Analyzing new possible mathematical connections between π , ζ(2), 4096, 1729, some Ramanujan mock theta functions, parameters of Number Theory and String Theory

Michele Nardelli[1], Antonio Nardelli

Abstract

In this paper we analyze new possible mathematical connections between π, ζ(2), 4096, 1729, some Ramanujan mock theta functions**,** *parameters of Number Theory and sectors of String Theory*

1[1] M.Nardelli studied at Dipartimento di Scienze della Terra Università degli Studi di Napoli Federico II, Largo S. Marcellino, 10 - 80138 Napoli, Dipartimento di Matematica ed Applicazioni "R. Caccioppoli" - Università degli Studi di Napoli "Federico II" – Polo delle Scienze e delle Tecnologie Monte S. Angelo, Via Cintia (Fuorigrotta), 80126 Napoli, Italy

A. Nardelli studies at the Università degli Studi di Napoli Federico II - Dipartimento di Studi Umanistici – **Sezione Filosofia - scholar of Theoretical Philosophy**

From:

Ramanujan's Last Letter to Hardy - *George E. Andrews & Bruce C. Berndt* - Chapter - First Online: 06 September 2018

We have:

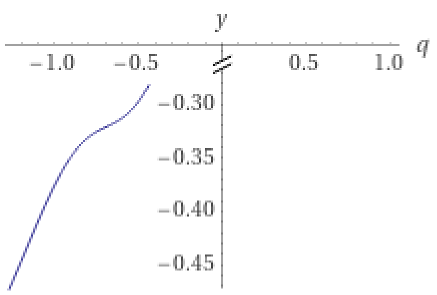
$$\psi_3(q) = \frac{q}{1-q} + \frac{q^4}{(1-q)(1-q^3)} + \frac{q^9}{(1-q)(1-q^3)(1-q^5)} + \cdots$$

q/(1-q) + (q^4)/((1-q)(1-q^3)) + (q^9)/((1-q)(1-q^3)(1-q^5))

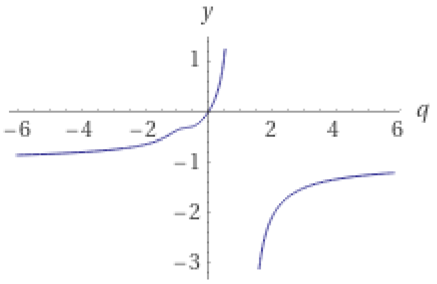
Input

$$\frac{q}{1-q} + \frac{q^4}{(1-q)\left(1-q^3\right)} + \frac{q^9}{(1-q)\left(1-q^3\right)\left(1-q^5\right)}$$

Plots **(figures that can be related to the open strings)**



(q from −1.3 to 1.1)



(q from −6 to 5.8)

Alternate forms

$$-\frac{q\left(q^8-q^5+1\right)}{(q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}$$

$$-\frac{q\left(q^8-q^5+1\right)}{(q-1)\left(q^3-1\right)\left(q^5-1\right)}$$

$$-\frac{q^9-q^6+q}{(q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}$$

Real root

$q = 0$

Complex roots

$q \approx -0.82774 - 0.44805\,i$

$q \approx -0.82774 + 0.44805\,i$

$q \approx -0.49056 - 1.02215\,i$

$q \approx -0.49056 + 1.02215\,i$

$q \approx 0.31454 - 0.84482\,i$

Series expansion at q=0

$$q+q^2+q^3+2q^4+2q^5+O(q^6)$$

(Taylor series)

Series expansion at q=∞

$$-1-\frac{1}{q}-\left(\frac{1}{q}\right)^2-\left(\frac{1}{q}\right)^3+O\left(\left(\frac{1}{q}\right)^4\right)$$

(Laurent series)

Derivative

$$\frac{d}{dq}\left(\frac{q}{1-q}+\frac{q^4}{(1-q)(1-q^3)}+\frac{q^9}{(1-q)(1-q^3)(1-q^5)}\right)=$$
$$\left(q^{14}+2q^{13}+3q^{12}+2q^{11}+6q^{10}+4q^9+3q^8+2q^6+4q^5+6q^4+6q^3+3q^2+2q+1\right)/\left((q-1)^4(q^2+q+1)^2(q^4+q^3+q^2+q+1)^2\right)$$

Indefinite integral

$$\int\left(\frac{q}{1-q}+\frac{q^4}{(1-q)(1-q^3)}+\frac{q^9}{(1-q)(1-q^3)(1-q^5)}\right)dq=$$
$$\frac{1}{90}\left(-18\sum_{\{\omega:\,\omega^4+\omega^3+\omega^2+\omega+1=0\}}\frac{\omega^3\log(q-\omega)-\log(q-\omega)}{4\omega^3+3\omega^2+2\omega+1}+5\log(q^2+q+1)-90q+\frac{6}{q-1}+\frac{3}{(q-1)^2}-82\log(1-q)-10\sqrt{3}\tan^{-1}\left(\frac{2q+1}{\sqrt{3}}\right)\right)+\text{constant}$$

(assuming a complex-valued logarithm)

$\tan^{-1}(x)$ is the inverse tangent function
$\log(x)$ is the natural logarithm

Limit

$$\lim_{q\to\pm\infty}\left(\frac{q}{1-q}+\frac{q^4}{(1-q)(1-q^3)}+\frac{q^9}{(1-q)(1-q^3)(1-q^5)}\right)=-1$$

From

$$-\frac{q^9 - q^6 + q}{(q-1)^3 \left(q^2+q+1\right) \left(q^4+q^3+q^2+q+1\right)}$$

we calculate

(-(q^9 - q^6 + q)/((q - 1)^3 (q^2 + q + 1) (q^4 + q^3 + q^2 + q + 1)))dxdy

Indefinite integral

$$\int\int -\frac{q^9 - q^6 + q}{(q-1)^3 \left(q^2+q+1\right) \left(q^4+q^3+q^2+q+1\right)} \, dx\, dy = c_1 x + c_2 - \frac{q\, x\, y}{(q-1)^3 \left(q^2+q+1\right) \left(q^4+q^3+q^2+q+1\right)} - \frac{q^9\, x\, y}{(q-1)^3 \left(q^2+q+1\right) \left(q^4+q^3+q^2+q+1\right)} + \frac{q^6\, x\, y}{(q-1)^3 \left(q^2+q+1\right) \left(q^4+q^3+q^2+q+1\right)}$$

Definite integral over a disk of radius R

$$\iint_{x^2+y^2<R^2} \frac{-q^9 + q^6 - q}{(q-1)^3 \left(q^2+q+1\right) \left(q^4+q^3+q^2+q+1\right)} \, dy\, dx = \frac{\pi \left(-q^9+q^6-q\right) R^2}{(q-1)^3 \left(q^2+q+1\right) \left(q^4+q^3+q^2+q+1\right)}$$

Definite integral over a square of edge length 2 L

$$\int_{-L}^{L}\int_{-L}^{L} \frac{-q + q^6 - q^9}{(-1+q)^3 \left(1+q+q^2\right) \left(1+q+q^2+q^3+q^4\right)} \, dx\, dy = \frac{4 L^2 \left(-q^9+q^6-q\right)}{(q-1)^3 \left(q^2+q+1\right) \left(q^4+q^3+q^2+q+1\right)}$$

Dividing the results of

$$\iint\limits_{x^2+y^2<R^2} \frac{-q^9+q^6-q}{(q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}\,dy\,dx = \frac{\pi\left(-q^9+q^6-q\right)R^2}{(q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}$$

and

$$\int_{-L}^{L}\int_{-L}^{L}\frac{-q+q^6-q^9}{(-1+q)^3\left(1+q+q^2\right)\left(1+q+q^2+q^3+q^4\right)}\,dx\,dy = \frac{4\,L^2\left(-q^9+q^6-q\right)}{(q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}$$

by

$$-\frac{q^9-q^6+q}{(q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}$$

we obtain:

((π (-q^9 + q^6 - q))/((q - 1)^3 (q^2 + q + 1) (q^4 + q^3 + q^2 + q + 1))) /((-(q^9 - q^6 + q)/((q - 1)^3 (q^2 + q + 1) (q^4 + q^3 + q^2 + q + 1))))

Input

$$-\frac{\frac{\pi\left(-q^9+q^6-q\right)}{(q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}}{\frac{q^9-q^6+q}{(q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}}$$

Result

π (for $q \neq 0$)

Decimal approximation

3.1415926535897932384626433832795028841971693993751058209749445923

…

3.141592653… = π

Property

π is a transcendental number

Alternative representations

$$-\frac{\pi\left(-q^9+q^6-q\right)}{\frac{\left(q^9-q^6+q\right)\left((q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)\right)}{(q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}}=\frac{180\,°\left(-q+q^6-q^9\right)}{\frac{\left((-1+q)^3\left(1+q+q^2\right)\left(1+q+q^2+q^3+q^4\right)\right)\left(-q+q^6-q^9\right)}{(-1+q)^3\left(1+q+q^2\right)\left(1+q+q^2+q^3+q^4\right)}}$$

$$-\frac{\pi\left(-q^9+q^6-q\right)}{\frac{\left(q^9-q^6+q\right)\left((q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)\right)}{(q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}}=-\frac{i\left(\log(-1)\left(-q+q^6-q^9\right)\right)}{\frac{\left((-1+q)^3\left(1+q+q^2\right)\left(1+q+q^2+q^3+q^4\right)\right)\left(-q+q^6-q^9\right)}{(-1+q)^3\left(1+q+q^2\right)\left(1+q+q^2+q^3+q^4\right)}}$$

$$-\frac{\pi\left(-q^9+q^6-q\right)}{\frac{\left(q^9-q^6+q\right)\left((q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)\right)}{(q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}}=\frac{\cos^{-1}(-1)\left(-q+q^6-q^9\right)}{\frac{\left((-1+q)^3\left(1+q+q^2\right)\left(1+q+q^2+q^3+q^4\right)\right)\left(-q+q^6-q^9\right)}{(-1+q)^3\left(1+q+q^2\right)\left(1+q+q^2+q^3+q^4\right)}}$$

$\log(x)$ is the natural logarithm

i is the imaginary unit

$\cos^{-1}(x)$ is the inverse cosine function

Series representations

$$-\frac{\pi\left(-q^9+q^6-q\right)}{\frac{\left(q^9-q^6+q\right)\left((q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)\right)}{(q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}}=4\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}$$

$$-\frac{\pi\left(-q^9+q^6-q\right)}{\frac{\left(q^9-q^6+q\right)\left((q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)\right)}{(q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}} = \sum_{k=0}^{\infty} -\frac{4\,(-1)^k\, 1195^{-1-2k}\left(5^{1+2k}-4\times 239^{1+2k}\right)}{1+2k}$$

$$-\frac{\pi\left(-q^9+q^6-q\right)}{\frac{\left(q^9-q^6+q\right)\left((q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)\right)}{(q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}} = \sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k}+\frac{2}{1+4k}+\frac{1}{3+4k}\right)$$

Integral representations

$$-\frac{\pi\left(-q^9+q^6-q\right)}{\frac{\left(q^9-q^6+q\right)\left((q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)\right)}{(q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}} = 4\int_0^1 \sqrt{1-t^2}\,dt$$

$$-\frac{\pi\left(-q^9+q^6-q\right)}{\frac{\left(q^9-q^6+q\right)\left((q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)\right)}{(q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}} = 2\int_0^1 \frac{1}{\sqrt{1-t^2}}\,dt$$

$$-\frac{\pi\left(-q^9+q^6-q\right)}{\frac{\left(q^9-q^6+q\right)\left((q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)\right)}{(q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}} = 2\int_0^\infty \frac{1}{1+t^2}\,dt$$

1/6(((π (-q^9 + q^6 - q))/((q - 1)^3 (q^2 + q + 1) (q^4 + q^3 + q^2 + q + 1)))/((-(q^9 - q^6 + q)/((q - 1)^3 (q^2 + q + 1) (q^4 + q^3 + q^2 + q + 1)))))^2

Input

$$\frac{1}{6}\left(-\frac{\frac{\pi(-q^9+q^6-q)}{(q-1)^3(q^2+q+1)(q^4+q^3+q^2+q+1)}}{\frac{q^9-q^6+q}{(q-1)^3(q^2+q+1)(q^4+q^3+q^2+q+1)}}\right)^2$$

Result

$$\frac{\pi^2}{6}$$

Decimal approximation

1.6449340668482264364724151666460251892189499012067984377355582293
…

1.644934066….

Property

$\frac{\pi^2}{6}$ is a transcendental number

Alternative representations

$$\frac{1}{6}\left(\frac{\pi(-q^9+q^6-q)}{\frac{((q-1)^3(q^2+q+1)(q^4+q^3+q^2+q+1))(-(q^9-q^6+q))}{(q-1)^3(q^2+q+1)(q^4+q^3+q^2+q+1)}}\right)^2 =$$

$$\frac{1}{6}\left(\frac{180\,°(-q+q^6-q^9)}{\frac{((-1+q)^3(1+q+q^2)(1+q+q^2+q^3+q^4))(-q+q^6-q^9)}{(-1+q)^3(1+q+q^2)(1+q+q^2+q^3+q^4)}}\right)^2$$

$$\frac{1}{6}\left(\frac{\pi\left(-q^9+q^6-q\right)}{\frac{\left((q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)\right)\left(-\left(q^9-q^6+q\right)\right)}{(q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}}\right)^2 = \frac{1}{6}\left(-\frac{i\left(\log(-1)\left(-q+q^6-q^9\right)\right)}{\frac{\left((-1+q)^3\left(1+q+q^2\right)\left(1+q+q^2+q^3+q^4\right)\right)\left(-q+q^6-q^9\right)}{(-1+q)^3\left(1+q+q^2\right)\left(1+q+q^2+q^3+q^4\right)}}\right)^2$$

$$\frac{1}{6}\left(\frac{\pi\left(-q^9+q^6-q\right)}{\frac{\left((q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)\right)\left(-\left(q^9-q^6+q\right)\right)}{(q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}}\right)^2 = \frac{1}{6}\left(\frac{\cos^{-1}(-1)\left(-q+q^6-q^9\right)}{\frac{\left((-1+q)^3\left(1+q+q^2\right)\left(1+q+q^2+q^3+q^4\right)\right)\left(-q+q^6-q^9\right)}{(-1+q)^3\left(1+q+q^2\right)\left(1+q+q^2+q^3+q^4\right)}}\right)^2$$

$\log(x)$ is the natural logarithm
i is the imaginary unit
$\cos^{-1}(x)$ is the inverse cosine function

Series representations

$$\frac{1}{6}\left(\frac{\pi\left(-q^9+q^6-q\right)}{\frac{\left((q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)\right)\left(-\left(q^9-q^6+q\right)\right)}{(q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}}\right)^2 = \sum_{k=1}^{\infty}\frac{1}{k^2}$$

$$\frac{1}{6}\left(\frac{\pi\left(-q^9+q^6-q\right)}{\frac{\left((q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)\right)\left(-\left(q^9-q^6+q\right)\right)}{(q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}}\right)^2 = -2\sum_{k=1}^{\infty}\frac{(-1)^k}{k^2}$$

$$\frac{1}{6}\left(\frac{\pi\left(-q^9+q^6-q\right)}{\frac{\left((q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)\right)\left(-\left(q^9-q^6+q\right)\right)}{(q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}}\right)^2=\frac{4}{3}\sum_{k=0}^{\infty}\frac{1}{(1+2k)^2}$$

Integral representations

$$\frac{1}{6}\left(\frac{\pi\left(-q^9+q^6-q\right)}{\frac{\left((q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)\right)\left(-\left(q^9-q^6+q\right)\right)}{(q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}}\right)^2=\frac{8}{3}\left(\int_0^1\sqrt{1-t^2}\,dt\right)^2$$

$$\frac{1}{6}\left(\frac{\pi\left(-q^9+q^6-q\right)}{\frac{\left((q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)\right)\left(-\left(q^9-q^6+q\right)\right)}{(q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}}\right)^2=\frac{2}{3}\left(\int_0^\infty\frac{1}{1+t^2}\,dt\right)^2$$

$$\frac{1}{6}\left(\frac{\pi\left(-q^9+q^6-q\right)}{\frac{\left((q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)\right)\left(-\left(q^9-q^6+q\right)\right)}{(q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}}\right)^2=\frac{2}{3}\left(\int_0^1\frac{1}{\sqrt{1-t^2}}\,dt\right)^2$$

We obtain also:

(((((4 (-q^9 + q^6 - q))/((q - 1)^3 (q^2 + q + 1) (q^4 + q^3 + q^2 + q + 1))))/((-(q^9 - q^6 + q)/((q - 1)^3 (q^2 + q + 1) (q^4 + q^3 + q^2 + q + 1)))))))^6

Input

$$\left(-\frac{\frac{4\left(-q^9+q^6-q\right)}{(q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}}{\frac{q^9-q^6+q}{(q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}}\right)^6$$

Result

4096

4096 = 64^2

27* sqrt(((((4 (-q^9 + q^6 - q))/((q - 1)^3 (q^2 + q + 1) (q^4 + q^3 + q^2 + q + 1))))/((-(q^9 - q^6 + q)/((q - 1)^3 (q^2 + q + 1) (q^4 + q^3 + q^2 + q + 1)))))))^6+1

Input

$$27 \sqrt{-\frac{\frac{4(-q^9+q^6-q)}{(q-1)^3 (q^2+q+1)(q^4+q^3+q^2+q+1)}}{\frac{q^9-q^6+q}{(q-1)^3 (q^2+q+1)(q^4+q^3+q^2+q+1)}}}^6 + 1$$

Result

1729

1729

This result is very near to the mass of candidate glueball $f_0(1710)$ scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. (1728 = $8^2 * 3^3$) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Series representations

$$27 \sqrt{-\frac{4\left(-q^9+q^6-q\right)}{\frac{(q^9-q^6+q)((q-1)^3 (q^2+q+1)(q^4+q^3+q^2+q+1))}{(q-1)^3 (q^2+q+1)(q^4+q^3+q^2+q+1)}}}^6 + 1 = 1 + 27 \sqrt{3}^6 \left(\sum_{k=0}^{\infty} 3^{-k} \binom{\frac{1}{2}}{k}\right)^6$$

$$27 \sqrt{-\frac{4\left(-q^9+q^6-q\right)}{\frac{(q^9-q^6+q)((q-1)^3 (q^2+q+1)(q^4+q^3+q^2+q+1))}{(q-1)^3 (q^2+q+1)(q^4+q^3+q^2+q+1)}}}^6 + 1 = 1 + 27 \sqrt{3}^6 \left(\sum_{k=0}^{\infty} \frac{\left(-\frac{1}{3}\right)^k \left(-\frac{1}{2}\right)_k}{k!}\right)^6$$

$$27\sqrt{-\frac{4\left(-q^9+q^6-q\right)}{\frac{(q^9-q^6+q)((q-1)^3(q^2+q+1)(q^4+q^3+q^2+q+1))}{(q-1)^3(q^2+q+1)(q^4+q^3+q^2+q+1)}}}^{6}+1 = 1+\frac{27\left(\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}3^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)\right)^6}{64\sqrt{\pi}^6}$$

$\binom{n}{m}$ is the binomial coefficient
$n!$ is the factorial function
$(a)_n$ is the Pochhammer symbol (rising factorial)
$\Gamma(x)$ is the gamma function
$\operatorname{Res}_{z=z_0} f$ is a complex residue

From:

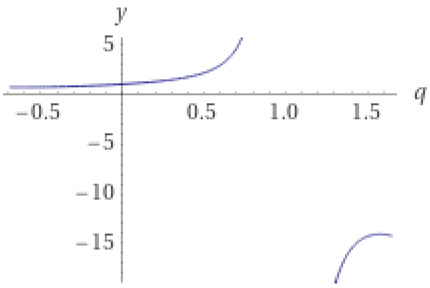
$$F_1(q)=\frac{1}{1-q}+\frac{q^4}{(1-q)(1-q^3)}+\frac{q^{12}}{(1-q)(1-q^3)(1-q^5)}+\cdots$$

1/(1-q) + (q^4)/((1-q)(1-q^3)) + (q^12)/((1-q)(1-q^3)(1-q^5))

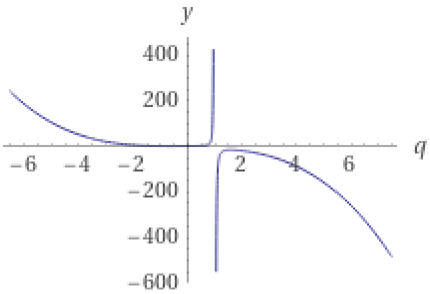
Input

$$\frac{1}{1-q}+\frac{q^4}{(1-q)\left(1-q^3\right)}+\frac{q^{12}}{(1-q)\left(1-q^3\right)\left(1-q^5\right)}$$

Plots **(figures that can be related to the open strings)**



(q from −0.7 to 1.6)



(q from −6.5 to 7.5)

Alternate forms

$$-\frac{q^{12}-q^9+q^8-q^5+q^4-q^3+1}{(q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}$$

$$-\frac{q^{12}-q^9+q^8-q^5+q^4-q^3+1}{(q-1)\left(q^3-1\right)\left(q^5-1\right)}$$

$$\frac{-q^{12}+q^9-q^8+q^5-q^4+q^3-1}{(q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}$$

Complex roots

$q \approx -0.91432 - 0.50203\,i$

$q \approx -0.91432 + 0.50203\,i$

$q \approx -0.56401 - 1.02530\,i$

$q \approx -0.56401 + 1.02530\,i$

$q \approx -0.54179 - 0.64330\,i$

Series expansion at q=0

$1 + q + q^2 + q^3 + 2\,q^4 + O(q^5)$

(Taylor series)

Series expansion at q=∞

$-q^3 - q^2 - q - 1 + O\left(\left(\frac{1}{q}\right)^1\right)$

(Taylor series)

Derivative

$$\frac{d}{dq}\left(\frac{1}{1-q} + \frac{q^4}{(1-q)\,(1-q^3)} + \frac{q^{12}}{(1-q)\,(1-q^3)\,(1-q^5)}\right) = \left(-3\,q^{18} - 2\,q^{17} - q^{16} + 6\,q^{15} + 6\,q^{14} + 14\,q^{13} + 10\,q^{12} + 8\,q^{11} - 5\,q^{10} - 6\,q^9 - 6\,q^8 + 2\,q^6 + 4\,q^5 + 6\,q^4 + 6\,q^3 + 3\,q^2 + 2\,q + 1\right) / \left((q-1)^4\,(q^2+q+1)^2\,(q^4+q^3+q^2+q+1)^2\right)$$

Indefinite integral

$$\int\left(\frac{1}{1-q} + \frac{q^4}{(1-q)\,(1-q^3)} + \frac{q^{12}}{(1-q)\,(1-q^3)\,(1-q^5)}\right)dq = \frac{1}{180}\left(36\sum_{\{\omega:\,\omega^4+5\,\omega^3+10\,\omega^2+10\,\omega+5=0\}}\frac{\omega^3\,\log(q-\omega-1)+3\,\omega^2\,\log(q-\omega-1)+2\,\omega\,\log(q-\omega-1)}{4\,\omega^3+15\,\omega^2+20\,\omega+10} - 45\,q^4 - 60\,q^3 - 90\,q^2 + 10\log(q^2+q+1) - 180\,q + \frac{48}{q-1} + \frac{6}{(q-1)^2} - 416\log(q-1) - 20\sqrt{3}\,\tan^{-1}\left(\frac{2\,q+1}{\sqrt{3}}\right) + 375\right) + \text{constant}$$

(assuming a complex-valued logarithm)

$\tan^{-1}(x)$ is the inverse tangent function log(x) is the natural logarithm

Local maximum

$$\max\left\{\frac{1}{1-q}+\frac{q^4}{(1-q)\left(1-q^3\right)}+\frac{q^{12}}{(1-q)\left(1-q^3\right)\left(1-q^5\right)}\right\}\approx -14.096 \text{ at } q\approx 1.5762$$

Local minimum

$$\min\left\{\frac{1}{1-q}+\frac{q^4}{(1-q)\left(1-q^3\right)}+\frac{q^{12}}{(1-q)\left(1-q^3\right)\left(1-q^5\right)}\right\}\approx 0.69241 \text{ at } q\approx -0.61689$$

Performing the following calculations

((-q^12 + q^9 - q^8 + q^5 - q^4 + q^3 - 1)/((q - 1)^3 (q^2 + q + 1) (q^4 + q^3 + q^2 + q + 1)))dxdy

we obtain:

Indefinite integral

$$\int\int \frac{-q^{12}+q^9-q^8+q^5-q^4+q^3-1}{(q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}\,dx\,dy =$$

$$c_1\,x + c_2 - \frac{q^4\,x\,y}{(q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)} + \frac{q^3\,x\,y}{(q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)} - \frac{x\,y}{(q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)} - \frac{q^{12}\,x\,y}{(q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)} + \frac{q^9\,x\,y}{(q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)} - \frac{q^8\,x\,y}{(q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)} + \frac{q^5\,x\,y}{(q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}$$

Definite integral over a disk of radius R

$$\iint\limits_{x^2+y^2<R^2} \frac{-q^{12}+q^9-q^8+q^5-q^4+q^3-1}{(q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}\,dy\,dx = \frac{\pi\left(-q^{12}+q^9-q^8+q^5-q^4+q^3-1\right)R^2}{(q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}$$

Definite integral over a square of edge length 2 L

$$\int_{-L}^{L}\int_{-L}^{L} \frac{-1+q^3-q^4+q^5-q^8+q^9-q^{12}}{(-1+q)^3\left(1+q+q^2\right)\left(1+q+q^2+q^3+q^4\right)}\,dx\,dy = \frac{4\,L^2\left(-q^{12}+q^9-q^8+q^5-q^4+q^3-1\right)}{(q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}$$

Dividing the two solutions of

$$\iint_{x^2+y^2<R^2} \frac{-q^{12}+q^9-q^8+q^5-q^4+q^3-1}{(q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}\,dy\,dx = \frac{\pi\left(-q^{12}+q^9-q^8+q^5-q^4+q^3-1\right)R^2}{(q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}$$

and

$$\int_{-L}^{L}\int_{-L}^{L} \frac{-1+q^3-q^4+q^5-q^8+q^9-q^{12}}{(-1+q)^3\left(1+q+q^2\right)\left(1+q+q^2+q^3+q^4\right)}\,dx\,dy = \frac{4L^2\left(-q^{12}+q^9-q^8+q^5-q^4+q^3-1\right)}{(q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}$$

by

$$\frac{-q^{12}+q^9-q^8+q^5-q^4+q^3-1}{(q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}$$

we obtain:

((π (-q^12 + q^9 - q^8 + q^5 - q^4 + q^3 - 1))/((q - 1)^3 (q^2 + q + 1) (q^4 + q^3 + q^2 + q + 1)))/(((-q^12 + q^9 - q^8 + q^5 - q^4 + q^3 - 1)/((q - 1)^3 (q^2 + q + 1) (q^4 + q^3 + q^2 + q + 1))))

Input

$$\frac{\frac{\pi\left(-q^{12}+q^9-q^8+q^5-q^4+q^3-1\right)}{(q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}}{\frac{-q^{12}+q^9-q^8+q^5-q^4+q^3-1}{(q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}}$$

Result

π

Decimal approximation

3.1415926535897932384626433832795028841971693993751058209749445923

...

