Deriving Field Rotation Rate for an Alt- Az Mounted Telescope

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Abstract

The field rotation rate is derived for an Alt-Az mounted telescope by considering two sources of rotation rate. The first source is related to the azimuth and altitude angle rates used in tracking the central target star with respect to the local frame. The second source is related to the rotation rate of the local coordinate frame with respect to the inertial frame. The field rotation rate is the sum of the two rotation rates along the telescope viewing axis.

1. Introduction

Long duration astronomical observation requires accurate tracking of the target star and ensuring that the star field does not rotate about the viewing axis due to the slewing of the telescope's pointing direction. Equatorially mounted telescopes do not suffer from field rotation, since the slewing motion is about an axis aligned with the Earth's spin axis. Simply slewing the telescope at a rate opposite to the Earth's spin rate results in an inertially fixed pointing direction of the telescope. In this manner, the target star is easily tracked and other objects in the FOV (field of view) remain in fixed relative positions, which permits long duration photographic exposure.

Telescopes that use Alt-Az mounts are aligned using azimuth and altitude angles, with the azimuth angle being about the local vertical axis and the altitude angle being about a local horizontal axis. Azimuth and altitude angles are varied so that the telescope tracks the target star across the sky. The angular rate of the azimuth angle has a projection along the telescope axis for non-zero values of altitude angle and results in a rotation rate of the telescope along its axis. In addition, the angular rate of the local frame due to the Earth's spin rate has a projection along the telescope axis and also contributes to the rotation of the telescope along its axis. When viewing through the telescope, the star field has a relative rotation rate opposite to that of the telescope known as field rotation [1 - 7]. Field rotation produces a smearing of objects in the FOV that are at angular distances from the telescope axis and the target star. Therefore, the star field must be de-rotated to keep all objects in the FOV in fixed relative locations, which is necessary for long duration photographic exposure. The current work provides a derivation of the rate of field rotation, which is needed for the necessary telescope de-rotation rate to compensate for the field rotation rate.

2. Analysis

The Earth's spin angular rate vector and the telescope pointing direction are defined in the local frame, where the x-axis is directed vertically, the z-axis is directed northward and the y-axis is directed easterly, as shown in Fig. 1. Eq. (1) defines the unit vector in the direction of the Earth's angular rate vector, where L is the latitude of the local frame.

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$$\boldsymbol{\omega} = \begin{bmatrix} \sin(L) \\ 0 \\ cos(L) \end{bmatrix} \tag{1}$$

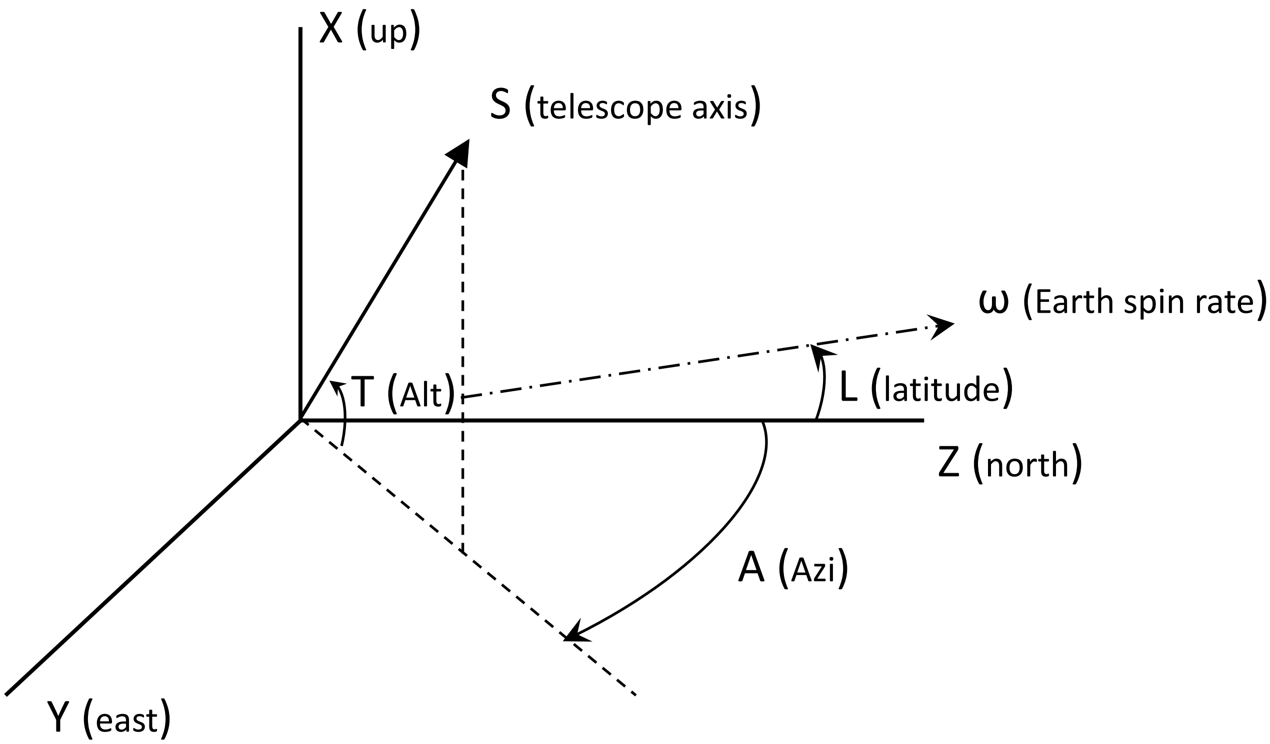

Fig. 1. The angles defining the Earth spin rate and Alt-Azi mounted telescope pointing axis relative to the local reference frame.

