energy density is same assuming that it has negative kinetic energy. In order to have negative kinetic energy, it must have negative inertial mass or imaginary velocity. But, because they assumed a positive inertial mass, it is a logical contradiction.

$$K = \frac{1}{2}mv^2 < 0 \tag{5}$$

$m < 0$ or $v = Vi$: negative mass or imaginary speed.

Negative mass contradicts the assumption of positive energy density, and energy density with imaginary speed is far from physical reality.

b) Size of the negative pressure [7]

From the analysis of the ideal gas, we obtain,

$$P = \frac{1}{3}(\frac{v^2}{c^2})\rho = \omega\rho \tag{6}$$

In the case of matter, $v \ll c$, So $P = \frac{1}{3}(\frac{v}{c})^2\rho \simeq 0$

In the case of radiation, v=c, So $P = \frac{1}{3}(\frac{v}{c})^2\rho = \frac{1}{3}\rho$

However, in the case of dark energy $P = \frac{1}{3}(\frac{v}{c})^2\rho = -\rho$

b)-1) Only considering the size (absolute value)

$$P = \frac{1}{3}(\frac{v}{c})^2\rho = |-\rho| \tag{7}$$

$$v = \sqrt{3}c \tag{8}$$

We need energy density with super-luminous speed.

b)-2) Considering even the negative sign

$$P = \frac{1}{3}(\frac{v}{c})^2\rho = -\rho \tag{9}$$

$$v = (\sqrt{3}c)i \tag{10}$$

We need energy density with imaginary super-luminous speed.

Above equation applies to "massless $\sim$ no upper limit" particles, and the velocity ranges from "0 $\sim$ c" are all included. If we seriously consider the physical presence of $P = -\rho = -3(\frac{1}{3}\rho)$, we will find that there is a serious problem.

c) Incorrect application of $dU = -PdV$

In the equation $dU = -PdV$, researchers claim that negative pressure exists if the energy density remains constant as the volume increases. Is this claim correct?

c)-1) This explanation is an inverted explanation

Since pressure is a property of an object, pressure exists first, and because of this pressure, changes in internal energy according to volume change appear.

That is, since pressure is positive, if $dV > 0$, then $dU < 0$.
Since the pressure is positive, if $dV < 0$, then $dU > 0$.
By the way, we use the logic "if $dV > 0$, $dU > 0$, then $P < 0$". How are you sure that this logic is correct?

c)-2) $\rho + 3P = \rho + 3(-\rho) = -2\rho$
Mass density ρ and pressure P are properties of the object to be analyzed. Both mass density ρ and pressure P are sources of gravity. It means that even if the region maintains a constant size without expanding or contracting, gravitational force is applied as much as $\rho + 3P = -2\rho$.

In other words, it suggests that the object (or energy density) has a gravity with a negative mass density of -2ρ. This is different from a vacuum with a positive energy density $+\rho$, which we think of.

c)-3) $dU = -PdV$ is the expression obtained when the law of conservation of energy is established
However, in the case of vacuum energy and the cosmological constant, energy conservation does not hold. As the universe expands, the total energy in the system increases. Therefore, we cannot guarantee that $dU = -PdV$ holds. $dU = -PdV$ is an equation that holds when the energy of the system is conserved. However, researchers are applying this equation to vacuum energy or cosmological constant where the energy of the system is not conserved. This is problematic.

I am not sure if $dU = -PdV$ holds even for negative pressure. In the case of negative pressure, there may be a sign reversal. However, although this equation holds even in the case of negative pressure, its interpretation is as follows.

This equation holds true when substances in radius r_1 expand from r_1 to r_2 ($r_2 > r_1$), and have the same uniform density in r_1 and r_2. In other words, it is argued that a negative pressure is required to create a uniform density effect **only with the material present in radius** r_1. But, vacuum energy is a form in which energy is newly generated by an increased volume. It is also energy that can be assumed to have an initial speed of $0 \sim c$. Considering the positive energy density, pressure term seems reasonable to assume $0 \sim \frac{1}{3}\rho$.

We can considered the vacuum energy density with $P = 0$. However, in order to have a uniform energy density even when space expands, does it have to suddenly have negative pressure? Shouldn't it be newly created, with $P = 0$, and filling the expanded space?

Can we not think of a vacuum energy density with $P = 0$? Can't there be a vacuum energy density with $0 \sim \frac{1}{3}\rho$? Even if vacuum energy with positive energy density is present, it is not certain whether it creates a negative pressure.

There is no physical entity that satisfies $\rho+3P = \rho+3(-\rho) = -2\rho$. Nothing has a kinetic energy component three times greater than that of light. Vacuum energy and cosmological constant has no negative pressure. **The negative pressure is due to incorrect application of** $dU = -PdV$.

Vacuum energy with positive energy density $+\rho$ does not have a negative pressure, but has a $0 \sim$ positive pressure. Because the source of pressure is related to momentum or kinetic energy.

Researchers, who could not accept negative mass density, try to somehow create an accelerated expansion through the equation $\rho+3P$, so assuming magic with positive energy density but negative pressure. As a result, we have created something imaginary, not a physical reality. It is not necessary for an entity to exist that exerts a negative pressure while having a positive energy density. It is sufficient that a positive energy component and a negative energy component exist. And, the negative energy component may be the gravitational potential energy or gravitational field's energy.

The vacuum energy model is a model with a positive mass density $+\rho$ and a negative pressure $3P = -3\rho$. On the other hand, the gravitational potential energy model is a model with a positive mass density $+\rho$ and an equivalent mass density $-\rho_{gs}$ of gravitational potential energy. The equivalent mass density of gravitational potential energy will replace the role of negative pressure in standard cosmology.

2. The energy of a gravitational field is negative
Alan Guth said: [8]

The energy of a gravitational field is negative!

The positive energy of the false vacuum was compensated by the negative energy of gravity.

