# Schnirelmann's Theorem Paradox

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**Abstract** In this paper it is going to be proved that Schnilermann's theorem can not hold. A disproof is based on fundamental theorem of arithmetic. However, since the proof of the theorem is widely accepted, that creates an unusual paradox that theorem can be proved and disproved at the same time.

## 1 Introduction

In elementary mathematics, a number line is a picture of the graduated straight line that serves as an abstraction to real numbers. Idea of number line was first introduced by John Napier [1], and later, John Wallis [2] used this graphical representation to explain operations of addition and subtraction in terms of moving backward and forward under the metaphor of a person walking. However, that type of graphical interpretation is not particularly suitable in other contexts of interest. In order to obtain another useful representation of natural numbers, a multiplication tensor or $\mathbf{M}_N$-tensor has been recently introduced [3]. Idea came from the fundamental theorem of arithmetic [4]. In [5] an addition tensor has been introduced. Multiplication and addition tensors are introduced as new ways of presentation of numbers that should simplify reasoning in some contexts.

In this paper, we are going to present a disproof of the Schnilermann's theorem [6] that uses the presentation of numbers introduced in [4] and [5].

<u>**Remark:**</u> ***In the text that follows, it is important to notice that there is no context in which it is***

***correct to state that number of natural numbers is equal to the number of odd or even numbers (or that number of natural numbers is equal to the number of numbers divisible by 3, or 5, and so on) – or in general, it is not considered possible that a set can have the same number of elements as the proper subset of that set (even in the case when a set contains infinite number of elements).***

***One way to understand the problem, for instance, is that it is possible to generate the same number of unique labels for odd or even numbers using the same number of unique labels for natural numbers. However, if we want to produce all those even and all those odd labels at the same time, obviously, it is necessary to have two sets of labels with natural numbers at the same time (it is necessary to clone the set of labels with natural numbers), which means that the number of labels with natural numbers on them, in that moment, is two times bigger than the number of labels with even or odd numbers on them. (In Cantor's set theory increase of the number of the elements of the set by cloning is ignored. In that theory, counting takes care only about unique elements of the set – it creates a completely different reality with all known unusual phenomena that look completely counter intuitive, if you are not all the time aware of the fact that cloning does not change the number of elements of infinite set. However, if we assume that Cantor is right (or that his theory belongs to classical and not alternative mathematics) we will have to accept that fundamental theorem of arithmetic can not hold, and that creates a paradox, since it is proved that fundamental theorem of arithmetic holds. That consequence can be expected having in mind fact that if you change a way of counting the numbers, you will quite probably have to change a way how you add the numbers - then you will have to change how you multiply numbers, and that quite predictably can lead toward problems with fundamental theorem of arithmetic. This problem will be addressed in separate paper, but the problem can easily be identified at the end of this paper.)***

***The analysis of the problem can also be made in quantum probabilistic context, but it is beyond the scope of this paper. What can be immediately understood is that quantum probabilistic approach will offer a different angle from which problem can be analyzed, since in that context cloning is not***

***allowed.***

## 2 Multiplication tensor

The fundamental theorem of arithmetic states that every integer greater than 1 can be uniquely represented by a product of powers of prime numbers, up to the order of the factors [4]. Having that in mind, an infinite dimensional tensor $\mathbf{M}_\mathbf{N}$ that contains all natural numbers only once, is going to be constructed [3]. In order to do that we are going to mark vector that contains all prime numbers with **p**. So, $p(1) = 2$, $p(2) = 3$, $p(3) = 5$, and so on. Tensor $\mathbf{M}_\mathbf{N}$ with elements $m_{i1\ i2\ i3\ \ldots}$ is defined by the following equation ($i_1$, $i_2$, $i_3$, … are natural numbers):

$$m_{i_1 i_2 i_3 \ldots} = p(1)^{i_1 - 1}\, p(2)^{i_2 - 1}\, p(3)^{i_3 - 1} \ldots \quad .$$

The alternative definition is also possible. Now, the following notation is going to be assumed for some infinite size vectors

$$\mathbf{2} = [2^0\ 2^1\ 2^2\ 2^3\ \ldots],\ \mathbf{3} = [3^0\ 3^1\ 3^2\ 3^3\ \ldots],\ \ \mathbf{5} = [5^0\ 5^1\ 5^2\ 5^3\ \ldots]\ \ldots$$