3.141592653…. = π

Property

π is a transcendental number

Alternative representations

$$\frac{\pi\left(-q^{12}+q^9-q^8+q^5-q^4+q^3-1\right)}{\frac{\left(-q^{12}+q^9-q^8+q^5-q^4+q^3-1\right)\left((q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)\right)}{(q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}} = \frac{180\,°\left(-1+q^3-q^4+q^5-q^8+q^9-q^{12}\right)}{\frac{\left((-1+q)^3\left(1+q+q^2\right)\left(1+q+q^2+q^3+q^4\right)\right)\left(-1+q^3-q^4+q^5-q^8+q^9-q^{12}\right)}{(-1+q)^3\left(1+q+q^2\right)\left(1+q+q^2+q^3+q^4\right)}}$$

$$\frac{\pi\left(-q^{12}+q^9-q^8+q^5-q^4+q^3-1\right)}{\frac{\left(-q^{12}+q^9-q^8+q^5-q^4+q^3-1\right)\left((q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)\right)}{(q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}} = -\frac{i\left(\log(-1)\left(-1+q^3-q^4+q^5-q^8+q^9-q^{12}\right)\right)}{\frac{\left((-1+q)^3\left(1+q+q^2\right)\left(1+q+q^2+q^3+q^4\right)\right)\left(-1+q^3-q^4+q^5-q^8+q^9-q^{12}\right)}{(-1+q)^3\left(1+q+q^2\right)\left(1+q+q^2+q^3+q^4\right)}}$$

$$\frac{\pi\left(-q^{12}+q^9-q^8+q^5-q^4+q^3-1\right)}{\frac{\left(-q^{12}+q^9-q^8+q^5-q^4+q^3-1\right)\left((q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)\right)}{(q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}} = \frac{\cos^{-1}(-1)\left(-1+q^3-q^4+q^5-q^8+q^9-q^{12}\right)}{\frac{\left((-1+q)^3\left(1+q+q^2\right)\left(1+q+q^2+q^3+q^4\right)\right)\left(-1+q^3-q^4+q^5-q^8+q^9-q^{12}\right)}{(-1+q)^3\left(1+q+q^2\right)\left(1+q+q^2+q^3+q^4\right)}}$$

$\log(x)$ is the natural logarithm

i is the imaginary unit

$\cos^{-1}(x)$ is the inverse cosine function

Series representations

$$\frac{\pi\left(-q^{12}+q^9-q^8+q^5-q^4+q^3-1\right)}{\frac{\left(-q^{12}+q^9-q^8+q^5-q^4+q^3-1\right)\left((q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)\right)}{(q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}} = 4\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}$$

$$\frac{\pi\left(-q^{12}+q^9-q^8+q^5-q^4+q^3-1\right)}{\frac{\left(-q^{12}+q^9-q^8+q^5-q^4+q^3-1\right)\left((q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)\right)}{(q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}} = \sum_{k=0}^{\infty}-\frac{4(-1)^k\,1195^{-1-2k}\left(5^{1+2k}-4\times 239^{1+2k}\right)}{1+2k}$$

$$\frac{\pi\left(-q^{12}+q^9-q^8+q^5-q^4+q^3-1\right)}{\frac{\left(-q^{12}+q^9-q^8+q^5-q^4+q^3-1\right)\left((q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)\right)}{(q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}} = \sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k}+\frac{2}{1+4k}+\frac{1}{3+4k}\right)$$

Integral representations

$$\frac{\pi\left(-q^{12}+q^9-q^8+q^5-q^4+q^3-1\right)}{\frac{\left(-q^{12}+q^9-q^8+q^5-q^4+q^3-1\right)\left((q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)\right)}{(q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}} = 4\int_0^1\sqrt{1-t^2}\,dt$$

$$\frac{\pi\left(-q^{12}+q^9-q^8+q^5-q^4+q^3-1\right)}{\frac{\left(-q^{12}+q^9-q^8+q^5-q^4+q^3-1\right)\left((q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)\right)}{(q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}} = 2\int_0^1 \frac{1}{\sqrt{1-t^2}}\,dt$$

$$\frac{\pi\left(-q^{12}+q^9-q^8+q^5-q^4+q^3-1\right)}{\frac{\left(-q^{12}+q^9-q^8+q^5-q^4+q^3-1\right)\left((q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)\right)}{(q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}} = 2\int_0^\infty \frac{1}{1+t^2}\,dt$$

1/6(((π (-q^12 + q^9 - q^8 + q^5 - q^4 + q^3 - 1))/((q - 1)^3 (q^2 + q + 1) (q^4 + q^3 + q^2 + q + 1)))/(((-q^12 + q^9 - q^8 + q^5 - q^4 + q^3 - 1)/((q - 1)^3 (q^2+q+1) (q^4+q^3+q^2+q+1)))))^2

Input

$$\frac{1}{6}\left(\frac{\frac{\pi\left(-q^{12}+q^9-q^8+q^5-q^4+q^3-1\right)}{(q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}}{\frac{-q^{12}+q^9-q^8+q^5-q^4+q^3-1}{(q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}}\right)^2$$

Result

$$\frac{\pi^2}{6}$$

Decimal approximation

1.6449340668482264364724151666460251892189499012067984377355582293

…

1.644934066….

Property

$\frac{\pi^2}{6}$ is a transcendental number

Alternative representations

$$\frac{1}{6}\left(\frac{\pi\left(-q^{12}+q^9-q^8+q^5-q^4+q^3-1\right)}{\frac{\left((q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)\right)\left(-q^{12}+q^9-q^8+q^5-q^4+q^3-1\right)}{(q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}}\right)^2 = \frac{1}{6}\left(\frac{180\,°\left(-1+q^3-q^4+q^5-q^8+q^9-q^{12}\right)}{\frac{\left((-1+q)^3\left(1+q+q^2\right)\left(1+q+q^2+q^3+q^4\right)\right)\left(-1+q^3-q^4+q^5-q^8+q^9-q^{12}\right)}{(-1+q)^3\left(1+q+q^2\right)\left(1+q+q^2+q^3+q^4\right)}}\right)^2$$

$$\frac{1}{6}\left(\frac{\pi\left(-q^{12}+q^9-q^8+q^5-q^4+q^3-1\right)}{\frac{\left((q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)\right)\left(-q^{12}+q^9-q^8+q^5-q^4+q^3-1\right)}{(q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}}\right)^2 = \frac{1}{6}\left(-\frac{i\left(\log(-1)\left(-1+q^3-q^4+q^5-q^8+q^9-q^{12}\right)\right)}{\frac{\left((-1+q)^3\left(1+q+q^2\right)\left(1+q+q^2+q^3+q^4\right)\right)\left(-1+q^3-q^4+q^5-q^8+q^9-q^{12}\right)}{(-1+q)^3\left(1+q+q^2\right)\left(1+q+q^2+q^3+q^4\right)}}\right)^2$$

$$\frac{1}{6}\left(\frac{\pi\left(-q^{12}+q^9-q^8+q^5-q^4+q^3-1\right)}{\frac{\left((q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)\right)\left(-q^{12}+q^9-q^8+q^5-q^4+q^3-1\right)}{(q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}}\right)^2 = \frac{1}{6}\left(\frac{\cos^{-1}(-1)\left(-1+q^3-q^4+q^5-q^8+q^9-q^{12}\right)}{\frac{\left((-1+q)^3\left(1+q+q^2\right)\left(1+q+q^2+q^3+q^4\right)\right)\left(-1+q^3-q^4+q^5-q^8+q^9-q^{12}\right)}{(-1+q)^3\left(1+q+q^2\right)\left(1+q+q^2+q^3+q^4\right)}}\right)^2$$

$\log(x)$ is the natural logarithm
i is the imaginary unit
$\cos^{-1}(x)$ is the inverse cosine function

Series representations

$$\frac{1}{6}\left(\frac{\pi\left(-q^{12}+q^9-q^8+q^5-q^4+q^3-1\right)}{\frac{\left((q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)\right)\left(-q^{12}+q^9-q^8+q^5-q^4+q^3-1\right)}{(q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}}\right)^2=\sum_{k=1}^{\infty}\frac{1}{k^2}$$

$$\frac{1}{6}\left(\frac{\pi\left(-q^{12}+q^9-q^8+q^5-q^4+q^3-1\right)}{\frac{\left((q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)\right)\left(-q^{12}+q^9-q^8+q^5-q^4+q^3-1\right)}{(q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}}\right)^2=-2\sum_{k=1}^{\infty}\frac{(-1)^k}{k^2}$$

$$\frac{1}{6}\left(\frac{\pi\left(-q^{12}+q^9-q^8+q^5-q^4+q^3-1\right)}{\frac{\left((q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)\right)\left(-q^{12}+q^9-q^8+q^5-q^4+q^3-1\right)}{(q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}}\right)^2=\frac{4}{3}\sum_{k=0}^{\infty}\frac{1}{(1+2k)^2}$$

Integral representations

$$\frac{1}{6}\left(\frac{\pi\left(-q^{12}+q^9-q^8+q^5-q^4+q^3-1\right)}{\frac{\left((q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)\right)\left(-q^{12}+q^9-q^8+q^5-q^4+q^3-1\right)}{(q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}}\right)^2=\frac{8}{3}\left(\int_0^1\sqrt{1-t^2}\,dt\right)^2$$

$$\frac{1}{6}\left(\frac{\pi\left(-q^{12}+q^9-q^8+q^5-q^4+q^3-1\right)}{\frac{\left((q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)\right)\left(-q^{12}+q^9-q^8+q^5-q^4+q^3-1\right)}{(q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}}\right)^2=\frac{2}{3}\left(\int_0^{\infty}\frac{1}{1+t^2}\,dt\right)^2$$

$$\frac{1}{6}\left(\frac{\pi\left(-q^{12}+q^9-q^8+q^5-q^4+q^3-1\right)}{\frac{\left((q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)\right)\left(-q^{12}+q^9-q^8+q^5-q^4+q^3-1\right)}{(q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}}\right)^2=\frac{2}{3}\left(\int_0^1\frac{1}{\sqrt{1-t^2}}\,dt\right)^2$$

We obtain also:

((((4(-q^12 + q^9 - q^8 + q^5 - q^4 + q^3 - 1))/((q - 1)^3 (q^2 + q + 1) (q^4 + q^3 + q^2 + q + 1))))/(((-q^12 + q^9 - q^8 + q^5 - q^4 + q^3 - 1)/((q - 1)^3 (q^2+q+1) (q^4+q^3+q^2+q+1)))))^6

Input

$$\left(\frac{\frac{4\left(-q^{12}+q^9-q^8+q^5-q^4+q^3-1\right)}{(q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}}{\frac{-q^{12}+q^9-q^8+q^5-q^4+q^3-1}{(q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}}\right)^6$$

Result

4096

$4096 = 64^2$

27*√((((4(-q^12 + q^9 - q^8 + q^5 - q^4 + q^3 - 1))/((q - 1)^3 (q^2 + q + 1) (q^4 + q^3 + q^2 + q + 1))))/(((-q^12 + q^9 - q^8 + q^5 - q^4 + q^3 - 1)/((q - 1)^3 (q^2+q+1) (q^4+q^3+q^2+q+1)))))^6+1

Input

$$27\sqrt{\frac{\frac{4\left(-q^{12}+q^9-q^8+q^5-q^4+q^3-1\right)}{(q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}}{\frac{-q^{12}+q^9-q^8+q^5-q^4+q^3-1}{(q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}}}^{\,6}+1$$

Result

1729

1729

This result is very near to the mass of candidate glueball $f_0(1710)$ scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. ($1728 = 8^2 * 3^3$) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Series representations

$$27 \sqrt{\frac{4\left(-q^{12}+q^9-q^8+q^5-q^4+q^3-1\right)}{\frac{\left(-q^{12}+q^9-q^8+q^5-q^4+q^3-1\right)\left((q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)\right)}{(q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}}}^6+1= 1+27\sqrt{3}^6\left(\sum_{k=0}^{\infty}3^{-k}\binom{\frac{1}{2}}{k}\right)^6$$

$$27 \sqrt{\frac{4\left(-q^{12}+q^9-q^8+q^5-q^4+q^3-1\right)}{\frac{\left(-q^{12}+q^9-q^8+q^5-q^4+q^3-1\right)\left((q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)\right)}{(q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}}}^6+1= 1+27\sqrt{3}^6\left(\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{3}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^6$$

$$27 \sqrt{\frac{4\left(-q^{12}+q^9-q^8+q^5-q^4+q^3-1\right)}{\frac{\left(-q^{12}+q^9-q^8+q^5-q^4+q^3-1\right)\left((q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)\right)}{(q-1)^3\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}}}^6+1= 1+\frac{27\left(\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}3^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)\right)^6}{64\sqrt{\pi}^6}$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function
$(a)_n$ is the Pochhammer symbol (rising factorial)
$\Gamma(x)$ is the gamma function
$\underset{z=z_0}{\text{Res}} f$ is a complex residue

From:

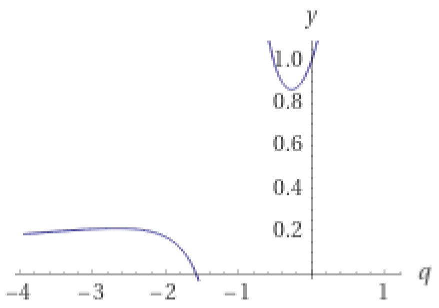
$$\frac{1}{1-q}+\frac{q^2}{(1-q^2)(1-q^3)}+\frac{q^6}{(1-q^3)(1-q^4)(1-q^5)}+\cdots$$

1/(1-q) + (q^2)/((1-q^2)(1-q^3)) + (q^6)/((1-q^3)(1-q^4)(1-q^5))

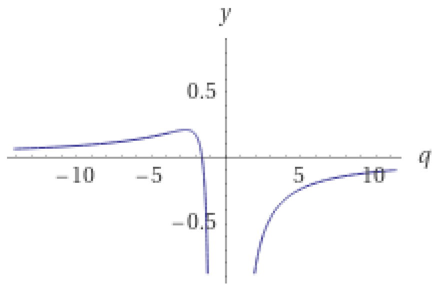
Input

$$\frac{1}{1-q}+\frac{q^2}{(1-q^2)(1-q^3)}+\frac{q^6}{(1-q^3)(1-q^4)(1-q^5)}$$

Plots **(figures that can be related to the open strings)**



(q from −3.9 to 1.1)



(q from −14.1 to 11.3)

Alternate forms

$$-\frac{q^{11}+q^{10}-2q^7-q^6-2q^5+2q^2+q+1}{(q-1)^3(q+1)\left(q^2+1\right)\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}$$

$$\frac{q^2}{q^5-q^3-q^2+1}-\frac{q^6}{\left(q^3-1\right)\left(q^4-1\right)\left(q^5-1\right)}+\frac{1}{1-q}$$

$$-\frac{q^{11}+q^{10}-2q^7-q^6-2q^5+2q^2+q+1}{(q-1)^3(q+1)\left(q^2+1\right)\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}$$

Partial fraction expansion

$$\frac{-2q-1}{9\left(q^2+q+1\right)}-\frac{1}{4\left(q^2+1\right)}+\frac{q-q^3}{5\left(q^4+q^3+q^2+q+1\right)}-\frac{641}{720(q-1)}+\frac{5}{16(q+1)}+\frac{17}{120(q-1)^2}-\frac{1}{60(q-1)^3}$$

Real root

$q \approx -1.5863$

Complex roots

$q \approx -0.71270 - 0.57224\,i$

$q \approx -0.71270 + 0.57224\,i$

$q \approx -0.30748 - 0.76350\,i$

$q \approx -0.30748 + 0.76350\,i$

$q \approx -0.15316 - 0.94261\,i$

Series expansion at q=0

$$1 + q + 2q^2 + q^3 + 2q^4 + O(q^5)$$

(Taylor series)

Series expansion at q=∞

$$-\frac{1}{q} - \left(\frac{1}{q}\right)^2 - \left(\frac{1}{q}\right)^4 + O\left(\left(\frac{1}{q}\right)^5\right)$$

(Laurent series)

Derivative

$$\frac{d}{dq}\left(\frac{1}{1-q} + \frac{q^2}{(1-q^2)(1-q^3)} + \frac{q^6}{(1-q^3)(1-q^4)(1-q^5)}\right) = \\ (q^{20} + 4q^{19} + 7q^{18} + 12q^{17} + 11q^{16} + 10q^{15} - \\ 2q^{14} - 16q^{13} - 37q^{12} - 42q^{11} - 36q^{10} - 10q^9 + 20q^8 + \\ 50q^7 + 59q^6 + 58q^5 + 42q^4 + 28q^3 + 14q^2 + 6q + 1) / \\ ((q-1)^4 (q+1)^2 (q^2+1)^2 (q^2+q+1)^2 (q^4+q^3+q^2+q+1)^2)$$

Indefinite integral

$$\int\left(\frac{1}{1-q} + \frac{q^2}{(1-q^2)(1-q^3)} + \frac{q^6}{(1-q^3)(1-q^4)(1-q^5)}\right)dq = \\ \frac{1}{720}\left(-144\sum_{\{\omega:\,\omega^4+\omega^3+\omega^2+\omega+1=0\}} \frac{\omega^3\log(q-\omega)-\omega\log(q-\omega)}{4\omega^3+3\omega^2+2\omega+1} - \\ 80\log(q^2+q+1) - \frac{102}{q-1} + \frac{6}{(q-1)^2} - 701\log(1-q) + \\ 60\log(q-1) + 225\log(q+1) - 180\tan^{-1}(q)\right) + \text{constant}$$

(assuming a complex-valued logarithm)

Local maximum

$$\max\left\{\frac{1}{1-q}+\frac{q^2}{(1-q^2)(1-q^3)}+\frac{q^6}{(1-q^3)(1-q^4)(1-q^5)}\right\}\approx 0.21177$$
at $q \approx -2.6782$

Local minimum

$$\min\left\{\frac{1}{1-q}+\frac{q^2}{(1-q^2)(1-q^3)}+\frac{q^6}{(1-q^3)(1-q^4)(1-q^5)}\right\}\approx 0.86488$$
at $q \approx -0.27319$

Limit

$$\lim_{q\to\pm\infty}\left(\frac{1}{1-q}+\frac{q^2}{(1-q^2)(1-q^3)}+\frac{q^6}{(1-q^3)(1-q^4)(1-q^5)}\right)=0$$

Performing the following calculations:

(-(q^11 + q^10 - 2 q^7 - q^6 - 2 q^5 + 2 q^2 + q + 1)/((q - 1)^3 (q + 1) (q^2 + 1) (q^2 + q + 1) (q^4 + q^3 + q^2 + q + 1)))dxdy

we obtain:

Indefinite integral

$$\int\int -\frac{q^{11}+q^{10}-2q^7-q^6-2q^5+2q^2+q+1}{(q-1)^3\,(q+1)\left(q^2+1\right)\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}\,dx\,dy =$$

$$c_1\,x+c_2-\frac{2q^2\,x\,y}{(q-1)^3\,(q+1)\left(q^2+1\right)\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}-\frac{q\,x\,y}{(q-1)^3\,(q+1)\left(q^2+1\right)\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}-\frac{x\,y}{(q-1)^3\,(q+1)\left(q^2+1\right)\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}-\frac{q^{11}\,x\,y}{(q-1)^3\,(q+1)\left(q^2+1\right)\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}-\frac{q^{10}\,x\,y}{(q-1)^3\,(q+1)\left(q^2+1\right)\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}+\frac{2q^7\,x\,y}{(q-1)^3\,(q+1)\left(q^2+1\right)\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}+\frac{q^6\,x\,y}{(q-1)^3\,(q+1)\left(q^2+1\right)\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}+\frac{2q^5\,x\,y}{(q-1)^3\,(q+1)\left(q^2+1\right)\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}$$

Definite integral over a disk of radius R

$$\iint\limits_{x^2+y^2<R^2}\frac{-q^{11}-q^{10}+2q^7+q^6+2q^5-2q^2-q-1}{(q-1)^3\,(q+1)\left(q^2+1\right)\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}\,dy\,dx =$$

$$\frac{\pi\left(-q^{11}-q^{10}+2q^7+q^6+2q^5-2q^2-q-1\right)R^2}{(q-1)^3\,(q+1)\left(q^2+1\right)\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}$$

Definite integral over a square of edge length 2 L

$$\int_{-L}^{L}\int_{-L}^{L}\frac{-1-q-2q^2+2q^5+q^6+2q^7-q^{10}-q^{11}}{(-1+q)^3\,(1+q)\left(1+q^2\right)\left(1+q+q^2\right)\left(1+q+q^2+q^3+q^4\right)}\,dx\,dy =$$

$$\frac{4L^2\left(-q^{11}-q^{10}+2q^7+q^6+2q^5-2q^2-q-1\right)}{(q-1)^3\,(q+1)\left(q^2+1\right)\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}$$

Dividing the two results of

$$\iint\limits_{x^2+y^2<R^2} \frac{-q^{11}-q^{10}+2q^7+q^6+2q^5-2q^2-q-1}{(q-1)^3(q+1)(q^2+1)(q^2+q+1)(q^4+q^3+q^2+q+1)}\,dy\,dx = \frac{\pi(-q^{11}-q^{10}+2q^7+q^6+2q^5-2q^2-q-1)R^2}{(q-1)^3(q+1)(q^2+1)(q^2+q+1)(q^4+q^3+q^2+q+1)}$$

and

$$\int_{-L}^{L}\int_{-L}^{L}\frac{-1-q-2q^2+2q^5+q^6+2q^7-q^{10}-q^{11}}{(-1+q)^3(1+q)(1+q^2)(1+q+q^2)(1+q+q^2+q^3+q^4)}\,dx\,dy = \frac{4L^2(-q^{11}-q^{10}+2q^7+q^6+2q^5-2q^2-q-1)}{(q-1)^3(q+1)(q^2+1)(q^2+q+1)(q^4+q^3+q^2+q+1)}$$

by

$$-\frac{q^{11}+q^{10}-2q^7-q^6-2q^5+2q^2+q+1}{(q-1)^3(q+1)(q^2+1)(q^2+q+1)(q^4+q^3+q^2+q+1)}$$

we obtain:

((π (-q^11-q^10+2q^7+q^6+2q^5-2q^2-q-1))/((q-1)^3 (q+1)(q^2+1)(q^2+q+1)(q^4+q^3+q^2+q+1))) / (-(q^11+q^10-2q^7-q^6-2q^5+2q^2+q+1)/((q-1)^3 (q+1)(q^2+1)(q^2+q+1)(q^4+q^3+q^2+q+1)))

Input

$$-\frac{\frac{\pi(-q^{11}-q^{10}+2q^7+q^6+2q^5-2q^2-q-1)}{(q-1)^3(q+1)(q^2+1)(q^2+q+1)(q^4+q^3+q^2+q+1)}}{\frac{q^{11}+q^{10}-2q^7-q^6-2q^5+2q^2+q+1}{(q-1)^3(q+1)(q^2+1)(q^2+q+1)(q^4+q^3+q^2+q+1)}}$$

Result

π

Decimal approximation

3.1415926535897932384626433832795028841971693993751058209749445923
…

3.141592653…. = π

Property

π is a transcendental number

Alternative representations

$$-\frac{\pi\left(-q^{11}-q^{10}+2q^7+q^6+2q^5-2q^2-q-1\right)}{\frac{\left(q^{11}+q^{10}-2q^7-q^6-2q^5+2q^2+q+1\right)\left((q-1)^3(q+1)\left(q^2+1\right)\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)\right)}{(q-1)^3(q+1)\left(q^2+1\right)\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}}=\frac{180\,°\left(-1-q-2q^2+2q^5+q^6+2q^7-q^{10}-q^{11}\right)}{\frac{\left((1+q)(-1+q)^3\left(1+q^2\right)\left(1+q+q^2\right)\left(1+q+q^2+q^3+q^4\right)\right)\left(-1-q-2q^2+2q^5+q^6+2q^7-q^{10}-q^{11}\right)}{(1+q)(-1+q)^3\left(1+q^2\right)\left(1+q+q^2\right)\left(1+q+q^2+q^3+q^4\right)}}$$

$$-\frac{\pi\left(-q^{11}-q^{10}+2q^7+q^6+2q^5-2q^2-q-1\right)}{\frac{\left(q^{11}+q^{10}-2q^7-q^6-2q^5+2q^2+q+1\right)\left((q-1)^3(q+1)\left(q^2+1\right)\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)\right)}{(q-1)^3(q+1)\left(q^2+1\right)\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}}=-\frac{i\left(\log(-1)\left(-1-q-2q^2+2q^5+q^6+2q^7-q^{10}-q^{11}\right)\right)}{\frac{\left((1+q)(-1+q)^3\left(1+q^2\right)\left(1+q+q^2\right)\left(1+q+q^2+q^3+q^4\right)\right)\left(-1-q-2q^2+2q^5+q^6+2q^7-q^{10}-q^{11}\right)}{(1+q)(-1+q)^3\left(1+q^2\right)\left(1+q+q^2\right)\left(1+q+q^2+q^3+q^4\right)}}$$

$$-\frac{\pi\left(-q^{11}-q^{10}+2q^7+q^6+2q^5-2q^2-q-1\right)}{\frac{\left(q^{11}+q^{10}-2q^7-q^6-2q^5+2q^2+q+1\right)\left((q-1)^3(q+1)\left(q^2+1\right)\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)\right)}{(q-1)^3(q+1)\left(q^2+1\right)\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}}=\frac{\cos^{-1}(-1)\left(-1-q-2q^2+2q^5+q^6+2q^7-q^{10}-q^{11}\right)}{\frac{\left((1+q)(-1+q)^3\left(1+q^2\right)\left(1+q+q^2\right)\left(1+q+q^2+q^3+q^4\right)\right)\left(-1-q-2q^2+2q^5+q^6+2q^7-q^{10}-q^{11}\right)}{(1+q)(-1+q)^3\left(1+q^2\right)\left(1+q+q^2\right)\left(1+q+q^2+q^3+q^4\right)}}$$

$\log(x)$ is the natural logarithm

i is the imaginary unit

$\cos^{-1}(x)$ is the inverse cosine function

Series representations

$$-\frac{\pi\left(-q^{11}-q^{10}+2q^7+q^6+2q^5-2q^2-q-1\right)}{\frac{\left(q^{11}+q^{10}-2q^7-q^6-2q^5+2q^2+q+1\right)\left((q-1)^3(q+1)\left(q^2+1\right)\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)\right)}{(q-1)^3(q+1)\left(q^2+1\right)\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}} = 4\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}$$

$$-\frac{\pi\left(-q^{11}-q^{10}+2q^7+q^6+2q^5-2q^2-q-1\right)}{\frac{\left(q^{11}+q^{10}-2q^7-q^6-2q^5+2q^2+q+1\right)\left((q-1)^3(q+1)\left(q^2+1\right)\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)\right)}{(q-1)^3(q+1)\left(q^2+1\right)\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}} = \sum_{k=0}^{\infty}-\frac{4(-1)^k\,1195^{-1-2k}\left(5^{1+2k}-4\times 239^{1+2k}\right)}{1+2k}$$

$$-\frac{\pi\left(-q^{11}-q^{10}+2q^7+q^6+2q^5-2q^2-q-1\right)}{\frac{\left(q^{11}+q^{10}-2q^7-q^6-2q^5+2q^2+q+1\right)\left((q-1)^3(q+1)\left(q^2+1\right)\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)\right)}{(q-1)^3(q+1)\left(q^2+1\right)\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}} = \sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k}+\frac{2}{1+4k}+\frac{1}{3+4k}\right)$$

Integral representations

$$-\frac{\pi\left(-q^{11}-q^{10}+2q^7+q^6+2q^5-2q^2-q-1\right)}{\frac{\left(q^{11}+q^{10}-2q^7-q^6-2q^5+2q^2+q+1\right)\left((q-1)^3(q+1)\left(q^2+1\right)\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)\right)}{(q-1)^3(q+1)\left(q^2+1\right)\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}} = 4\int_0^1\sqrt{1-t^2}\,dt$$

$$-\frac{\pi\left(-q^{11}-q^{10}+2q^7+q^6+2q^5-2q^2-q-1\right)}{\frac{\left(q^{11}+q^{10}-2q^7-q^6-2q^5+2q^2+q+1\right)\left((q-1)^3(q+1)\left(q^2+1\right)\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)\right)}{(q-1)^3(q+1)\left(q^2+1\right)\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}} = 2\int_0^1\frac{1}{\sqrt{1-t^2}}\,dt$$

$$-\frac{\pi\left(-q^{11}-q^{10}+2q^7+q^6+2q^5-2q^2-q-1\right)}{\frac{\left(q^{11}+q^{10}-2q^7-q^6-2q^5+2q^2+q+1\right)\left((q-1)^3(q+1)\left(q^2+1\right)\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)\right)}{(q-1)^3(q+1)\left(q^2+1\right)\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}} =$$

$$2\int_0^\infty \frac{1}{1+t^2}\,dt$$

1/6(((π (-q^11-q^10+2q^7+q^6+2q^5-2q^2-q-1))/((q-1)^3 (q+1)(q^2+1)(q^2+q+1)(q^4+q^3+q^2+q+1))) / (-(q^11+q^10-2q^7-q^6-2q^5+2q^2+q+1)/((q-1)^3 (q+1)(q^2+1)(q^2+q+1)(q^4+q^3+q^2+q+1))))^2

Input

$$\frac{1}{6}\left(-\frac{\frac{\pi\left(-q^{11}-q^{10}+2q^7+q^6+2q^5-2q^2-q-1\right)}{(q-1)^3(q+1)\left(q^2+1\right)\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}}{\frac{q^{11}+q^{10}-2q^7-q^6-2q^5+2q^2+q+1}{(q-1)^3(q+1)\left(q^2+1\right)\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}}\right)^2$$

Result

$$\frac{\pi^2}{6}$$

Decimal approximation

1.644934066848226436472415166646025189218949901206798437735558229 3

…

1.644934066….