Stephen Hawking also argued that [9]

> *The matter in the universe is made out of positive energy. However, the matter is all attracting itself by gravity. Two pieces of matter that are close to each other have less energy than the same two pieces a long way apart, because you have to expend energy to separate them against the gravitational force that is pulling them together. Thus, in a sense, the gravitational field has negative energy. In the case of a universe that is approximately uniform in space, one can show that this negative gravitational energy exactly cancels the positive energy represented by the matter. So the total energy of the universe is zero.*
>
> *Now twice zero is also zero. Thus the universe can double the amount of positive matter energy and also double the negative gravitational energy without violation of the conservation of energy.*

Both argued that gravitational potential energy is negative energy and is the true energy that can cancel positive mass energy.

3. Gravitational self-energy

The concept of gravitational self-energy (U_{gs}) is the total of gravitational potential energy possessed by a certain object M itself. Since a certain object M itself is a binding state of infinitesimal mass dMs, it involves the existence of gravitational potential energy among these dMs and is the value of adding up these. $M = \sum dM$.

Gravitational self-energy or Gravitational binding energy($-U_{gs}$) in case of spherical uniform distribution is given by

$$U_{gs} = -\frac{3}{5}\frac{GM^2}{R} \tag{11}$$

> *Treating the Earth as a continuous, classical mass distribution (with no gravitational self-energy in the elementary, subatomic particles), we find that its gravitational self-energy is about* 4.6×10^{-10} *times its rest-mass energy. The gravitational self-energy of the Moon is smaller, only about* 0.2×10^{-10} *times its rest-mass energy. - Gravitation and Spacetime* [1]

All energy is a gravitational source, and gravitational potential energy is also a gravitational source. Thus, the gravitational field is self-interacting. This makes the problem very complicated. However, gravitational self-energy is the first set of gravitational potential energies produced by a mass. Thus, the gravitational self-energy will be in the first term when developing the solution of gravitational interaction. That is, it is the largest and most important term. Since gravitational self-energy is the largest and most important first-order term, we need to utilize it.

The gravitational self-energy is proportional to $-\frac{M^2}{R}$. Therefore, as the mass increases, the gravitational self-energy value increases.

In the case of Moon, $U_{gs-Moon} = (-1.89 \times 10^{-11})M_{Moon}c^2$

In the case of Earth, $U_{gs-Earth} = (-4.17 \times 10^{-10})M_{Earth}c^2$

In the case of the Sun, $U_{gs-Sun} = (-1.27 \times 10^{-4})M_{Sun}c^2$

In case of a Black hole, $U_{gs-Black-hole} = (-3.0 \times 10^{-1})M_{Black-hole}c^2$

$$U_{gs-Black-hole} = -\frac{3}{5}\frac{GM^2}{R} = -\frac{3}{5}\frac{GM^2}{(\frac{2GM}{c^2})} = -\frac{3}{10}Mc^2 \tag{12}$$

It can be seen that as the mass increases, the ratio of gravitational self-energy increases. **So, now we can ask the following question. What about the universe with much greater mass?**

4. Binding energy in the mass defect problem

Consider situations a) and c).

In a), the total mass of the two particle system is 2m, and the total energy is $E_T = 2mc^2$

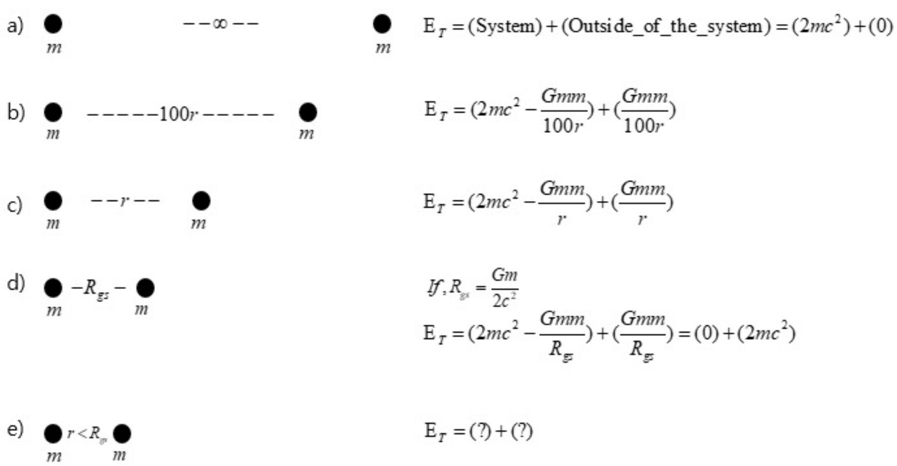

Figure 1: In order for a system to form a bond state, a mass defect equal to the binding energy must occur.

In c), is the total energy of the two particle system $E_T = 2mc^2$?

In c), when the two particle system act gravitationally on an external particle, will they act gravitationally with a gravitational mass of $2m$?

Are a), b) and c) the same mass or same state?

If you do not apply negative binding energy, the conclusions you get will not match the actual results.

In c), the total energy of the two particle system is

$$E_T = 2mc^2 - \frac{Gmm}{r} \tag{13}$$

In the dimensional analysis of energy, E has $kg(m/s)^2$, so all energy can be expressed in the form of $(mass)X(velocity)^2$. So, $E = mc^2$ holds true for all kinds of energy. Here, m is the equivalent mass. If we introduce the negative equivalent mass $-m_{gp}$ for the negative gravitational potential energy,

$$-\frac{Gmm}{r} = -m_{gp}c^2 \tag{14}$$

$$E_T = 2mc^2 - \frac{Gmm}{r} = 2mc^2 - m_{gp}c^2 = (2m - m_{gp})c^2 = m^*c^2 \tag{15}$$

The gravitational force acting on a relatively distant third mass m_3 is

$$F = -\frac{Gm^*m_3}{R^2} = -\frac{G(2m - m_{gp})m_3}{R^2} = -\frac{G(2m)m_3}{R^2} - \frac{G(-m_{gp})m_3}{R^2} \tag{16}$$

That is, **when considering the gravitational action of a bind system, not only the mass in its free state but also the binding energy term ($-m_{gp}$) should be considered.** Alternatively, the gravitational force acting on the bind system can be decomposed into a free-state mass term and an equivalent mass term of binding energy.