It is simple to be seen that every vector is marked by bold number that is equal to some prime number and that components of the vector are defined as powers of that prime number, including power zero (it can be seen that every vector represents infinite cyclic semi group defined by a primitive that is one of the prime numbers). Now, the $\mathbf{M}_\mathbf{N}$-tensor can be defined as

$$\mathbf{M}_\mathbf{N} = \mathbf{2} \circ \mathbf{3} \circ \mathbf{5} \circ \mathbf{7} \circ \ldots,$$

where ○ stands for outer product.

The tensor $\mathbf{M}_\mathbf{N}$ is of infinite dimension (equal to number of prime numbers) and size, and contains all natural numbers exactly ones. It is easy to understand why it is so, having in mind the fundamental theorem of arithmetic. This type of infinite tensor is called a half infinite tensor [3].

## 3 Addition tensor

Now, recently introduced [5] addition tensor, or $\mathbf{A_N}$–tensor, is going to be presented. The tensor $\mathbf{A_N}$ with elements $\boldsymbol{a}_{i1\ i2\ i3\ \ldots}$ is defined by the following equation ($i_1$, $i_2$, $i_3$, … are natural numbers):

$$a_{i_1 i_2 i_3 \ldots} = (i_1 - 1)\, p(1) + (i_2 - 1)\, p(2) + (i_3 - 1)\, p(3) + \ldots \quad .$$

The edges of that tensor will contain the following vectors

$$\mathbf{2a} = [0\ 2\ 4\ 6\ \ldots],\ \mathbf{3a} = [0\ 3\ 6\ 9\ \ldots],\ \mathbf{5a} = [0\ 5\ 10\ 15\ \ldots]\ \ldots$$

It is simple to be seen that every vector is defined by some prime number and that components of the vector represent all non-negative integer multiples of that prime number. Now, the $\mathbf{A_N}$-tensor can be defined as

$$\mathbf{A_N} = \mathbf{2a} \circ^{+} \mathbf{3a} \circ^{+} \mathbf{5a} \circ^{+} \mathbf{7a} \circ^{+} \ldots,$$

where $\circ^{+}$ stands for outer sum, which is analogous to outer product where operation of interest is addition..

It is interesting to notice that the tensor $\mathbf{M_N}$ does not contain number 0 that is neutral element for addition, while, on the other hand, the tensor $\mathbf{A_N}$ does not contain number 1 that is neutral number for multiplication.

## 4 A disproof of Schnirelmann's theorem

Schnirelmann theorem [6] states that any natural number bigger than 1 can be written as the sum of not more than $C$ prime numbers, where $C$ is effectively computable constant (estimated by Schnirelmann as $C < 800000$). By comparing sub-tensor $\mathbf{A}_N(1{:}C+1, 1{:}C+1, 1{:}C+1, \ldots)$ and sub-tensor $\mathbf{M}_N(1{:}C+1, 1{:}C+1, 1{:}C+1, \ldots)$, where $C$ is any finite number, it is simple to understand that those two sub-tensors have the same number of elements by construction. Sub-tensor $\mathbf{A}_N(1{:}C+1, 1{:}C+1, 1{:}C+1, \ldots)$ contains all natural numbers that can be created as sums of at most $C$ primes (together with many other numbers). The sub-tensor $\mathbf{M}_N(1{:}C+1, 1{:}C+1, 1{:}C+1, \ldots)$ obviously does not contain all natural numbers and we can conclude that exists infinitely many natural numbers that are not contained in sub-tensor $\mathbf{M}_N(1{:}C+1, 1{:}C+1, 1{:}C+1, \ldots)$ - it is not difficult to be seen, that if it is not the case then fundamental theorem of arithmetic cannot hold. Now, it is not difficult to be understood that Schnirelmann's theorem cannot hold, since there will be infinitely many natural numbers that are not contained in sub-tensor $\mathbf{A}_N(1{:}C+1, 1{:}C+1, 1{:}C+1, \ldots)$ and cannot be represented as the sum of at most $C$ prime numbers.

This completes this paper and creates a paradox that theorem can be proved and disproved at the same time.

## References

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