Property

$\frac{\pi^2}{6}$ is a transcendental number

Alternative representations

$$\frac{1}{6}\left(\frac{\pi\left(-q^{11}-q^{10}+2q^7+q^6+2q^5-2q^2-q-1\right)}{\frac{((q-1)^3(q+1)(q^2+1)(q^2+q+1)(q^4+q^3+q^2+q+1))(-(q^{11}+q^{10}-2q^7-q^6-2q^5+2q^2+q+1))}{(q-1)^3(q+1)(q^2+1)(q^2+q+1)(q^4+q^3+q^2+q+1)}}\right)^2 = \frac{1}{6}\left(\frac{180°\left(-1-q-2q^2+2q^5+q^6+2q^7-q^{10}-q^{11}\right)}{\frac{((1+q)(-1+q)^3(1+q^2)(1+q+q^2)(1+q+q^2+q^3+q^4))(-1-q-2q^2+2q^5+q^6+2q^7-q^{10}-q^{11})}{(1+q)(-1+q)^3(1+q^2)(1+q+q^2)(1+q+q^2+q^3+q^4)}}\right)^2$$

$$\frac{1}{6}\left(\frac{\pi\left(-q^{11}-q^{10}+2q^7+q^6+2q^5-2q^2-q-1\right)}{\frac{((q-1)^3(q+1)(q^2+1)(q^2+q+1)(q^4+q^3+q^2+q+1))(-(q^{11}+q^{10}-2q^7-q^6-2q^5+2q^2+q+1))}{(q-1)^3(q+1)(q^2+1)(q^2+q+1)(q^4+q^3+q^2+q+1)}}\right)^2 = \frac{1}{6}\left(-\frac{i\left(\log(-1)\left(-1-q-2q^2+2q^5+q^6+2q^7-q^{10}-q^{11}\right)\right)}{\frac{((1+q)(-1+q)^3(1+q^2)(1+q+q^2)(1+q+q^2+q^3+q^4))(-1-q-2q^2+2q^5+q^6+2q^7-q^{10}-q^{11})}{(1+q)(-1+q)^3(1+q^2)(1+q+q^2)(1+q+q^2+q^3+q^4)}}\right)^2$$

$$\frac{1}{6}\left(\frac{\pi\left(-q^{11}-q^{10}+2q^7+q^6+2q^5-2q^2-q-1\right)}{\frac{((q-1)^3(q+1)(q^2+1)(q^2+q+1)(q^4+q^3+q^2+q+1))(-(q^{11}+q^{10}-2q^7-q^6-2q^5+2q^2+q+1))}{(q-1)^3(q+1)(q^2+1)(q^2+q+1)(q^4+q^3+q^2+q+1)}}\right)^2 = \frac{1}{6}\left(\frac{\cos^{-1}(-1)\left(-1-q-2q^2+2q^5+q^6+2q^7-q^{10}-q^{11}\right)}{\frac{((1+q)(-1+q)^3(1+q^2)(1+q+q^2)(1+q+q^2+q^3+q^4))(-1-q-2q^2+2q^5+q^6+2q^7-q^{10}-q^{11})}{(1+q)(-1+q)^3(1+q^2)(1+q+q^2)(1+q+q^2+q^3+q^4)}}\right)^2$$

$\log(x)$ is the natural logarithm

i is the imaginary unit

$\cos^{-1}(x)$ is the inverse cosine function

Series representations

$$\frac{1}{6}\left(\frac{\pi\left(-q^{11}-q^{10}+2q^7+q^6+2q^5-2q^2-q-1\right)}{\frac{((q-1)^3(q+1)(q^2+1)(q^2+q+1)(q^4+q^3+q^2+q+1))(-(q^{11}+q^{10}-2q^7-q^6-2q^5+2q^2+q+1))}{(q-1)^3(q+1)(q^2+1)(q^2+q+1)(q^4+q^3+q^2+q+1)}}\right)^2 = \sum_{k=1}^{\infty}\frac{1}{k^2}$$

$$\frac{1}{6}\left(\frac{\pi\left(-q^{11}-q^{10}+2q^7+q^6+2q^5-2q^2-q-1\right)}{\frac{\left((q-1)^3(q+1)\left(q^2+1\right)\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)\right)\left(-\left(q^{11}+q^{10}-2q^7-q^6-2q^5+2q^2+q+1\right)\right)}{(q-1)^3(q+1)\left(q^2+1\right)\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}}\right)^2 = -2\sum_{k=1}^{\infty}\frac{(-1)^k}{k^2}$$

$$\frac{1}{6}\left(\frac{\pi\left(-q^{11}-q^{10}+2q^7+q^6+2q^5-2q^2-q-1\right)}{\frac{\left((q-1)^3(q+1)\left(q^2+1\right)\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)\right)\left(-\left(q^{11}+q^{10}-2q^7-q^6-2q^5+2q^2+q+1\right)\right)}{(q-1)^3(q+1)\left(q^2+1\right)\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}}\right)^2 = \frac{4}{3}\sum_{k=0}^{\infty}\frac{1}{(1+2k)^2}$$

Integral representations

$$\frac{1}{6}\left(\frac{\pi\left(-q^{11}-q^{10}+2q^7+q^6+2q^5-2q^2-q-1\right)}{\frac{\left((q-1)^3(q+1)\left(q^2+1\right)\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)\right)\left(-\left(q^{11}+q^{10}-2q^7-q^6-2q^5+2q^2+q+1\right)\right)}{(q-1)^3(q+1)\left(q^2+1\right)\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}}\right)^2 = \frac{8}{3}\left(\int_0^1\sqrt{1-t^2}\,dt\right)^2$$

$$\frac{1}{6}\left(\frac{\pi\left(-q^{11}-q^{10}+2q^7+q^6+2q^5-2q^2-q-1\right)}{\frac{\left((q-1)^3(q+1)\left(q^2+1\right)\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)\right)\left(-\left(q^{11}+q^{10}-2q^7-q^6-2q^5+2q^2+q+1\right)\right)}{(q-1)^3(q+1)\left(q^2+1\right)\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}}\right)^2 = \frac{2}{3}\left(\int_0^\infty\frac{1}{1+t^2}\,dt\right)^2$$

$$\frac{1}{6}\left(\frac{\pi\left(-q^{11}-q^{10}+2q^7+q^6+2q^5-2q^2-q-1\right)}{\frac{\left((q-1)^3 (q+1)\left(q^2+1\right)\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)\right)\left(-\left(q^{11}+q^{10}-2q^7-q^6-2q^5+2q^2+q+1\right)\right)}{(q-1)^3 (q+1)\left(q^2+1\right)\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}}\right)^2 = \frac{2}{3}\left(\int_0^1 \frac{1}{\sqrt{1-t^2}}\,dt\right)^2$$

We obtain also:

((4 (-q^11-q^10+2q^7+q^6+2q^5-2q^2-q-1))/((q-1)^3 (q+1)(q^2+1)(q^2+q+1)(q^4+q^3+q^2+q+1))/ (-(q^11+q^10-2q^7-q^6-2q^5+2q^2+q+1)/((q-1)^3 (q+1)(q^2+1)(q^2+q+1)(q^4+q^3+q^2+q+1))))^6

Input

$$\left(-\frac{\frac{4\left(-q^{11}-q^{10}+2q^7+q^6+2q^5-2q^2-q-1\right)}{(q-1)^3 (q+1)\left(q^2+1\right)\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}}{\frac{q^{11}+q^{10}-2q^7-q^6-2q^5+2q^2+q+1}{(q-1)^3 (q+1)\left(q^2+1\right)\left(q^2+q+1\right)\left(q^4+q^3+q^2+q+1\right)}}\right)^6$$

Result

4096

$4096 = 64^2$

27√((4 (-q^11-q^10+2q^7+q^6+2q^5-2q^2-q-1))/((q-1)^3 (q+1)(q^2+1)(q^2+q+1)(q^4+q^3+q^2+q+1))/ (-(q^11+q^10-2q^7-q^6-2q^5+2q^2+q+1)/((q-1)^3 (q+1)(q^2+1)(q^2+q+1)(q^4+q^3+q^2+q+1))))^6+1

Input

$$27\sqrt[6]{-\frac{\frac{4(-q^{11}-q^{10}+2q^7+q^6+2q^5-2q^2-q-1)}{(q-1)^3(q+1)(q^2+1)(q^2+q+1)(q^4+q^3+q^2+q+1)}}{\frac{q^{11}+q^{10}-2q^7-q^6-2q^5+2q^2+q+1}{(q-1)^3(q+1)(q^2+1)(q^2+q+1)(q^4+q^3+q^2+q+1)}}}+1$$

Result

1729

1729

This result is very near to the mass of candidate glueball $f_0(1710)$ scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. ($1728 = 8^2 * 3^3$) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Series representations

$$27\sqrt[6]{-\frac{4(-q^{11}-q^{10}+2q^7+q^6+2q^5-2q^2-q-1)}{\frac{(q^{11}+q^{10}-2q^7-q^6-2q^5+2q^2+q+1)((q-1)^3(q+1)(q^2+1)(q^2+q+1)(q^4+q^3+q^2+q+1))}{(q-1)^3(q+1)(q^2+1)(q^2+q+1)(q^4+q^3+q^2+q+1)}}}+1 = 1+27\sqrt[6]{3}\left(\sum_{k=0}^{\infty}3^{-k}\binom{\frac{1}{2}}{k}\right)^6$$

$$27\sqrt[6]{-\frac{4(-q^{11}-q^{10}+2q^7+q^6+2q^5-2q^2-q-1)}{\frac{(q^{11}+q^{10}-2q^7-q^6-2q^5+2q^2+q+1)((q-1)^3(q+1)(q^2+1)(q^2+q+1)(q^4+q^3+q^2+q+1))}{(q-1)^3(q+1)(q^2+1)(q^2+q+1)(q^4+q^3+q^2+q+1)}}}+1 = 1+27\sqrt[6]{3}\left(\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{3}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^6$$

$$27\sqrt{-\frac{4\left(-q^{11}-q^{10}+2q^{7}+q^{6}+2q^{5}-2q^{2}-q-1\right)}{\frac{\left(q^{11}+q^{10}-2q^{7}-q^{6}-2q^{5}+2q^{2}+q+1\right)\left((q-1)^{3}(q+1)(q^{2}+1)(q^{2}+q+1)(q^{4}+q^{3}+q^{2}+q+1)\right)}{(q-1)^{3}(q+1)(q^{2}+1)(q^{2}+q+1)(q^{4}+q^{3}+q^{2}+q+1)}}}^{\,6}+1=$$

$$1+\frac{27\left(\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}3^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)\right)^{6}}{64\sqrt{\pi}^{6}}$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\Gamma(x)$ is the gamma function

$\operatorname{Res}_{z=z_0} f$ is a complex residue

From

$$f_3(q)=1+\frac{q}{(1+q)^2}+\frac{q^4}{(1+q)^2(1+q^2)^2}+\cdots$$

$$f_3(q)+\sqrt{\frac{\pi}{t}}\exp\left(\frac{\pi^2}{24t}-\frac{t}{24}\right)\to 4.$$

$$t\to 0$$

$q = e^{-t} \to 1$; $q < 1$

1+(q)/((1+q)^2) + (q)^4/((1+q)^2((1+q^2)^2)) + sqrt(Pi/(t)) * exp((Pi^2)/(24*(t))-(t)/(24)) = 4

Input

$$1+\frac{q}{(1+q)^2}+\frac{q^4}{(1+q)^2\left(1+q^2\right)^2}+\sqrt{\frac{\pi}{t}}\exp\left(\frac{\pi^2}{24\,t}-\frac{t}{24}\right)=4$$

Result

$$\frac{q^4}{(q+1)^2\left(q^2+1\right)^2}+\frac{q}{(q+1)^2}+\sqrt{\pi}\;e^{\pi^2/(24\,t)-t/24}\sqrt{\frac{1}{t}}+1=4$$

Alternate forms

$$\frac{q^4}{\left(q^3+q^2+q+1\right)^2}+\frac{q}{(q+1)^2}+\sqrt{\pi}\;e^{((\pi-t)\,(t+\pi))/(24\,t)}\sqrt{\frac{1}{t}}+1=4$$

$$\frac{q^6+3\,q^5+4\,q^4+6\,q^3+3\,q^2+3\,q+1}{(q+1)^2\left(q^2+1\right)^2}+\sqrt{\pi}\;e^{\pi^2/(24\,t)-t/24}\sqrt{\frac{1}{t}}=4$$

$$\frac{1}{(q+1)^2\left(q^2+1\right)^2}\,e^{-t/24}\left(q^6\,e^{t/24}+\sqrt{\pi}\;q^6\,e^{\pi^2/(24\,t)}\sqrt{\frac{1}{t}}+3\,q^5\,e^{t/24}+2\sqrt{\pi}\;q^5\,e^{\pi^2/(24\,t)}\sqrt{\frac{1}{t}}+4\,q^4\,e^{t/24}+3\sqrt{\pi}\;q^4\,e^{\pi^2/(24\,t)}\sqrt{\frac{1}{t}}+6\,q^3\,e^{t/24}+4\sqrt{\pi}\;q^3\,e^{\pi^2/(24\,t)}\sqrt{\frac{1}{t}}+3\,q^2\,e^{t/24}+3\sqrt{\pi}\;q^2\,e^{\pi^2/(24\,t)}\sqrt{\frac{1}{t}}+3\,q\,e^{t/24}+2\sqrt{\pi}\;q\,e^{\pi^2/(24\,t)}\sqrt{\frac{1}{t}}+e^{t/24}+\sqrt{\pi}\;e^{\pi^2/(24\,t)}\sqrt{\frac{1}{t}}\right)=4$$

For q = 0.417613 and t = 0.873199 :

1+(0.417613)/((1+0.417613)^2) + (0.417613)^4/((1+0.417613)^2((1+0.417613^2)^2)) + sqrt(Pi/(0.873199)) * exp((Pi^2)/(24*(0.873199))-(0.873199)/(24))

Input interpretation

$$1+\frac{0.417613}{(1+0.417613)^2}+\frac{0.417613^4}{(1+0.417613)^2\left(1+0.417613^2\right)^2}+\sqrt{\frac{\pi}{0.873199}}\exp\left(\frac{\pi^2}{24\times 0.873199}-\frac{0.873199}{24}\right)$$

Result

4.14798…

4.14798…. → 4

Series representations

$$1+\frac{0.417613}{(1+0.417613)^2}+\frac{0.417613^4}{(1+0.417613)^2\left(1+0.417613^2\right)^2}+\sqrt{\frac{\pi}{0.873199}}\exp\left(\frac{\pi^2}{24\times 0.873199}-\frac{0.873199}{24}\right)=1.21878+\exp\left(-0.0363833+0.0477173\,\pi^2\right)\sqrt{-1+1.14521\,\pi}\sum_{k=0}^{\infty}(-1+1.14521\,\pi)^{-k}\binom{\frac{1}{2}}{k}$$

$$1+\frac{0.417613}{(1+0.417613)^2}+\frac{0.417613^4}{(1+0.417613)^2\left(1+0.417613^2\right)^2}+\sqrt{\frac{\pi}{0.873199}}\exp\left(\frac{\pi^2}{24\times 0.873199}-\frac{0.873199}{24}\right)=$$
$$1.21878+\exp\left(-0.0363833+0.0477173\,\pi^2\right)\sqrt{-1+1.14521\,\pi}\sum_{k=0}^{\infty}\frac{(-1)^k\,(-1+1.14521\,\pi)^{-k}\left(-\frac{1}{2}\right)_k}{k!}$$

$$1+\frac{0.417613}{(1+0.417613)^2}+\frac{0.417613^4}{(1+0.417613)^2\left(1+0.417613^2\right)^2}+\sqrt{\frac{\pi}{0.873199}}\exp\left(\frac{\pi^2}{24\times 0.873199}-\frac{0.873199}{24}\right)=1.21878+$$
$$\exp\left(-0.0363833+0.0477173\,\pi^2\right)\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(1.14521\,\pi-z_0)^k\,z_0^{-k}}{k!}$$

for (not ($z_0\in\mathbb{R}$ and $-\infty<z_0\le 0$))

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

Now we analyze again

$$1+\frac{q}{(1+q)^2}+\frac{q^4}{(1+q)^2\left(1+q^2\right)^2}+\sqrt{\frac{\pi}{t}}\exp\left(\frac{\pi^2}{24\,t}-\frac{t}{24}\right)=4$$

omitting the result. We obtain:

1+(q)/((1+q)^2) + (q)^4/((1+q)^2((1+q^2)^2)) + sqrt(Pi/(t)) * exp((Pi^2)/(24*(t))-(t)/(24))

Input

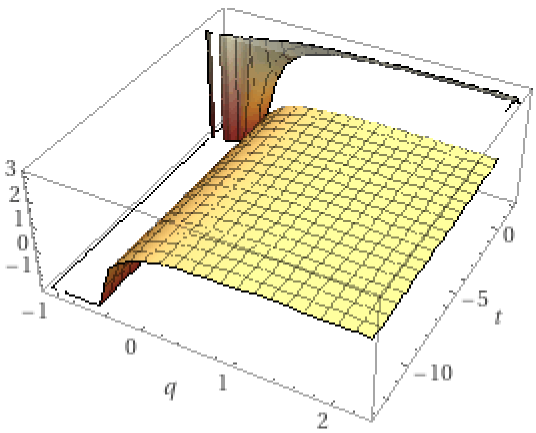
$$1+\frac{q}{(1+q)^2}+\frac{q^4}{(1+q)^2\left(1+q^2\right)^2}+\sqrt{\frac{\pi}{t}}\exp\left(\frac{\pi^2}{24\,t}-\frac{t}{24}\right)$$

Exact result

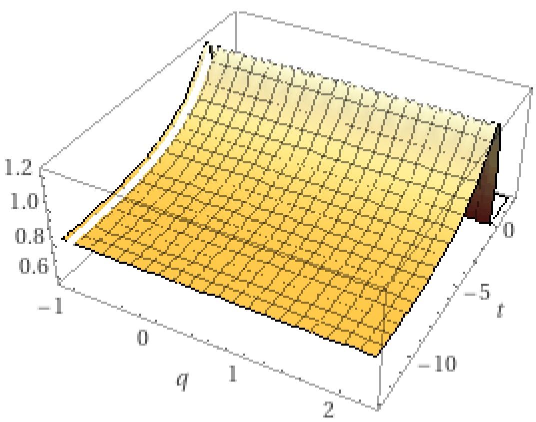
$$\frac{q^4}{(q+1)^2\left(q^2+1\right)^2}+\frac{q}{(q+1)^2}+\sqrt{\pi}\,e^{\pi^2/(24\,t)-t/24}\sqrt{\frac{1}{t}}+1$$

3D plots

Real part **(figures that can be related to the D-branes/Instantons)**

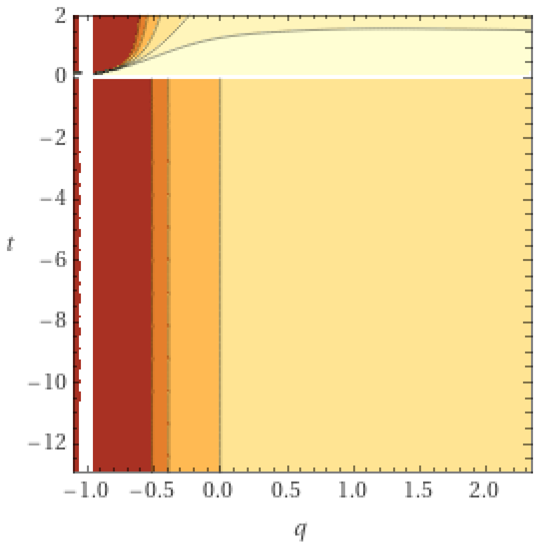


Imaginary part

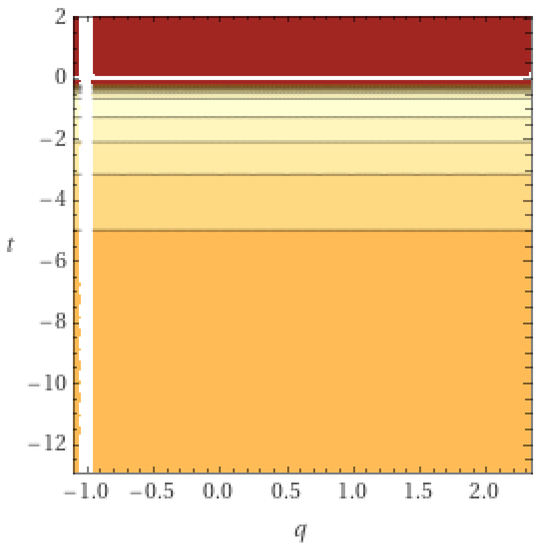


Contour plots

Real part



Imaginary part



Alternate forms

$$\frac{q^4}{(q^3+q^2+q+1)^2}+\frac{q}{(q+1)^2}+\sqrt{\pi}\, e^{((\pi-t)(t+\pi))/(24t)}\sqrt{\frac{1}{t}}+1$$

$$\frac{q^6+3q^5+4q^4+6q^3+3q^2+3q+1}{(q+1)^2\left(q^2+1\right)^2}+\sqrt{\pi}\, e^{\pi^2/(24t)-t/24}\sqrt{\frac{1}{t}}$$

$$\frac{1}{(q+1)^2\left(q^2+1\right)^2}\left(q^6\left(\sqrt{\pi}\, e^{\pi^2/(24t)-t/24}\sqrt{\frac{1}{t}}+1\right)+\right.$$
$$q^5\left(2\sqrt{\pi}\, e^{\pi^2/(24t)-t/24}\sqrt{\frac{1}{t}}+3\right)+q^4\left(3\sqrt{\pi}\, e^{\pi^2/(24t)-t/24}\sqrt{\frac{1}{t}}+4\right)+$$
$$q^3\left(4\sqrt{\pi}\, e^{\pi^2/(24t)-t/24}\sqrt{\frac{1}{t}}+6\right)+q^2\left(3\sqrt{\pi}\, e^{\pi^2/(24t)-t/24}\sqrt{\frac{1}{t}}+3\right)+$$
$$\left.q\left(2\sqrt{\pi}\, e^{\pi^2/(24t)-t/24}\sqrt{\frac{1}{t}}+3\right)+\sqrt{\pi}\, e^{\pi^2/(24t)-t/24}\sqrt{\frac{1}{t}}+1\right)$$

Series expansion at t=0

$$e^{\left(\pi^2-t^2\right)/(24t)}\left(\frac{\sqrt{\pi}\sqrt{\frac{1}{t}}\sqrt{t}}{\sqrt{t}}+O\left(t^{11/2}\right)\right)+\frac{q^4}{(q+1)^2\left(q^2+1\right)^2}+\frac{q}{(q+1)^2}+1$$

Series expansion at t=∞

$$e^{\left(\pi^2-t^2\right)/(24t)}\left(\sqrt{\pi}\sqrt{\frac{1}{t}}+O\left(\left(\frac{1}{t}\right)^{11/2}\right)\right)+\frac{q^4}{(q+1)^2\left(q^2+1\right)^2}+\frac{q}{(q+1)^2}+1$$

Derivative

$$\frac{\partial}{\partial t}\left(1+\frac{q}{(1+q)^2}+\frac{q^4}{(1+q)^2\left(1+q^2\right)^2}+\sqrt{\frac{\pi}{t}}\exp\left(\frac{\pi^2}{24\,t}-\frac{t}{24}\right)\right)=$$
$$-\frac{1}{24}\sqrt{\pi}\, e^{\left(\pi^2-t^2\right)/(24t)}\left(\frac{1}{t}\right)^{5/2}\left(t\,(t+12)+\pi^2\right)$$

Indefinite integral

$$\int\left(1+\frac{q}{(1+q)^2}+\frac{q^4}{(1+q)^2\left(1+q^2\right)^2}+e^{\pi^2/(24t)-t/24}\sqrt{\pi}\sqrt{\frac{1}{t}}\right)dt =$$
$$-(-1)^{11/12}\sqrt{6}\,\pi\sqrt{\frac{1}{t}}\sqrt{t}\left(\operatorname{erf}\left(\frac{t-i\pi}{2\sqrt{6}\sqrt{t}}\right)+\sqrt[6]{-1}\operatorname{erf}\left(\frac{t+i\pi}{2\sqrt{6}\sqrt{t}}\right)-\sqrt[6]{-1}+1\right)+$$
$$\frac{q^4\,t}{(q+1)^2\left(q^2+1\right)^2}+\frac{q\,t}{(q+1)^2}+t+\text{constant}$$

$\operatorname{erf}(x)$ is the error function

Limit

$$\lim_{q\to-\infty}\left(1+\frac{q}{(1+q)^2}+\frac{q^4}{(1+q)^2\left(1+q^2\right)^2}+e^{\pi^2/(24t)-t/24}\sqrt{\pi}\sqrt{\frac{1}{t}}\right)=$$
$$\sqrt{\pi}\,e^{\pi^2/(24t)-t/24}\sqrt{\frac{1}{t}}+1\approx 1.77245\times 2.71828^{0.411234/t-0.0416667t}\sqrt{\frac{1}{t}}+1$$

$$\lim_{q\to\infty}\left(1+\frac{q}{(1+q)^2}+\frac{q^4}{(1+q)^2\left(1+q^2\right)^2}+e^{\pi^2/(24t)-t/24}\sqrt{\pi}\sqrt{\frac{1}{t}}\right)=$$
$$\sqrt{\pi}\,e^{\left(\pi^2-t^2\right)/(24t)}\sqrt{\frac{1}{t}}+1\approx 1.77245\sqrt{\frac{1}{t}}\,2.71828^{\left(0.0416667\left(9.8696-t^2\right)\right)/t}+1$$

$$\lim_{t\to\infty}\left(1+\frac{q}{(1+q)^2}+\frac{q^4}{(1+q)^2\left(1+q^2\right)^2}+e^{\pi^2/(24t)-t/24}\sqrt{\pi}\sqrt{\frac{1}{t}}\right)=$$
$$\frac{q^4}{(q+1)^2\left(q^2+1\right)^2}+\frac{q}{(q+1)^2}+1$$

We perform the following calculations:

((1+(q)/((1+q)^2) + (q)^4/((1+q)^2((1+q^2)^2)) + sqrt(Pi/(t)) * exp((Pi^2)/(24*(t))-(t)/(24))))dxdy

Indefinite integral

$$\int\int\left(1+\frac{q}{(1+q)^2}+\frac{q^4}{(1+q)^2\left(1+q^2\right)^2}+\sqrt{\frac{\pi}{t}}\ \exp\left(\frac{\pi^2}{24\,t}-\frac{t}{24}\right)\right)dx\,dy =$$
$$c_1\,x+c_2+\frac{q^4\,x\,y}{(q+1)^2\left(q^2+1\right)^2}+\frac{q\,x\,y}{(q+1)^2}+\sqrt{\pi}\ e^{\pi^2/(24\,t)-t/24}\,\sqrt{\frac{1}{t}}\ x\,y+x\,y$$

Definite integral over a disk of radius R

$$\iint_{x^2+y^2<R^2}\left(\frac{q^4}{(q+1)^2\left(q^2+1\right)^2}+\frac{q}{(q+1)^2}+\sqrt{\pi}\ e^{\pi^2/(24\,t)-t/24}\,\sqrt{\frac{1}{t}}\ +1\right)dy\,dx =$$
$$\pi\,R^2\left(\frac{q^4}{(q+1)^2\left(q^2+1\right)^2}+\frac{q}{(q+1)^2}+\sqrt{\pi}\ e^{\pi^2/(24\,t)-t/24}\,\sqrt{\frac{1}{t}}\ +1\right)$$

Definite integral over a square of edge length 2 L

$$\int_{-L}^{L}\int_{-L}^{L}\left(1+\frac{q}{(1+q)^2}+\frac{q^4}{(1+q)^2\left(1+q^2\right)^2}+e^{\pi^2/(24\,t)-t/24}\,\sqrt{\pi}\,\sqrt{\frac{1}{t}}\right)dx\,dy =$$
$$4\,L^2\left(\frac{q^4}{(q+1)^2\left(q^2+1\right)^2}+\frac{q}{(q+1)^2}+\sqrt{\pi}\ e^{\pi^2/(24\,t)-t/24}\,\sqrt{\frac{1}{t}}\ +1\right)$$

