A Disproof of Weak Goldbach's Conjecture

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Abstract In this paper it is going to be proved that weak Goldbach's conjecture can not hold. The proof is based on fundamental theorem of arithmetic. This paper presents the idea that is simple modification (extension) of the idea used to prove that strong Goldbach's conjecture cannot hold.

1 Introduction

In elementary mathematics, a number line is a picture of the graduated straight line that serves as an abstraction to real numbers. Idea of number line was first introduced by John Napier [1], and later, John Wallis [2] used this graphical representation to explain operations of addition and subtraction in terms of moving backward and forward under the metaphor of a person walking. However, that type of graphical interpretation is not particularly suitable in other contexts of interest. In order to obtain another useful representation of natural numbers, a multiplication tensor or $\mathbf{M}_N$-tensor has been recently introduced [3]. Idea came from the fundamental theorem of arithmetic [4]. In [5] an addition tensor has been introduced and that representation is used to show that strong Goldbach's conjecture [6] cannot hold. In [5, 7] an elementary disproof of the strong Goldbach's conjecture has been presented, too. Multiplication and addition are introduced as new ways of presentation of numbers that should simplify reasoning in some contexts.

In this paper, we are going to present a simple extension (modification) of the idea in [5] in order to show that weak Goldbach's conjecture [6] can not hold, too.

Remark: ***In the text that follows, it is important to notice that there is no context in which it is correct to state that number of natural numbers is equal to the number of odd or even numbers (or that number of natural numbers is equal to the number of numbers divisible by 3, or 5, and so on) – or in general, it is not considered possible that a set can have the same number of elements as the proper subset of that set (even in the case when a set contains infinite number of elements).***

One way to understand the problem, for instance, is that it is possible to generate the same number of unique labels for odd or even numbers using the same number of unique labels for natural numbers. However, if we want to produce all those even and all those odd labels at the same time, obviously, it is necessary to have two sets of labels with natural numbers at the same time (it is necessary to clone the set of labels with natural numbers), which means that the number of labels with natural numbers on them, in that moment, is two times bigger than the number of labels with even or odd numbers on them. (In Cantor's set theory increase of the number of the elements of the set by cloning is ignored. In that theory, counting takes care only about unique elements of the set – it creates a completely different reality with all known unusual phenomena that look completely counter intuitive, if you are not all the time aware of the fact that cloning does not change the number of elements of infinite set. However, if we assume that Cantor is right (or that his theory belongs to classical and not alternative mathematics) we will have to accept that fundamental theorem of arithmetic can not hold, and that creates a paradox, since it is proved that fundamental theorem of arithmetic holds. That consequence can be expected having in mind fact that if you change a way of counting the numbers, you will quite probably have to change a way how you add the numbers - then you will have to change how you multiply numbers, and that quite predictably can lead toward problems with fundamental theorem of arithmetic. This problem will be addressed in separate paper, but the problem can easily be identified at the end of this paper.)

The analysis of the problem can also be made in quantum probabilistic context, but it is beyond the scope of this paper. What can be immediately understood is that quantum probabilistic approach

will offer a different angle from which problem can be analyzed, since in that context cloning is not allowed.

2 Multiplication tensor

The fundamental theorem of arithmetic states that every integer greater than 1 can be uniquely represented by a product of powers of prime numbers, up to the order of the factors [4]. Having that in mind, an infinite dimensional tensor $\mathbf{M_N}$ that contains all natural numbers only once, is going to be constructed [3]. In order to do that we are going to mark vector that contains all prime numbers with **p**. So, $p(1) = 2$, $p(2) = 3$, $p(3) = 5$, and so on. Tensor $\mathbf{M_N}$ with elements $m_{i1\ i2\ i3\ \ldots}$ is defined by the following equation (i_1, i_2, i_3, … are natural numbers):

$$m_{i_1 i_2 i_3 \ldots} = p(1)^{i_1 - 1}\, p(2)^{i_2 - 1}\, p(3)^{i_3 - 1} \ldots \quad .$$

The alternative definition is also possible. Now, the following notation is going to be assumed for some infinite size vectors

$$\mathbf{2} = [2^0\ 2^1\ 2^2\ 2^3\ \ldots],\ \mathbf{3} = [3^0\ 3^1\ 3^2\ 3^3\ \ldots],\ \mathbf{5} = [5^0\ 5^1\ 5^2\ 5^3\ \ldots]\ \ldots$$

It is simple to be seen that every vector is marked by bold number that is equal to some prime number and that components of the vector are defined as powers of that prime number, including power zero (it can be seen that every vector represents infinite cyclic semi group defined by a primitive that is one of the prime numbers). Now, the $\mathbf{M_N}$-tensor can be defined as

$$\mathbf{M_N} = \mathbf{2} \circ \mathbf{3} \circ \mathbf{5} \circ \mathbf{7} \circ \ldots,$$

where ○ stands for outer product.