Two coordinate transformations are used to define the telescope reference frame with respect to the local reference frame. The first transformation is the azimuth angle rotation about the x-axis in the clockwise direction given by **R**(**x**, -A), as shown in eq. (2). The negative A rotation in eq. (2) is used to indicate a clockwise rather than the standard counterclockwise rotation.

$$\mathbf{R}(\mathbf{x}, -A) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(A) & \sin(A) \\ 0 & -\sin(A) & \cos(A) \end{bmatrix} \tag{2}$$

The second transformation is the altitude angle rotation angle about the y-axis in the counterclockwise direction given by **R**(**y**, T), as shown in eq. (3).

$$\mathbf{R}(\mathbf{y},T) = \begin{bmatrix} \cos(T) & 0 & \sin(T) \\ 0 & 1 & 0 \\ -\sin(T) & 0 & \cos(T) \end{bmatrix} \tag{3}$$

The transformation from the telescope frame to the local frame, **U**, is given by the product of the azimuth and altitude transformations, as given in eq. (4).

$$\mathbf{U} = \mathbf{R}(\mathbf{x},-A)\,\mathbf{R}(\mathbf{y},T) = \begin{bmatrix} \cos(T) & 0 & \sin(T) \\ -\sin(T)\sin(A) & \cos(A) & \cos(T)\sin(A) \\ -\sin(T)\cos(A) & -\sin(A) & \cos(T)\cos(A) \end{bmatrix} \tag{4}$$

The unit vector directed to the target star is in the z direction in the telescope frame and it can be transformed to the local frame, as shown in eq. (5).

$$\mathbf{U}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \sin(T) \\ \cos(T)\,Sin(A) \\ \cos(T)\cos(A) \end{bmatrix} = \mathbf{s} \tag{5}$$

The telescope pointing direction, **s**, in the local frame does not remain in a fixed orientation, since the local frame has an angular rate given by eq. (1). The rate of change of **s** in the local frame is given by the cross product shown in eq. (6), where Ω is the magnitude of the Earth spin rate.

$$\frac{\mathbf{ds}}{\mathbf{dt}} = [\mathbf{s} \times \boldsymbol{\omega}]\,\Omega = \begin{bmatrix} \cos(T)\sin(A)\cos(L) \\ \cos(T)\cos(A)\sin(L) - \sin(T)\cos(L) \\ -\cos(T)\sin(A)\sin(L) \end{bmatrix}\Omega \tag{6}$$

The rate of change of **s** can also be obtained by its derivative, as shown in eq. (7).

$$\frac{\mathbf{ds}}{\mathbf{dt}} = \begin{bmatrix} \cos(T)\frac{dT}{dt} \\ -\sin(T)\sin(A)\frac{dT}{dt} + \cos(T)\cos(A)\frac{dA}{dt} \\ -\sin(t)\cos(A)\frac{dT}{dt} - \cos(T)\sin(A)\frac{dA}{dt} \end{bmatrix} \tag{7}$$

The rate of change of A and T can be found by equating the x, y, z components in eq. (6) with the respective components in eq. (7). One finds the rate of change of T by equating the x components in eqs. (6) and (7), as shown in eq. (8).

$$\frac{dT}{dt} = \Omega\sin(A)\cos(L) \tag{8}$$

Eq. (9) is obtained by equating y components in eqs. (6) and (7).

$$-\sin(T)\sin(A)\frac{dT}{dt} + \cos(T)\cos(A)\frac{dA}{dt} = \cos(T)\,cos(A)\sin(L)\Omega - \sin(T)\,cos(L)\Omega \tag{9}$$

Using eq. (8) in eq. (9) and solving for dA/dt, one obtains eq. (10).

$$\frac{\mathrm{dA}}{\mathrm{dt}} = \left[sin(L) - \frac{\cos(A)\sin(T)\cos(L)}{\cos(T)}\right]\Omega \tag{10}$$

The rate of change of the azimuth angle is in the negative x direction and it has a component along the **s** axis, which rotates the telescope about its pointing axis. The rate of rotation of the telescope about its axis is given by eq. (11), where eqs. (5) and (10) have been used.

$$\frac{\mathbf{dA}}{\mathbf{dt}} \cdot \mathbf{s} = [\frac{sin^2(T)\cos(L)\cos(A) - \cos(T)\sin(L)\sin(T)}{\cos(T)}]\, \Omega \tag{11}$$