Dividing the two results

$$\iint_{x^2+y^2<R^2}\left(\frac{q^4}{(q+1)^2\left(q^2+1\right)^2}+\frac{q}{(q+1)^2}+\sqrt{\pi}\ e^{\pi^2/(24\,t)-t/24}\,\sqrt{\frac{1}{t}}\ +1\right)dy\,dx =$$
$$\pi\,R^2\left(\frac{q^4}{(q+1)^2\left(q^2+1\right)^2}+\frac{q}{(q+1)^2}+\sqrt{\pi}\ e^{\pi^2/(24\,t)-t/24}\,\sqrt{\frac{1}{t}}\ +1\right)$$

and

$$\int_{-L}^{L}\int_{-L}^{L}\left(1+\frac{q}{(1+q)^2}+\frac{q^4}{(1+q)^2\left(1+q^2\right)^2}+e^{\pi^2/(24t)-t/24}\sqrt{\pi}\sqrt{\frac{1}{t}}\right)dx\,dy = 4L^2\left(\frac{q^4}{(q+1)^2\left(q^2+1\right)^2}+\frac{q}{(q+1)^2}+\sqrt{\pi}\,e^{\pi^2/(24t)-t/24}\sqrt{\frac{1}{t}}+1\right)$$

by

$$\frac{q^4}{(q+1)^2\left(q^2+1\right)^2}+\frac{q}{(q+1)^2}+\sqrt{\pi}\,e^{\pi^2/(24t)-t/24}\sqrt{\frac{1}{t}}+1$$

we obtain:

((π (q^4/((q + 1)^2 (q^2 + 1)^2) + q/(q + 1)^2 + sqrt(π) e^(π^2/(24 t) - t/24) sqrt(1/t) + 1))) / ((q^4/((q + 1)^2 (q^2 + 1)^2) + q/(q + 1)^2 + sqrt(π) e^(π^2/(24 t) - t/24) sqrt(1/t) + 1))

Input

$$\frac{\pi\left(\frac{q^4}{(q+1)^2\left(q^2+1\right)^2}+\frac{q}{(q+1)^2}+\sqrt{\pi}\,e^{\pi^2/(24t)-t/24}\sqrt{\frac{1}{t}}+1\right)}{\frac{q^4}{(q+1)^2\left(q^2+1\right)^2}+\frac{q}{(q+1)^2}+\sqrt{\pi}\,e^{\pi^2/(24t)-t/24}\sqrt{\frac{1}{t}}+1}$$

Exact result

π

Decimal approximation

3.1415926535897932384626433832795028841971693993751058209749445923

…

3.141592653…. = π

Property

π is a transcendental number

Alternative representations

$$\frac{\pi\left(\frac{q^4}{(q+1)^2\,(q^2+1)^2}+\frac{q}{(q+1)^2}+\sqrt{\pi}\;e^{\pi^2/(24t)-t/24}\sqrt{\frac{1}{t}}+1\right)}{\frac{q^4}{(q+1)^2\,(q^2+1)^2}+\frac{q}{(q+1)^2}+\sqrt{\pi}\;e^{\pi^2/(24t)-t/24}\sqrt{\frac{1}{t}}+1}=\pi$$

$$\frac{\pi\left(\frac{q^4}{(q+1)^2\,(q^2+1)^2}+\frac{q}{(q+1)^2}+\sqrt{\pi}\;e^{\pi^2/(24t)-t/24}\sqrt{\frac{1}{t}}+1\right)}{\frac{q^4}{(q+1)^2\,(q^2+1)^2}+\frac{q}{(q+1)^2}+\sqrt{\pi}\;e^{\pi^2/(24t)-t/24}\sqrt{\frac{1}{t}}+1}=\pi \quad \text{for } z=e$$

$$\frac{\pi\left(\frac{q^4}{(q+1)^2\,(q^2+1)^2}+\frac{q}{(q+1)^2}+\sqrt{\pi}\;e^{\pi^2/(24t)-t/24}\sqrt{\frac{1}{t}}+1\right)}{\frac{q^4}{(q+1)^2\,(q^2+1)^2}+\frac{q}{(q+1)^2}+\sqrt{\pi}\;e^{\pi^2/(24t)-t/24}\sqrt{\frac{1}{t}}+1}=\pi \quad \text{for } a=\frac{\pi^2-t^2}{24\,t\,\log(w)}$$

$$\frac{\pi\left(\frac{q^4}{(q+1)^2\,(q^2+1)^2}+\frac{q}{(q+1)^2}+\sqrt{\pi}\;e^{\pi^2/(24t)-t/24}\sqrt{\frac{1}{t}}+1\right)}{\frac{q^4}{(q+1)^2\,(q^2+1)^2}+\frac{q}{(q+1)^2}+\sqrt{\pi}\;e^{\pi^2/(24t)-t/24}\sqrt{\frac{1}{t}}+1}=\pi$$

$$\text{for } -12\pi\le \operatorname{Im}\left(\frac{\pi^2}{t}-t\right)\le 0$$

$\log(x)$ is the natural logarithm
$\operatorname{Im}(z)$ is the imaginary part of z

Series representations

$$\frac{\pi\left(\frac{q^4}{(q+1)^2(q^2+1)^2}+\frac{q}{(q+1)^2}+\sqrt{\pi}\,e^{\pi^2/(24t)-t/24}\sqrt{\frac{1}{t}}+1\right)}{\frac{q^4}{(q+1)^2(q^2+1)^2}+\frac{q}{(q+1)^2}+\sqrt{\pi}\,e^{\pi^2/(24t)-t/24}\sqrt{\frac{1}{t}}+1}=4\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}$$

$$\frac{\pi\left(\frac{q^4}{(q+1)^2(q^2+1)^2}+\frac{q}{(q+1)^2}+\sqrt{\pi}\,e^{\pi^2/(24t)-t/24}\sqrt{\frac{1}{t}}+1\right)}{\frac{q^4}{(q+1)^2(q^2+1)^2}+\frac{q}{(q+1)^2}+\sqrt{\pi}\,e^{\pi^2/(24t)-t/24}\sqrt{\frac{1}{t}}+1}=\sum_{k=0}^{\infty}-\frac{4(-1)^k\,1195^{-1-2k}\left(5^{1+2k}-4\times239^{1+2k}\right)}{1+2k}$$

$$\frac{\pi\left(\frac{q^4}{(q+1)^2(q^2+1)^2}+\frac{q}{(q+1)^2}+\sqrt{\pi}\,e^{\pi^2/(24t)-t/24}\sqrt{\frac{1}{t}}+1\right)}{\frac{q^4}{(q+1)^2(q^2+1)^2}+\frac{q}{(q+1)^2}+\sqrt{\pi}\,e^{\pi^2/(24t)-t/24}\sqrt{\frac{1}{t}}+1}=\sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k}+\frac{2}{1+4k}+\frac{1}{3+4k}\right)$$

1/6((((π (q^4/((q + 1)^2 (q^2 + 1)^2) + q/(q + 1)^2 + sqrt(π) e^(π^2/(24 t) - t/24) sqrt(1/t) + 1))) / ((q^4/((q + 1)^2 (q^2 + 1)^2) + q/(q + 1)^2 + sqrt(π) e^(π^2/(24 t) - t/24) sqrt(1/t) + 1))))^2

Input

$$\frac{1}{6}\left(\frac{\pi\left(\frac{q^4}{(q+1)^2\,(q^2+1)^2}+\frac{q}{(q+1)^2}+\sqrt{\pi}\;e^{\pi^2/(24\,t)-t/24}\sqrt{\frac{1}{t}}+1\right)}{\frac{q^4}{(q+1)^2\,(q^2+1)^2}+\frac{q}{(q+1)^2}+\sqrt{\pi}\;e^{\pi^2/(24\,t)-t/24}\sqrt{\frac{1}{t}}+1}\right)^2$$

Exact result

$$\frac{\pi^2}{6}$$

Decimal approximation

1.6449340668482264364724151666460251892189499012067984377355582293 …

1.644934066….

Property

$\frac{\pi^2}{6}$ is a transcendental number

Alternative representations

$$\frac{1}{6}\left(\frac{\pi\left(\frac{q^4}{(q+1)^2\,(q^2+1)^2}+\frac{q}{(q+1)^2}+\sqrt{\pi}\;e^{\pi^2/(24\,t)-t/24}\sqrt{\frac{1}{t}}+1\right)}{\frac{q^4}{(q+1)^2\,(q^2+1)^2}+\frac{q}{(q+1)^2}+\sqrt{\pi}\;e^{\pi^2/(24\,t)-t/24}\sqrt{\frac{1}{t}}+1}\right)^2=\frac{\pi^2}{6}$$

$$\frac{1}{6}\left(\frac{\pi\left(\frac{q^4}{(q+1)^2\,(q^2+1)^2}+\frac{q}{(q+1)^2}+\sqrt{\pi}\;e^{\pi^2/(24\,t)-t/24}\sqrt{\frac{1}{t}}+1\right)}{\frac{q^4}{(q+1)^2\,(q^2+1)^2}+\frac{q}{(q+1)^2}+\sqrt{\pi}\;e^{\pi^2/(24\,t)-t/24}\sqrt{\frac{1}{t}}+1}\right)^2=\frac{\pi^2}{6}\quad\text{for } z=e$$

$$\frac{1}{6}\left(\frac{\pi\left(\frac{q^4}{(q+1)^2 (q^2+1)^2}+\frac{q}{(q+1)^2}+\sqrt{\pi}\ e^{\pi^2/(24t)-t/24}\sqrt{\frac{1}{t}}+1\right)}{\frac{q^4}{(q+1)^2 (q^2+1)^2}+\frac{q}{(q+1)^2}+\sqrt{\pi}\ e^{\pi^2/(24t)-t/24}\sqrt{\frac{1}{t}}+1}\right)^2=\frac{\pi^2}{6}$$

for $a=\frac{\pi^2-t^2}{24\,t\log(w)}$

$$\frac{1}{6}\left(\frac{\pi\left(\frac{q^4}{(q+1)^2 (q^2+1)^2}+\frac{q}{(q+1)^2}+\sqrt{\pi}\ e^{\pi^2/(24t)-t/24}\sqrt{\frac{1}{t}}+1\right)}{\frac{q^4}{(q+1)^2 (q^2+1)^2}+\frac{q}{(q+1)^2}+\sqrt{\pi}\ e^{\pi^2/(24t)-t/24}\sqrt{\frac{1}{t}}+1}\right)^2=\frac{\pi^2}{6}$$

for $-12\pi\le\operatorname{Im}\left(\frac{\pi^2}{t}-t\right)\le 0$

$\log(x)$ is the natural logarithm
$\operatorname{Im}(z)$ is the imaginary part of z

Series representations

$$\frac{1}{6}\left(\frac{\pi\left(\frac{q^4}{(q+1)^2 (q^2+1)^2}+\frac{q}{(q+1)^2}+\sqrt{\pi}\ e^{\pi^2/(24t)-t/24}\sqrt{\frac{1}{t}}+1\right)}{\frac{q^4}{(q+1)^2 (q^2+1)^2}+\frac{q}{(q+1)^2}+\sqrt{\pi}\ e^{\pi^2/(24t)-t/24}\sqrt{\frac{1}{t}}+1}\right)^2=\sum_{k=1}^{\infty}\frac{1}{k^2}$$

$$\frac{1}{6}\left(\frac{\pi\left(\frac{q^4}{(q+1)^2 (q^2+1)^2}+\frac{q}{(q+1)^2}+\sqrt{\pi}\ e^{\pi^2/(24t)-t/24}\sqrt{\frac{1}{t}}+1\right)}{\frac{q^4}{(q+1)^2 (q^2+1)^2}+\frac{q}{(q+1)^2}+\sqrt{\pi}\ e^{\pi^2/(24t)-t/24}\sqrt{\frac{1}{t}}+1}\right)^2=-2\sum_{k=1}^{\infty}\frac{(-1)^k}{k^2}$$

$$\frac{1}{6}\left(\frac{\pi\left(\frac{q^4}{(q+1)^2 (q^2+1)^2}+\frac{q}{(q+1)^2}+\sqrt{\pi}\ e^{\pi^2/(24t)-t/24}\sqrt{\frac{1}{t}}+1\right)}{\frac{q^4}{(q+1)^2 (q^2+1)^2}+\frac{q}{(q+1)^2}+\sqrt{\pi}\ e^{\pi^2/(24t)-t/24}\sqrt{\frac{1}{t}}+1}\right)^2=\frac{4}{3}\sum_{k=0}^{\infty}\frac{1}{(1+2k)^2}$$

Integral representations

$$\frac{1}{6}\left(\frac{\pi\left(\frac{q^4}{(q+1)^2\,(q^2+1)^2}+\frac{q}{(q+1)^2}+\sqrt{\pi}\;e^{\pi^2/(24\,t)-t/24}\sqrt{\frac{1}{t}}+1\right)}{\frac{q^4}{(q+1)^2\,(q^2+1)^2}+\frac{q}{(q+1)^2}+\sqrt{\pi}\;e^{\pi^2/(24\,t)-t/24}\sqrt{\frac{1}{t}}+1}\right)^2=\frac{8}{3}\left(\int_0^1\sqrt{1-t^2}\,dt\right)^2$$

$$\frac{1}{6}\left(\frac{\pi\left(\frac{q^4}{(q+1)^2\,(q^2+1)^2}+\frac{q}{(q+1)^2}+\sqrt{\pi}\;e^{\pi^2/(24\,t)-t/24}\sqrt{\frac{1}{t}}+1\right)}{\frac{q^4}{(q+1)^2\,(q^2+1)^2}+\frac{q}{(q+1)^2}+\sqrt{\pi}\;e^{\pi^2/(24\,t)-t/24}\sqrt{\frac{1}{t}}+1}\right)^2=\frac{2}{3}\left(\int_0^\infty\frac{1}{1+t^2}\,dt\right)^2$$

$$\frac{1}{6}\left(\frac{\pi\left(\frac{q^4}{(q+1)^2\,(q^2+1)^2}+\frac{q}{(q+1)^2}+\sqrt{\pi}\;e^{\pi^2/(24\,t)-t/24}\sqrt{\frac{1}{t}}+1\right)}{\frac{q^4}{(q+1)^2\,(q^2+1)^2}+\frac{q}{(q+1)^2}+\sqrt{\pi}\;e^{\pi^2/(24\,t)-t/24}\sqrt{\frac{1}{t}}+1}\right)^2=\frac{2}{3}\left(\int_0^1\frac{1}{\sqrt{1-t^2}}\,dt\right)^2$$

We obtain also:

(((((4(q^4/((q + 1)^2 (q^2 + 1)^2) + q/(q + 1)^2 + sqrt(π) e^(π^2/(24 t) - t/24) sqrt(1/t) + 1))) / ((q^4/((q + 1)^2 (q^2 + 1)^2) + q/(q + 1)^2 + sqrt(π) e^(π^2/(24 t) - t/24) sqrt(1/t) + 1)))))^6

Input

$$\left(\frac{4\left(\frac{q^4}{(q+1)^2\,(q^2+1)^2}+\frac{q}{(q+1)^2}+\sqrt{\pi}\;e^{\pi^2/(24\,t)-t/24}\sqrt{\frac{1}{t}}+1\right)}{\frac{q^4}{(q+1)^2\,(q^2+1)^2}+\frac{q}{(q+1)^2}+\sqrt{\pi}\;e^{\pi^2/(24\,t)-t/24}\sqrt{\frac{1}{t}}+1}\right)^6$$

Result

4096

$4096 = 64^2$

27*√((((((4(q^4/((q+1)^2 (q^2+1)^2) + q/(q+1)^2 + sqrt(π) e^(π^2/(24 t) - t/24) sqrt(1/t) + 1))) / ((q^4/(((q + 1)^2 (q^2 + 1)^2) + q/(q + 1)^2+sqrt(π) e^(π^2/(24 t)-t/24) sqrt(1/t)+1)))))^6)+1

Input

$$27 \sqrt{\left(\frac{4\left(\frac{q^4}{(q+1)^2 (q^2+1)^2} + \frac{q}{(q+1)^2} + \sqrt{\pi}\, e^{\pi^2/(24 t) - t/24} \sqrt{\frac{1}{t}} + 1\right)}{\frac{q^4}{(q+1)^2 (q^2+1)^2} + \frac{q}{(q+1)^2} + \sqrt{\pi}\, e^{\pi^2/(24 t) - t/24} \sqrt{\frac{1}{t}} + 1}\right)^6} + 1$$

Result

1729

1729

This result is very near to the mass of candidate glueball $f_0(1710)$ scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. ($1728 = 8^2 * 3^3$) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Alternative representations

$$27 \sqrt{\left(\frac{4\left(\frac{q^4}{(q+1)^2 (q^2+1)^2} + \frac{q}{(q+1)^2} + \sqrt{\pi}\, e^{\pi^2/(24 t) - t/24} \sqrt{\frac{1}{t}} + 1\right)}{\frac{q^4}{(q+1)^2 (q^2+1)^2} + \frac{q}{(q+1)^2} + \sqrt{\pi}\, e^{\pi^2/(24 t) - t/24} \sqrt{\frac{1}{t}} + 1}\right)^6} + 1 = 1 + 27 \sqrt{4^6}$$

$$27 \sqrt{\left(\frac{4\left(\frac{q^4}{(q+1)^2 (q^2+1)^2} + \frac{q}{(q+1)^2} + \sqrt{\pi}\, e^{\pi^2/(24 t) - t/24} \sqrt{\frac{1}{t}} + 1\right)}{\frac{q^4}{(q+1)^2 (q^2+1)^2} + \frac{q}{(q+1)^2} + \sqrt{\pi}\, e^{\pi^2/(24 t) - t/24} \sqrt{\frac{1}{t}} + 1}\right)^6} + 1 = 27 \sqrt{4^6} + 1$$

for $z = e$

$$27\sqrt{\left(\frac{4\left(\frac{q^4}{(q+1)^2(q^2+1)^2}+\frac{q}{(q+1)^2}+\sqrt{\pi}\ e^{\pi^2/(24t)-t/24}\sqrt{\frac{1}{t}}+1\right)}{\frac{q^4}{(q+1)^2(q^2+1)^2}+\frac{q}{(q+1)^2}+\sqrt{\pi}\ e^{\pi^2/(24t)-t/24}\sqrt{\frac{1}{t}}+1}\right)^6}+1=27\sqrt{4^6}+1$$

for $a=\frac{\pi^2-t^2}{24\,t\log(w)}$

$$27\sqrt{\left(\frac{4\left(\frac{q^4}{(q+1)^2(q^2+1)^2}+\frac{q}{(q+1)^2}+\sqrt{\pi}\ e^{\pi^2/(24t)-t/24}\sqrt{\frac{1}{t}}+1\right)}{\frac{q^4}{(q+1)^2(q^2+1)^2}+\frac{q}{(q+1)^2}+\sqrt{\pi}\ e^{\pi^2/(24t)-t/24}\sqrt{\frac{1}{t}}+1}\right)^6}+1=27\sqrt{4^6}+1$$

for $-12\pi\leq\operatorname{Im}\left(\frac{\pi^2}{t}-t\right)\leq 0$

log(x) is the natural logarithm

Series representations

$$27\sqrt{\left(\frac{4\left(\frac{q^4}{(q+1)^2(q^2+1)^2}+\frac{q}{(q+1)^2}+\sqrt{\pi}\ e^{\pi^2/(24t)-t/24}\sqrt{\frac{1}{t}}+1\right)}{\frac{q^4}{(q+1)^2(q^2+1)^2}+\frac{q}{(q+1)^2}+\sqrt{\pi}\ e^{\pi^2/(24t)-t/24}\sqrt{\frac{1}{t}}+1}\right)^6}+1=$$
$$1+27\sqrt{4095}\sum_{k=0}^{\infty}4095^{-k}\binom{\frac{1}{2}}{k}$$

$$27\sqrt{\left(\frac{4\left(\frac{q^4}{(q+1)^2(q^2+1)^2}+\frac{q}{(q+1)^2}+\sqrt{\pi}\ e^{\pi^2/(24t)-t/24}\sqrt{\frac{1}{t}}+1\right)}{\frac{q^4}{(q+1)^2(q^2+1)^2}+\frac{q}{(q+1)^2}+\sqrt{\pi}\ e^{\pi^2/(24t)-t/24}\sqrt{\frac{1}{t}}+1}\right)^6}+1=$$
$$1+27\sqrt{4095}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4095}\right)^k\left(-\frac{1}{2}\right)_k}{k!}$$

$$27\sqrt{\left(\frac{4\left(\frac{q^4}{(q+1)^2(q^2+1)^2}+\frac{q}{(q+1)^2}+\sqrt{\pi}\, e^{\pi^2/(24t)-t/24}\sqrt{\frac{1}{t}}+1\right)}{\frac{q^4}{(q+1)^2(q^2+1)^2}+\frac{q}{(q+1)^2}+\sqrt{\pi}\, e^{\pi^2/(24t)-t/24}\sqrt{\frac{1}{t}}+1}\right)^6+1} =$$

$$1+\frac{27\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j} 4095^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\Gamma(x)$ is the gamma function

$\operatorname{Res}_{z=z_0} f$ is a complex residue

From:

Harmonic Maass forms and mock modular forms : theory and applications – *Kathrin Bringmann, Amanda Folsom, Ken Ono, Larry Rolen.* -

We have:

$$q = e^{2\pi i\tau} \text{ with } \tau \in \mathbb{H}.$$

for τ = 4 + i :

e^(2Pi*i*(4+i))

Input

$e^{2\pi i(4+i)}$

i is the imaginary unit

Exact result

$e^{-2\pi}$

Decimal approximation

0.0018674427317079888144302129348270303934228050024753171993815386

…

(using the principal branch of the logarithm for complex exponentiation)

0.0018674427….

Property

$e^{-2\pi}$ is a transcendental number

Alternative representations

$$e^{2\pi i(4+i)} = e^{360\,°\, i(4+i)}$$

$$e^{2\pi i(4+i)} = e^{-2 i^2 (4+i) \log(-1)}$$

$$e^{2\pi i(4+i)} = \exp^{2\pi i(4+i)}(z) \quad \text{for } z = 1$$

$\log(x)$ is the natural logarithm

Series representations

$$e^{2\pi i(4+i)} = e^{-8 \sum_{k=0}^{\infty} (-1)^k/(1+2k)}$$

$$e^{2\pi i(4+i)} = \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{-2\pi}$$

$$e^{2\pi i(4+i)} = \left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}}\right)^{-2\pi}$$

$n!$ is the factorial function

Integral representations

$$e^{2\pi i(4+i)} = e^{-8\int_0^1 \sqrt{1-t^2}\, dt}$$

$$e^{2\pi i(4+i)} = e^{-4\times\int_0^1 1/\sqrt{1-t^2}\, dt}$$

$$e^{2\pi i(4+i)} = e^{-4\times\int_0^\infty 1/(1+t^2) dt}$$

We have:

A.4. Order 6 mock theta functions

From:

$$\begin{aligned}
\mu(q) &:= \frac{1}{2} + \frac{1}{2}\sum_{n=0}^{\infty} \frac{(-1)^n q^{n+1}\left(1+q^n\right)\left(q;q^2\right)_n}{(-q;q)_{n+1}} \\
&= -2q^{\frac{1}{2}} g_2\left(q^{\frac{1}{2}};q^3\right) + \frac{(q;q)_\infty^3 \left(q^{\frac{3}{2}};q^{\frac{3}{2}}\right)_\infty^2 \left(q^6;q^6\right)_\infty^3}{\left(q^{\frac{1}{2}};q^{\frac{1}{2}}\right)_\infty^2 \left(q^3;q^3\right)_\infty^3 \left(q^4;q^4\right)_\infty \left(q^{12};q^{12}\right)_\infty} \\
&\quad - \frac{1}{2}\frac{(q;q)_\infty^2 \left(q^3;q^3\right)_\infty^2}{\left(q^2;q^2\right)_\infty^2 \left(q^6;q^6\right)_\infty} \\
&= 2iq^{-\frac{1}{4}}\mu\left(2\tau+\frac{1}{2},\frac{1}{2};6\tau\right) - \frac{1}{2}q^{\frac{1}{12}}\frac{\eta(\tau)^2\eta(3\tau)^2}{\eta(2\tau)^2\eta(6\tau)} \\
&= \frac{q^{\frac{3}{4}}\eta(6\tau)}{2\eta(12\tau)^2}\Theta^{+}_{\left(\begin{smallmatrix}1&1\\1&0\end{smallmatrix}\right),\left(\begin{smallmatrix}0\\1\end{smallmatrix}\right),\left(\begin{smallmatrix}-1\\1\end{smallmatrix}\right)}\left(2\tau+\frac{1}{2},\tau+\frac{1}{2};6\tau\right) - \frac{1}{2}q^{\frac{1}{12}}\frac{\eta(\tau)^2\eta(3\tau)^2}{\eta(2\tau)^2\eta(6\tau)} \\
&= \frac{1}{2} + q - \frac{3}{2}q^2 + 2q^3 - 2q^4 + 3q^5 - \frac{11}{2}q^6 + 7q^7 - \frac{15}{2}q^8 + 11q^9 + \cdots,
\end{aligned}$$

for: $q = e^{-2\pi}$, from the last result

1/2 + q – 3/2 q^2 + 2q^3 – 2q^4 + 3q^5 – 11/2 q^6 + 7q^7 – 15/2 q^8 + 11q^9

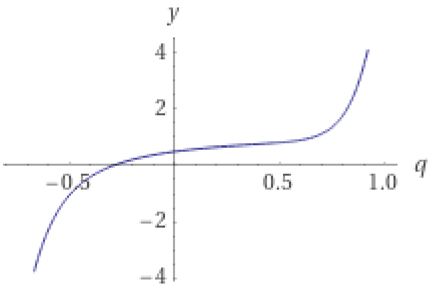
Input

$$\frac{1}{2} + q - \frac{3}{2}q^2 + 2q^3 - 2q^4 + 3q^5 - \frac{11}{2}q^6 + 7q^7 - \frac{15}{2}q^8 + 11q^9$$

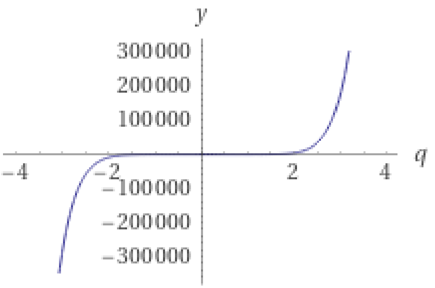
Result

$$11q^9 - \frac{15q^8}{2} + 7q^7 - \frac{11q^6}{2} + 3q^5 - 2q^4 + 2q^3 - \frac{3q^2}{2} + q + \frac{1}{2}$$

Plots **(figures that can be related to the open strings)**



(q from −0.8 to 1)



(q from −4.1 to 4.1)

Alternate forms

$$\frac{1}{2}\left(22\,q^9 - 15\,q^8 + 14\,q^7 - 11\,q^6 + 6\,q^5 - 4\,q^4 + 4\,q^3 - 3\,q^2 + 2\,q + 1\right)$$

$$\frac{1}{2}\,(q\,(q\,(q\,(q\,(q\,(q\,(q\,(q\,(22\,q - 15) + 14) - 11) + 6) - 4) + 4) - 3) + 2) + 1)$$

Real root

$q \approx -0.293474$

Complex roots

$q \approx -0.55490 - 0.50425\,i$

$q \approx -0.55490 + 0.50425\,i$

$q \approx -0.09938 - 0.84472\,i$

$q \approx -0.09938 + 0.84472\,i$

$q \approx 0.38775 - 0.66529\,i$

Polynomial discriminant

$\Delta = 24\,236\,294\,784\,979\,394$

Derivative

$$\frac{d}{dq}\left(\frac{1}{2} + q - \frac{3q^2}{2} + 2q^3 - 2q^4 + 3q^5 - \frac{11q^6}{2} + 7q^7 - \frac{15q^8}{2} + 11q^9\right) = 99q^8 - 60q^7 + 49q^6 - 33q^5 + 15q^4 - 8q^3 + 6q^2 - 3q + 1$$

Indefinite integral

$$\int\left(\frac{1}{2} + q - \frac{3q^2}{2} + 2q^3 - 2q^4 + 3q^5 - \frac{11q^6}{2} + 7q^7 - \frac{15q^8}{2} + 11q^9\right)dq = \frac{11q^{10}}{10} - \frac{5q^9}{6} + \frac{7q^8}{8} - \frac{11q^7}{14} + \frac{q^6}{2} - \frac{2q^5}{5} + \frac{q^4}{2} - \frac{q^3}{2} + \frac{q^2}{2} + \frac{q}{2} + \text{constant}$$

From the result

$$11q^9 - \frac{15q^8}{2} + 7q^7 - \frac{11q^6}{2} + 3q^5 - 2q^4 + 2q^3 - \frac{3q^2}{2} + q + \frac{1}{2}$$

for $q = e^{-2\pi}$

11 (e^(-2Pi))^9 - (15 (e^(-2Pi))^8)/2 + 7 (e^(-2Pi))^7 - (11 (e^(-2Pi))^6)/2 + 3 (e^(-2Pi))^5 - 2 (e^(-2Pi))^4 + 2 (e^(-2Pi))^3 - (3 (e^(-2Pi))^2)/2 + (e^(-2Pi)) + 1/2

Input

$$11\left(e^{-2\pi}\right)^9-\frac{1}{2}\left(15\left(e^{-2\pi}\right)^8\right)+7\left(e^{-2\pi}\right)^7-\frac{1}{2}\left(11\left(e^{-2\pi}\right)^6\right)+3\left(e^{-2\pi}\right)^5-2\left(e^{-2\pi}\right)^4+2\left(e^{-2\pi}\right)^3-\frac{1}{2}\left(3\left(e^{-2\pi}\right)^2\right)+e^{-2\pi}+\frac{1}{2}$$

Exact result

$$\frac{1}{2}+11e^{-18\pi}-\frac{15e^{-16\pi}}{2}+7e^{-14\pi}-\frac{11e^{-12\pi}}{2}+3e^{-10\pi}-2e^{-8\pi}+2e^{-6\pi}-\frac{3e^{-4\pi}}{2}+e^{-2\pi}$$

Decimal approximation

0.5018622247187427343842112954034246017823330077432687527282233068
…

0.50186222471….