While we usually use the mass m^* of the bind system, we forget that m^* is "$m - m_{binding-energy}$". Gravitational potential energy is also a kind of binding energy. Therefore, the gravitational potential energy, which is the binding energy, must also be considered in the universe.

When a macroscopic object is subjected to gravitational force, the equation includes the gravitational term of the negative gravitational potential energy. In a normal case, since the term of gravitational potential energy is negligible compared to mass energy, it can be neglected. Alternatively, you can think of the mass of the gravitational source as the equivalent mass m^* including the gravitational self-energy, not the free mass.

However, since the gravitational self-energy has a fairly large value near the Schwarzschild radius, it must be considered. The same is true of the universe.

II. The sources of dark energy are gravitational potential energy and the expansion of the particle horizon

The universe is almost flat, and its mass density is also very low. [4] [10] Thus, Newtonian mechanics approximation can be applied. And, the following reasoning should not be denied by the assertion that "it is difficult to define the total energy in general relativity."

Whether or not it is an appropriate approximation is not determined by current field equation and current general relativity. This is because the current field equation is an incomplete field equation that does not include the gravitational action of the gravitational field. When it is difficult to find a complete solution, we have found numerous solutions through approximation. The success of this approximation or inference must be determined by the model's predictions and observations of the universe.

> *Equation (29.51, acceleration equation) is an illustration of Birkhoff's theorem. In 1923 the American mathematician G.D.Birkhoff (1884-1944) proved quite generally that for a spherically symmetric distribution of matter, Einstein's field equations have a unique solution. As a corollary,* ***the acceleration of an expanding shell in our fluid universe is determined solely by the fluid lying within the shell. Equation (29.51) shows that the acceleration does not depend on any factors other than ρ, P, and R.*** *Because Birkhoff's theorem holds even when general relativity is included, it is quite important in the study of cosmology.* [4]

By Birkhoff's theorem, only the mass-energy component in the particle horizon is important. Since the particle horizon is the range of interaction, if we find the mass energy (Mc^2) and gravitational self-energy ($(-M_{gs})c^2$) values at each particle horizon, mass energy is an attractive component, and the equivalent mass of gravitational self-energy is a repulsive component. Critical density value $\rho_c = 8.50 \times 10^{-27}[kgm^{-3}]$ [10] was used. In practice the density changes, but we just need to look at the trend.

1. Comparison of magnitudes of mass energy and gravitational self-energy in the cosmic event horizon

1) Total mass energy in the cosmic event horizon $R = 16.7Gly$

Cosmic event horizon 16.7Gly. Since the universe is almost flat spacetime, the total mass energy (include radiation energy) in the particle horizon is

$$Mc^2 = \frac{4\pi r^3 \rho c^2}{3} = 1.28 \times 10^{70}[kgm^2s^{-2}] \tag{17}$$

2) Gravitational self-energy in the cosmic event horizon

$$U = -\frac{3}{5}\frac{GM^2}{R} = -\frac{16\pi^2 GR^5\rho^2}{15} = -4.99 \times 10^{69}[kgm^2s^{-2}] \tag{18}$$

3) Comparison of magnitudes of total mass energy and gravitational self-energy in the cosmic event horizon

$$\frac{U}{Mc^2} = \frac{-4.99 \times 10^{69}}{1.28 \times 10^{70}} = -0.39 \tag{19}$$

In the calculation, the current critical density value was used, but when the particle horizon is 16.7Gly, the density is different from now. So, just look at the logic.

In the cosmic event horizon (16.7Gly), the repulsive component is smaller than the attractive component. In this period, the universe is decelerating expansion. That is, when the particle horizon is 16.7Gly, the dark energy component is smaller than that of matter. This period is a period of decelerated expansion.

2. The inflection point that changes from decelerated expansion to accelerated expansion

The inflection point is that the particle horizon at the transition from material dominance to dark energy dominance. The particle horizon at the transition from a period of predominant attraction to a period of predominant repulsion.

If the mass energy and the gravitational self-energy are equal, then

$$Mc^2 = |-\frac{3}{5}\frac{GM^2}{R_{gs}}| \tag{20}$$

$$R_{gs} = \sqrt{\frac{5c^2}{4\pi G\rho}} \tag{21}$$

We only need to understand the general flow and possibility, so let's get R_{gs} by putting in the current critical density value. If we know the exact density function, we can find the exact inflection point.

$$R_{gs} = \sqrt{\frac{5c^2}{4\pi G\rho}} = 26.2Gly \tag{22}$$

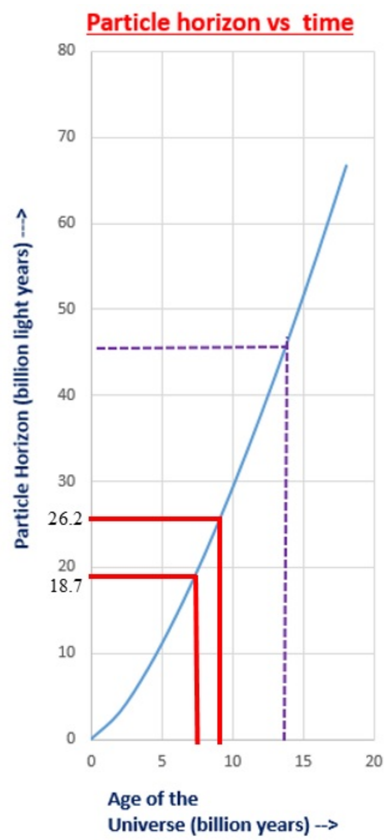

Figure 2: Particle horizon vs time. [11] **About 5-7 billion years ago, it passed the inflection point 18.7 ∼ 26.2Gly.** About 5 to 7 billion years ago, it can be said that the universe entered a period of accelerated expansion. The inflection point is affected by the density.