Eq. (11) accounts for the azimuth angle rate, but an additional term due to the angular rate of the local frame must be added to eq. (11) to obtain the total rotation of the telescope about its axis. The additional term is the component of the Earth's spin angular rate vector along the telescope axis, which is given by eq. (12), where eqs. (1) and (5) have been used.

$$(\boldsymbol{\omega} \cdot \mathbf{s})(\Omega) \; = [\cos(T)\cos(A)\cos(L) + \sin(T)\sin(L)]\; (\Omega) \tag{12}$$

The total rotation rate of the telescope axis is given by the addition of the rates in eqs. (11) and (12), as shown in eq. (13). Using a trigonometric identity and combining terms, eq. (13) is reduced to eq. (15).

$$\frac{R}{\Omega} = \frac{sin^2(T)\cos(L)\cos(A) - \cos(T)\sin(L)\sin(T) + cos^2(T)\,cos(L)\cos(A) + \sin(T)\sin(L)\cos(T)}{\cos(T)} \tag{13}$$

$$\frac{R}{\Omega} = \frac{sin^2(T)\cos(L)\cos(A) + cos^2(T)\,cos(L)\cos(A)}{\cos(T)} = \frac{[sin^2(T) + \cos^2(T)]\cos(L)\cos(A)}{\cos(T)} \tag{14}$$

$$R = \left[\frac{\cos(L)\cos(A)}{\cos(T)}\right](\Omega) \tag{15}$$

If the z components in eqs. (6) and (7) were used instead of the y components, one would also find that the rotation rate of the telescope about its axis is given by eq. (16), where the unit vector **S** is used to indicate the direction of telescope rotation.

$$\mathbf{R} = \left[\frac{\cos(A)\cos(L)}{\cos(T)}\right](\Omega)\;\mathbf{s} \tag{16}$$

With respect to the telescope reference frame, the star field appears to rotate in the opposite direction when viewed through the telescope. Therefore, the field rotation rate, **FR**, is given by eq. (17), which agrees with published results [6], [7].

$$\mathbf{FR} = -\mathbf{R} = -\left[\frac{\cos(A)\cos(L)}{\cos(T)}\right](\Omega)\;\mathbf{s} \tag{17}$$

3. Validation

The derived angular rates of T and A in eq. (8) and eq. (10) were validated numerically by using them to propagate the orientation of the telescope axis and comparing results to known values. Using the input values of A and T, the initial orientation of the telescope axis is given by eq. (5). With the latitude of the local frame given by L to establish the direction of **ω**, the axis was propagated for time τ using the standard method shown in eq. (17). The negative value of Ω was used in eq. (17) to achieve inertial frame to local frame transformation rather than local frame to inertial frame transformation.

$$\mathbf{sf} = \mathbf{R}(\boldsymbol{\omega}, -\Omega\,\tau)\;\mathbf{s} \tag{17}$$

The telescope axis was propagated numerically from its initial orientation to its final orientation. For each time increment, A and T were updated using eqs. (18) and (19), where dA/dt and dT/dt were obtained from eq. (8) and eq. (10).

$$\mathrm{A}(\tau + \Delta\tau) = \mathrm{A}(\tau) + \frac{\mathrm{dA}}{\mathrm{dt}}\Delta\tau \tag{18}$$

$$\mathrm{T}(\tau + \Delta\tau) = \mathrm{T}(\tau) + \frac{\mathrm{dT}}{\mathrm{dt}}\Delta\tau \tag{19}$$

Once the final values of A and T are found by repeated use of eqs. (18) and (19), the propagated value of the telescope pointing axis, **sfp,** is obtained using eq. (20).

$$\mathbf{sfp} = \begin{bmatrix} sin\,(T) \\ cos(T)\,sin(A) \\ cos(T)\,cos\,(A) \end{bmatrix} \tag{20}$$

Using the vector dot product, the angular separation angle between **sf** and **sfp** was found for values L = 25 deg., A = 105 deg., T = 36 deg, for a propagation time of 5 hours using an integration frequency of 10 Hz, as shown in eq. (21).

$$angle(5\,hr.) = cos^{-1}(\mathbf{sf} \cdot \mathbf{sfp}) = 4.8489 \times 10^{-4}\,deg. \tag{21}$$

The small angular error in eq. (21) indicates that eqs. (8) and (10) are correct and that a star can be tracked across the sky by using eqs. (5), (8), (10), (18), and (19). Accuracy can be increased by increasing propagation frequency. If even greater accuracy is desired, a more accurate integration scheme can be used to replace eqs. (18) and (19). Nevertheless, the azimuth angle rate and resulting field rotation rate become very large near the singularity at T=90 deg, as noted in [4] and [5]. This causes difficulty in tracking the target star and compensating for the high field rotation rate. Therefore, a keep-out region near T = 90 deg may be needed.

4. Conclusion

The field rotation rate of an Alt-Az mounted telescope was found as the sum of two rates. The first rate is the projection of the azimuth angle rate along the telescope axis and the second rate is the projection of the Earth's spin rate along the telescope axis. The derived field rotation equation is in agreement with published results. The validity of the derived altitude and azimuth angle rates was established via numerical propagation.

References

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