Property

$$\frac{1}{2}+11e^{-18\pi}-\frac{15e^{-16\pi}}{2}+7e^{-14\pi}-\frac{11e^{-12\pi}}{2}+3e^{-10\pi}-2e^{-8\pi}+2e^{-6\pi}-\frac{3e^{-4\pi}}{2}+e^{-2\pi}$$ is a transcendental number

Alternate forms

$$\frac{1}{2}\left(1+22e^{-18\pi}-15e^{-16\pi}+14e^{-14\pi}-11e^{-12\pi}+6e^{-10\pi}-4e^{-8\pi}+4e^{-6\pi}-3e^{-4\pi}+2e^{-2\pi}\right)$$

$$e^{-18\pi}\left(11+e^{10\pi}\left(-2-\sinh(2\pi)+4\sinh(4\pi)-6\sinh(6\pi)+8\sinh(8\pi)+5\cosh(2\pi)-7\cosh(4\pi)+8\cosh(6\pi)-7\cosh(8\pi)\right)\right)$$

$$\frac{1}{2}e^{-18\pi}\left(22-15e^{2\pi}+14e^{4\pi}-11e^{6\pi}+6e^{8\pi}-4e^{10\pi}+4e^{12\pi}-3e^{14\pi}+2e^{16\pi}+e^{18\pi}\right)$$

$\cosh(x)$ is the hyperbolic cosine function
$\sinh(x)$ is the hyperbolic sine function

Alternative representations

$$11\left(e^{-2\pi}\right)^9-\frac{15}{2}\left(e^{-2\pi}\right)^8+7\left(e^{-2\pi}\right)^7-\frac{11}{2}\left(e^{-2\pi}\right)^6+3\left(e^{-2\pi}\right)^5-2\left(e^{-2\pi}\right)^4+2\left(e^{-2\pi}\right)^3-\frac{3}{2}\left(e^{-2\pi}\right)^2+e^{-2\pi}+\frac{1}{2}=11\exp^{-2\cos^{-1}(-1)}(z)^9-\frac{15}{2}\exp^{-2\cos^{-1}(-1)}(z)^8+7\exp^{-2\cos^{-1}(-1)}(z)^7-\frac{11}{2}\exp^{-2\cos^{-1}(-1)}(z)^6+3\exp^{-2\cos^{-1}(-1)}(z)^5-2\exp^{-2\cos^{-1}(-1)}(z)^4+2\exp^{-2\cos^{-1}(-1)}(z)^3-\frac{3}{2}\exp^{-2\cos^{-1}(-1)}(z)^2+\exp^{-2\cos^{-1}(-1)}(z)+\frac{1}{2}\quad\text{for } z=1$$

$$11\left(e^{-2\pi}\right)^9-\frac{15}{2}\left(e^{-2\pi}\right)^8+7\left(e^{-2\pi}\right)^7-\frac{11}{2}\left(e^{-2\pi}\right)^6+3\left(e^{-2\pi}\right)^5-2\left(e^{-2\pi}\right)^4+2\left(e^{-2\pi}\right)^3-\frac{3}{2}\left(e^{-2\pi}\right)^2+e^{-2\pi}+\frac{1}{2}=11\exp^{-2\pi}(z)^9-\frac{15}{2}\exp^{-2\pi}(z)^8+7\exp^{-2\pi}(z)^7-\frac{11}{2}\exp^{-2\pi}(z)^6+3\exp^{-2\pi}(z)^5-2\exp^{-2\pi}(z)^4+2\exp^{-2\pi}(z)^3-\frac{3}{2}\exp^{-2\pi}(z)^2+\exp^{-2\pi}(z)+\frac{1}{2}\quad\text{for } z=1$$

$$11\left(e^{-2\pi}\right)^9-\frac{15}{2}\left(e^{-2\pi}\right)^8+7\left(e^{-2\pi}\right)^7-\frac{11}{2}\left(e^{-2\pi}\right)^6+3\left(e^{-2\pi}\right)^5-2\left(e^{-2\pi}\right)^4+2\left(e^{-2\pi}\right)^3-\frac{3}{2}\left(e^{-2\pi}\right)^2+e^{-2\pi}+\frac{1}{2}=\frac{1}{2}+e^{-360^\circ}-\frac{3}{2}\left(e^{-360^\circ}\right)^2+2\left(e^{-360^\circ}\right)^3-2\left(e^{-360^\circ}\right)^4+3\left(e^{-360^\circ}\right)^5-\frac{11}{2}\left(e^{-360^\circ}\right)^6+7\left(e^{-360^\circ}\right)^7-\frac{15}{2}\left(e^{-360^\circ}\right)^8+11\left(e^{-360^\circ}\right)^9$$

$\cos^{-1}(x)$ is the inverse cosine function

Series representations

$$11\left(e^{-2\pi}\right)^9-\frac{15}{2}\left(e^{-2\pi}\right)^8+7\left(e^{-2\pi}\right)^7-\frac{11}{2}\left(e^{-2\pi}\right)^6+3\left(e^{-2\pi}\right)^5-2\left(e^{-2\pi}\right)^4+2\left(e^{-2\pi}\right)^3-\frac{3}{2}\left(e^{-2\pi}\right)^2+e^{-2\pi}+\frac{1}{2}=\frac{1}{2}+11\,e^{-72\sum_{k=0}^{\infty}(-1)^k/(1+2k)}-\frac{15}{2}\,e^{-64\sum_{k=0}^{\infty}(-1)^k/(1+2k)}+7\,e^{-56\sum_{k=0}^{\infty}(-1)^k/(1+2k)}-\frac{11}{2}\,e^{-48\sum_{k=0}^{\infty}(-1)^k/(1+2k)}+3\,e^{-40\sum_{k=0}^{\infty}(-1)^k/(1+2k)}-2\,e^{-32\sum_{k=0}^{\infty}(-1)^k/(1+2k)}+2\,e^{-24\sum_{k=0}^{\infty}(-1)^k/(1+2k)}-\frac{3}{2}\,e^{-16\sum_{k=0}^{\infty}(-1)^k/(1+2k)}+e^{-8\sum_{k=0}^{\infty}(-1)^k/(1+2k)}$$

$$11\left(e^{-2\pi}\right)^9-\frac{15}{2}\left(e^{-2\pi}\right)^8+7\left(e^{-2\pi}\right)^7-\frac{11}{2}\left(e^{-2\pi}\right)^6+3\left(e^{-2\pi}\right)^5-2\left(e^{-2\pi}\right)^4+2\left(e^{-2\pi}\right)^3-\frac{3}{2}\left(e^{-2\pi}\right)^2+e^{-2\pi}+\frac{1}{2}=\frac{1}{2}\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{-18\pi}\left(22-15\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{2\pi}+14\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{4\pi}-11\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{6\pi}+6\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{8\pi}-4\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{10\pi}+4\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{12\pi}-3\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{14\pi}+2\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{16\pi}+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{18\pi}\right)$$

$$11\left(e^{-2\pi}\right)^9-\frac{15}{2}\left(e^{-2\pi}\right)^8+7\left(e^{-2\pi}\right)^7-\frac{11}{2}\left(e^{-2\pi}\right)^6+3\left(e^{-2\pi}\right)^5-2\left(e^{-2\pi}\right)^4+2\left(e^{-2\pi}\right)^3-\frac{3}{2}\left(e^{-2\pi}\right)^2+e^{-2\pi}+\frac{1}{2}=\frac{1}{2}\left(22-15\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{2\pi}+14\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{4\pi}-11\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{6\pi}+6\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{8\pi}-4\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{10\pi}+4\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{12\pi}-3\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{14\pi}+2\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{16\pi}+\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{18\pi}\right)\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{-18\pi}$$

$n!$ is the factorial function

Integral representations

$$11\left(e^{-2\pi}\right)^9-\frac{15}{2}\left(e^{-2\pi}\right)^8+7\left(e^{-2\pi}\right)^7-\frac{11}{2}\left(e^{-2\pi}\right)^6+3\left(e^{-2\pi}\right)^5-2\left(e^{-2\pi}\right)^4+2\left(e^{-2\pi}\right)^3-\frac{3}{2}\left(e^{-2\pi}\right)^2+e^{-2\pi}+\frac{1}{2}=$$
$$\frac{1}{2}e^{-36\int_0^\infty \sin(t)/t\,dt}\left(22-15e^{4\int_0^\infty \sin(t)/t\,dt}+14e^{8\int_0^\infty \sin(t)/t\,dt}-11e^{12\int_0^\infty \sin(t)/t\,dt}+6e^{16\int_0^\infty \sin(t)/t\,dt}-4e^{20\int_0^\infty \sin(t)/t\,dt}+4e^{24\int_0^\infty \sin(t)/t\,dt}-3e^{28\int_0^\infty \sin(t)/t\,dt}+2e^{32\int_0^\infty \sin(t)/t\,dt}+e^{36\int_0^\infty \sin(t)/t\,dt}\right)$$

$$11\left(e^{-2\pi}\right)^9-\frac{15}{2}\left(e^{-2\pi}\right)^8+7\left(e^{-2\pi}\right)^7-\frac{11}{2}\left(e^{-2\pi}\right)^6+3\left(e^{-2\pi}\right)^5-2\left(e^{-2\pi}\right)^4+2\left(e^{-2\pi}\right)^3-\frac{3}{2}\left(e^{-2\pi}\right)^2+e^{-2\pi}+\frac{1}{2}=$$
$$\frac{1}{2}e^{-36\times\int_0^\infty 1/(1+t^2)\,dt}\left(22-15e^{4\times\int_0^\infty 1/(1+t^2)\,dt}+14e^{8\times\int_0^\infty 1/(1+t^2)\,dt}-11e^{12\times\int_0^\infty 1/(1+t^2)\,dt}+6e^{16\times\int_0^\infty 1/(1+t^2)\,dt}-4e^{20\times\int_0^\infty 1/(1+t^2)\,dt}+4e^{24\times\int_0^\infty 1/(1+t^2)\,dt}-3e^{28\times\int_0^\infty 1/(1+t^2)\,dt}+2e^{32\times\int_0^\infty 1/(1+t^2)\,dt}+e^{36\times\int_0^\infty 1/(1+t^2)\,dt}\right)$$

$$11\left(e^{-2\pi}\right)^9-\frac{15}{2}\left(e^{-2\pi}\right)^8+7\left(e^{-2\pi}\right)^7-\frac{11}{2}\left(e^{-2\pi}\right)^6+3\left(e^{-2\pi}\right)^5-2\left(e^{-2\pi}\right)^4+2\left(e^{-2\pi}\right)^3-\frac{3}{2}\left(e^{-2\pi}\right)^2+e^{-2\pi}+\frac{1}{2}=\frac{1}{2}e^{-36\int_0^\infty \sin^2(t)/t^2\,dt}$$
$$\left(22-15e^{4\int_0^\infty \sin^2(t)/t^2\,dt}+14e^{8\int_0^\infty \sin^2(t)/t^2\,dt}-11e^{12\int_0^\infty \sin^2(t)/t^2\,dt}+6e^{16\int_0^\infty \sin^2(t)/t^2\,dt}-4e^{20\int_0^\infty \sin^2(t)/t^2\,dt}+4e^{24\int_0^\infty \sin^2(t)/t^2\,dt}-3e^{28\int_0^\infty \sin^2(t)/t^2\,dt}+2e^{32\int_0^\infty \sin^2(t)/t^2\,dt}+e^{36\int_0^\infty \sin^2(t)/t^2\,dt}\right)$$

From the result

$$11q^9-\frac{15q^8}{2}+7q^7-\frac{11q^6}{2}+3q^5-2q^4+2q^3-\frac{3q^2}{2}+q+\frac{1}{2}$$

we obtain also:

(11 q^9 - (15 q^8)/2 + 7 q^7 - (11 q^6)/2 + 3 q^5 - 2 q^4 + 2 q^3 - (3 q^2)/2 + q + 1/2)dxdy

Indefinite integral

$$\int\int\left(11 q^9 - \frac{15 q^8}{2} + 7 q^7 - \frac{11 q^6}{2} + 3 q^5 - 2 q^4 + 2 q^3 - \frac{3 q^2}{2} + q + \frac{1}{2}\right) dx\, dy = c_1 x + c_2 + 11 q^9 x y - \frac{15}{2} q^8 x y + 7 q^7 x y - \frac{11}{2} q^6 x y + 3 q^5 x y - 2 q^4 x y + 2 q^3 x y - \frac{3}{2} q^2 x y + q x y + \frac{x y}{2}$$

Definite integral over a disk of radius R

$$\iint_{x^2+y^2<R^2} \left(11 q^9 - \frac{15 q^8}{2} + 7 q^7 - \frac{11 q^6}{2} + 3 q^5 - 2 q^4 + 2 q^3 - \frac{3 q^2}{2} + q + \frac{1}{2}\right) dy\, dx = \pi\left(11 q^9 - \frac{15 q^8}{2} + 7 q^7 - \frac{11 q^6}{2} + 3 q^5 - 2 q^4 + 2 q^3 - \frac{3 q^2}{2} + q + \frac{1}{2}\right) R^2$$

Definite integral over a square of edge length 2 L

$$\int_{-L}^{L}\int_{-L}^{L}\left(\frac{1}{2} + q - \frac{3 q^2}{2} + 2 q^3 - 2 q^4 + 3 q^5 - \frac{11 q^6}{2} + 7 q^7 - \frac{15 q^8}{2} + 11 q^9\right) dx\, dy = 4 L^2\left(11 q^9 - \frac{15 q^8}{2} + 7 q^7 - \frac{11 q^6}{2} + 3 q^5 - 2 q^4 + 2 q^3 - \frac{3 q^2}{2} + q + \frac{1}{2}\right)$$

Dividing the results of the

$$\iint_{x^2+y^2<R^2} \left(11 q^9 - \frac{15 q^8}{2} + 7 q^7 - \frac{11 q^6}{2} + 3 q^5 - 2 q^4 + 2 q^3 - \frac{3 q^2}{2} + q + \frac{1}{2}\right) dy\, dx = \pi \left(11 q^9 - \frac{15 q^8}{2} + 7 q^7 - \frac{11 q^6}{2} + 3 q^5 - 2 q^4 + 2 q^3 - \frac{3 q^2}{2} + q + \frac{1}{2}\right) R^2$$

and

$$\int_{-L}^{L}\int_{-L}^{L}\left(\frac{1}{2} + q - \frac{3 q^2}{2} + 2 q^3 - 2 q^4 + 3 q^5 - \frac{11 q^6}{2} + 7 q^7 - \frac{15 q^8}{2} + 11 q^9\right) dx\, dy = 4 L^2 \left(11 q^9 - \frac{15 q^8}{2} + 7 q^7 - \frac{11 q^6}{2} + 3 q^5 - 2 q^4 + 2 q^3 - \frac{3 q^2}{2} + q + \frac{1}{2}\right)$$

by

$$11 q^9 - \frac{15 q^8}{2} + 7 q^7 - \frac{11 q^6}{2} + 3 q^5 - 2 q^4 + 2 q^3 - \frac{3 q^2}{2} + q + \frac{1}{2}$$

we obtain:

(π (11 q^9 - (15 q^8)/2 + 7 q^7 - (11 q^6)/2 + 3 q^5 - 2 q^4 + 2 q^3 - (3 q^2)/2 + q + 1/2)) / (11 q^9 - (15 q^8)/2 + 7 q^7 - (11 q^6)/2 + 3 q^5 - 2 q^4 + 2 q^3 - (3 q^2)/2 + q + 1/2)

Input

$$\frac{\pi \left(11 q^9 - \frac{1}{2}\left(15 q^8\right) + 7 q^7 - \frac{1}{2}\left(11 q^6\right) + 3 q^5 - 2 q^4 + 2 q^3 - \frac{1}{2}\left(3 q^2\right) + q + \frac{1}{2}\right)}{11 q^9 - \frac{1}{2}\left(15 q^8\right) + 7 q^7 - \frac{1}{2}\left(11 q^6\right) + 3 q^5 - 2 q^4 + 2 q^3 - \frac{1}{2}\left(3 q^2\right) + q + \frac{1}{2}}$$

Result

π

Decimal approximation

3.1415926535897932384626433832795028841971693993751058209749445923
…

3.141592653…. = π

Property

π is a transcendental number

Alternative representations

$$\frac{\pi\left(11 q^9-\frac{15 q^8}{2}+7 q^7-\frac{11 q^6}{2}+3 q^5-2 q^4+2 q^3-\frac{3 q^2}{2}+q+\frac{1}{2}\right)}{11 q^9-\frac{15 q^8}{2}+7 q^7-\frac{11 q^6}{2}+3 q^5-2 q^4+2 q^3-\frac{3 q^2}{2}+q+\frac{1}{2}}=180^{\circ}$$

$$\frac{\pi\left(11 q^9-\frac{15 q^8}{2}+7 q^7-\frac{11 q^6}{2}+3 q^5-2 q^4+2 q^3-\frac{3 q^2}{2}+q+\frac{1}{2}\right)}{11 q^9-\frac{15 q^8}{2}+7 q^7-\frac{11 q^6}{2}+3 q^5-2 q^4+2 q^3-\frac{3 q^2}{2}+q+\frac{1}{2}}=-i \log(-1)$$

$$\frac{\pi\left(11 q^9-\frac{15 q^8}{2}+7 q^7-\frac{11 q^6}{2}+3 q^5-2 q^4+2 q^3-\frac{3 q^2}{2}+q+\frac{1}{2}\right)}{11 q^9-\frac{15 q^8}{2}+7 q^7-\frac{11 q^6}{2}+3 q^5-2 q^4+2 q^3-\frac{3 q^2}{2}+q+\frac{1}{2}}=\cos^{-1}(-1)$$

$\log(x)$ is the natural logarithm
i is the imaginary unit
$\cos^{-1}(x)$ is the inverse cosine function

Series representations

$$\frac{\pi\left(11 q^9-\frac{15 q^8}{2}+7 q^7-\frac{11 q^6}{2}+3 q^5-2 q^4+2 q^3-\frac{3 q^2}{2}+q+\frac{1}{2}\right)}{11 q^9-\frac{15 q^8}{2}+7 q^7-\frac{11 q^6}{2}+3 q^5-2 q^4+2 q^3-\frac{3 q^2}{2}+q+\frac{1}{2}}=4 \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2 k}$$

$$\frac{\pi\left(11 q^9-\frac{15 q^8}{2}+7 q^7-\frac{11 q^6}{2}+3 q^5-2 q^4+2 q^3-\frac{3 q^2}{2}+q+\frac{1}{2}\right)}{11 q^9-\frac{15 q^8}{2}+7 q^7-\frac{11 q^6}{2}+3 q^5-2 q^4+2 q^3-\frac{3 q^2}{2}+q+\frac{1}{2}}=$$

$$\sum_{k=0}^{\infty}-\frac{4(-1)^k\, 1195^{-1-2k}\left(5^{1+2k}-4\times 239^{1+2k}\right)}{1+2k}$$

$$\frac{\pi\left(11 q^9-\frac{15 q^8}{2}+7 q^7-\frac{11 q^6}{2}+3 q^5-2 q^4+2 q^3-\frac{3 q^2}{2}+q+\frac{1}{2}\right)}{11 q^9-\frac{15 q^8}{2}+7 q^7-\frac{11 q^6}{2}+3 q^5-2 q^4+2 q^3-\frac{3 q^2}{2}+q+\frac{1}{2}}=$$

$$\sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k}+\frac{2}{1+4k}+\frac{1}{3+4k}\right)$$

1/6((π (11 q^9 - (15 q^8)/2 + 7 q^7 - (11 q^6)/2 + 3 q^5 - 2 q^4 + 2 q^3 - (3 q^2)/2 + q + 1/2)) / (11 q^9 - (15 q^8)/2 + 7 q^7 - (11 q^6)/2 + 3 q^5 - 2 q^4 + 2 q^3 - (3 q^2)/2 + q + 1/2))^2

Input

$$\frac{1}{6}\left(\frac{\pi\left(11 q^9-\frac{1}{2}\left(15 q^8\right)+7 q^7-\frac{1}{2}\left(11 q^6\right)+3 q^5-2 q^4+2 q^3-\frac{1}{2}\left(3 q^2\right)+q+\frac{1}{2}\right)}{11 q^9-\frac{1}{2}\left(15 q^8\right)+7 q^7-\frac{1}{2}\left(11 q^6\right)+3 q^5-2 q^4+2 q^3-\frac{1}{2}\left(3 q^2\right)+q+\frac{1}{2}}\right)^2$$

Result

$$\frac{\pi^2}{6}$$

Decimal approximation

1.6449340668482264364724151666460251892189499012067984377355582293

…

1.644934066….

Property

$\frac{\pi^2}{6}$ is a transcendental number

Alternative representations

$$\frac{1}{6}\left(\frac{\pi\left(11q^9-\frac{15q^8}{2}+7q^7-\frac{11q^6}{2}+3q^5-2q^4+2q^3-\frac{3q^2}{2}+q+\frac{1}{2}\right)}{11q^9-\frac{15q^8}{2}+7q^7-\frac{11q^6}{2}+3q^5-2q^4+2q^3-\frac{3q^2}{2}+q+\frac{1}{2}}\right)^2=\frac{1}{6}(180\,°)^2$$

$$\frac{1}{6}\left(\frac{\pi\left(11q^9-\frac{15q^8}{2}+7q^7-\frac{11q^6}{2}+3q^5-2q^4+2q^3-\frac{3q^2}{2}+q+\frac{1}{2}\right)}{11q^9-\frac{15q^8}{2}+7q^7-\frac{11q^6}{2}+3q^5-2q^4+2q^3-\frac{3q^2}{2}+q+\frac{1}{2}}\right)^2=\frac{1}{6}(-i\log(-1))^2$$

$$\frac{1}{6}\left(\frac{\pi\left(11q^9-\frac{15q^8}{2}+7q^7-\frac{11q^6}{2}+3q^5-2q^4+2q^3-\frac{3q^2}{2}+q+\frac{1}{2}\right)}{11q^9-\frac{15q^8}{2}+7q^7-\frac{11q^6}{2}+3q^5-2q^4+2q^3-\frac{3q^2}{2}+q+\frac{1}{2}}\right)^2=\frac{1}{6}\cos^{-1}(-1)^2$$

$\log(x)$ is the natural logarithm
i is the imaginary unit
$\cos^{-1}(x)$ is the inverse cosine function

Series representations

$$\frac{1}{6}\left(\frac{\pi\left(11q^9-\frac{15q^8}{2}+7q^7-\frac{11q^6}{2}+3q^5-2q^4+2q^3-\frac{3q^2}{2}+q+\frac{1}{2}\right)}{11q^9-\frac{15q^8}{2}+7q^7-\frac{11q^6}{2}+3q^5-2q^4+2q^3-\frac{3q^2}{2}+q+\frac{1}{2}}\right)^2=\sum_{k=1}^{\infty}\frac{1}{k^2}$$

$$\frac{1}{6}\left(\frac{\pi\left(11 q^9-\frac{15 q^8}{2}+7 q^7-\frac{11 q^6}{2}+3 q^5-2 q^4+2 q^3-\frac{3 q^2}{2}+q+\frac{1}{2}\right)}{11 q^9-\frac{15 q^8}{2}+7 q^7-\frac{11 q^6}{2}+3 q^5-2 q^4+2 q^3-\frac{3 q^2}{2}+q+\frac{1}{2}}\right)^2 = -2\sum_{k=1}^{\infty}\frac{(-1)^k}{k^2}$$

$$\frac{1}{6}\left(\frac{\pi\left(11 q^9-\frac{15 q^8}{2}+7 q^7-\frac{11 q^6}{2}+3 q^5-2 q^4+2 q^3-\frac{3 q^2}{2}+q+\frac{1}{2}\right)}{11 q^9-\frac{15 q^8}{2}+7 q^7-\frac{11 q^6}{2}+3 q^5-2 q^4+2 q^3-\frac{3 q^2}{2}+q+\frac{1}{2}}\right)^2 = \frac{4}{3}\sum_{k=0}^{\infty}\frac{1}{(1+2k)^2}$$

Integral representations

$$\frac{1}{6}\left(\frac{\pi\left(11 q^9-\frac{15 q^8}{2}+7 q^7-\frac{11 q^6}{2}+3 q^5-2 q^4+2 q^3-\frac{3 q^2}{2}+q+\frac{1}{2}\right)}{11 q^9-\frac{15 q^8}{2}+7 q^7-\frac{11 q^6}{2}+3 q^5-2 q^4+2 q^3-\frac{3 q^2}{2}+q+\frac{1}{2}}\right)^2 = \frac{8}{3}\left(\int_0^1\sqrt{1-t^2}\,dt\right)^2$$

$$\frac{1}{6}\left(\frac{\pi\left(11 q^9-\frac{15 q^8}{2}+7 q^7-\frac{11 q^6}{2}+3 q^5-2 q^4+2 q^3-\frac{3 q^2}{2}+q+\frac{1}{2}\right)}{11 q^9-\frac{15 q^8}{2}+7 q^7-\frac{11 q^6}{2}+3 q^5-2 q^4+2 q^3-\frac{3 q^2}{2}+q+\frac{1}{2}}\right)^2 = \frac{2}{3}\left(\int_0^{\infty}\frac{1}{1+t^2}\,dt\right)^2$$

$$\frac{1}{6}\left(\frac{\pi\left(11 q^9-\frac{15 q^8}{2}+7 q^7-\frac{11 q^6}{2}+3 q^5-2 q^4+2 q^3-\frac{3 q^2}{2}+q+\frac{1}{2}\right)}{11 q^9-\frac{15 q^8}{2}+7 q^7-\frac{11 q^6}{2}+3 q^5-2 q^4+2 q^3-\frac{3 q^2}{2}+q+\frac{1}{2}}\right)^2 = \frac{2}{3}\left(\int_0^1\frac{1}{\sqrt{1-t^2}}\,dt\right)^2$$

We obtain also:

(((4 (11 q^9 - (15 q^8)/2 + 7 q^7 - (11 q^6)/2 + 3 q^5 - 2 q^4 + 2 q^3 - (3 q^2)/2 + q + 1/2)) / (11 q^9 - (15 q^8)/2 + 7 q^7 - (11 q^6)/2 + 3 q^5 - 2 q^4 + 2 q^3 - (3 q^2)/2 + q + 1/2)))^6

Input

$$\left(\frac{4\left(11 q^9-\frac{1}{2}\left(15 q^8\right)+7 q^7-\frac{1}{2}\left(11 q^6\right)+3 q^5-2 q^4+2 q^3-\frac{1}{2}\left(3 q^2\right)+q+\frac{1}{2}\right)}{11 q^9-\frac{1}{2}\left(15 q^8\right)+7 q^7-\frac{1}{2}\left(11 q^6\right)+3 q^5-2 q^4+2 q^3-\frac{1}{2}\left(3 q^2\right)+q+\frac{1}{2}}\right)^6$$

Exact result

4096

$4096 = 64^2$

27(((4 (11 q^9 - (15 q^8)/2 + 7 q^7 - (11 q^6)/2 + 3 q^5 - 2 q^4 + 2 q^3 - (3 q^2)/2 + q + 1/2)) / (11 q^9 - (15 q^8)/2 + 7 q^7 - (11 q^6)/2 + 3 q^5 - 2 q^4 + 2 q^3 - (3 q^2)/2 + q + 1/2)))^3+1

Input

$$27\left(\left(4\left(11 q^9-\frac{1}{2}\left(15 q^8\right)+7 q^7-\frac{1}{2}\left(11 q^6\right)+3 q^5-2 q^4+2 q^3-\frac{1}{2}\left(3 q^2\right)+q+\frac{1}{2}\right)\right)\Big/\left(11 q^9-\frac{1}{2}\left(15 q^8\right)+7 q^7-\frac{1}{2}\left(11 q^6\right)+3 q^5-2 q^4+2 q^3-\frac{1}{2}\left(3 q^2\right)+q+\frac{1}{2}\right)\right)^3+1$$

Exact result

1729

1729

This result is very near to the mass of candidate glueball $f_0(1710)$ scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. ($1728 = 8^2 * 3^3$) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

From

(**Ramanujan's Last Letter to Hardy** – *Srinivasa Ramanujan* - 1920):

Mock ϑ-functions (of 5th order)

$$F_1(q) = \frac{1}{1-q} + \frac{q^4}{(1-q)(1-q^3)} + \frac{q^{12}}{(1-q)(1-q^3)(1-q^5)} + \cdots$$

1/(1-(e^(-2Pi)))+((e^(-2Pi))^4)/((((1-(e^(-2Pi))))(1-(e^(-2Pi))^3)))+((e^(-2Pi))^12)/((((1-(e^(-2Pi))))(1-(e^(-2Pi))^3)(1-(e^(-2Pi))^5)))

Input

$$\frac{1}{1-e^{-2\pi}} + \frac{\left(e^{-2\pi}\right)^4}{\left(1-e^{-2\pi}\right)\left(1-\left(e^{-2\pi}\right)^3\right)} + \frac{\left(e^{-2\pi}\right)^{12}}{\left(1-e^{-2\pi}\right)\left(1-\left(e^{-2\pi}\right)^3\right)\left(1-\left(e^{-2\pi}\right)^5\right)}$$

Exact result

$$\frac{1}{1-e^{-2\pi}} + \frac{e^{-8\pi}}{\left(1-e^{-6\pi}\right)\left(1-e^{-2\pi}\right)} + \frac{e^{-24\pi}}{\left(1-e^{-10\pi}\right)\left(1-e^{-6\pi}\right)\left(1-e^{-2\pi}\right)}$$

Decimal approximation

1.0018709366108449543910792249858779544360878172458310798751983079

…

1.00187093661….