Assuming that the average density is approximately 1.25 times the current average density,
we get $R_{gs} = 23.7Gly$.
Assuming that the average density is approximately 1.5 times the current average density,
we get $R_{gs} = 21.4Gly$.

Assuming that the average density is approximately 2 times the current average density,
we get $R_{gs} = 18.7Gly$.

Comparing the data from the existing particle horizon graph, it is estimated that it is approximately 5 to 7 billion years ago from the present. This is similar to the result of standard cosmology. [4] This model includes the transition of the universe to the period of decelerated expansion, the inflection point, and the period of accelerated expansion.

3. Comparison of magnitudes of mass energy and gravitational self-energy in the observable universe

1) Total mass energy of the observable universe $R = 46.5Gly$

$$Mc^2 = \frac{4\pi r^3 \rho c^2}{3} = 2.75 \times 10^{71}[kgm^2s^{-2}] \tag{23}$$

2) Gravitational self-energy of the observable universe

$$U = -\frac{3}{5}\frac{GM^2}{R} = -\frac{16\pi^2 G R^5 \rho^2}{15} = -8.35 \times 10^{71}[kgm^2s^{-2}] \tag{24}$$

3) In the observable universe, the ratio of total mass energy to gravitational self-energy

$$\frac{U}{Mc^2} = \frac{-8.35 \times 10^{71}}{2.75 \times 10^{71}} = -3.04 \tag{25}$$

The repulsive force component is approximately 3.04 times the attractive force component. So, the universe is accelerating expansion.

4) Result summary

At particle horizon R=16.7Gly, $(-M_{gs})c^2 = (-0.39M)c^2$
$|(-M_{gs})c^2| < Mc^2$: Decelerated expansion period

At particle horizon R=26.2Gly, $(-M_{gs})c^2 = (-1.00M)c^2$
$|(-M_{gs})c^2| = Mc^2$: Inflection point (About 5-7 billion years ago, consistent with standard cosmology.)

At particle horizon R=46.5Gly, $(-M_{gs})c^2 = (-3.04M)c^2$
$|(-M_{gs})c^2| > Mc^2$: Accelerated expansion period

5) In the standard cosmology, the ratio of attractive and repulsive components

The following ratio is approximate ratio obtained from standard cosmology. [10]

$$\frac{M_{repulsive}}{M_{attractive}} = \frac{-3\rho_\Lambda}{\rho_m + \rho_\Lambda} = \frac{-3(0.683\rho)}{0.317\rho + 0.683\rho} = -2.05 \tag{26}$$

At 16.7Gly, the attraction component is larger than the repulsive component, whereas at 46.5Gly, the repulsive component is 3.04 times larger than the attractive component. In the middle, there is an inflection point that changes from decelerated expansion to accelerated expansion. About 5-7 billion years ago, similar to the predictions of standard cosmology.

Although there is a slight difference between the two values, it is not an astronomical error, so it is likely to be resolved later. It is possible that kinetic energy, general relativity, cosmological effects, assumption of uniformly distribution, or differences in cosmological models may explain the difference between the two values.

4. Regarding the cause of the error

1) New formula for calculating gravitational self-energy

According to the principle of general relativity, gravitational potential energy is also a source of gravity. Therefore, there is a problem in the conventional expression of gravitational self-energy.

Looking at the formula we use to find gravitational self-energy,

$$M' = \frac{4\pi}{3} r^3 \rho \tag{27}$$

$$dm = \rho r^2 dr \sin\theta d\theta d\varphi \tag{28}$$

$$dU_{gs} = -G\frac{M'm}{r} \tag{29}$$

In the existing gravitational self-energy equation, the gravitational action due to negative gravitational self-energy is omitted from the internal mass M' term. The equivalent mass of gravitational self-energy must also be reflected in the M' term.

$$M^* = M' - {M_{gs}}' = M' - \frac{3}{5}\frac{GM'^2}{rc^2} = \frac{4\pi}{3}r^3\rho - \frac{16\pi^2 G}{15c^2}r^5\rho^2 \tag{30}$$

$$dU_{gs} = -G\frac{M^* dm}{r} = -G\frac{(M' - {M_{gs}}')(\rho r^2 dr \sin\theta d\theta d\varphi)}{r} = -(\frac{4\pi G\rho^2}{3}r^4 - \frac{16\pi^2 G^2\rho^3}{15c^2}r^6)dr\sin\theta d\theta d\varphi \tag{31}$$

$$U_{gs} = -\frac{4\pi G\rho^2}{3}\int_0^R r^4 dr\int_0^\pi \sin\theta d\theta\int_0^{2\pi} d\varphi + \frac{16\pi^2 G^2\rho^3}{15c^2}\int_0^R r^6 dr\int_0^\pi \sin\theta d\theta\int_0^{2\pi} d\varphi \tag{32}$$

$$U_{gs} = -\frac{(4\pi\rho)^2 G}{3}\int_0^R r^4 dr + \frac{(4\pi\rho)^3 G^2}{15c^2}\int_0^R r^6 dr = -\frac{3}{5}\frac{GM^2}{R} + \frac{(4\pi\rho)^3 G^2}{15c^2}\frac{1}{7}R^7 \tag{33}$$

$$U_{gs} = -\frac{3}{5}\frac{GM^2}{R}(1 - \frac{3}{7}\frac{GM}{Rc^2}) \tag{34}$$

$$U_{gs-T} = U_{gs-M} + U_{gs-gs} = (-\frac{3}{5}\frac{GM^2}{R}) + (\frac{3}{7}\frac{GM}{Rc^2}\frac{3}{5}\frac{GM^2}{R}) \tag{35}$$

The first term is the gravitational self-energy term produced by the positive mass M, and the second term is the gravitational self-energy term created by the equivalent mass of the gravitational self-energy of the internal mass M'. The second term is very small in the case of ordinary matter and can be neglected, but it cannot be neglected when there are sufficiently many substances such as the universe.