The tensor $\mathbf{M}_{\mathbf{N}}$ is of infinite dimension (equal to number of prime numbers) and size, and contains all natural numbers exactly ones. It is easy to understand why it is so, having in mind the fundamental theorem of arithmetic. This type of infinite tensor is called a half infinite tensor [3].

The tensor that represents all odd numbers, $\mathbf{M}_{\mathbf{NO}}$, contains elements defined as

$$m_{i_1 i_2 \ldots} = p(2)^{i_1 - 1}\, p(3)^{i_2 - 1} \ldots \quad ,$$

or

$$\mathbf{M}_{\mathbf{NO}} = \mathbf{3} \circ \mathbf{5} \circ \mathbf{7} \circ \ldots,$$

where ○ stands for outer product.

3 Addition tensor

Now, recently introduced [5] addition tensor, or $\mathbf{A}_{\mathbf{N}}$–tensor, is going to be presented. The tensor $\mathbf{A}_{\mathbf{N}}$ with elements $\boldsymbol{a}_{i1\, i2\, i3\, \ldots}$ is defined by the following equation (i_1, i_2, i_3, … are natural numbers):

$$a_{i_1 i_2 i_3 \ldots} = (i_1 - 1)\, p(1) + (i_2 - 1)\, p(2) + (i_3 - 1)\, p(3) + \ldots \quad .$$

The edges of that tensor will contain the following vectors

2a = [0 2 4 6 …], **3a** = [0 3 6 9 …], **5a** = [0 5 10 15 …] …

It is simple to be seen that every vector is defined by some prime number and that components of the vector represent all non-negative integer multiples of that prime number. Now, the $\mathbf{A}_{\mathbf{N}}$-tensor can be defined as

$$\mathbf{A_N} = \mathbf{2a} \circ^+ \mathbf{3a} \circ^+ \mathbf{5a} \circ^+ \mathbf{7a} \circ^+ \ldots,$$

where $\circ^+$ stands for outer sum, which is analogous to outer product where operation of interest is addition..

It is interesting to notice that the tensor $\mathbf{M_N}$ does not contain number 0 that is neutral element for addition, while, on the other hand, the tensor $\mathbf{A_N}$ does not contain number 1 that is neutral number for multiplication.

Here we will present an additional addition tensor $\mathbf{A_{NO}}$ that is created by odd prime numbers, where elements of that tensor are defined as

$$a_{i_1 i_2 i_3 \ldots} = (i_1 - 1)\,p(2) + (i_2 - 1)\,p(3) + (i_3 - 1)\,p(4) + \ldots \quad .$$

or

$$\mathbf{A_{NO}} = \mathbf{3a} \circ^+ \mathbf{5a} \circ^+ \mathbf{7a} \circ^+ \ldots,$$

where $\circ^+$ stands for outer sum.

4 A proof that weak Goldbach's conjecture cannot hold

Goldbach's conjecture, weak version, states that every odd integer bigger than 7 can be expressed as the sum of three odd prime numbers [6]. From introduction of addition tensor it is trivial to understand that sub-tensor $\mathbf{A_{NO}}$(1:4, 1:4, 1:4, …) contains all possible sums of three prime numbers (plus many other sums). It is trivial to understand that sub-tensor $\mathbf{A_{NO}}$(1:4, 1:4, 1:4, …) and sub-tensor $\mathbf{M_{NO}}$(1:4, 1:4, 1:4, …) have the same number of elements. Also, it is simple to understand that sub-tensor $\mathbf{M}_{\mathbf{NO}}$(1:4, 1:4, 1:4, …) does not contain all odd numbers – it does not contain a single odd number whose single prime factor has exponents bigger than 3, and we know that must be infinite number of such numbers (since there are infinitely many odd primes whose factorization contains at least one prime factor with exponent bigger than 3) – if it is not the case then fundamental theorem of arithmetic cannot hold. Now, it can easily be understood that the number of sums created by three odd prime numbers is smaller than the number of odd numbers, which means that weak Goldbach's conjecture can not hold. That completes the proof.

References

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