Property

$$\frac{1}{1-e^{-2\pi}}+\frac{e^{-8\pi}}{\left(1-e^{-6\pi}\right)\left(1-e^{-2\pi}\right)}+\frac{e^{-24\pi}}{\left(1-e^{-10\pi}\right)\left(1-e^{-6\pi}\right)\left(1-e^{-2\pi}\right)}$$
is a transcendental number

Alternate forms

$$\frac{1}{2}(1+\coth(\pi))\left(1+\frac{1}{e^{8\pi}-e^{14\pi}-e^{18\pi}+e^{24\pi}}+\frac{1}{2}e^{-5\pi}\operatorname{csch}(3\pi)\right)$$

$$\frac{1+\frac{e^{-2\pi}}{e^{6\pi}-1}+\frac{1}{e^{8\pi}-e^{14\pi}-e^{18\pi}+e^{24\pi}}}{1-e^{-2\pi}}$$

$$\frac{-1-e^{-24\pi}-e^{-16\pi}+e^{-10\pi}+e^{-6\pi}+e^{-8\pi}\left(e^{-10\pi}-1\right)}{\left(e^{-10\pi}-1\right)\left(e^{-6\pi}-1\right)\left(e^{-2\pi}-1\right)}$$

$\coth(x)$ is the hyperbolic cotangent function
$\operatorname{csch}(x)$ is the hyperbolic cosecant function

Alternative representations

$$\frac{1}{1-e^{-2\pi}}+\frac{\left(e^{-2\pi}\right)^4}{\left(1-e^{-2\pi}\right)\left(1-\left(e^{-2\pi}\right)^3\right)}+\frac{\left(e^{-2\pi}\right)^{12}}{\left(1-e^{-2\pi}\right)\left(1-\left(e^{-2\pi}\right)^3\right)\left(1-\left(e^{-2\pi}\right)^5\right)}=\frac{1}{1-\exp^{-2\cos^{-1}(-1)}(z)}+\frac{\exp^{-2\cos^{-1}(-1)}(z)^4}{\left(1-\exp^{-2\cos^{-1}(-1)}(z)\right)\left(1-\exp^{-2\cos^{-1}(-1)}(z)^3\right)}+\frac{\exp^{-2\cos^{-1}(-1)}(z)^{12}}{\left(1-\exp^{-2\cos^{-1}(-1)}(z)\right)\left(1-\exp^{-2\cos^{-1}(-1)}(z)^3\right)\left(1-\exp^{-2\cos^{-1}(-1)}(z)^5\right)}$$ for $z=1$

$$\frac{1}{1-e^{-2\pi}}+\frac{(e^{-2\pi})^4}{(1-e^{-2\pi})(1-(e^{-2\pi})^3)}+\frac{(e^{-2\pi})^{12}}{(1-e^{-2\pi})(1-(e^{-2\pi})^3)(1-(e^{-2\pi})^5)}=$$
$$\frac{1}{1-\exp^{-2\pi}(z)}+\frac{\exp^{-2\pi}(z)^4}{(1-\exp^{-2\pi}(z))(1-\exp^{-2\pi}(z)^3)}+$$
$$\frac{\exp^{-2\pi}(z)^{12}}{(1-\exp^{-2\pi}(z))(1-\exp^{-2\pi}(z)^3)(1-\exp^{-2\pi}(z)^5)} \quad \text{for } z=1$$

$$\frac{1}{1-e^{-2\pi}}+\frac{(e^{-2\pi})^4}{(1-e^{-2\pi})(1-(e^{-2\pi})^3)}+\frac{(e^{-2\pi})^{12}}{(1-e^{-2\pi})(1-(e^{-2\pi})^3)(1-(e^{-2\pi})^5)}=$$
$$\frac{1}{1-e^{-360^\circ}}+\frac{(e^{-360^\circ})^4}{(1-e^{-360^\circ})(1-(e^{-360^\circ})^3)}+$$
$$\frac{(e^{-360^\circ})^{12}}{(1-e^{-360^\circ})(1-(e^{-360^\circ})^3)(1-(e^{-360^\circ})^5)}$$

$\cos^{-1}(x)$ is the inverse cosine function

Series representations

$$\frac{1}{1-e^{-2\pi}}+\frac{(e^{-2\pi})^4}{(1-e^{-2\pi})(1-(e^{-2\pi})^3)}+\frac{(e^{-2\pi})^{12}}{(1-e^{-2\pi})(1-(e^{-2\pi})^3)(1-(e^{-2\pi})^5)}=$$
$$\frac{1}{1-e^{-8\sum_{k=0}^{\infty}(-1)^k/(1+2k)}}+\frac{e^{-32\sum_{k=0}^{\infty}(-1)^k/(1+2k)}}{\left(1-e^{-24\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)\left(1-e^{-8\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)}+$$
$$\frac{e^{-96\sum_{k=0}^{\infty}(-1)^k/(1+2k)}}{\left(1-e^{-40\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)\left(1-e^{-24\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)\left(1-e^{-8\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)}$$

$$\frac{1}{1-e^{-2\pi}}+\frac{\left(e^{-2\pi}\right)^4}{\left(1-e^{-2\pi}\right)\left(1-\left(e^{-2\pi}\right)^3\right)}+\frac{\left(e^{-2\pi}\right)^{12}}{\left(1-e^{-2\pi}\right)\left(1-\left(e^{-2\pi}\right)^3\right)\left(1-\left(e^{-2\pi}\right)^5\right)}=$$

$$\frac{1}{1-\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{-8\sum_{k=0}^{\infty}(-1)^k/(1+2k)}}+$$

$$\frac{\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{-32\sum_{k=0}^{\infty}(-1)^k/(1+2k)}}{\left(1-\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{-24\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)\left(1-\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{-8\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)}+$$

$$\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{-96\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\Bigg/\left(\left(1-\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{-40\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)\left(1-\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{-24\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)\left(1-\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{-8\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)\right)$$

$$\frac{1}{1-e^{-2\pi}}+\frac{\left(e^{-2\pi}\right)^4}{\left(1-e^{-2\pi}\right)\left(1-\left(e^{-2\pi}\right)^3\right)}+\frac{\left(e^{-2\pi}\right)^{12}}{\left(1-e^{-2\pi}\right)\left(1-\left(e^{-2\pi}\right)^3\right)\left(1-\left(e^{-2\pi}\right)^5\right)}=$$

$$\frac{1}{1-\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{-8\sum_{k=0}^{\infty}(-1)^k/(1+2k)}}+$$

$$\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{-96\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\Bigg/\left(\left(1-\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{-40\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)\left(1-\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{-24\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)\left(1-\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{-8\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)\right)+$$

$$\frac{\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{-32\sum_{k=0}^{\infty}(-1)^k/(1+2k)}}{\left(1-\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{-24\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)\left(1-\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{-8\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)}$$

$n!$ is the factorial function

Integral representations

$$\frac{1}{1-e^{-2\pi}}+\frac{(e^{-2\pi})^4}{(1-e^{-2\pi})(1-(e^{-2\pi})^3)}+\frac{(e^{-2\pi})^{12}}{(1-e^{-2\pi})(1-(e^{-2\pi})^3)(1-(e^{-2\pi})^5)}=$$
$$\frac{1}{1-e^{-6\int_0^\infty \sin^4(t)/t^4\,dt}}+\frac{e^{-24\int_0^\infty \sin^4(t)/t^4\,dt}}{\left(1-e^{-18\int_0^\infty \sin^4(t)/t^4\,dt}\right)\left(1-e^{-6\int_0^\infty \sin^4(t)/t^4\,dt}\right)}+$$
$$\frac{e^{-72\int_0^\infty \sin^4(t)/t^4\,dt}}{\left(1-e^{-30\int_0^\infty \sin^4(t)/t^4\,dt}\right)\left(1-e^{-18\int_0^\infty \sin^4(t)/t^4\,dt}\right)\left(1-e^{-6\int_0^\infty \sin^4(t)/t^4\,dt}\right)}$$

$$\frac{1}{1-e^{-2\pi}}+\frac{(e^{-2\pi})^4}{(1-e^{-2\pi})(1-(e^{-2\pi})^3)}+\frac{(e^{-2\pi})^{12}}{(1-e^{-2\pi})(1-(e^{-2\pi})^3)(1-(e^{-2\pi})^5)}=$$
$$\frac{1}{1-e^{-8\int_0^1 \sqrt{1-t^2}\,dt}}+\frac{e^{-32\int_0^1 \sqrt{1-t^2}\,dt}}{\left(1-e^{-24\int_0^1 \sqrt{1-t^2}\,dt}\right)\left(1-e^{-8\int_0^1 \sqrt{1-t^2}\,dt}\right)}+$$
$$\frac{e^{-96\int_0^1 \sqrt{1-t^2}\,dt}}{\left(1-e^{-40\int_0^1 \sqrt{1-t^2}\,dt}\right)\left(1-e^{-24\int_0^1 \sqrt{1-t^2}\,dt}\right)\left(1-e^{-8\int_0^1 \sqrt{1-t^2}\,dt}\right)}$$

$$\frac{1}{1-e^{-2\pi}}+\frac{(e^{-2\pi})^4}{(1-e^{-2\pi})(1-(e^{-2\pi})^3)}+\frac{(e^{-2\pi})^{12}}{(1-e^{-2\pi})(1-(e^{-2\pi})^3)(1-(e^{-2\pi})^5)}=$$
$$\frac{1}{1-e^{-4\times\int_0^1 1/\sqrt{1-t^2}\,dt}}+\frac{e^{-16\times\int_0^1 1/\sqrt{1-t^2}\,dt}}{\left(1-e^{-12\times\int_0^1 1/\sqrt{1-t^2}\,dt}\right)\left(1-e^{-4\times\int_0^1 1/\sqrt{1-t^2}\,dt}\right)}+$$
$$\frac{e^{-48\times\int_0^1 1/\sqrt{1-t^2}\,dt}}{\left(1-e^{-20\times\int_0^1 1/\sqrt{1-t^2}\,dt}\right)\left(1-e^{-12\times\int_0^1 1/\sqrt{1-t^2}\,dt}\right)\left(1-e^{-4\times\int_0^1 1/\sqrt{1-t^2}\,dt}\right)}$$

We have:

Order 5 mock theta functions

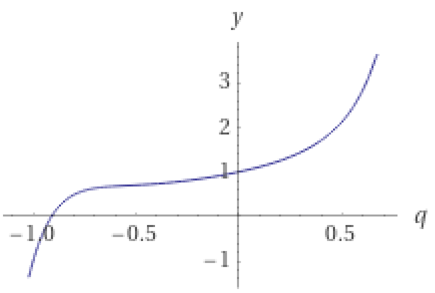
$$
\begin{aligned}
F_1(q) &:= \sum_{n=0}^{\infty} \frac{q^{2n(n+1)}}{(q;q^2)_{n+1}} \\
&= q g_3\left(q^2;q^5\right) + \frac{\left(-q^5;q^{20}\right)_\infty \left(-q^{15};q^{20}\right)_\infty \left(q^{20};q^{20}\right)_\infty}{\left(q^2;q^{10}\right)_\infty \left(q^8;q^{10}\right)_\infty} \\
&= -iq^{-\frac{5}{8}}\mu\left(3\tau,4\tau;15\tau\right) - iq^{-\frac{9}{8}}\mu\left(8\tau,4\tau;15\tau\right) - iq^{-\frac{19}{24}}\frac{\eta(10\tau)^3}{\eta(5\tau)\vartheta(2\tau;10\tau)} \\
&= \frac{q^{-\frac{71}{120}}}{2\eta(2\tau)}\Theta^{+}_{\left(\begin{smallmatrix}5 & 0\\ 0 & -2\end{smallmatrix}\right),\left(\begin{smallmatrix}2/5\\ 1/4\end{smallmatrix}\right),\left(\begin{smallmatrix}1/2\\ 2\end{smallmatrix}\right),\left(\begin{smallmatrix}2\\ 5\end{smallmatrix}\right),\left(\begin{smallmatrix}-2\\ 5\end{smallmatrix}\right)}(2\tau) \\
&= 1 + q + q^2 + q^3 + 2q^4 + 2q^5 + 2q^6 + 3q^7 + 3q^8 + 3q^9 + \cdots,
\end{aligned}
$$

(1 + q + q^2 + q^3 + 2q^4 + 2q^5 + 2q^6 + 3q^7 + 3q^8 + 3q^9)

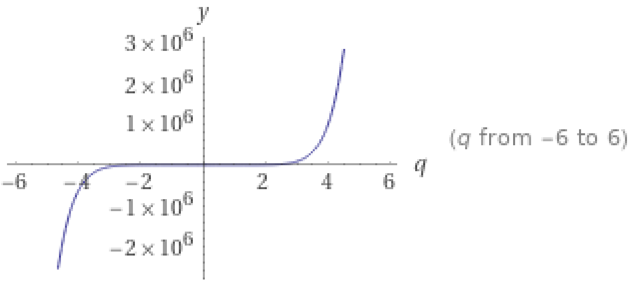
Input

$1 + q + q^2 + q^3 + 2q^4 + 2q^5 + 2q^6 + 3q^7 + 3q^8 + 3q^9$

Plots **(figures that can be related to the open strings)**



(q from −1.1 to 0.7)



Alternate form
$q(q^2+q+1)(3q^6+2q^3+1)+1$

Real root
$q \approx -0.910054$

Complex roots

$q \approx -0.64441 - 0.55876\,i$

$q \approx -0.64441 + 0.55876\,i$

$q \approx -0.37827 - 0.87918\,i$

$q \approx -0.37827 + 0.87918\,i$

$q \approx 0.30062 - 0.83198\,i$

Polynomial discriminant
$\Delta = 560009680641$

Derivative

$$\frac{d}{dq}(1+q+q^2+q^3+2q^4+2q^5+2q^6+3q^7+3q^8+3q^9) = 27q^8+24q^7+21q^6+12q^5+10q^4+8q^3+3q^2+2q+1$$

Indefinite integral

$$\int (1+q+q^2+q^3+2q^4+2q^5+2q^6+3q^7+3q^8+3q^9)\,dq = \frac{3q^{10}}{10}+\frac{q^9}{3}+\frac{3q^8}{8}+\frac{2q^7}{7}+\frac{q^6}{3}+\frac{2q^5}{5}+\frac{q^4}{4}+\frac{q^3}{3}+\frac{q^2}{2}+q + \text{constant}$$

1+(e^(-2Pi))+(e^(-2Pi))^2+(e^(-2Pi))^3+2(e^(-2Pi))^4+2(e^(-2Pi))^5+2(e^(-2Pi))^6+3(e^(-2Pi))^7+3(e^(-2Pi))^8+3(e^(-2Pi))^9

Input

$$1+e^{-2\pi}+(e^{-2\pi})^2+(e^{-2\pi})^3+2(e^{-2\pi})^4+2(e^{-2\pi})^5+2(e^{-2\pi})^6+3(e^{-2\pi})^7+3(e^{-2\pi})^8+3(e^{-2\pi})^9$$

Exact result

$$1+3e^{-18\pi}+3e^{-16\pi}+3e^{-14\pi}+2e^{-12\pi}+2e^{-10\pi}+2e^{-8\pi}+e^{-6\pi}+e^{-4\pi}+e^{-2\pi}$$

Decimal approximation

1.0018709366108449543910792229188560823522627454656105498762915486…

1.00187093661….

Property

$1+3e^{-18\pi}+3e^{-16\pi}+3e^{-14\pi}+2e^{-12\pi}+2e^{-10\pi}+2e^{-8\pi}+e^{-6\pi}+e^{-4\pi}+e^{-2\pi}$ is a transcendental number

Alternate forms

$$e^{-18\pi}\left(3+3e^{2\pi}+3e^{4\pi}+2e^{6\pi}+e^{8\pi}(1+e^{2\pi})(2+e^{4\pi}+e^{8\pi})\right)$$

$$e^{-18\pi}\left(3+3e^{2\pi}+3e^{4\pi}+2e^{6\pi}+2e^{8\pi}+2e^{10\pi}+e^{12\pi}+e^{14\pi}+e^{16\pi}+e^{18\pi}\right)$$

Alternative representations

$$1+e^{-2\pi}+\left(e^{-2\pi}\right)^2+\left(e^{-2\pi}\right)^3+2\left(e^{-2\pi}\right)^4+2\left(e^{-2\pi}\right)^5+2\left(e^{-2\pi}\right)^6+3\left(e^{-2\pi}\right)^7+3\left(e^{-2\pi}\right)^8+3\left(e^{-2\pi}\right)^9 = 1+\exp^{-2\cos^{-1}(-1)}(z)+\exp^{-2\cos^{-1}(-1)}(z)^2+\exp^{-2\cos^{-1}(-1)}(z)^3+2\exp^{-2\cos^{-1}(-1)}(z)^4+2\exp^{-2\cos^{-1}(-1)}(z)^5+2\exp^{-2\cos^{-1}(-1)}(z)^6+3\exp^{-2\cos^{-1}(-1)}(z)^7+3\exp^{-2\cos^{-1}(-1)}(z)^8+3\exp^{-2\cos^{-1}(-1)}(z)^9 \quad \text{for } z=1$$

$$1+e^{-2\pi}+\left(e^{-2\pi}\right)^2+\left(e^{-2\pi}\right)^3+2\left(e^{-2\pi}\right)^4+2\left(e^{-2\pi}\right)^5+2\left(e^{-2\pi}\right)^6+3\left(e^{-2\pi}\right)^7+3\left(e^{-2\pi}\right)^8+3\left(e^{-2\pi}\right)^9 = 1+\exp^{-2\pi}(z)+\exp^{-2\pi}(z)^2+\exp^{-2\pi}(z)^3+2\exp^{-2\pi}(z)^4+2\exp^{-2\pi}(z)^5+2\exp^{-2\pi}(z)^6+3\exp^{-2\pi}(z)^7+3\exp^{-2\pi}(z)^8+3\exp^{-2\pi}(z)^9 \quad \text{for } z=1$$

$\cos^{-1}(x)$ is the inverse cosine function

Series representations

$$1+e^{-2\pi}+\left(e^{-2\pi}\right)^2+\left(e^{-2\pi}\right)^3+2\left(e^{-2\pi}\right)^4+2\left(e^{-2\pi}\right)^5+2\left(e^{-2\pi}\right)^6+3\left(e^{-2\pi}\right)^7+3\left(e^{-2\pi}\right)^8+3\left(e^{-2\pi}\right)^9 = e^{-72\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\left(3+3e^{8\sum_{k=0}^{\infty}(-1)^k/(1+2k)}+3e^{16\sum_{k=0}^{\infty}(-1)^k/(1+2k)}+2e^{24\sum_{k=0}^{\infty}(-1)^k/(1+2k)}+2e^{32\sum_{k=0}^{\infty}(-1)^k/(1+2k)}+2e^{40\sum_{k=0}^{\infty}(-1)^k/(1+2k)}+e^{48\sum_{k=0}^{\infty}(-1)^k/(1+2k)}+e^{56\sum_{k=0}^{\infty}(-1)^k/(1+2k)}+e^{64\sum_{k=0}^{\infty}(-1)^k/(1+2k)}+e^{72\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)$$

$$1+e^{-2\pi}+\left(e^{-2\pi}\right)^2+\left(e^{-2\pi}\right)^3+2\left(e^{-2\pi}\right)^4+2\left(e^{-2\pi}\right)^5+2\left(e^{-2\pi}\right)^6+3\left(e^{-2\pi}\right)^7+3\left(e^{-2\pi}\right)^8+3\left(e^{-2\pi}\right)^9 = \left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{-18\pi}\left(3+3\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{2\pi}+3\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{4\pi}+2\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{6\pi}+2\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{8\pi}+2\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{10\pi}+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{12\pi}+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{14\pi}+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{16\pi}+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{18\pi}\right)$$

$$1 + e^{-2\pi} + \left(e^{-2\pi}\right)^2 + \left(e^{-2\pi}\right)^3 + 2\left(e^{-2\pi}\right)^4 + 2\left(e^{-2\pi}\right)^5 + 2\left(e^{-2\pi}\right)^6 + 3\left(e^{-2\pi}\right)^7 + 3\left(e^{-2\pi}\right)^8 + 3\left(e^{-2\pi}\right)^9 = \left(3 + 3\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{2\pi} + 3\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{4\pi} + 2\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{6\pi} + 2\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{8\pi} + 2\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{10\pi} + \left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{12\pi} + \left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{14\pi} + \left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{16\pi} + \left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{18\pi}\right)\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{-18\pi}$$

$n!$ is the factorial function

Integral representations

$$1 + e^{-2\pi} + \left(e^{-2\pi}\right)^2 + \left(e^{-2\pi}\right)^3 + 2\left(e^{-2\pi}\right)^4 + 2\left(e^{-2\pi}\right)^5 + 2\left(e^{-2\pi}\right)^6 + 3\left(e^{-2\pi}\right)^7 + 3\left(e^{-2\pi}\right)^8 + 3\left(e^{-2\pi}\right)^9 = e^{-36\int_0^\infty \sin(t)/t\,dt}\left(3 + 3e^{4\int_0^\infty \sin(t)/t\,dt} + 3e^{8\int_0^\infty \sin(t)/t\,dt} + 2e^{12\int_0^\infty \sin(t)/t\,dt} + 2e^{16\int_0^\infty \sin(t)/t\,dt} + 2e^{20\int_0^\infty \sin(t)/t\,dt} + e^{24\int_0^\infty \sin(t)/t\,dt} + e^{28\int_0^\infty \sin(t)/t\,dt} + e^{32\int_0^\infty \sin(t)/t\,dt} + e^{36\int_0^\infty \sin(t)/t\,dt}\right)$$

$$1 + e^{-2\pi} + \left(e^{-2\pi}\right)^2 + \left(e^{-2\pi}\right)^3 + 2\left(e^{-2\pi}\right)^4 + 2\left(e^{-2\pi}\right)^5 + 2\left(e^{-2\pi}\right)^6 + 3\left(e^{-2\pi}\right)^7 + 3\left(e^{-2\pi}\right)^8 + 3\left(e^{-2\pi}\right)^9 = e^{-36\times\int_0^\infty 1/(1+t^2)\,dt}\left(3 + 3e^{4\times\int_0^\infty 1/(1+t^2)\,dt} + 3e^{8\times\int_0^\infty 1/(1+t^2)\,dt} + 2e^{12\times\int_0^\infty 1/(1+t^2)\,dt} + 2e^{16\times\int_0^\infty 1/(1+t^2)\,dt} + 2e^{20\times\int_0^\infty 1/(1+t^2)\,dt} + e^{24\times\int_0^\infty 1/(1+t^2)\,dt} + e^{28\times\int_0^\infty 1/(1+t^2)\,dt} + e^{32\times\int_0^\infty 1/(1+t^2)\,dt} + e^{36\times\int_0^\infty 1/(1+t^2)\,dt}\right)$$

$$1 + e^{-2\pi} + \left(e^{-2\pi}\right)^2 + \left(e^{-2\pi}\right)^3 + 2\left(e^{-2\pi}\right)^4 + 2\left(e^{-2\pi}\right)^5 + 2\left(e^{-2\pi}\right)^6 + 3\left(e^{-2\pi}\right)^7 + 3\left(e^{-2\pi}\right)^8 + 3\left(e^{-2\pi}\right)^9 = e^{-36\int_0^\infty \sin^2(t)/t^2\,dt}\left(3 + 3e^{4\int_0^\infty \sin^2(t)/t^2\,dt} + 3e^{8\int_0^\infty \sin^2(t)/t^2\,dt} + 2e^{12\int_0^\infty \sin^2(t)/t^2\,dt} + 2e^{16\int_0^\infty \sin^2(t)/t^2\,dt} + 2e^{20\int_0^\infty \sin^2(t)/t^2\,dt} + e^{24\int_0^\infty \sin^2(t)/t^2\,dt} + e^{28\int_0^\infty \sin^2(t)/t^2\,dt} + e^{32\int_0^\infty \sin^2(t)/t^2\,dt} + e^{36\int_0^\infty \sin^2(t)/t^2\,dt}\right)$$

Performing the following calculations,

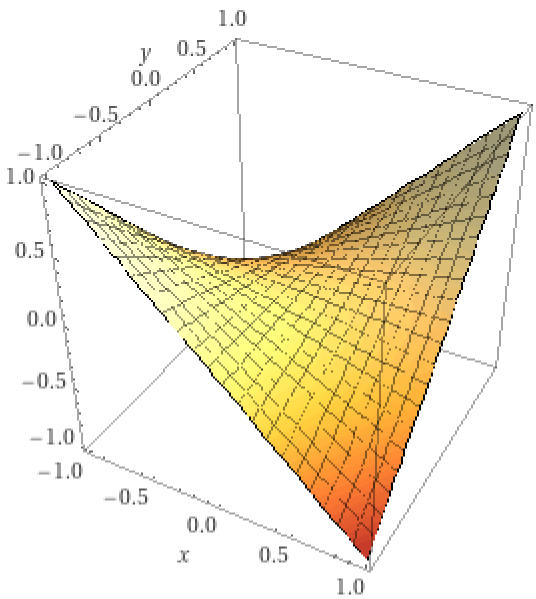
(1+(e^(-2Pi))+(e^(-2Pi))^2+(e^(-2Pi))^3+2(e^(-2Pi))^4+2(e^(-2Pi))^5+2(e^(-2Pi))^6+3(e^(-2Pi))^7+3(e^(-2Pi))^8+3(e^(-2Pi))^9)dxdy

we obtain:

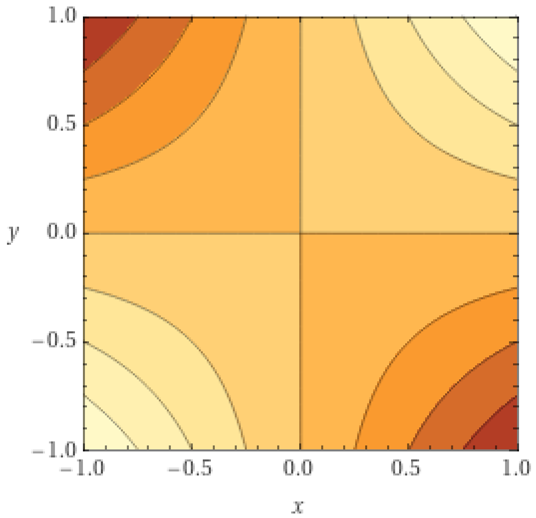
Indefinite integral

$$\int\int\left(1+e^{-2\pi}+\left(e^{-2\pi}\right)^2+\left(e^{-2\pi}\right)^3+2\left(e^{-2\pi}\right)^4+2\left(e^{-2\pi}\right)^5+2\left(e^{-2\pi}\right)^6+3\left(e^{-2\pi}\right)^7+3\left(e^{-2\pi}\right)^8+3\left(e^{-2\pi}\right)^9\right)dx\,dy = c_1 x+c_2+e^{-2\pi}x y+e^{-4\pi}x y+e^{-6\pi}x y+2e^{-8\pi}x y+2e^{-10\pi}x y+2e^{-12\pi}x y+3e^{-14\pi}x y+3e^{-16\pi}x y+3e^{-18\pi}x y+x y$$