The above equation depends on the process or pathway that collects the matters. However, since the present cosmic situation does not collect the materials in the free state, the application of this formula does not seem reasonable. The present universe is a case in which, as the universe ages, the range of gravitational interaction expands, and objects that were outside the gravitational interaction suddenly participate in gravitational interaction, and gravitational potential energy is generated.

Comparing the first and second terms, At $R = 46.5Gly$,

$$U_{gs} = -\frac{3}{5}\frac{GM^2}{R}(1 - \frac{3}{7}\frac{GM}{Rc^2}) = (1.2)(\frac{3}{5}\frac{GM^2}{R}) \tag{36}$$

If we look for the case where $(1 - \frac{3}{7}\frac{GM}{Rc^2}) = 0$,

$$R = \frac{3}{7}\frac{GM}{c^2} = \frac{3}{14}R_S = \frac{5}{7}R_{gs} \tag{37}$$

2) Universe structure correction variable or Gravitational self-energy correction variable $\beta(t)$

Another possibility related to the cause of the error is the mass problem of strongly bound objects, such as stars, black holes and galaxies. The mass of a star or galaxy observed from the outside will already be the total mass reflecting the value of its own gravitational energy. On the other hand, the gravitational self-energy equation assumes a uniform distribution and calculates all gravitational potential energy terms. In other words, there is a possibility that the gravitational self-energy term is over-calculated.

By introducing the universe structure (correction) variable $\beta(t)$, if we correct the gravitational self-energy value,

$$\beta(t) = \frac{2.05}{3.04} = 0.674 \tag{38}$$

$$U_{gs}{}' = \beta(t) U_{gs} = (0.674)(-\frac{3}{5}\frac{GM^2}{R}) \tag{39}$$

Although the universe structure correction variable $\beta(t)$ is a variable in all time, it is thought that it can be treated almost like a constant since the change will be small after the galactic structure is formed. Therefore, β can be called a universe structure constant. It is a name corresponding to the fine structure constant of elementary particle physics, and can also be called a macro structure constant.

3) Because of the propagation velocity of the field, it is possible that some matter within the particle horizon is not participating in gravitational interaction

That is, it is possible that the gravitational force has not yet been transmitted from one end to the other. There is a possibility of overcalculation of gravitational potential energy.

In addition to 1) - 3) mentioned here, there may be factors that need to be corrected. In summary, I will use the universe structure correction variable $\beta(t)$.

5. New Friedmann equations and cosmological constant function obtained by the gravitational potential energy model

The Friedmann equation can be obtained from the field equation. The basic form can also be obtained through Newtonian mechanics. If the object to be analyzed has spherical symmetry, from the second Newton's law, $R(t) = a(t)R$

$$m\frac{d^2R(t)}{dt^2} = -\frac{GM(t)m}{R(t)^2} \tag{40}$$

$$m\frac{d^2a(t)R}{dt^2} = -Gm\frac{\frac{4\pi}{3}a^3(t)R^3\rho(t)}{a^2(t)R^2} = -Gm\frac{4\pi}{3}a(t)R\rho(t) \tag{41}$$

$$\frac{\ddot{a}(t)}{a(t)} = -\frac{4\pi G}{3}\rho(t) \tag{42}$$

By adding pressure, we can create an acceleration equation.

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + \frac{3P}{c^2}) \tag{43}$$

Returning to the starting point, we put in the internal mass $M(t)$. However, according to the gravitational potential energy (gravitational self-energy) model, this mass M only needs to be replaced by $(M) + (-M_{gs})$. The gravitational self-energy of the particles making up the mass $M(t)$ and the mass $M(t)$ have the same center of mass. That is, when the internal mass M is entered, the gravitational self-energy also has the same center of mass, so it can be applied immediately.

Even when using the FLRW metric, there is a relative distance $a(t)R$ or radius defined by the scale factor $a(t)$, and there is a mass density. If we multiply the mass (energy) density by the coefficient, it will be restored to the mass in $a(t)R$. As long as internal masses are applied, the gravitational self-energy between their internal mass terms always seems applicable.

The current cosmological constant model is

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu} \tag{44}$$

$$(\frac{\dot{a}}{a})^2 = \frac{8\pi G\rho}{3} - \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3} \tag{45}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + \frac{3P}{c^2}) + \frac{\Lambda c^2}{3} \tag{46}$$

1) In the gravitational potential energy model, Friedmann's equations

In the gravitational potential energy model, if we derive the Friedmann equation,

$$-M_{gs} = -\frac{3}{5}\frac{GM^2}{Rc^2}\beta(t) = -\frac{16\pi^2 G R^5(t)\rho^2}{15c^2}\beta(t) \tag{47}$$

$$-\rho_{gs} = \frac{-M_{gs}}{V} = -\frac{4\pi G a^2 R^2 \rho^2}{5c^2}\beta(t) \tag{48}$$

$-\rho_{gs}$ is the equivalent mass density of gravitational self-energy, and $\beta(t)$ is the universe structure correction variable. Instead of energy density ρ, we have $(\rho) + (-\rho_{gs})$. $R(t) = a(t)R$. $a(t)$ is the scale factor.