3D plot **(figure that can be related to a D-brane/Instanton)**



Contour plot



Definite integral over a disk of radius R

$$\iint_{x^2+y^2<R^2} \left(1 + 3\,e^{-18\pi} + 3\,e^{-16\pi} + 3\,e^{-14\pi} + 2\,e^{-12\pi} + 2\,e^{-10\pi} + 2\,e^{-8\pi} + e^{-6\pi} + e^{-4\pi} + e^{-2\pi}\right) dy\,dx = \left(1 + 3\,e^{-18\pi} + 3\,e^{-16\pi} + 3\,e^{-14\pi} + 2\,e^{-12\pi} + 2\,e^{-10\pi} + 2\,e^{-8\pi} + e^{-6\pi} + e^{-4\pi} + e^{-2\pi}\right) \pi R^2$$

Definite integral over a square of edge length 2 L

$$\int_{-L}^{L}\int_{-L}^{L}\left(1 + 3\,e^{-18\pi} + 3\,e^{-16\pi} + 3\,e^{-14\pi} + 2\,e^{-12\pi} + 2\,e^{-10\pi} + 2\,e^{-8\pi} + e^{-6\pi} + e^{-4\pi} + e^{-2\pi}\right) dx\,dy = 4\left(1 + 3\,e^{-18\pi} + 3\,e^{-16\pi} + 3\,e^{-14\pi} + 2\,e^{-12\pi} + 2\,e^{-10\pi} + 2\,e^{-8\pi} + e^{-6\pi} + e^{-4\pi} + e^{-2\pi}\right) L^2$$

Dividing the two results of

$$\iint\limits_{x^2+y^2<R^2} \left(1 + 3 e^{-18\pi} + 3 e^{-16\pi} + 3 e^{-14\pi} + 2 e^{-12\pi} + 2 e^{-10\pi} + 2 e^{-8\pi} + e^{-6\pi} + e^{-4\pi} + e^{-2\pi}\right) dy\, dx = \left(1 + 3 e^{-18\pi} + 3 e^{-16\pi} + 3 e^{-14\pi} + 2 e^{-12\pi} + 2 e^{-10\pi} + 2 e^{-8\pi} + e^{-6\pi} + e^{-4\pi} + e^{-2\pi}\right) \pi R^2$$

and

$$\int_{-L}^{L}\int_{-L}^{L}\left(1 + 3 e^{-18\pi} + 3 e^{-16\pi} + 3 e^{-14\pi} + 2 e^{-12\pi} + 2 e^{-10\pi} + 2 e^{-8\pi} + e^{-6\pi} + e^{-4\pi} + e^{-2\pi}\right) dx\, dy = 4\left(1 + 3 e^{-18\pi} + 3 e^{-16\pi} + 3 e^{-14\pi} + 2 e^{-12\pi} + 2 e^{-10\pi} + 2 e^{-8\pi} + e^{-6\pi} + e^{-4\pi} + e^{-2\pi}\right) L^2$$

by

$$1 + 3 e^{-18\pi} + 3 e^{-16\pi} + 3 e^{-14\pi} + 2 e^{-12\pi} + 2 e^{-10\pi} + 2 e^{-8\pi} + e^{-6\pi} + e^{-4\pi} + e^{-2\pi}$$

we obtain:

((1+3e^(-18π)+3e^(-16π)+3e^(-14π)+2e^(-12π)+2e^(-10π)+2e^(-8π)+e^(-6π)+e^(-4π)+e^(-2π))π)/(1+3e^(-18π)+3e^(-16π)+3e^(-14π)+2e^(-12π)+2e^(-10π)+2e^(-8π)+e^(-6π)+e^(-4π)+e^(-2 π))

Input

$$\frac{\left(1 + 3 e^{-18\pi} + 3 e^{-16\pi} + 3 e^{-14\pi} + 2 e^{-12\pi} + 2 e^{-10\pi} + 2 e^{-8\pi} + e^{-6\pi} + e^{-4\pi} + e^{-2\pi}\right)\pi}{1 + 3 e^{-18\pi} + 3 e^{-16\pi} + 3 e^{-14\pi} + 2 e^{-12\pi} + 2 e^{-10\pi} + 2 e^{-8\pi} + e^{-6\pi} + e^{-4\pi} + e^{-2\pi}}$$

Exact result

π

Decimal approximation

3.1415926535897932384626433832795028841971693993751058209749445923

…

3.141592653…. = π

Property

π is a transcendental number

Alternative representations

$$\frac{\left(1+3\,e^{-18\pi}+3\,e^{-16\pi}+3\,e^{-14\pi}+2\,e^{-12\pi}+2\,e^{-10\pi}+2\,e^{-8\pi}+e^{-6\pi}+e^{-4\pi}+e^{-2\pi}\right)\pi}{1+3\,e^{-18\pi}+3\,e^{-16\pi}+3\,e^{-14\pi}+2\,e^{-12\pi}+2\,e^{-10\pi}+2\,e^{-8\pi}+e^{-6\pi}+e^{-4\pi}+e^{-2\pi}} = \pi \text{ for } z = 1$$

$$\frac{\left(1+3\,e^{-18\pi}+3\,e^{-16\pi}+3\,e^{-14\pi}+2\,e^{-12\pi}+2\,e^{-10\pi}+2\,e^{-8\pi}+e^{-6\pi}+e^{-4\pi}+e^{-2\pi}\right)\pi}{1+3\,e^{-18\pi}+3\,e^{-16\pi}+3\,e^{-14\pi}+2\,e^{-12\pi}+2\,e^{-10\pi}+2\,e^{-8\pi}+e^{-6\pi}+e^{-4\pi}+e^{-2\pi}} = 180\,°$$

$$\frac{\left(1+3\,e^{-18\pi}+3\,e^{-16\pi}+3\,e^{-14\pi}+2\,e^{-12\pi}+2\,e^{-10\pi}+2\,e^{-8\pi}+e^{-6\pi}+e^{-4\pi}+e^{-2\pi}\right)\pi}{1+3\,e^{-18\pi}+3\,e^{-16\pi}+3\,e^{-14\pi}+2\,e^{-12\pi}+2\,e^{-10\pi}+2\,e^{-8\pi}+e^{-6\pi}+e^{-4\pi}+e^{-2\pi}} = 180\,° \text{ for } z = 1$$

Series representations

$$\frac{\left(1+3\,e^{-18\pi}+3\,e^{-16\pi}+3\,e^{-14\pi}+2\,e^{-12\pi}+2\,e^{-10\pi}+2\,e^{-8\pi}+e^{-6\pi}+e^{-4\pi}+e^{-2\pi}\right)\pi}{1+3\,e^{-18\pi}+3\,e^{-16\pi}+3\,e^{-14\pi}+2\,e^{-12\pi}+2\,e^{-10\pi}+2\,e^{-8\pi}+e^{-6\pi}+e^{-4\pi}+e^{-2\pi}} = 4\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2\,k}$$

$$\frac{\left(1+3\,e^{-18\pi}+3\,e^{-16\pi}+3\,e^{-14\pi}+2\,e^{-12\pi}+2\,e^{-10\pi}+2\,e^{-8\pi}+e^{-6\pi}+e^{-4\pi}+e^{-2\pi}\right)\pi}{1+3\,e^{-18\pi}+3\,e^{-16\pi}+3\,e^{-14\pi}+2\,e^{-12\pi}+2\,e^{-10\pi}+2\,e^{-8\pi}+e^{-6\pi}+e^{-4\pi}+e^{-2\pi}} = \sum_{k=0}^{\infty} -\frac{4\,(-1)^k\,1195^{-1-2k}\left(5^{1+2k}-4\times 239^{1+2k}\right)}{1+2k}$$

$$\frac{\left(1+3\,e^{-18\pi}+3\,e^{-16\pi}+3\,e^{-14\pi}+2\,e^{-12\pi}+2\,e^{-10\pi}+2\,e^{-8\pi}+e^{-6\pi}+e^{-4\pi}+e^{-2\pi}\right)\pi}{1+3\,e^{-18\pi}+3\,e^{-16\pi}+3\,e^{-14\pi}+2\,e^{-12\pi}+2\,e^{-10\pi}+2\,e^{-8\pi}+e^{-6\pi}+e^{-4\pi}+e^{-2\pi}} = \sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k}+\frac{2}{1+4k}+\frac{1}{3+4k}\right)$$

Integral representations

$$\frac{\left(1+3\,e^{-18\pi}+3\,e^{-16\pi}+3\,e^{-14\pi}+2\,e^{-12\pi}+2\,e^{-10\pi}+2\,e^{-8\pi}+e^{-6\pi}+e^{-4\pi}+e^{-2\pi}\right)\pi}{1+3\,e^{-18\pi}+3\,e^{-16\pi}+3\,e^{-14\pi}+2\,e^{-12\pi}+2\,e^{-10\pi}+2\,e^{-8\pi}+e^{-6\pi}+e^{-4\pi}+e^{-2\pi}} = 4\int_0^1 \sqrt{1-t^2}\,dt$$

$$\frac{\left(1+3\,e^{-18\pi}+3\,e^{-16\pi}+3\,e^{-14\pi}+2\,e^{-12\pi}+2\,e^{-10\pi}+2\,e^{-8\pi}+e^{-6\pi}+e^{-4\pi}+e^{-2\pi}\right)\pi}{1+3\,e^{-18\pi}+3\,e^{-16\pi}+3\,e^{-14\pi}+2\,e^{-12\pi}+2\,e^{-10\pi}+2\,e^{-8\pi}+e^{-6\pi}+e^{-4\pi}+e^{-2\pi}} = 2\int_0^1 \frac{1}{\sqrt{1-t^2}}\,dt$$

$$\frac{\left(1+3\,e^{-18\pi}+3\,e^{-16\pi}+3\,e^{-14\pi}+2\,e^{-12\pi}+2\,e^{-10\pi}+2\,e^{-8\pi}+e^{-6\pi}+e^{-4\pi}+e^{-2\pi}\right)\pi}{1+3\,e^{-18\pi}+3\,e^{-16\pi}+3\,e^{-14\pi}+2\,e^{-12\pi}+2\,e^{-10\pi}+2\,e^{-8\pi}+e^{-6\pi}+e^{-4\pi}+e^{-2\pi}} = 2\int_0^\infty \frac{1}{1+t^2}\,dt$$

1/6(((1+3e^(-18π)+3e^(-16π)+3e^(-14π)+2e^(-12π)+2e^(-10π)+2e^(-8π)+e^(-6π)+e^(-4π)+e^(-2π))π)/(1+3e^(-18π)+3e^(-16π)+3e^(-14π)+2e^(-12π)+2e^(-10π)+2e^(-8π)+e^(-6π)+e^(-4π)+e^(-2 π)))^2

Input

$$\frac{1}{6}\left(\left(\left(1+3 e^{-18\pi}+3 e^{-16\pi}+3 e^{-14\pi}+2 e^{-12\pi}+2 e^{-10\pi}+2 e^{-8\pi}+e^{-6\pi}+e^{-4\pi}+e^{-2\pi}\right)\pi\right)/\left(1+3 e^{-18\pi}+3 e^{-16\pi}+3 e^{-14\pi}+2 e^{-12\pi}+2 e^{-10\pi}+2 e^{-8\pi}+e^{-6\pi}+e^{-4\pi}+e^{-2\pi}\right)\right)^2$$

Exact result

$$\frac{\pi^2}{6}$$

Decimal approximation

1.6449340668482264364724151666460251892189499012067984377355582293
…

1.644934066….

Property

$\frac{\pi^2}{6}$ is a transcendental number

Alternative representations

$$\frac{1}{6}\left(\left(\left(1+3 e^{-18\pi}+3 e^{-16\pi}+3 e^{-14\pi}+2 e^{-12\pi}+2 e^{-10\pi}+2 e^{-8\pi}+e^{-6\pi}+e^{-4\pi}+e^{-2\pi}\right)\pi\right)/\left(1+3 e^{-18\pi}+3 e^{-16\pi}+3 e^{-14\pi}+2 e^{-12\pi}+2 e^{-10\pi}+2 e^{-8\pi}+e^{-6\pi}+e^{-4\pi}+e^{-2\pi}\right)\right)^2=\frac{\pi^2}{6} \quad \text{for } z=1$$

$$\frac{1}{6}\left(\left(\left(1+3 e^{-18\pi}+3 e^{-16\pi}+3 e^{-14\pi}+2 e^{-12\pi}+2 e^{-10\pi}+2 e^{-8\pi}+e^{-6\pi}+e^{-4\pi}+e^{-2\pi}\right)\pi\right)/\left(1+3 e^{-18\pi}+3 e^{-16\pi}+3 e^{-14\pi}+2 e^{-12\pi}+2 e^{-10\pi}+2 e^{-8\pi}+e^{-6\pi}+e^{-4\pi}+e^{-2\pi}\right)\right)^2=\frac{1}{6}\left(180\,^\circ\right)^2$$

$$\frac{1}{6}\left(\left(\left(1+3e^{-18\pi}+3e^{-16\pi}+3e^{-14\pi}+2e^{-12\pi}+2e^{-10\pi}+2e^{-8\pi}+e^{-6\pi}+e^{-4\pi}+e^{-2\pi}\right)\pi\right)/\left(1+3e^{-18\pi}+3e^{-16\pi}+3e^{-14\pi}+2e^{-12\pi}+2e^{-10\pi}+2e^{-8\pi}+e^{-6\pi}+e^{-4\pi}+e^{-2\pi}\right)\right)^2=\frac{1}{6}(180°)^2 \quad \text{for } z=1$$

Series representations

$$\frac{1}{6}\left(\left(\left(1+3e^{-18\pi}+3e^{-16\pi}+3e^{-14\pi}+2e^{-12\pi}+2e^{-10\pi}+2e^{-8\pi}+e^{-6\pi}+e^{-4\pi}+e^{-2\pi}\right)\pi\right)/\left(1+3e^{-18\pi}+3e^{-16\pi}+3e^{-14\pi}+2e^{-12\pi}+2e^{-10\pi}+2e^{-8\pi}+e^{-6\pi}+e^{-4\pi}+e^{-2\pi}\right)\right)^2=\sum_{k=1}^{\infty}\frac{1}{k^2}$$

$$\frac{1}{6}\left(\left(\left(1+3e^{-18\pi}+3e^{-16\pi}+3e^{-14\pi}+2e^{-12\pi}+2e^{-10\pi}+2e^{-8\pi}+e^{-6\pi}+e^{-4\pi}+e^{-2\pi}\right)\pi\right)/\left(1+3e^{-18\pi}+3e^{-16\pi}+3e^{-14\pi}+2e^{-12\pi}+2e^{-10\pi}+2e^{-8\pi}+e^{-6\pi}+e^{-4\pi}+e^{-2\pi}\right)\right)^2=-2\sum_{k=1}^{\infty}\frac{(-1)^k}{k^2}$$

$$\frac{1}{6}\left(\left(\left(1+3e^{-18\pi}+3e^{-16\pi}+3e^{-14\pi}+2e^{-12\pi}+2e^{-10\pi}+2e^{-8\pi}+e^{-6\pi}+e^{-4\pi}+e^{-2\pi}\right)\pi\right)/\left(1+3e^{-18\pi}+3e^{-16\pi}+3e^{-14\pi}+2e^{-12\pi}+2e^{-10\pi}+2e^{-8\pi}+e^{-6\pi}+e^{-4\pi}+e^{-2\pi}\right)\right)^2=\frac{4}{3}\sum_{k=0}^{\infty}\frac{1}{(1+2k)^2}$$

Integral representations

$$\frac{1}{6}\left(\left(\left(1+3e^{-18\pi}+3e^{-16\pi}+3e^{-14\pi}+2e^{-12\pi}+2e^{-10\pi}+2e^{-8\pi}+e^{-6\pi}+e^{-4\pi}+e^{-2\pi}\right)\pi\right)/\left(1+3e^{-18\pi}+3e^{-16\pi}+3e^{-14\pi}+2e^{-12\pi}+2e^{-10\pi}+2e^{-8\pi}+e^{-6\pi}+e^{-4\pi}+e^{-2\pi}\right)\right)^2=\frac{8}{3}\left(\int_0^1\sqrt{1-t^2}\,dt\right)^2$$

$$\frac{1}{6}\left(\left(\left(1+3e^{-18\pi}+3e^{-16\pi}+3e^{-14\pi}+2e^{-12\pi}+2e^{-10\pi}+2e^{-8\pi}+e^{-6\pi}+e^{-4\pi}+e^{-2\pi}\right)\pi\right)/\left(1+3e^{-18\pi}+3e^{-16\pi}+3e^{-14\pi}+2e^{-12\pi}+2e^{-10\pi}+2e^{-8\pi}+e^{-6\pi}+e^{-4\pi}+e^{-2\pi}\right)\right)^2=\frac{2}{3}\left(\int_0^\infty\frac{1}{1+t^2}\,dt\right)^2$$

$$\frac{1}{6}\left(\left(\left(1+3e^{-18\pi}+3e^{-16\pi}+3e^{-14\pi}+2e^{-12\pi}+2e^{-10\pi}+2e^{-8\pi}+e^{-6\pi}+e^{-4\pi}+e^{-2\pi}\right)\pi\right)/\left(1+3e^{-18\pi}+3e^{-16\pi}+3e^{-14\pi}+2e^{-12\pi}+2e^{-10\pi}+2e^{-8\pi}+e^{-6\pi}+e^{-4\pi}+e^{-2\pi}\right)\right)^2=\frac{2}{3}\left(\int_0^1\frac{1}{\sqrt{1-t^2}}\,dt\right)^2$$

We obtain also:

(((4(1+3e^(-18π)+3e^(-16π)+3e^(-14π)+2e^(-12π)+2e^(-10π)+2e^(-8π)+e^(-6π)+e^(-4π)+e^(-2π)))/(1+3e^(-18π)+3e^(-16π)+3e^(-14π)+2e^(-12π)+2e^(-10π)+2e^(-8π)+e^(-6π)+e^(-4π)+e^(-2 π))))^6

Input

$$\left(\left(4\left(1+3e^{-18\pi}+3e^{-16\pi}+3e^{-14\pi}+2e^{-12\pi}+2e^{-10\pi}+2e^{-8\pi}+e^{-6\pi}+e^{-4\pi}+e^{-2\pi}\right)\right)/\left(1+3e^{-18\pi}+3e^{-16\pi}+3e^{-14\pi}+2e^{-12\pi}+2e^{-10\pi}+2e^{-8\pi}+e^{-6\pi}+e^{-4\pi}+e^{-2\pi}\right)\right)^6$$

Result

4096

$4096 = 64^2$

27sqrt(((4(1+3e^(-18π)+3e^(-16π)+3e^(-14π)+2e^(-12π)+2e^(-10π)+2e^(-8π)+e^(-6π)+e^(-4π)+e^(-2π)))/(1+3e^(-18π)+3e^(-16π)+3e^(-14π)+2e^(-12π)+2e^(-10π)+2e^(-8π)+e^(-6π)+e^(-4π)+e^(-2 π))))^6+1

Input

$$27\sqrt{\left(\left(4\left(1+3e^{-18\pi}+3e^{-16\pi}+3e^{-14\pi}+2e^{-12\pi}+2e^{-10\pi}+2e^{-8\pi}+e^{-6\pi}+e^{-4\pi}+e^{-2\pi}\right)\right)/\left(1+3e^{-18\pi}+3e^{-16\pi}+3e^{-14\pi}+2e^{-12\pi}+2e^{-10\pi}+2e^{-8\pi}+e^{-6\pi}+e^{-4\pi}+e^{-2\pi}\right)\right)^6}+1$$

Result

1729

1729

This result is very near to the mass of candidate glueball f_0(1710) scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. ($1728 = 8^2 * 3^3$) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Series representations

$$27\sqrt{\left(\left(4\left(1+3e^{-18\pi}+3e^{-16\pi}+3e^{-14\pi}+2e^{-12\pi}+2e^{-10\pi}+2e^{-8\pi}+e^{-6\pi}+e^{-4\pi}+e^{-2\pi}\right)\right)/\left(1+3e^{-18\pi}+3e^{-16\pi}+3e^{-14\pi}+2e^{-12\pi}+2e^{-10\pi}+2e^{-8\pi}+e^{-6\pi}+e^{-4\pi}+e^{-2\pi}\right)\right)}^6+1=1+27\sqrt{3}^6\left(\sum_{k=0}^{\infty}3^{-k}\binom{\frac{1}{2}}{k}\right)^6$$

$$27\sqrt{\left(\left(4\left(1+3e^{-18\pi}+3e^{-16\pi}+3e^{-14\pi}+2e^{-12\pi}+2e^{-10\pi}+2e^{-8\pi}+e^{-6\pi}+e^{-4\pi}+e^{-2\pi}\right)\right)/\left(1+3e^{-18\pi}+3e^{-16\pi}+3e^{-14\pi}+2e^{-12\pi}+2e^{-10\pi}+2e^{-8\pi}+e^{-6\pi}+e^{-4\pi}+e^{-2\pi}\right)\right)}^6+1=1+27\sqrt{3}^6\left(\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{3}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^6$$

$$27\sqrt{\left(\left(4\left(1+3e^{-18\pi}+3e^{-16\pi}+3e^{-14\pi}+2e^{-12\pi}+2e^{-10\pi}+2e^{-8\pi}+e^{-6\pi}+e^{-4\pi}+e^{-2\pi}\right)\right)/\left(1+3e^{-18\pi}+3e^{-16\pi}+3e^{-14\pi}+2e^{-12\pi}+2e^{-10\pi}+2e^{-8\pi}+e^{-6\pi}+e^{-4\pi}+e^{-2\pi}\right)\right)}^6+1=1+\frac{27\left(\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}3^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)\right)^6}{64\sqrt{\pi}^6}$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\Gamma(x)$ is the gamma function

$\operatorname{Res}_{z=z_0} f$ is a complex residue

RAMANUJAN-NARDELLI MOCK GENERAL FORMULA

Inserting the mass of Supermassive Black hole M87 = $13.12806*10^{39}$ Kg in the Hawking radiation calculator, we obtain:

Mass = 1.31281e+40
Radius = 1.94973e+13
Temperature = 9.34606e-18

From the Ramanujan-Nardelli mock general formula, we obtain:

sqrt(((1/((((4*1.962364415e+19)/(5*0.0864055^2)))*1/(1.312841e+40)* sqrt((-((((9.34606e-18 * 4*Pi*(1.94973e+13)^3-(1.94973e+13)^2))))) / ((6.67408e-11)))))))

Input interpretation

$$\sqrt{\left(1\Big/\left(\frac{4\times1.962364415\times10^{19}}{5\times0.0864055^2}\times\frac{1}{1.312841\times10^{40}}\sqrt{-\frac{9.34606\times10^{-18}\times4\,\pi\,(1.94973\times10^{13})^3-(1.94973\times10^{13})^2}{6.67408\times10^{-11}}}\right)\right)}$$

Result

1.6183488761977978262552287851136172021586573991542479287151681323
…

1.618348876…. result that is a very good approximation to the value of the golden ratio 1.618033988749...

Performing the following calculations

(sqrt(((1/((((4*1.962364415e+19)/(5*0.0864055^2)))*1/(1.312841e+40)* sqrt((-((((9.34606e-18 * 4*Pi*(1.94973e+13)^3-(1.94973e+13)^2))))) / ((6.67408e-11))))))))dxdy

we obtain:

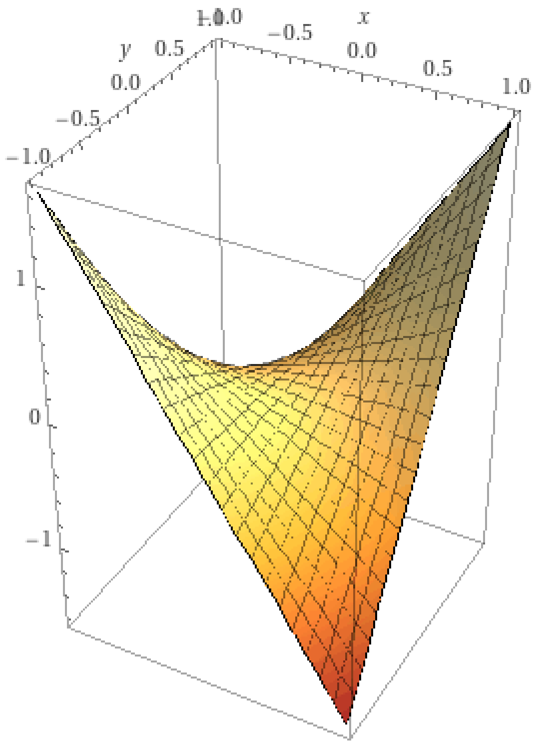
Input interpretation

$$\int\int\sqrt{\left(1\Bigg/\left(\frac{4\times 1.962364415\times 10^{19}}{5\times 0.0864055^2}\times\frac{1}{1.312841\times 10^{40}}\sqrt{-\frac{9.34606\times 10^{-18}\times 4\,\pi\,(1.94973\times 10^{13})^3-(1.94973\times 10^{13})^2}{6.67408\times 10^{-11}}}\right)\right)}\,dx\,dy$$

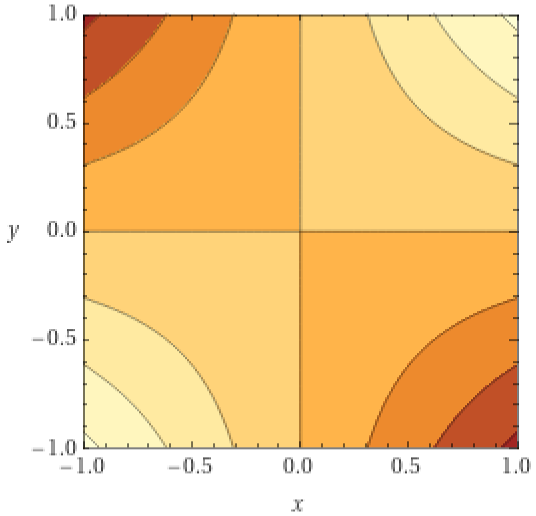
Result

$1.61835\,x\,y$

3D plot **(figure that can be related to a D-brane/Instanton)**



Contour plot



Indefinite integral assuming all variables are real

$0.809174\, x^2\, y$ + constant

Definite integral over a disk of radius R

$$\iint_{x^2+y^2<R^2} 1.61835\, dy\, dx = 5.08419\, R^2$$

Definite integral over a square of edge length 2 L

$$\int_{-L}^{L}\int_{-L}^{L} 1.61835\, dx\, dy = 6.4734\, L^2$$

Dividing the two solutions

$$\iint_{x^2+y^2<R^2} 1.61835\, dy\, dx = 5.08419\, R^2$$

and

$$\int_{-L}^{L}\int_{-L}^{L} 1.61835\, dx\, dy = 6.4734\, L^2$$

by

1.6183488761977978262552287851136172021586573991542479287151681323
…

we obtain, for R = 1 and L = 1 :

(5.08419)/(1.618348876197)

Input interpretation

$$\frac{5.08419}{1.618348876197}$$

Result

3.1415908366726647667381486419079051021283884741800923465321588716

...

3.1415908366…. ≈ π

from which:

1/6((5.08419)/(1.618348876197))^2

Input interpretation

$$\frac{1}{6}\left(\frac{5.08419}{1.618348876197}\right)^2$$

Result

1.6449321641776089717036281097430547288529399379269472075485991824

...

1.6449321641776…. ≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape)

Furthermore, we obtain also:

((6.4734)/(1.618348876197))^6

Input interpretation

$$\left(\frac{6.4734}{1.618348876197}\right)^6$$

Result

4096.0170659311357662338211472736995957938951134788909483337840809

...

4096.01706593…. ≈ 4096 = 64^2

from which:

27(((6.4734)/(1.618348876197))^3)+1

Input interpretation

$$27\left(\frac{6.4734}{1.618348876197}\right)^3 + 1$$

Result

1729.0035998410992817258037477741572756432263621674974235395162162...

1729.003599841….