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}((\rho - \rho_{gs}) + \frac{3P}{c^2}) = -\frac{4\pi G}{3}(\rho + \frac{3P}{c^2}) + \frac{4\pi G}{3}\rho_{gs} \tag{49}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + \frac{3P}{c^2}) + \frac{\beta(t)}{15}(\frac{4\pi G a R \rho}{c})^2 \tag{50}$$

Dark energy term and Cosmological constant term

$$\frac{\Lambda c^2}{3} = \frac{\beta(t)}{15}(\frac{4\pi G a R \rho}{c})^2 \tag{51}$$

$$\Lambda(t) = \beta(t)(\frac{4\pi G a(t) R \rho(t)}{\sqrt{5}c^2})^2 = \beta(\frac{4\pi G a R \rho}{\sqrt{5}c^2})^2 \tag{52}$$

First Friedmann equation

$$(\frac{\dot{a}}{a})^2 = \frac{8\pi G \rho}{3} - \frac{kc^2}{a^2} + \frac{\beta(t)}{15}(\frac{4\pi G a R \rho}{c})^2 \tag{53}$$

Second Friedmann equation, acceleration equation

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + \frac{3P}{c^2}) + \frac{\beta(t)}{15}(\frac{4\pi G a R \rho}{c})^2 \tag{54}$$

To see if the derived equation is probable, let's find the value of the cosmological constant.

$$\Lambda = \frac{\beta(t)}{5}(\frac{4(3.14)(6.67\times 10^{-11} m^3 kg^{-1} s^{-2})(46.5\times 9.46\times 10^{24} m)(8.50\times 10^{-27} kgm^{-3})}{(2.99\times 10^8 ms^{-1})^2})^2 \tag{55}$$

$$\Lambda = \frac{\beta(t)}{5}(\frac{4\pi G a R \rho}{c^2})^2 = (2.455\times 10^{-52} m^{-2})\beta(t) \tag{56}$$

The value obtained from the Planck satellite is, [10]

$$\Lambda = 3(\frac{H_0}{c})^2 \Omega_\Lambda = 1.1056\times 10^{-52} m^{-2} \tag{57}$$

Now, we get the universe structure variable (or constant)

$$\beta(t_{now}) = \frac{1.1056\times 10^{-52} m^{-2}}{2.455\times 10^{-52} m^{-2}} = 0.450 \tag{58}$$

First Friedmann equation

$$(\frac{\dot{a}}{a})^2 = \frac{8\pi G \rho}{3} - \frac{kc^2}{a^2} + 3(\frac{2\pi G a R \rho}{5c})^2 \tag{59}$$

Second Friedmann equation, acceleration equation

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + \frac{3P}{c^2}) + 3(\frac{2\pi G a R \rho}{5c})^2 \tag{60}$$

Dark energy term

$$\frac{\Lambda(t)c^2}{3} = \frac{\beta(t)}{15}(\frac{4\pi G a(t) R \rho(t)}{c})^2 = 3(\frac{2\pi G a R \rho}{5c})^2 \tag{61}$$

$a(t)R$ is the particle horizon at the point in time to be analyzed. In the gravitational self-energy model, if $\rho(t) \propto \frac{1}{a(t)R}$, it is possible that the density of dark energy seems to be constant. However, I think that the density of dark energy will change.

Note that the gravitational self-energy model and the cosmological constant model have different elements. For example, the gravitational self-energy model has a negative energy density and does not assume negative pressure. In addition, the dark energy density is a constant in the cosmological constant model, but the dark energy density is a variable in the gravitational self-energy model. Since $a(t)$ and $\rho(t)$ are functions of time, **the dark energy (or cosmological constant term) is a function of time**.

2) In the gravitational field's energy model, Friedmann's equations

From the similarity of gravity and electromagnetic force, the energy density of the gravitational field is

$$E(r) = \frac{1}{4\pi\varepsilon_0}\frac{Qr}{R^3} \tag{62}$$

$$g(r) = \frac{GMr}{R^3} \tag{63}$$

$$\varepsilon(r) = -\frac{g(r)^2}{8\pi G} = -\frac{GM^2 r^2}{8\pi R^6} \tag{64}$$

$$U_g = \int_0^R \varepsilon(r) 4\pi r^2 dr = \int_0^R (-\frac{GM^2}{2R^6}) r^4 dr = -\frac{1}{10}\frac{GM^2}{R} = \frac{1}{6}U_{gs} \tag{65}$$

Based on the current calculation results, gravitational potential energy seems more suitable as a source of dark energy.

The equivalent mass density of the gravitational field is

$$-\rho_g = \frac{1}{6}(-\rho_{gs}) \tag{66}$$

In the classical model, in the case of spherical symmetry, the gravitational field's energy and the gravitational potential energy have the same shape, only the coefficient is different. Therefore, only the $\beta(t)$ value is different, and the **Friedmann equations are of the same form.**

$$\beta(t) = 2.70 \tag{67}$$

$$\Lambda(t) = \frac{1}{6}\frac{\beta(t)}{5}(\frac{4\pi G R(t)\rho(t)}{c^2})^2 = (\frac{6\pi G a(t) R \rho(t)}{5c^2})^2 \tag{68}$$

6. One way to explain why dark energy density appears to be a constant

According to the observation results so far, the dark energy density seems to be a constant. One way to explain these results is to hypothesize the introduction or creation of new substances. Since the cosmological constant model explains the phenomenon by assuming a uniform energy density, let's try to explain it through dynamics here.

The dark energy term suggested by gravitational potential energy model is as follows.

$$\frac{1}{3}\Lambda(t)c^2 = \frac{\beta(t)}{15}(\frac{4\pi G a(t) R \rho(t)}{c})^2 = 3(\frac{2\pi G a(t) R \rho(t)}{5c})^2 \tag{69}$$

Then, in order for this dark energy term to look like a constant, the following relationship needs to be established.

$$\rho(t) \propto \frac{1}{a(t)R} \tag{70}$$

When a matter within a radius R expands to $2R$, its density drops to $1/2^3$. However, the above equation suggests that when R is changed to $2R$, the density drops only by 1/2. Therefore, other kinetic property must exist.

Let's assume the following situation. Matter and galaxies are moving according to the Hubble-Lemaitre law. Assume that the gravitational field is moving at a higher speed than these. The propagation speed of the gravitational field will have "speed of space expansion" + "speed of light".