This result is very near to the mass of candidate glueball $f_0(1710)$ scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. ($1728 = 8^2 * 3^3$) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Inserting the mass of Supermassive Black hole Sagittarius A* = $8.15531 * 10^{36}$ Kg in the Hawking radiation calculator, we obtain:

Mass = 8.15531e+36
Radius = 1.21120e+10
Temperature = 1.50449e-14

From the Ramanujan-Nardelli mock general formula, we obtain:

sqrt(((1/((((4*1.962364415e+19)/(5*0.0864055^2)))*1/(8.15531e+36)* sqrt((-((((1.50449e-14 * 4*Pi*(1.21120e+10)^3-(1.21120e+10)^2))))) / ((6.67408e-11)))))))

Input interpretation

$$\sqrt{\frac{1}{\frac{4\times 1.962364415\times 10^{19}}{5\times 0.0864055^2}\times \frac{1}{8.15531\times 10^{36}}\sqrt{-\frac{1.50449\times 10^{-14}\times 4\pi (1.21120\times 10^{10})^3-(1.21120\times 10^{10})^2}{6.67408\times 10^{-11}}}}}$$

Result

1.618324575218357093780789687325972623758640536909311165451127647 8
...

1.618324575218…. result that is a very good approximation to the value of the golden ratio 1.618033988749...

Performing the following calculations, we obtain:

((sqrt(((1/((((4*1.962364415e+19)/(5*0.0864055^2)))*1/(8.15531e+36)* sqrt((-((((1.50449e-14 * 4*Pi*(1.21120e+10)^3-(1.21120e+10)^2))))) / ((6.67408e-11)))))))))dxdy

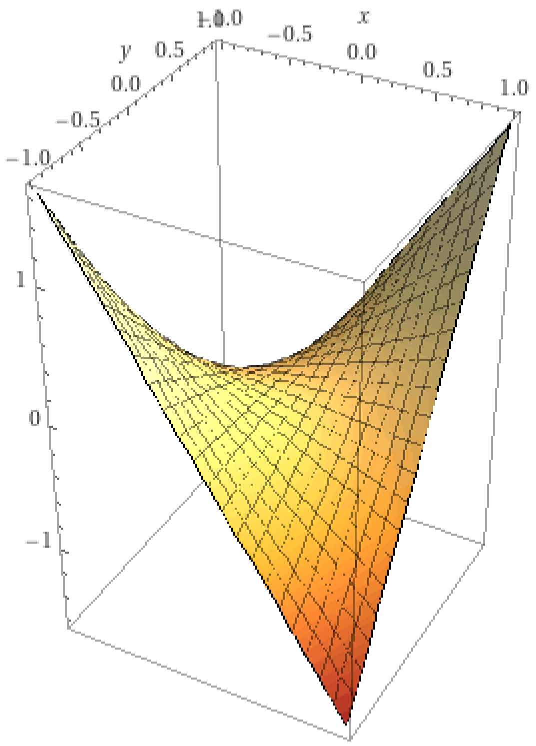
Input interpretation

$$\int\int\sqrt{\left(1\Big/\left(\frac{4\times 1.962364415\times 10^{19}}{5\times 0.0864055^2}\times \frac{1}{8.15531\times 10^{36}}\sqrt{-\frac{1.50449\times 10^{-14}\times 4\pi (1.21120\times 10^{10})^3-(1.21120\times 10^{10})^2}{6.67408\times 10^{-11}}}\right)\right)}\,dx\,dy$$

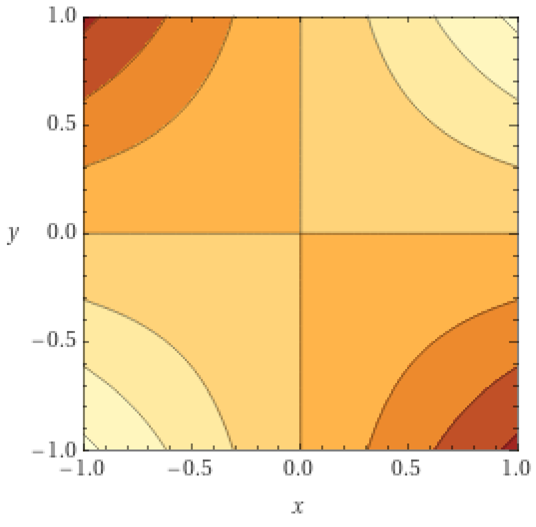
Result

$1.61832\,x\,y$

3D plot **(figure that can be related to a D-brane/Instanton)**



Contour plot



Indefinite integral assuming all variables are real

$0.809162\, x^2\, y$ + constant

Definite integral over a disk of radius R

$$\iint_{x^2+y^2<R^2} 1.61832\, dy\, dx = 5.08412\, R^2$$

Definite integral over a square of edge length 2 L

$$\int_{-L}^{L}\int_{-L}^{L} 1.61832\, dx\, dy = 6.4733\, L^2$$

Dividing

$$\iint_{x^2+y^2<R^2} 1.61832\, dy\, dx = 5.08412\, R^2$$

and

$$\int_{-L}^{L}\int_{-L}^{L} 1.61832\, dx\, dy = 6.4733\, L^2$$

by

1.618324575218357093780789687325972623758640536909311165451127647 8
…

we obtain:

(5.08412)/(1.618324575218)

Input interpretation

$$\frac{5.08412}{1.618324575218}$$

Result

3.1415947566112516971811709217939205669226924496347421814190927703

...

3.1415947566…. ≈ π

1/6((5.08412)/(1.618324575218))^2

Input interpretation

$$\frac{1}{6}\left(\frac{5.08412}{1.618324575218}\right)^2$$

Result

1.6449362691278849648824269561669659678782924464799424737773089781

...

1.6449362691… ≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape)

and also:

((6.4733)/(1.618324575218))^6

Input interpretation

$$\left(\frac{6.4733}{1.618324575218}\right)^6$$

Result

4096.0064507759705900955502783310420297633409611636562318920436201

...

4096.00645077…. ≈ 4096 = 64^2

27((6.4733)/(1.618324575218))^3+1

Input interpretation

$$27\left(\frac{6.4733}{1.618324575218}\right)^3+1$$

Result

1729.0013607100205522047525958978955413929299451723477074773883989
…

1729.00136071….

This result is very near to the mass of candidate glueball f_0(1710) scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. ($1728 = 8^2 * 3^3$) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Now, we consider a black hole with a mass equal to 125.35 GeV/c^2 = 2.235×10^-25 Kg. We obtain:

Mass = 2.235e-25
Radius = 3.31934e-52
Temperature = 5.48974e+47

From the Ramanujan-Nardelli mock general formula, we obtain:

sqrt(((1/((((4*1.962364415e+19)/(5*0.0864055^2)))*1/(2.235e-25)* sqrt((-((((5.48974e+47 * 4*Pi*(3.31934e-52)^3-(3.31934e-52)^2))))) / ((6.67408e-11))))))

Input interpretation

$$\sqrt{\frac{1}{\frac{4\times 1.962364415\times 10^{19}}{5\times 0.0864055^2}\times\frac{1}{2.235\times 10^{-25}}\sqrt{-\frac{5.48974\times 10^{47}\times 4\pi(3.31934\times 10^{-52})^3-(3.31934\times 10^{-52})^2}{6.67408\times 10^{-11}}}}}$$

Result

1.618326762847872059164136673144423470495070831890776559500272545
...

1.6183267628…. result that is a very good approximation to the value of the golden ratio 1.618033988749...

Performing the following calculations

(sqrt(((1/((((4*1.962364415e+19)/(5*0.0864055^2)))*1/(2.235e-25)* sqrt((-((((5.48974e+47 * 4*Pi*(3.31934e-52)^3-(3.31934e-52)^2))))) / ((6.67408e-11))))))))dxdy

we obtain:

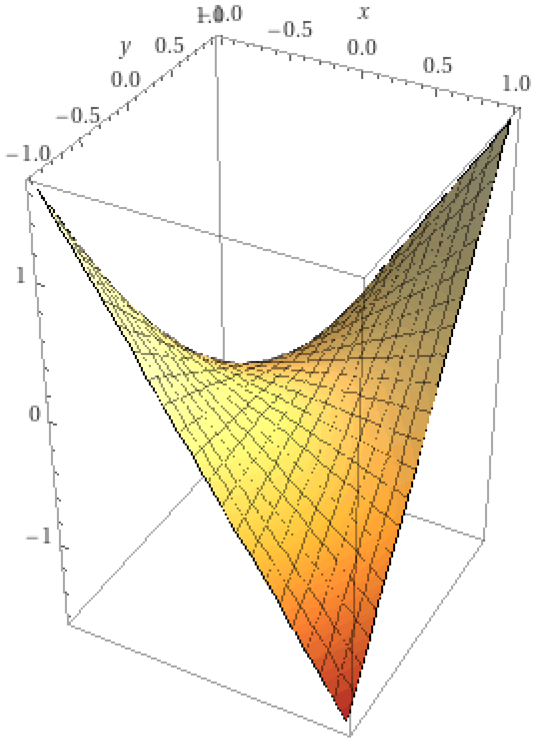
Input interpretation

$$\int\int\sqrt{\left(1\middle/\left(\frac{4\times 1.962364415\times 10^{19}}{5\times 0.0864055^2}\times\frac{1}{2.235\times 10^{-25}}\sqrt{-\frac{5.48974\times 10^{47}\times 4\pi(3.31934\times 10^{-52})^3-(3.31934\times 10^{-52})^2}{6.67408\times 10^{-11}}}\right)\right)}$$

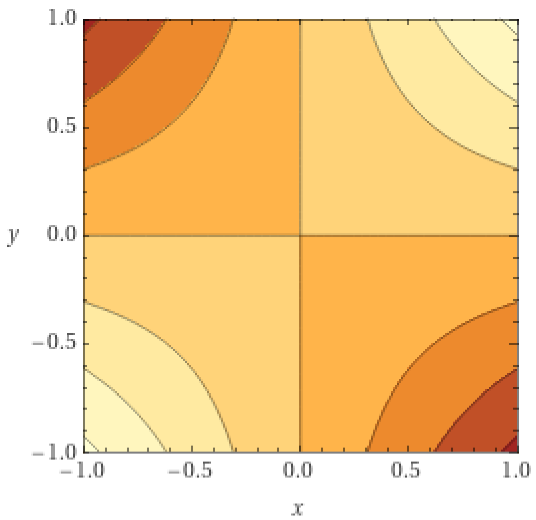
dx
dy

Result
1.61833 x y

3D plot **(figure that can be related to a D-brane/Instanton)**



Contour plot



Indefinite integral assuming all variables are real

$0.809163\, x^2\, y + \text{constant}$

Definite integral over a disk of radius R

$$\iint_{x^2+y^2<R^2} 1.61833\, dy\, dx = 5.08412\, R^2$$

Definite integral over a square of edge length 2 L

$$\int_{-L}^{L}\int_{-L}^{L} 1.61833\, dx\, dy = 6.47331\, L^2$$

Dividing the two results of

$$\iint_{x^2+y^2<R^2} 1.61833\, dy\, dx = 5.08412\, R^2$$

and

$$\int_{-L}^{L}\int_{-L}^{L} 1.61833\, dx\, dy = 6.47331\, L^2$$

by

1.6183267628478720591641366731444234704950708318907765595002725545
...

we obtain, for R = 1 and L = 1 :

((5.08412)/(1.6183267628478))

Input interpretation

$$\frac{5.08412}{1.6183267628478}$$

Result

3.141590509850667331267061031616175953342978843203374360765949699 4 …

3.14159050985…. ≈ π

1/6((5.08412)/(1.6183267628478))^2

Input interpretation

$$\frac{1}{6}\left(\frac{5.08412}{1.6183267628478}\right)^2$$

Result

1.644931821930629318362258445163879727137009655852040151586850577 6 …

1.64493182193…. ≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape)

Furthermore, we obtain also:

((6.47331)/(1.6183267628478))^6

Input interpretation

$$\left(\frac{6.47331}{1.6183267628478}\right)^6$$

Result

4096.0111944469517005863150586467147159840265808997360979275657062
…

4096.01119444…. ≈ 4096 = 64^2

and

27((6.47331)/(1.6183267628478))^3+1

Input interpretation

$$27\left(\frac{6.47331}{1.6183267628478}\right)^3+1$$

Result

1729.0023613270404873655343915867394951838228453523266483019865340
…

1729.002361327….

This result is very near to the mass of candidate glueball $f_0(1710)$ scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. (1728 = 8^2 * 3^3) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Now, we consider a black hole with a mass equal to 9.1093837015×10^−31 Kg. We obtain:

Mass = 9.10938e-31
Radius = 1.35289e-57
Temperature = 1.34691e+53

From the Ramanujan-Nardelli mock general formula, we obtain:

sqrt(((1/((((4*1.962364415e+19)/(5*0.0864055^2)))*1/(9.10938e-31)* sqrt((-((((1.34691e+53 * 4*Pi*(1.35289e-57)^3-(1.35289e-57)^2))))) / ((6.67408e-11)))))))

Input interpretation

$$\sqrt{\left(1/\left(\frac{4\times 1.962364415\times 10^{19}}{5\times 0.0864055^2}\times\frac{1}{9.10938\times 10^{-31}}\sqrt{-\frac{1.34691\times 10^{53}\times 4\,\pi\left(1.35289\times 10^{-57}\right)^3-\left(1.35289\times 10^{-57}\right)^2}{6.67408\times 10^{-11}}}\right)\right)}$$

Result
1.6183277720791551875718947186373873468171829984026934808907523349
...

1.61832777207915…. result that is a very good approximation to the value of the golden ratio 1.618033988749...

Performing the following calculations:

(sqrt(((1/((((4*1.962364415e+19)/(5*0.0864055^2)))*1/(9.10938e-31)* sqrt((-((((1.34691e+53 * 4*Pi*(1.35289e-57)^3-(1.35289e-57)^2))))) / ((6.67408e-11))))))))dxdy

we obtain:

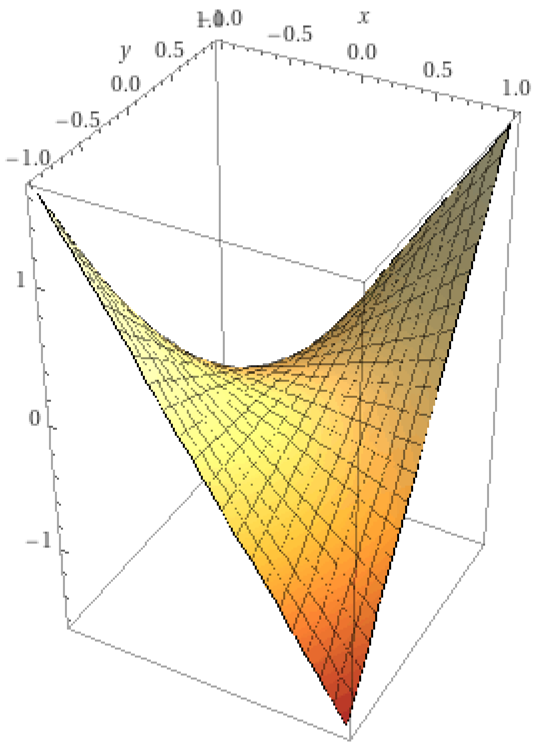
Input interpretation

$$\int\int\sqrt{\left(1\Big/\left(\frac{4\times 1.962364415\times 10^{19}}{5\times 0.0864055^2}\times\frac{1}{9.10938\times 10^{-31}}\sqrt{-\frac{1.34691\times 10^{53}\times 4\,\pi\,(1.35289\times 10^{-57})^3-(1.35289\times 10^{-57})^2}{6.67408\times 10^{-11}}}\right)\right)}\,dx\,dy$$

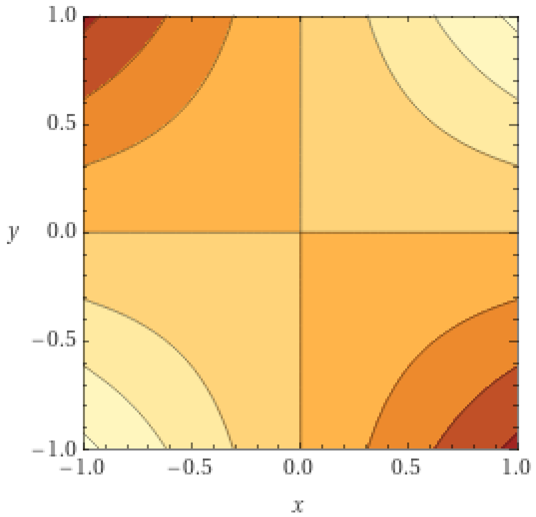
Result

$1.61833\,x\,y$

3D plot **(figure that can be related to a D-brane/Instanton)**



Contour plot



Indefinite integral assuming all variables are real

$0.809164\, x^2\, y$ + constant

Definite integral over a disk of radius R

$$\iint_{x^2+y^2<R^2} 1.61833\, d\,y\, d\,x = 5.08413\, R^2$$

Definite integral over a square of edge length 2 L

$$\int_{-L}^{L}\int_{-L}^{L} 1.61833\, d\,x\, d\,y = 6.47331\, L^2$$

Dividing the two results

$$\iint_{x^2+y^2<R^2} 1.61833\, d\,y\, d\,x = 5.08413\, R^2$$

and

$$\int_{-L}^{L}\int_{-L}^{L} 1.61833\, dx\, dy = 6.47331\, L^2$$

by

1.6183277720791551875718947186373873468171829984026934808907523349
…

we obtain:

(5.08413)/(1.61832777207915)

Input interpretation

$$\frac{5.08413}{1.61832777207915}$$

Result
3.1415947298909375554664412522266417454067279677451259122017794081
…

3.14159472989…. ≈ π

1/6((5.08413)/(1.61832777207915))^2

Input interpretation

$$\frac{1}{6}\left(\frac{5.08413}{1.61832777207915}\right)^2$$

Result

1.6449362411464188163394672794178787540532875893359836962853503331
…

1.6449362411…. ≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape)

and

((6.47331)/(1.61832777207915))^6

Input interpretation

$$\left(\frac{6.47331}{1.61832777207915}\right)^6$$

Result

4095.9958681952473916984957059676112476385275739872583738303969807
…

4095.99586819…. ≈ 4096 = 64^2

27((6.47331)/(1.61832777207915))^3+1

Input interpretation

$$27\left(\frac{6.47331}{1.61832777207915}\right)^3 + 1$$

Result

1728.9991284472152037880547851616731916733519695829350104140916594
…

1728.999128447….

This result is very near to the mass of candidate glueball $f_0(1710)$ scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. ($1728 = 8^2 * 3^3$) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

From:

Modular equations and approximations to π - *Srinivasa Ramanujan*
Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

We have that:

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \cdots, \\ 64g_{22}^{-24} &= \qquad\qquad 4096e^{-\pi\sqrt{22}} + \cdots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\ldots.$$

Again

$$G_{37} = (6+\sqrt{37})^{\frac{1}{4}},$$

$$\begin{aligned} 64G_{37}^{24} &= e^{\pi\sqrt{37}} + 24 + 276e^{-\pi\sqrt{37}} + \cdots, \\ 64G_{37}^{-24} &= \qquad\qquad 4096e^{-\pi\sqrt{37}} - \cdots, \end{aligned}$$

so that

$$64(G_{37}^{24} + G_{37}^{-24}) = e^{\pi\sqrt{37}} + 24 + 4372e^{-\pi\sqrt{37}} - \cdots = 64\{(6+\sqrt{37})^{6} + (6-\sqrt{37})^{6}\}.$$

Hence

$$e^{\pi\sqrt{37}} = 199148647.999978\ldots.$$

Similarly, from

$$g_{58} = \sqrt{\left(\frac{5+\sqrt{29}}{2}\right)},$$

we obtain

$$64(g_{58}^{24} + g_{58}^{-24}) = e^{\pi\sqrt{58}} - 24 + 4372e^{-\pi\sqrt{58}} + \cdots = 64\left\{\left(\frac{5+\sqrt{29}}{2}\right)^{12} + \left(\frac{5-\sqrt{29}}{2}\right)^{12}\right\}.$$

Hence

$$e^{\pi\sqrt{58}} = 24591257751.99999982\ldots.$$

From:

An Update on Brane Supersymmetry Breaking
J. Mourad and A. Sagnotti - arXiv:1711.11494v1 [hep-th] 30 Nov 2017

From the following vacuum equations:

$$T\,e^{\gamma_E\,\phi} \;=\; -\,\frac{\beta_E^{(p)}\,h^2}{\gamma_E}\;e^{\,-\,2\,(8\,-\,p)\,C\,+\,2\,\beta_E^{(p)}\,\phi}$$

$$16\,k'\,e^{\,-\,2\,C} \;=\; \frac{h^2\left(p\,+\,1\,-\,\frac{2\,\beta_E^{(p)}}{\gamma_E}\right)e^{\,-\,2\,(8\,-\,p)\,C\,+\,2\,\beta_E^{(p)}\,\phi}}{(7\,-\,p)}$$

$$(A')^2 \;=\; k\,e^{\,-\,2\,A} \,+\, \frac{h^2}{16(p\,+\,1)}\left(7\,-\,p\,+\,\frac{2\,\beta_E^{(p)}}{\gamma_E}\right)\,e^{\,-\,2\,(8\,-\,p)\,C\,+\,2\,\beta_E^{(p)}\,\phi}$$

we have obtained, from the results almost equals of the equations, putting

$4096\,e^{-\pi\sqrt{18}}$ instead of

$$e^{\,-\,2\,(8\,-\,p)\,C\,+\,2\,\beta_E^{(p)}\,\phi}$$

a new possible mathematical connection between the two exponentials. Thence, also the values concerning p, C, β_E and ϕ correspond to the exponents of e (i.e. of exp). Thence we obtain for p = 5 and $\beta_E = 1/2$:

$e^{-6C+\phi} = 4096\,e^{-\pi\sqrt{18}}$

Therefore, with respect to the exponentials of the vacuum equations, the Ramanujan's exponential has a coefficient of 4096 which is equal to 64^2, while -6C+ϕ is equal to -$\pi\sqrt{18}$. From this it follows that it is possible to establish mathematically, the dilaton value.

For

exp((-Pi*sqrt(18)) we obtain:

Input:

$\exp\left(-\pi\sqrt{18}\right)$

Exact result:

$e^{-3\sqrt{2}\,\pi}$

Decimal approximation:

$1.627201622607250929294215673911797954183858113695 4016\ldots \times 10^{-6}$

1.6272016… * 10^{-6}

Property:

$e^{-3\sqrt{2}\,\pi}$ is a transcendental number

Series representations:

$$e^{-\pi\sqrt{18}} = e^{-\pi\sqrt{17}\sum_{k=0}^{\infty} 17^{-k}\binom{1/2}{k}}$$

$$e^{-\pi\sqrt{18}} = \exp\left(-\pi\sqrt{17}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{17}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$e^{-\pi\sqrt{18}} = \exp\left(-\frac{\pi\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j} 17^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}\right)$$

Now, we have the following calculations:

$$e^{-6C+\phi}=4096\,e^{-\pi\sqrt{18}}$$

$e^{-\pi\sqrt{18}}$ = 1.6272016… * 10^-6

from which:

$\frac{1}{4096}e^{-6C+\phi}$ = 1.6272016… * 10^-6

0.000244140625 $e^{-6C+\phi}$ = $e^{-\pi\sqrt{18}}$ = 1.6272016… * 10^-6

Now:

$$\ln\left(e^{-\pi\sqrt{18}}\right)=-13.328648814475=-\pi\sqrt{18}$$

And:

(1.6272016* 10^-6) *1/ (0.000244140625)

Input interpretation:

$$\frac{1.6272016}{10^6}\times\frac{1}{0.000244140625}$$

Result:

0.0066650177536

0.006665017...

Thence:

$$0.000244140625\, e^{-6C+\phi} = e^{-\pi\sqrt{18}}$$

Dividing both sides by 0.000244140625, we obtain:

$$\frac{0.000244140625}{0.000244140625} e^{-6C+\phi} = \frac{1}{0.000244140625} e^{-\pi\sqrt{18}}$$

$$e^{-6C+\phi} = 0.0066650177536$$

((((exp((-Pi*sqrt(18)))))))*1/0.000244140625

Input interpretation:

$$\exp\left(-\pi\sqrt{18}\right) \times \frac{1}{0.000244140625}$$

Result:

0.00666501785…

0.00666501785…

Series representations:

$$\frac{\exp\left(-\pi\sqrt{18}\right)}{0.000244141} = 4096 \exp\left(-\pi\sqrt{17} \sum_{k=0}^{\infty} 17^{-k} \binom{\frac{1}{2}}{k}\right)$$

$$\frac{\exp\left(-\pi\sqrt{18}\right)}{0.000244141} = 4096 \exp\left(-\pi\sqrt{17} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{17}\right)^k \left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$\frac{\exp(-\pi\sqrt{18})}{0.000244141} = 4096 \exp\left(-\frac{\pi \sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 17^{-s}\, \Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}\right)$$

Now:

$$e^{-6C+\phi} = 0.0066650177536$$

$$\exp\left(-\pi\sqrt{18}\right) \times \frac{1}{0.000244140625} =$$

$$e^{-\pi\sqrt{18}} \times \frac{1}{0.000244140625}$$

$$= 0.00666501785\ldots$$

From:

ln(0.00666501784619)

Input interpretation:

log(0.00666501784619)

Result:

−5.010882647757…

-5.010882647757…

Alternative representations:

$$\log(0.006665017846190000) = \log_e(0.006665017846190000)$$

$$\log(0.006665017846190000) = \log(a)\log_a(0.006665017846190000)$$

$$\log(0.00666501784619000) = -\text{Li}_1(0.99333498215381000)$$

Series representations:

$$\log(0.00666501784619000) = -\sum_{k=1}^{\infty} \frac{(-1)^k (-0.99333498215381000)^k}{k}$$

$$\log(0.00666501784619000) = 2 i \pi \left\lfloor \frac{\arg(0.00666501784619000 - x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (0.00666501784619000 - x)^k x^{-k}}{k} \quad \text{for } x < 0$$

$$\log(0.00666501784619000) = \left\lfloor \frac{\arg(0.00666501784619000 - z_0)}{2\pi} \right\rfloor \log\left(\frac{1}{z_0}\right) + \log(z_0) + \left\lfloor \frac{\arg(0.00666501784619000 - z_0)}{2\pi} \right\rfloor \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (0.00666501784619000 - z_0)^k z_0^{-k}}{k}$$

Integral representation:

$$\log(0.00666501784619000) = \int_1^{0.00666501784619000} \frac{1}{t}\, dt$$

In conclusion:

$$-6C+\phi=-5.010882647757\ldots$$

and for C = 1, we obtain:

$\phi=-5.010882647757+6=0.989117352243 = \phi$

Note that the values of n_s (spectral index) 0.965, of the average of the Omega mesons Regge slope 0.987428571 and of the dilaton 0.989117352243, are also connected to the following two Rogers-Ramanujan continued fractions:

$$\frac{e^{-\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}}-\varphi+1}=1-\cfrac{e^{-\pi}}{1+\cfrac{e^{-2\pi}}{1+\cfrac{e^{-3\pi}}{1+\cfrac{e^{-4\pi}}{1+\ldots}}}}\approx 0.9568666373$$

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\dfrac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\cfrac{e^{-\pi\sqrt{5}}}{1+\cfrac{e^{-2\pi\sqrt{5}}}{1+\cfrac{e^{-3\pi\sqrt{5}}}{1+\cfrac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}}\approx 0.9991104684$$

(http://www.bitman.name/math/article/102/109/)

Also performing the 512th root of the inverse value of the Pion meson rest mass 139.57, we obtain:

((1/(139.57)))^1/512

Input interpretation:

$$\sqrt[512]{\frac{1}{139.57}}$$

Result:

0.990400732708644027550973755713301415460732796178555551684...

0.99040073.... result very near to the dilaton value $0.989117352243=\phi$ and to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1} = 1-\cfrac{e^{-\pi\sqrt{5}}}{1+\cfrac{e^{-2\pi\sqrt{5}}}{1+\cfrac{e^{-3\pi\sqrt{5}}}{1+\cfrac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}} \approx 0.9991104684$$

Acknowledgments

We would like to thank Professor **Augusto Sagnotti** theoretical physicist at Scuola Normale Superiore (Pisa – Italy) for his very useful explanations and his availability

References

Ramanujan's Last Letter to Hardy - *George E. Andrews & Bruce C. Berndt* - Chapter - First Online: 06 September 2018

Harmonic Maass forms and mock modular forms : theory and applications – *Kathrin Bringmann, Amanda Folsom, Ken Ono, Larry Rolen.* - © 2017 by the American Mathematical Society. All rights reserved.

Modular equations and approximations to π - *Srinivasa Ramanujan* Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

An Update on Brane Supersymmetry Breaking
Jihad Mourad and Augusto Sagnotti - arXiv:1711.11494v1 [hep-th] 30 Nov 2017