Then, over time, the amount of matter and galaxies that enter into gravitational interactions will increase. This increase in matter and galaxies has the effect of increasing gravitational self-energy. When R_0 is changed to $2R_0$, if the mass is increased by 8, the density remains the same. When R_0 is changed to $2R_0$, if the mass is 4 times, the density is halved.

Therefore, the point we need to find is R_x, which has a mass 4 times greater than its mass at R_0.

$$M_x = \frac{4\pi {R_x}^3}{3}\rho_0 = 4(\frac{4\pi {R_0}^3}{3}\rho_0) \tag{71}$$

$$R_x = (4)^{\frac{1}{3}} R_0 \simeq 1.587 R_0 \tag{72}$$

When R_0 expands by k times, the value of R_x whose mass is k times: $R_x = (\frac{k^3}{2})R_0$

Comparing the average speed of the field with the average speed of the matter in R_x,

$$V_{R_0-Field} = \frac{\Delta R}{\Delta t} = \frac{2R_0 - R_0}{\Delta t} = \frac{R_0}{\Delta t} \tag{73}$$

$$V_{R_x-Matter} = \frac{\Delta R}{\Delta t} = \frac{2R_0 - R_x}{\Delta t} = (0.413)\frac{R_0}{\Delta t} = (0.413)V_{R_0-Field} \tag{74}$$

$$V_{R_0-Field} = 2.42 V_{R_x-Matter} \tag{75}$$

At a time when $a(t)R$ is about half of the current universe, if the speed of the field is about 2.4 times faster than the speed of matter, dark energy appears to be a constant. In addition, it can be explained through the addition of new energy such as vacuum energy, not the dynamic explanation.

7. Increase in dark energy

7-1. The ratio of increase in gravitational self-energy to increase in mass energy

To simplify the calculation, assuming a uniform density ρ,

$$\frac{d(Mc^2)}{dR} = 4\pi R^2 \rho c^2 \tag{76}$$

$$\frac{d(U_{gs})}{dR} = -\frac{16\pi^2 G}{3} R^4 \rho^2 = (-\frac{4\pi R^3 \rho G}{3Rc^2})(4\pi R^2 \rho c^2) = -\frac{GM}{Rc^2}(\frac{d(Mc^2)}{dR}) \tag{77}$$

$$\frac{d(U_{gs})}{dR} = -\frac{R_S}{2R}\frac{d(Mc^2)}{dR} \tag{78}$$

R_S is the Schwarzschild radius of the black hole formed by Particle Horizon.

The size of the event horizon formed by the mass distribution of 46.5 Gly is 477.8 Gly.

$$\frac{d(U_{gs})}{dR} = -\frac{R_S}{2R}(\frac{d(Mc^2)}{dR}) = -\frac{477.8Gly}{2(46.5Gly)}(\frac{d(Mc^2)}{dR}) = -(5.14)(\frac{d(Mc^2)}{dR}) \tag{79}$$

If the particle horizon increases and a positive mass is increases by ΔM, the equivalent mass of negative gravitational potential energy is increases by $-5.14\Delta M$. This value is not a fixed value, it depends on the density and the size of the particle horizon.

To find the ratio $-\frac{R_S}{2R}$ according to R, if $R = k \times Gly$,

$$\frac{\frac{d(U_{gs})}{dR}}{\frac{d(Mc^2)}{dR}} = -\frac{R_S}{2R} \tag{80}$$

$$R_S(k \times Gly) = \frac{2GM}{c^2} = (0.00475 \times Gly)(k^3) \tag{81}$$

$R(Gly)$	$R_S(Gly)$	$-R_S/2R$
10	4.80	-0.238
15	16.0	-0.533
20	38.0	-0.950
25	74.2	-1.48
30	128	-2.13
35	204	-2.91
40	304	-3.80
45	433	-4.81
50	594	-5.94

The density used the current critical density, but the density is a variable. Please see the approximate trend. The rate of increase of gravitational self-energy tends to be greater than the rate of increase of mass energy. Therefore, at some point, a situation arises in which dark energy becomes larger than matter and dark matter.

By the way, the negative mass ratio of 5.14 produced is very similar to the ratio of matter : dark matter.

7-2. Increase in dark energy (gravitational self-energy) due to increase in particle horizon

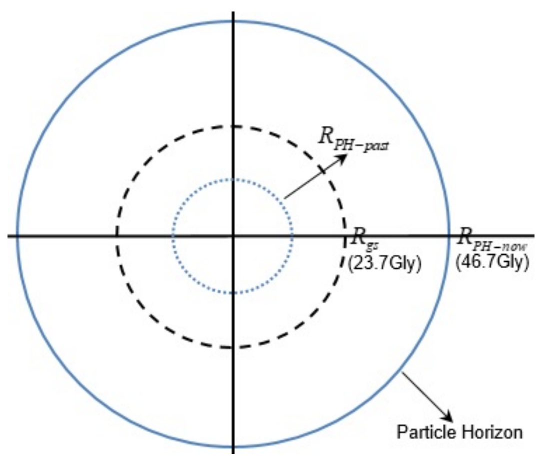

Figure 3: The past particle horizon, the present particle horizon, and R_{gs}= 23.7Gly (The inflection point calculated assuming 1.25 times the current average density), the inflection point at which the magnitudes of repulsive and attractive forces are equal. When the particle horizon is smaller than R_{gs}, the attractive component is dominant, and when the particle horizon is larger than R_{gs}, the repulsive component (gravitational self-energy) is dominant.

1) Matters and galaxies spread almost uniformly throughout the universe through the inflation process.

2) Matters and galaxies move according to the Hubble-Lemaitre law.

3) On the other hand, the propagation speed of the field, the range of interaction (particle horizon), has the fastest speed, the speed of light in expanding space.

4) Thus, over time, many new substances (matters and galaxies) enter the particle horizon. In other words, the newly entering materials undergo gravitational interaction, resulting in an increase in mass and an increase in gravitational potential energy in the region within the particle horizon.

5) By the way, **while mass energy is proportional to M, total gravitational potential energy (gravitational self-energy) is proportional to $-\frac{M^2}{R}$.** As M increases, the gravitational potential energy increases faster. Accordingly, the repulsive force component increases faster than the attractive force component.

6) The increase in gravitational potential energy due to the newly incorporated matter into the particle horizon is results in an increase in dark energy. The same principle is applicable to the birth of energy within a particle horizon. That is, when the mass energy increases by M, the gravitational self-energy increases by $-\frac{M^2}{R}$. In this model, even if a mass is born while satisfying energy conservation in a local area, as the new gravitational field propagates, the gravitational interaction with other matter will increase. Accordingly, an increase in gravitational self-energy may occur. The same principle can be applied to vacuum energy. Vacuum energy may also be present in this model. At this time, vacuum energy with positive energy density does not have negative pressure, but pressure has positive pressure and negative gravitational potential energy.

7) In the present universe, it is predicted that the dark energy effect (repulsive effect) surpassed the gravitational effect of matter and dark matter about 5 billion years ago. According to this model, it is the point at which the positive mass energy and the negative gravitational self-energy are equal. Knowing the average density function, we can get the exact value.

8) Gravitational potential energy is a concept that already exists and is negative energy that can create repulsive force. **This model produces similar results to the phenomenon of applying negative pressure while having positive inertial mass.** As the particle horizon expands, the positive mass increases (new influx or birth of matter), but the negative gravitational potential energy created by these positive masses is greater. While having a positive inertial mass, it is creating a negative gravitational mass that is larger than the positive inertial mass.

8. The future of the universe

In the standard cosmological ΛCDM model [4], dark energy is an object with uniform energy density. Thus, this universe will forever accelerating expansion.

In the gravitational self-energy model, the source of dark energy is the energy of the gravitational field or gravitational potential energy. The gravitational self-energy is proportional to $-M^2/R$, and if there is no inflow of mass from outside the system, this value can decrease. From the point where the velocity of the field and the velocity of matter become the same, there is no inflow of matter from the outside of the system. On the other hand, as R increases, the gravitational self-energy decreases.

Therefore, in the gravitational self-energy model, the universe does not accelerate forever, but at a certain point in the future, it stops the accelerated expansion and enters the period of decelerated expansion. However, the universe will not shrink back to a very small area like the time of the Big Bang, but will maintain a certain size or more. Its size depends on the $R_{gs}(or R_{gs-vir})$ that the entire universe produces by its total mass.

If there is something like vacuum energy (with positive pressure, not negative pressure) in the universe, it also has the effect of increasing the mass of the system. In this case, the expansion of the universe can go on forever.

9. How to validate the dark energy model that gravitational potential energy is the dark energy

1) Find the expressions of $\rho(t)$ and $R_{ph}(t)$

$\rho(t)$ is the average density inside the particle horizon. $R_{ph}(t)$ is the particle horizon.

2) At each time, within the particle horizon
Find $E = M(t)c^2$ and $U_{gs} = -\frac{3}{5}\frac{GM(t)^2}{R_{ph}(t)}\beta(t)$
$E = M(t)c^2$ is the attractive energy component and U_{gs} is the repulsive energy component.

3) Compare $\frac{U_{gs}}{E}$ with the observations
Compare $\frac{U_{gs}}{E}$ with the observations. And for the inflection point transitioning from decelerated expansion to accelerated expansion, the theoretical value and the observed value are compared.

4) It is necessary to verify the Friedmann equations and the dark energy function
In the gravitational self-energy model, the dark energy is a function of time. Therefore, if the dark energy effect is precisely observed, it is possible to determine whether the model is right or wrong.

III. Conclusion

The fundamental principle of general relativity states that "all energy is a source of gravity". However, the field equation created by Einstein did not fully realize this principle. Gravitational potential energy or gravitational field's energy must also function as a gravitational source. However, because it was difficult to define the energy of the gravitational field in general relativity, In this paper, gravitational potential energy and gravitational self-energy are used.

When the bound system acts on gravity, the gravitational action of gravitational potential energy is also included. Therefore, even in the case of the universe, we have to calculate the gravitational action of gravitational potential energy or gravitational field's energy. Since gravitational potential energy has a negative equivalent mass, gravitational potential energy creates a repulsive force. If the gravitational potential energy or gravitational field's energy is greater than the positive mass energy, it can be a candidate for dark energy to explain the current accelerated expansion. And, when we calculated the gravitational potential energy for the observable universe, it is approximately three times larger than the mass energy, so it can explain the accelerated expansion of the universe.

Mass energy is an attractive component, and the equivalent mass of gravitational potential energy is a repulsive component. Therefore, if $|(-M_{gs})c^2| < Mc^2$, there is a decelerated expansion, and if $|(-M_{gs})c^2| > Mc^2$, accelerated expansion is performed. $|(-M_{gs})c^2| = Mc^2$ is the inflection point from the decelerated expansion to the accelerated expansion.

The effect of dark energy occurs because all positive energy (matter, radiation, (if present) vacuum energy...) entering the particle horizon produces negative gravitational potential energy. Since the total mass in the particle horizon is proportional to M, whereas the gravitational self-energy is proportional to $-\frac{M^2}{R}$, the repulsive force component increases faster and accelerated expansion occurs. This model can be verified because it points to the gravitational self-energy and the particle horizon as the causes. This model predicts an inflection point where dark energy becomes larger and more important than the energy of matter and radiation. Also, dark energy is presented as a function of time.

There is no cosmological constant. Dark energy does not have a uniform energy density, and the density of dark energy is a function of time. The source of dark energy is the gravitational potential energy or the energy-momentum of the gravitational field. Because the observable universe is almost flat and very dense, gravitational potential energy or gravitational self-energy can be good approximation to the dark energy.

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